Natural frequencies of a vibrating repaired panel in an ocean structure

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Abstract

The present work arose from the need of repairing steel plates badly damaged by corrosion in a portion of the structural element of the naval vehicle. The possibility of removing the portion and replacing it by a patch of a composite material was considered. Its dynamic behavior is altered by the introduction of the patch and the prediction of its new behavior is of great interest in many situations. This condition would appear in other real-life situations like as a localized orthotropic effect caused in the panel by a welding procedure or a metallurgical process. The first four natural frequency coefficients of the composite repaired panel with different types of boundary conditions are determined by means of a variational approach. The displacement function is approximated making use of complete sets of beam functions. The eigenvalues have been computed from (225×225) secular determinants. An independent solution is obtained using the finite element method and a reasonably good agreement with the analytical solution is encountered.
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1. Introduction

The study of flexural vibrations of isotropic panels with an orthotropic inclusion is of interest from an ocean engineering viewpoint since the localized orthotropic effect changes the dynamic behavior of the original structure. There are several situations where this condition may appear. One situation is that where in a damaged vibrating isotropic plate-like-structure the deteriorated (e.g. corrosion) portion acquires orthotropic-like characteristics. The orthotropic effect may be also caused by a welding procedure or by repairing an isotropic panel when a damaged portion is replaced by an orthotropic patch, adding a composite material (usually epoxy). Previous investigators (Cornwell et al., 1999; Laura et al., 2000; Laura and Gutiérrez, 1984) considered only the fundamental frequency of the structural system and they are extended here in order to determine the higher frequencies and also to deal with other boundary condition.

The approach may be also of interest when dealing with offshore platforms.

An excellent survey of previous theoretical and experimental studies dealing with vibrating fully isotropic or completely orthotropic plates is available in Leissa's classical treatise (Leissa, 1969).

2. Analytical solution

The vibrating mechanical idealized model is depicted in Fig. 1. The displacement amplitude is expressed in terms of series of beam functions, where each coordinate function satisfies identically the essential boundary conditions at the outer edge of the plate.
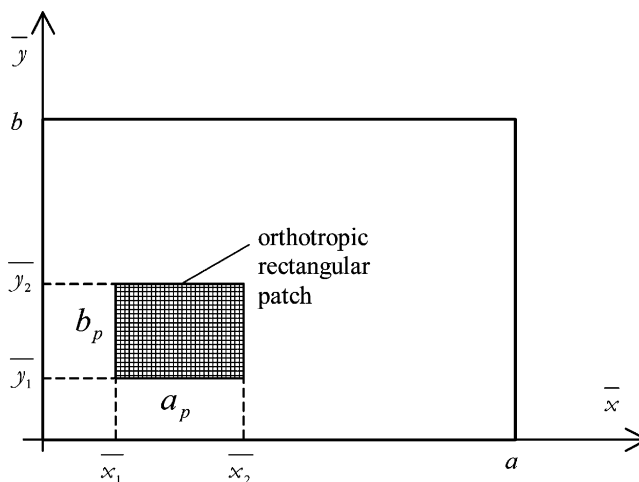

Fig. 1.   Isotropic rectangular panel with orthotropic patch.

$$W_a(\bar{x},\bar{y}) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}X_m(\bar{x})Y_n(\bar{y}) \tag{1}$$

where $W_a(\bar{x},\bar{y})$ is the approximate displacement amplitude, $X_m(\bar{x})$, $Y_n(\bar{y})$ are the beam functions and A_{mn} are the undetermined coefficients.

The Rayleigh–Ritz method requires minimization of the functional:

$$J(W) = U - T \tag{2}$$

where U and T are the maximum strain energy and maximum kinetic energy of the plate with the inclusion, respectively.

Using Lekhnitskii's notation, presented in his famous book *Anisotropic Plates*(1968), the dynamic functional can be written as:

$$
\begin{aligned}
J(W) = \frac{D}{2}\iint_A \{ \ (\ W_{\bar{x}\bar{x}} + W_{\bar{y}\bar{y}} \ )^2 \ &- 2 \, (1{-}v) \ [\ W_{\bar{x}\bar{x}}W_{\bar{y}\bar{y}} \\
-(W_{\bar{x}\bar{y}})^2] \ \} \ d\bar{x} \, d\bar{y} + \frac{1}{2}\iint_{A_p}&[D_1(\ W_{\bar{x}\bar{x}})^2 + 2D_1v_2W_{\bar{x}\bar{x}}W_{\bar{y}\bar{y}} \\
+ \ D_2 \, (W_{\bar{y}\bar{y}} \ )^2 &+ 4D_k(W_{\bar{x}\bar{y}})^2] \ d\bar{x} \, d\bar{y} \\
-\frac{1}{2}\rho \ \omega^2 \iint_A h \ W^2 d\bar{x} \, d\bar{y} &-\frac{1}{2}\rho_p\omega^2 \iint_{A_p} hW^2d\bar{x} \, d\bar{y}.
\end{aligned}
\tag{3}
$$

where $W_{\bar{x}\bar{x}} = \dfrac{\partial^2 W}{\partial \ \bar{x}^2}$; $W_{\bar{y}\bar{y}} = \dfrac{\partial^2 W}{\partial \ \bar{y}^2}$; $W_{\bar{x}\bar{y}} = \dfrac{\partial^2 W}{\partial\bar{x}\partial \ \bar{y}}$; and D, v, ρ are bending rigidity, Poisson's ratio and density of the isotropic domain: A, respectively, and D_1, D_2, D_k, v_2, ρ_p are bending and twisting rigidities for the principal directions of elasticity, one of Poisson's ratios and density of the orthotropic domain: A_p, respectively.

The principal directions of elasticity of the orthotropic inclusion are considered parallel to the plate's sides, as it is shown in Fig. 1.

The Eq. (3) can be conveniently non dimensionalized by introducing: $x = \dfrac{\bar{x}}{a}$; $x_1 = \dfrac{\bar{x}_1}{a}$; $x_2 = \dfrac{\bar{x}_2}{a}$; $y \ = \dfrac{\bar{y}}{b}$; $y_1 = \dfrac{\bar{y}_1}{b}$; $y_2 = \dfrac{\bar{y}_2}{b}$.

Substituting the Eq. (1) into Eq. (3), integrating and requiring that $J(W)$ be a minimum with respect to the A_{mn}:

$$\frac{\partial J}{\partial A_{mn}} = 0 \tag{4}$$

one obtains a determinantal equation, whose roots are the natural frequency coefficients:

$$\Omega_i = \sqrt{\frac{\rho h}{D}} \, \omega_i \, a^2; \text{ with } i = 1, 2,... \tag{5}$$

In the case of the simply supported plate the beam functions are (Blevins, 1979):

$$X_m(x) = \sin k_m x \tag{6a}$$

$$Y_n(y) = \sin k_n y \tag{6b}$$

both of which satisfy the governing boundary conditions when

$$k_m = m\pi, \tag{7a}$$

$$k_n = n\pi \tag{7b}$$

When the plate is fully clamped the corresponding beam functions are (Blevins, 1979):

$$X_m(x) = (\sin k_m x - \sinh k_m x) \, r_m + \cosh k_m x - \cos k_m x \tag{8a}$$

$$Y_n(y) = (\sin k_n y - \sinh k_n y) r_n + \cosh k_n y - \cos k_n y \tag{8b}$$

with

$$r_m = \frac{\cos k_m - \cosh k_m}{\sin k_m - \sinh k_m} \tag{9a}$$

;

$$r_n = \frac{\cos k_n - \cosh k_n}{\sin k_n - \sinh k_n} \tag{9b}$$

where k_m and k_n are now the roots of the transcendental equation:

$$\cos k \cosh k = 1 \tag{10}$$

3. Numerical results

Calculation of natural frequency coefficients has been performed for simply supported and clamped rectangular plates with different relation of sides $b/a = \lambda$. The Poisson's ratio v is assumed to be equal to 0.30 for the isotropic material. In all cases the patch constitutive characteristics are specified in the corresponding Tables.

In general the eigenvalues have been evaluated from a (225×225) determinantal equation.

Table 1 deals with a study of the convergence of the fundamental frequency coefficient as the number of terms, m, n, is increased in Eq. (1). The case under study in Table 1 corresponds to the simply supported rectangular plate with orthotropic

Table 1
Analysis of convergence of the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho\, h/D}\ \omega\, a^2$ for simply supported rectangular plates ($v = 0.30$), with an orthotropic inclusion $\lambda = b/a = \lambda_p = b_p/a_p$. ($D_1/D = 2.856$. $v_2 = 0.757$. $D_k/D = 0.765$. $D_2/D = 2.256$. $\rho_p/\rho = 2.95$). Fig. 1

		$\Omega_1 = \sqrt{\rho\, h/D}\ \omega_1\, a^2 (N = M, x_1 = y_1 = 0.1)$					
$\lambda = b/a$	$x_2 = y_2$	$M = 1$	$M = 2$	$M = 3$	$M = 5$	$M = 10$	$M = 15$
1	0.3	19.954	19.904	19.869	19.847	19.805	19.792
	0.4	19.870	19.628	19.605	19.455	19.381	19.361
	0.5	19.607	19.130	19.074	18.865	18.712	18.644
	0.6	19.359	18.924	18.510	18.413	18.163	18.107
	0.7	19.239	19.060	18.595	18.404	18.182	18.076
	0.8	19.268	19.242	19.088	18.798	18.620	18.565
	0.9	19.468	19.468	19.439	19.396	19.283	19.223
2/3	0.3	32.320	32.257	32.206	32.175	32.111	32.091
	0.4	32.045	31.733	31.696	31.472	31.356	31.324
	0.5	31.427	30.802	30.731	30.418	30.181	30.074
	0.6	30.848	30.269	29.695	29.549	29.167	29.080
	0.7	30.523	30.281	29.621	29.354	29.017	28.854
	0.8	30.485	30.449	30.226	29.798	29.529	29.445
	0.9	30.741	30.741	30.696	30.632	30.458	30.363
1/2	0.3	49.538	49.464	49.392	49.351	49.259	49.228
	0.4	48.948	48.556	48.497	48.178	48.005	47.954
	0.5	47.801	47.006	46.920	46.476	46.124	45.961
	0.6	46.725	45.998	45.254	45.045	44.489	44.359
	0.7	46.079	45.776	44.916	44.567	44.081	43.842
	0.8	45.896	45.852	45.560	44.975	44.592	44.469
	0.9	46.146	46.146	46.088	45.999	45.748	45.609

inclusions of different dimensions. In these cases the patch does posses the same aspect ratio as the plate, $\lambda_p = b_p/a_p = b/a = \lambda$.

In Table 2 a comparison is presented of fundamental frequency values between those available in the literature, Ercoli et al. (1992), and the ones determined using the approximate expression Eq. (1) with $M = N = 15$.

In this case the orthotropic inclusion is centered in the plate (Fig. 2) and its aspect ratio is the same as the plate, $\lambda_p = \lambda$.

Table 3 depicts the fundamental frequency coefficient (a) for the complete isotropic plate, (b) for the plate with a hole, $\lambda_p = 2/3\,\lambda$, (c) and (d) the plate with two different types of orthotropic patches, $\lambda_p = 2/3\,\lambda$.

Table 4 shows the four lower natural frequency coefficients for simply supported rectangular plates with an orthotropic rectangular inclusion compared with finite element values. In this Table the plate's aspect ratio λ and the rectangular inclusion's aspect ratio are the same: $\lambda_p = \lambda$. The analytical results are compared with numerical values obtained by means of a well known finite element code, Algor Professional Mech./V. E. (1999). The meshes employed were of 50×50 elements for the case of $\lambda = 1$, 60×40 elements for the case of $\lambda = 2/3$ and 80×40 elements for $\lambda = 1/2$; being $\lambda = b/a$ the aspect ratio of the plate.

Table 2
Comparison of fundamental frequency coefficients of the simply supported rectangular plate ($v = 0.30$), with an orthotropic centered inclusion. ($\lambda = \lambda_p$). ($D_1/D = 2.856$. $v_2 = 0.757$. $D_k/D = 0.765$. $D_2/D = 2.256$. $\rho_p/\rho = 2.95$). Fig. 2

$b/a = b_p/a_p$	$x_1 = y_1$	$\Omega_1 = \sqrt{\rho h/D}\,\omega_1\,a^2$ $x_2 = y_2$	Ercoli et al. (1992)[*]	Present study
1	0.3	0.7	17.45	17.66
	0.2	0.8	17.90	18.07
	0.1	0.9	19.14	19.22
2/3	0.3	0.7	28.07	28.38
	0.2	0.8	28.50	28.77
	0.1	0.9	30.23	30.36
1/2	0.3	0.7	42.94	43.42
	0.2	0.8	43.23	43.64
	0.1	0.9	45.42	45.61

[*] Obtained by means of a powerful finite element code (SAMCEF)
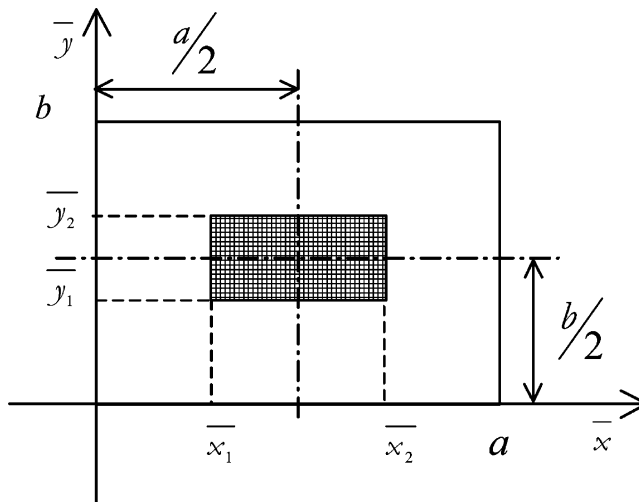

Fig. 2. Centered orthotropic patches in the isotropic simply supported or clamped rectangular panel.

Table 5 depicts a similar comparison for the fully clamped plate, with: $\lambda_p = \lambda$. Both Tables, 4 and 5, show an adequate agreement between the analytical method and the finite element method.

4. Conclusions

The results are in good agreement with those available in the literature, (Ercoli et al., 1992). The maximum difference between them is of the order of 1.1% for the case ($x_1 = y_1 = 0.3$; $x_2 = y_2 = 0.7$).

Table 3
Fundamental frequency coefficients for rectangular plates. $\Omega_1 = \sqrt{\rho\,h/D}\,\omega_1\,a^2$. Hole or inclusion dimensions: $x_1 = 0.3$, $y_1 = 0.1$; $x_2 = 0.6$, $y_2 = 0.3$. $\lambda_p = 2/3\lambda$. Material I: $D_1/D = 2.856$. $v_2 = 0.757$. $D_k/D = 0.765$. $D_2/D = 2.256$. $\rho_p/\rho = 2.95$. Material II: $D_1/D = 1.164$. $v_2 = 0.451$. $D_k/D = 0.492$. $D_2/D = 0.228$. $\rho_p/\rho = 1.75$

| | b/a | $\Omega_1 = \sqrt{\rho\,h/D}\,\omega_1\,a^2$ | | | | |
		(a) Solid Plate	b_p/a_p	(b) Plate with a hole	(c) Inclusion Material I	(d) Inclusion Material II
Simply supported panel	2.0	12.337	4/3	12.098	12.004	12.045
	1.5	14.256	1	14.020	13.862	13.897
	1.0	19.739	2/3	19.940	19.160	19.092
	2/3	32.076	4/9	30.539	31.051	30.533
	1/2	49.348	1/3	45.592	47.679	46.272
Clamped panel	2.0	24.578	4/3	24.771	23.539	23.919
	1.5	27.005	1	27.082	25.936	26.347
	1.0	35.985	2/3	35.329	34.454	34.970
	2/3	60.761	4/9	57.502	58.126	58.491
	1/2	98.311	1/3	90.507	94.009	94.069

Table 4
The four first natural frequency coefficients for simply supported rectangular plates ($v = 0.30$), with an orthotropic inclusion. ($\lambda = \lambda_p$), ($D_1/D = 2.856$. $v_2 = 0.757$. $D_k/D = 0.765$. $D_2/D = 2.256$. $\rho_p/\rho = 2.95$). (1) Analytical method; (2) Finite element technique

| $b/a = b_p/a_p$ | $(x_1 = y_1 = 0.1)\ x_2 = y_2$ | | $\Omega_i = \sqrt{\rho\,h/D}\,\omega_i\,a^2;\ i = 1, 2, 3, 4$ | | | |
			Ω_1	Ω_2	Ω_3	Ω_4
1	0.3	(1)	19.7921	48.0247	49.2518	77.2083
		(2)	19.4231	47.5597	49.1775	76.8069
	0.6	(1)	18.1071	46.3881	48.6905	79.4222
		(2)	17.6742	45.1601	47.7025	75.5841
	0.8	(1)	18.5650	44.5278	46.7526	74.8224
		(2)	18.1388	43.4016	45.4923	70.9274
2/3	0.3	(1)	32.0909	60.8707	96.9704	110.3120
		(2)	31.5731	60.4586	96.5279	109.5351
	0.6	(1)	29.0804	60.0053	93.1890	107.8250
		(2)	28.4523	58.1825	91.6603	105.8564
	0.8	(1)	29.4446	58.3664	87.3141	106.7190
		(2)	28.8351	56.3258	85.7706	103.3371
1/2	0.3	(1)	49.2286	77.8527	127.3600	164.9280
		(2)	48.6088	77.3192	126.2948	164.2072
	0.6	(1)	44.3588	76.3894	124.0480	158.0730
		(2)	43.5420	73.8752	121.4296	155.9128
	0.8	(1)	44.4690	74.2272	123.4660	147.2980
		(2)	43.7080	71.3904	118.5908	145.1816

Table 5
The four first natural frequency coefficients for clamped rectangular plates, with an orthotropic inclusion, ($\lambda = \lambda_p$), ($D_1/D = 2.856$. $v_2 = 0.757$. $D_k/D = 0.765$. $D_2/D = 2.256$. $\rho_p/\rho = 2.95$). (1) Analytical method; (2) Finite element technique

| $b/a = b_p/a_p$ | ($x_1 = y_1 = 0.1$) $x_2 = y_2$ | | $\Omega_i = \sqrt{\rho\,h/D}\,\omega_i\,a^2;\ i = 1, 2, 3, 4$ | | | |
			Ω_1	Ω_2	Ω_3	Ω_4
1	0.3	(1)	36.043	71.081	73.334	104.725
		(2)	36.031	70.879	73.329	104.315
	0.6	(1)	30.643	67.107	69.145	106.760
		(2)	30.196	66.281	68.347	106.073
	0.8	(1)	29.441	62.136	65.240	99.181
		(2)	29.322	61.528	64.364	97.106
2/3	0.3	(1)	60.715	92.344	144.159	149.202
		(2)	60.695	92.226	143.686	149.157
	0.6	(1)	50.826	87.903	134.427	144.494
		(2)	50.115	86.912	133.033	141.962
	0.8	(1)	48.073	82.772	122.832	139.145
		(2)	47.841	81.664	121.351	136.625
1/2	0.3	(1)	97.944	125.112	176.314	247.662
		(2)	97.901	124.968	176.064	246.922
	0.6	(1)	80.596	117.738	170.761	229.292
		(2)	79.738	116.444	168.430	226.639
	0.8	(1)	75.418	109.135	164.680	208.064
		(2)	75.046	107.686	161.972	206.181

The accuracy can be considered satisfactory from a practical engineering point of view. On the other hand, it is also possible, in principle, to increase the number of terms in the summation on Eq. (1) to reduce this difference.

One concludes that in the case of clamped edges the agreement between analytical and finite element results is better than in the case of simply supported edges. Future work will consider different combinations of boundary conditions and the presence of concentrated masses.

Acknowledgements

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