Modeling the sloshing problem in a rectangular tank with submerged incomplete baffles

Abstract

This work presents an experimental and numerical study of a sloshing problem in rectangular tanks with baffles immersed at different distance from the bottom. The numerical simulation is performed using a generalized streamline formulation for the solution of the Navier Stokes equations in conjunction with a Lagrangian approach to describe the free surface motion. The physical problem consists of a tank with baffles attached at its walls. The tank is subjected to cyclic horizontal motions with diverse amplitudes and frequencies. Baffles of different sizes are used to evaluate its effects on the waves. Steady state forced sloshing and free sloshing regimes are reported. The water level evolution is successfully validated by comparison with those registered in the experiments for the several cases analyzed. Additionally, the effectiveness of baffles in limiting the wave height and its propagation is satisfactorily described and quantified.

1 Introduction

Free surface flows are commonly presented in nature as well as in industrial applications. From ocean science to liquid storage, they present a wide range of technical problems that require the description of free surface evolution under variable acceleration effects in order to take advantage of either the swell (i.e., energy generation) or to reduce the nocive wave effect on coast or containers. Simple devices that act as dampers and deflectors are called baffles and they are designed to reduce the wave amplitude and to limit its evolution. Baffles are commonly installed on trucks or tanks used to transport or to storage liquids. Applications like those previously referred encourage experimental as well as numerical studies of waves and their evolution in presence of baffles [1-6].

Different numerical techniques to deal with free surface analyses have been reported in the literature. They could be basically classified according to the free surface is either considered as boundary of the domain (in such a case the computational domain varies in time) or is embedded in a fixed domain filled with two fluids (typically representing air and water). The Arbitrary Lagrangian–Eulerian (ALE) [7-9] or Deformable-Spatial-Domain/Stabilized-Space-Time formulation (DSD/SST) [10,11] are well known moving domain techniques that have been extensively applied. The pioneering and well reported Marker-and-Cell (MAC) [12,13], the pseudo-concentration method [14-16], Volume Of Fluid (VOF) [17-19], and Level Set (LS) [20-23] methods have been successfully used on fixed discretizations. Many of such formulations have been also coupled with remeshing techniques to make the resulting numerical schemes more powerful [24-33]. An important aspect in modeling is the verification and validation of the results. In fact, several efforts have been done to contrast results [1-6, 33-37] that make even nowadays the benchmarking relevant. This task requires the definition of experiments able to deal with the necessary data for numerical comparison. Regarding free surface flow problems, experiments are reported in the literature (see, e.g., [1-6, 34-37]) where some works are specifically devoted to baffled tanks [3,5]. Baffles are usually used to reduce the effect of waves. Their location could be decisive to achieve such objective and, as they are added to a structure, their lightness is also a goal. Hence, non massive baffles are practically designed to fulfill such requirements.

The present study encompasses experiments and numerical simulations with the objective of analyzing the behavior of the free surface during the sloshing of water in rectangular tanks with baffles submerged at different depths. The simulations are performed using a fixed mesh finite element formulation reported in [37, 39-41] where, in contrast to the strategy described in [39], the volume control algorithm used in this analysis is not iterative. To assess the numerical behavior of the referred formulation, this work is focused on the validation of a numerical model of the proposed problem by comparing the computed free surface transient responses with experimental data. To this end, an experiment of sloshing in rectangular tanks subjected to cyclic acceleration including the effect of baffles with different geometries is conducted within the context of the present work and its collected data is novelty reported. Taken into account experimental observations, only 2D free surface evolutions will be simulated. In particular, the numerical analysis reports the sloshing of water in rectangular tanks under different shake conditions, varying the amplitude and frequency of the imposed motion. Steady state forced sloshing (time-periodic regime) and free sloshing (damped decaying regime) are analyzed for the different cases of study. The efficiency to diminish the wave height is evaluated varying baffles sizes and positions. Moreover, primary resonance conditions are also tested.

The remainder of the paper presents: the experiments, a brief description of the governing equations and the numerical formulation, the modeling and its validation. Finally, we summarize the conclusions of the reported investigation.

2 Experimental Work

The experiment consists of an acrylic box of rectangular section (called *tank* from here onward, with dimensions 388 *mm* x 183 *mm* x 245 *mm*, see Figure 1.a) mounted on a shake table [45] able to move under controlled horizontal motion (Figure 1.b). The laboratory facilities are similar to those reported in [37]. Nevertheless, in the present work, the tank is designed to attach baffles of different sizes (*Cort*) at different distance from the bottom (*Hc*), see Figure 1.c. The dimensions and positions of the baffle in the experiments are shown in Table 1.

The tank is filled with water. In the present work only experiments using a water level of H = 100 *mm* will be reported. The present work is focused on sloshing near the first resonant mode. Hence, considering the frequency at the primary resonance of the tank without baffles (named f_n) as a reference, the tank is shaken at frequencies of 0.58 *Hz*, 0.87 *Hz*, 1.16 *Hz*, 1.45 *Hz*, 1.97 *Hz*, i.e. 0.5 f_n, 0.75 f_n , f_n , 1.25 f_n and 1.7 f_n based on f_n computed from the analytical expression [38]:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\pi \cdot g}{l} \cdot \tanh\frac{\pi \cdot H}{l}} \qquad (1)$$

where l is the length of the tank with respect to the direction of motion, g is the gravity and H is the water level at rest, resulting the reference frequency as $f_n = 1.16$ *Hz*. Moreover, motion amplitudes of 2.5 *mm*, 5 *mm*, 7.5 *mm* and 15 *mm* are considered. In order to prevent overflows, only a frequency of 0.75 f_n for an amplitude of 15 *mm* was applied. Table 2 summarizes the frequency and amplitude applied to obtain the imposed motion. Therefore, considering that the 5 baffle configurations shown in Table 1 (see Figure 1.c) are tested at each frequency and amplitude of the imposed motion as those reported in Table 2, a set of 55 cases in total were analyzed.

The free surface evolution is registered by ultrasonic sensors located in the z-middle plane at 20 *mm* apart from the tank walls. Additionally, sensors placed at the tank corners help to evaluate the evolution of 3D free surface effects. The error in the experimental measurements is estimated as ±0.5 *mm*.

The experimental evolutions of the free surface relative to the water level at rest, named h in the present work, are reported together with the computed responses in Section 4. In particular, free surface evolutions are reported during time-periodic (forced vibrations) and damped decay (free sloshing once the shake table is stopped) regimes.

The experiments show that 3D effects can evolve at frequencies closely to primary resonance depending on the amplitude of the imposed motion as it was also shown in [37] where a configuration without baffles was used. Moreover, although the system used to fasten the baffles can induce disorder in the free surface it does not promote, however, strong 3D effects in the cases reported. In spite of this, the experimental observations presented in this work reveal that the motion evolves practically 2D in the X-Y plane for the cases analyzed.

3 Governing Equations and Numerical Formulation

The Navier-Stokes equations are used to describe a Newtonian viscous incompressible fluid flow:

$$\rho\frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p - \nabla\cdot(2\mu\boldsymbol{\varepsilon}) = \rho\mathbf{b} \qquad (2)$$

$$\nabla\cdot\mathbf{v} = 0 \qquad (3)$$

where $\rho, \mathbf{v}, p, \mu, \boldsymbol{\varepsilon}$ and $\boldsymbol{b}$ are the density, velocity, pressure, dynamic viscosity, strain-rate tensor and body force, respectively. The gradient operator is denoted by ∇. The set of equations of motion (2) and the incompressibility constraint (3) are solved in a fixed domain of analysis Ω filled with fluid 1 (Ω_1) and fluid 2 (Ω_2) such that $\Omega = \Omega_1 \cup \Omega_2$ and $\Omega_1 \cap \Omega_2 = 0$. The discrete form of the equations is obtained in the context of the finite element method using a generalized streamline operator technique to stabilize the solution (see [43,44] and references therein). The stabilised nature of the technique allows the use of equal-order interpolation functions for velocity and pressure. In the context of flow analysis with moving interfaces, all the matrices and vectors derived from the discrete form are computed including the discontinuity in material properties. The time integration is performed using a standard backward Euler scheme. The problem description is completed with appropriate initial and boundary conditions.

An additional equation is required to determine the free surface evolution. In this work, the interface between fluids 1 and 2 is identified by marker points. The positions of such markers are updated from time *t* to *t+Δt* using the following Lagrangian formulation [39,42]:

$$X^{t+\Delta t} = X^{t} + \Delta t \cdot V^{t+\alpha\cdot\Delta t} \tag{4}$$

where *X* and *V* are respectively the position and velocity of each marker point and α is the time integration parameter. The new set of markers′ positions $X^{t+\Delta t}$ needs to be volume preserving for incompressible flows. To ensure this, the following algorithm to compute volume preserving coordinates at time *t+Δt* is adopted:

$$X^{t+\Delta t}_{vol-pres} = X^{t+\Delta t} + \beta\, \delta X \tag{5}$$

such that δX is chosen to satisfy mass fluxes balance along the interface with unit normal *n* as:

$$\delta X = -sign(V \cdot n) V\, \Delta t \tag{6}$$

and β is a minimum parameter computed to fulfill the condition

$$\frac{1}{n_{\dim}} \int_{\Gamma_1} (X^{t+\Delta t}_{vol-pres} \cdot n)\, d\Gamma_1 = V_{1-known} \tag{7}$$

where Γ_1 is the boundary of the current liquid domain Ω_1 , n_{dim} is the space dimension and $V_{1-known}$ is the volume to be preserved (see references [39-42] for further details). Moreover, due to the Lagrangian nature of equation (4), redistribution of markers is required when time goes by. Hence, an interface remeshing technique based on preserving curvatures needs to be applied as it was reported in [40] and [41] for 2D and 3D cases, respectively. Material properties are distributed at every time according to the interface position and, in addition, the discrete Navier–Stokes equations are integrated using subelements [39].

Finally, the numerical algorithm from time *t* to *t+Δt* reads:

1. From X^{t} , compute properties ρ_1 , μ_1 , ρ_2 and μ_2.
2. Calculate v and p with the system of equations (2) and (3).
3. Update the interface using equation (4); $X^{t+\Delta t}$ is obtained.
4. Determine a volume preserving markers' positions $X^{t+\Delta t}_{vol-pres}$ such that equations 5 to 7 are fulfilled.
5. Re-distribute markers accordingly geometric conservation laws (i.e., curvature preservation); ${}^{remeshed}X^{t+\Delta t}_{vol-pres}$ is obtained.
6. Advance in time, $t = t + \Delta t$ and $X^{t} = {}^{remeshed}X^{t+\Delta t}_{vol-pres}$; go to 1.

Remark 1: Numerical tests have shown stable results when using bigger time steps in comparison with level set type techniques (see [39]).

Remark 2: The described volume preserving algorithm has an excellent performance without loss of mass. In contrast to the original strategy reported in [39], note that the present volume control algorithm is not iterative.

Remark 3: The value of $V_{1\text{-}known}$ is fixed in closed systems (as in the present analysis). In cases of open systems, it could be computed including the mass flux balance along the domain interface (see, e.g., [42]).

Finally, as it was reported in previous works [37], a simple turbulence model is defined based on a well known mixing length law by replacing μ in equation (2) by $\mu_t = \min(\mu + l_{mix}^2 \rho \sqrt{2\varepsilon : \varepsilon}; \mu_{\max})$, l_{mix} being a characteristic mixing length and μ_{max} a cutoff value. This model also acts as discontinuity-capturing viscosity terms of more complicated turbulence models since the increased viscosity near the interface serves essentially to stabilize the jump in shear stresses.

4 Modeling the Experiments: Presentation and Discussion of the Results

Figure 2 sketches typical meshes used in the present analysis (not all meshes are shown, the meshes need to fit the different configurations). The meshes are composed by four noded elements. To obtain a better accuracy of the interface representation, the meshes are refined at the zone where the interface motion is expected by using a *y* element size equal to 1 *mm*, which is twice the value of the water level experimental error. The refined zone is commonly taken as 20 mm below and above the interface position at rest. Nevertheless, to analyze the problems where high waves are detected, this region needs to be extended where the interface is expected to be. Outside this zone, the *y* mesh size is 4 *mm*. Based on the mesh size, the numerical error can be bounded by ±0.5 for the finer mesh and ±2 mm in the coarser one. The time step used in the analysis is 0.005 *s*.

The properties used in the simulations are: ρ_1 = 998 kg/m^3 and μ_1 = 0.001 *kg/(m·s)* for the water and ρ_2 = 1.2 kg/m^3 and μ_2 = 1.8 E-5 *kg/(m·s)* for the air. The turbulent viscosity is computed using a global mixing length chosen proportional to the freesurface wave length (instead of a typically-used local mixing length, see also [33] for models comparison) l_{mix} = 0.15 *m* and μ_{max} = 0.1 *kg/(m·s)*.

To illustrate the effect of the baffles on the free surface evolution, the results computed with and without baffles using a frequency $f = f_n$ = 1.16 *Hz* and amplitude *A* = 5 *mm* for the imposed motion are plotted in Figure 3 for the first 6 *s* of the analysis. Baffles are located at different positions from the bottom of the tank *Hc* and their heights *Cort* are also varied (see Figure 1c). The simulation without baffle demonstrates a constant increase in the sloshing height in accordance to the expected analytical behavior. On the other hand, it is observed that the presence of baffles contributes to decrease the wave heights and a time-periodic regime is reached in shorter times. Greater reductions on the water level is obtained for the highest baffles, especially for the baffle of *Cort* = 56.25 *mm* positioned at *Hc* = 75 *mm* from the bottom of the tank, which is closer to the water level at rest.

The experimental data obtained during time-periodic regime for an imposed motion of $f = f_n$ = 1.16 *Hz* and *A* = 5 *mm* and baffles positioned at *Hc* = 50 *mm* and *Hc* = 75 *mm,* are reported together with the numerical prediction in Figure 4. As it can be seen, the simulations satisfactorily match the experiments. Experimental data obtained for a baffle of *Cort* = 25 *mm* positioned at *Hc* = 50 *mm* could not be satisfactorily registered; due to this fact the corresponding results are not shown.

Moreover, the free surface evolution without baffle does not reach a time-periodic regime since a resonant mode evolves at the imposed frequency.

The free surface evolution during time-periodic regimes for an imposed motion of different amplitudes and frequencies are shown in Figures 5 to 9 (A = 5 *mm* at f = 0.87 *Hz* and f = 1.45 *Hz* in Figures 5 and 6, respectively; A = 7.5 *mm* at f = 0.87 *Hz* and f = 1.45 *Hz* in Figures 7 and 8, respectively; A = 15 *mm* at f = 0.87 *Hz* in Figure 9). Experiments were not conducted at this higher amplitude at f = 1.45 *Hz* to prevent overflows. As it can be seen from Figures 5 to 9, the simulations are able to reasonably capture the sloshing for different baffle configurations. At frequencies under the first resonance the numerical results match the experiments for the imposed motions of the three amplitudes analyzed. For the frequency over the first resonance, numerical results exhibit discrepancies. This could be attributable to a poor numerical discretization when the wave height has a magnitude over 20 *mm* which increases the numerical error to at least ±2 *mm*. Moreover, a Fast Fourier Transform (FFT) analysis reveals that numerical and experimental signals reported in Figures 5, 7 and 9 present as main frequencies the imposed one and its second harmonic, i.e., 0.87 *Hz* and 1.76 *Hz*. The FFT study for signals plotted in Figures 6 and 8 shows that the main frequencies for the experiments are: the imposed (1.45 *Hz*), its second harmonic (2.90 *Hz*) and second resonance mode (1.9 *Hz*). Nevertheless, the FFTs obtained from the numerical solutions report as main frequencies the same but in the following order of relevance: 1.45 *Hz*, 1.9 *Hz* and 2.9 *Hz*. The discrepancies observed in Figures 6 and 8 could be explained for such differences.

To evaluate similarities and discrepancies of the reported amplitude signals during time-periodic regimes, upper and lower bounds for maximum and minimum wave heights are reported in Tables 3 and 4 for imposed motion frequencies of 0.87 *Hz* and 1.45 *Hz*. These tables summarize experimental values and numerical ones for different baffles configurations and amplitudes of the imposed motion. To assess the influence of the sensor location (ds), results computed at ds = 20 *mm* and 30 *mm* from the wall are also included. These results illustrate that higher wave heights occur closer to the wall. Higher differences between the water levels registered in points at 20 *mm* and 30 *mm* from the wall are found when strong imposed motions are applied. The modeling at ds = 20 *mm* from the wall satisfactorily approaches the experimental data (registered at the same point).

Figures 10 and 11 plot maximum and minimum wave heights reached during time-periodic regimes for different imposed amplitudes at frequencies 0.87 *Hz* and 1.45 *Hz*, respectively. They demonstrate a good match between experimental and numerical values. However, important differences for an imposed amplitude of 15 *mm* are observed. On the other hand, a major effect on decreasing the wave heights when the baffles were used for these frequencies could not be found.

Figure 12 shows the frequency scan using an imposed amplitude of 5 *mm* for baffles positioned at 50 *mm* and 75 *mm* from the bottom of the tank. As in resonance without baffles, the wave height increases continuously. The values reported are at 6 cycles in the time evolution and they are taken as a qualitative reference. Tables 5 and 6 report more detailed information of the frequency scan analysis. It is seen that the baffles play a relevant role at frequencies near resonance. In addition, a good agreement can be found between numerical and experimental data. Major differences are found at frequency 1.16 Hz between upper and lower bound values for both the maximum and minimum wave heights illustrating the phenomenon of beating that appear when the imposed motion has a frequency near (but not exatly equal) to the resonance frequency of the system. The difference between upper and lower bound limits at 1.16 Hz (and or at higher imposed amplitude motion) could be amplified due to the 2D model adopted, neverthless strong 3D effects are not confirmed as it was mentioned in Section 2 when baffles are used in the cases analyzed.

Figures 13 to 15 encompass the numerical and experimental results obtained during damped decaying regime for the studies without baffle, *Cort* = 25 and *Cort* = 37.5 *mm* at *Hc* = 50 *mm* and *Cort* = 37.5 and *Cort* = 56.25 *mm* at *Hc* = 75 *mm*. The free sloshing behavior evolves when the shake table is stopped from different imposed motions: amplitude of 7.5 *mm* and frequency 0.87 *Hz* (Figure 13); amplitude of 7.5 *mm* and frequency 1.45 *Hz* (Figure 14); and amplitude of 15 *mm* and frequency 0.87 *Hz* (Figure 15). A global good agreement can be found in the prediction of the decaying time. Major discrepancies are obtained in the computed amplitudes. Such differences can be attributable to the conditions developed at the beginning of the damped decaying regime as it can be also observed from the figures. Impulsive behaviors are present in the experiments at early stages of those periods. These aspects are not reproduced in the numerical model. Nevertheless, the FFT analysis reported in Table 7 demonstrates a satisfactory description of frequencies (only the first two representative frequencies are reported).

In addition, Figure 16 shows the numerical evolution of the free surface during the damped decaying regime that progresses after an imposed motion of amplitude 7.5 *mm* at frequency 1.45 *Hz*. These results show a more rapid decrease of the wave height with baffles. The highest decrease occurs when the taller baffle (*Cort* = 56.25 *mm*) is used placed at 75 *mm* from the bottom of the tank.

5 Conclusions

This paper reports on simulations and experiments conducted to evaluate the effect of baffles in rectangular tanks during sloshing at different frequencies and amplitudes of the imposed motion. Water level evolutions were registered with ultrasonic sensors. The simulations were performed using a fixed mesh finite element formulation where the free surface was updated via a Lagrangian scheme.

The water level evolution was properly captured by the model. In particular, wave heights and frequencies response for the several cases analyzed satisfactorily compared with the experimental data during forced and free sloshing. The major discrepancies were found at imposed motions with higher amplitudes. Nevertheless, the numerical solution globally described the free surface behavior in such cases.

The effectiveness of the baffles increases near resonance conditions. From the set of experiments studied, the major reduction of the wave height was obtained when larger baffles were positioned closer to the water level at rest.

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