

A SELECTIVE GENETIC ALGORITHM FOR MULTIOBJECTIVE OPTIMIZATION OF CROSS SECTIONS IN 3D TRUSSED STRUCTURES BASED ON A SPATIAL SENSITIVITY ANALYSIS

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ABSTRACT

There are many problems in civil or mechanical engineering related to structural design. In such a case the solution techniques which lead to deterministic results are no longer valid due to the heuristic nature of design problems. In this article a computational tool based on genetic algorithms, applied to the optimal design of cross sections (solid tubes) of 3D truss structures is proposed. The main feature of this genetic algorithm approach is the introduction of a selective-smart method developed in order to improve the convergence rate of large optimization problems. This selective genetic algorithm is based on a preliminary sensitivity analysis performed over each variable, in order to reduce the search space of the evolutionary process. In order to account for the optimization of the total weight, the displacement (of a specific section) and the internal stresses distribution of the structure a multiobjective optimization function was proposed. The numerical results presented in this article show a significant improvement in the convergence rate as well as an important reduction in the relative error, compared to the exact solution.

KEYWORD: Multiobjective optimization; Genetic algorithm; Sensitivity analysis; 3D bars structure

1 INTRODUCTION

Usually, building design methodology is closely related to structural engineer experience which in turn seizes the general guidelines taken from engineering project or architectural design, and mainly this is the way in which restrictions and allowance took over the initial sketch out. However, one new and at the time important tool is generally underestimated, i.e., the scientific optimization analysis. An extended utilization of the above-mentioned, would clearly improve the mechanical behaviour of the structural system as well as the residents comfort.

In addition, many engineering problems concerning the structural design may be addressed with simplified or classic theories of Materials Mechanics as well as by numerical approach based on the Finite Element Method (FEM), leading in both cases to deterministic solutions. However, the subset of optimal structural design does not remain included in the set of deterministic field because of its heuristic dependence. In these cases, the numerical solution can be obtained by iterative algorithms, such as Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Ant Colony (AC), etc. In such a way, the non-deterministic numerical solution obtained defines a bounded domain where the unique real solution is enclosed (Balamurugan *et al.*, 2014; Belegundu and Chandrupatla, 1999; Daei and Mirmohammadi, 2015).

The engineering problems related to structural design comprehend a wide number of situations: trussed systems optimization in both two-dimensional and three-dimensional (Coello and

Cristiansen, 2000; Huang and Wang, 2008; Mroginski *et al.*, 2009; Talaslioglu, 2009; Thein and Liu, 2012; Torii *el al.*, 2012), topology analysis of plane frames structures (Stromberg *et al.*, 2012), design of composite structural sections (Lopez *et al.*, 2009), micromechanics optimization (Huang *et al.*, 2011), performance of thermodynamic machines (Ahmadi *et al.*, 2014; Sadatsakkak *et al.*, 2015), fiber reinforced polymeric (Cai and Aref, 2015), among others.

In this article a computational tool based on genetic algorithms (GA), applied to the 3D structural design is proposed. Though for the time being the analysis is bounded to 3D truss based structures, it is perfectly possible to extend the general conclusions hereafter posed to cope with many other stress-involved problems, namely, geometrically induced nonlinearities (Di Rado *et al.*, 2008), geo-mechanical stress determination (Mroginski *et al.*, 2010; Beneyto *et al.*, 2015), nonlinear finite element analysis (Mroginski and Etse, 2013), etc.

The main feature of this GA is the inclusion of a selective-smart method developed in order to improve the convergence rate of large optimization problems. This Selective Genetic Algorithm (SGA) is based on a preliminary sensitivity analysis performed over each variable, in order to reduce the search space of the evolutionary process. Along with the previous, the SGA was tested, on the one hand, with three different objective functions in order to minimize the total weight, the displacement and/or the Euclidean norm of internal stress. On the other hand, a multiobjective function was introduced in order to simultaneously optimize the single objective functions described before.

Two numerical examples were considered. The first one is a small optimization problem with 24 variables for each individual. The second one consists in a large optimization problem of a 3D tower structure with 80 variables for each individual. Both examples were analyzed with the classical Non-Selective Genetic Algorithm (NGA) and the SGA proposed in this article. The main difference between the NGA and the SGA is that the preliminary sensitivity analysis is performed only in the SGA algorithm. In order to get a similar relative error the required number of iterations is quite different in both cases. Furthermore, when the proposed SGA is compared with the NGA a significant improvement in the convergence rate as well as a good approximation of the deterministic solution (in case of being known) can be observed, from the numerical results presented in this article.

2 ALGORITHM DESCRIPTION

The theoretical framework of GA arises from the evolution theory of Darwin. Therefore, the individuals with major aptitude in a population have a greater chance of survival. In the optimization problem, each individual represents a space of different solutions and their aptitude is defined by the evaluation function. This function is known as objective function and its extreme value is sought during the GA procedure.

2.1 Main Features

The main features of the proposed algorithm are

- Individuals are made up by the diameter of the bars
- An elitist type of algorithm is used. Thus, most adapted individuals are directly chosen out of the population and moved to the next generation without going through the crossing process.
- The upper and lower variables bounds are the only constraints imposed.
- A selective iterative procedure was introduced in order to focus the search on the most sensitive variables in the first step of the analysis.
- A partial population renovation is implemented in order to avoid a saturation with the best individuals. Therefore, the mutation step is no longer needed (Mroginski *et al.*, 2009)
- A penalty function is introduced in order to avoid truss plastification (Fancello and Pereira, 2003; Thein and Liu, 2012)

2.2 Particular Features

Variable initializations

The initial values of the subsequent command variables are prescribed. The population size (*PopSize*), the maximum numbers of generations (*ngen*), the number of variables (*numvars*), the percentage of individuals which will not be renewed (*porpas*) and the number of individuals of the elite group (*Nelit*). To clarify, the pseudo-code of the GA proposed in this work is showed in Fig. 1.

```
BEGIN
Variable Initialization
SENSITIVITY Sensitivity Analysis
FOR Loop over groups of variables
  Initial population generation
  FOR Loop over generations
     Evaluation of the objective function
     Definition of the elite group
     Selection
     IF Odd generation
        Smart Crossover
     ELSE Pair generation
       Multiple Crossover
     END IF
     Population = Crossover Population + Elite
     New population
  END FOR
END FOR
END
```

Figure 1: Elementary pseudo-code of this selective genetic algorithm

Sensitivity analysis

At the beginning of the evolutionary iterative algorithm a sensitivity analysis of the involved variables corresponding to each individual is carried out. The main aim of this study is to set up groups of variables (*ngrup*) according to their dependence with the adopted objective function.

The easiest way to perform the sensitivity analysis consists in imposing the minimum value (or maximum) in almost all variables, except those in which the analysis is carried out, and evaluate the objective function. This allows the characterization of the influence of each variable according to its importance with regards to the adopted objective function. Firstly, each group is analyzed individually in order to avoid spurious solutions. Then, the whole variables groups are simultaneously considered in the iterative GA procedure. In seeking the global optimum with classical NGA algorithm and variables with very low sensitivity is usually observed that the solution found correspond to the groups most sensitive while those with lower sensitivity adopt random or spurious values.

This selective optimization technique magnifies the influence of minor importance variables on the objective function. Thereby, the variables groups with too low influence on the global objective function can be optimized as well. The recurrent application of this optimization technique to the remaining variable groups allows the saturation of the less influent ones up to their corresponding optimum value which, whereby, are now hardly missed afterwards the global iterative process (i.e., involving the whole universe of variables) was carried out.

Initial population generation

The initial population is created by a heuristic algorithm specially designed to satisfy the constraints of the problem regarding the upper and lower bounds of the variables. It must be underscored that the population consists in a set of individuals, which in turn are arrays of (numvars, 1) dimension, formed by the diameters of tubular sections (with 2 mm thickness) corresponding to the three-dimensional truss structure. Due to the commercial viability of tubular bars, the diameters are assumed to be discrete variables. The maximum and minimum imposed boundaries are 50 mm and 10 mm, respectively.

Multiobjective evaluation

In this work the numerical evaluation of the objective function was carried out by a Finite Element (FE) software based on an open source platform (Scilab) developed by the authors. In the FE code linear elasticity was assumed and 3D bars elements (three degree of freedom for each node) were implemented.

The optimization procedure is carried out considering the following objective functions:

 f_1 : Minimization of the structure weight [kg]

f₂: Minimization of displacement [m]

 f_3 : Minimization of the Euclidean norm of the stress [Kpa]

The multiobjective analysis is performed using a linear combination of f_1 and f_2 , through the interpolation function of Eq. (1) (Belegundu and Chandrupatla, 1999; Mroginski *et al.*, 2009)

$$F = \sum \omega_i f_i \tag{1}$$

being ω_i the corresponding weights coefficients.

The function f_3 is used to compare the multiobjective optimization with the single optimization approach, using a function with an equivalent physical meaning.

Besides, in order to avoid truss plastification a penalty function was introduced (Fancello and Pereira, 2003). According to the actual stress state, the second invariant of the deviatory stress tensor (J2) in each bar is obtained and compared with the elastic-plastic limit (Von Mises material model). Thus, in case of plastification, the corresponding objective function is increased by a weighted factor.

Selection

A Simple Roulette selection process is adopted. Thus, the individuals selection is based directly on the probabilities of each one through a random shot. In order to reduce the saturation of the population with the most adapted individuals, the probabilities are obtained using the scaling function of Eq. (2) instead any of those more simple (direct quotient, etc.) commonly employed elsewhere (Belegundu and Chandrupatla, 1999).

$$P = \frac{F + (0.1F_{max} - 1.1F_{mir})}{max[1.5 1.1*(F_{max} - F_{mir})]}$$
(2)

being F the objective function obtained from Eq. (1), and F_{min} and F_{max} are its extreme values.

Crossover

In this subroutine a mixed crossover algorithm is proposed. On the one hand, a particular crossing process for structural optimization is used for odd generations, called Smart Crossover, since the combination between variables represent a physical meaning, i.e. the variable array is conveniently classified in diagonals, pillars, etc. (see Figure 2). The variables groups adoption of the smart-crossover operator are based on subjective criterions derived from researcher's experience (or engineer's). A remarkable aspect is that these groups must not be mistaken with those obtained in the aforementioned sensibility analysis.

The smart crossover operator exchanges the entire groups of variables between two individuals who are crossing; in fact, the main goal is to preserve the invariance of the group as a whole after the crossing process. In some structures, the assessment of a single bar impact in the overall structure behavior is too difficult a task whenever it is not performed regarding the structural component in a

complete manner (i.e., bottom chord, top chord, webs, etc.). Specifically, one feature of the smart crossover operator is precisely to grasp the aforesaid overall behavior of each variable group.

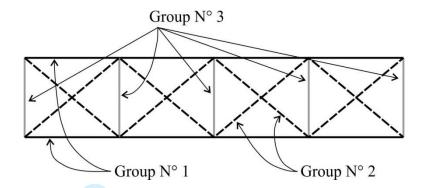


Figure 2: Group definition for smart crossover method considered in odd generations

On the other hand, a Multiple Crossover technique is implemented for pair generations. Therefore in order to perform the combination between the selected individuals groups within each generation (see Figure 3) a random binary vector of dimension *numvars* with the crossover positions is created.

Population renewal

If spurious solutions regarding to locals minimum are to be avoided, some partial renovation technique over the population should be included in the proposed GA. This technique involves a partial replacement that must not exceed 30% of the original one to prevent the GA becomes a simple random search algorithm. This technique increases the search field of the GA and removes less adapted individuals of the evolutionary process. Consequently, the additional step of mutation is not longer required.

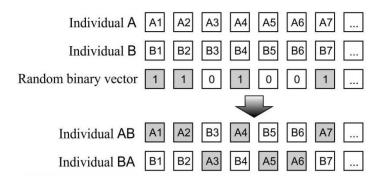


Figure 3: Illustration of the Multiple Crossover technique employed in this article

Stopping criterion

For the present multiobjective GA, the stopped criterion is reached when the 95% of the chromosomes (variables) have the same value. Furthermore, towards reducing the computational cost of the evolutionary process, this test is performed once every 100 iterations. It is worth to remark that the individuals resulting from the selection-crossover process define the new population; therefore, its size remains unchanged. In such a way, this algorithm can be classified as generational (Castro *et al.*, 2006).

3 RESULTS

The proposed selective Genetic Algorithm was applied to three-dimensional structural optimization and some numerical outcomes are hereinafter presented. Two examples of 3D bars structures with known and closed solution are analyzed in order to evaluate the convergence rate and the estimated error (obtained with the Euclidean norm) of the proposed Selective Genetic Algorithm (SGA).

3.1 Small problem.

The first example is a 3D bridge optimization problem. The material properties are: Young module E=210 MPa, Poisson module v=0.27 and density $\delta=7850$ Kg/m³. Also, the geometry, boundary condition and the adopted enumeration of each variable are presented in Figure 4. Two vertical external forces were considered, P=10KN.

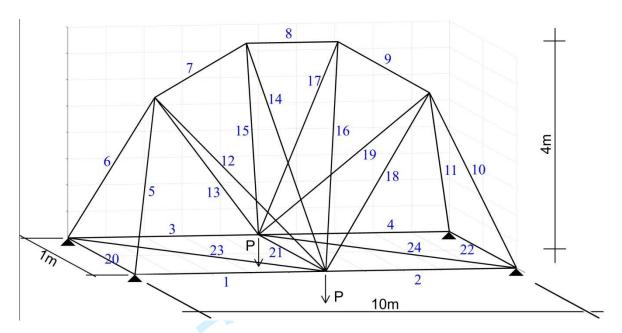


Figure 4: Geometry and bars identification of the small structural optimization problem

The controlling variables adopted are: *numvars* = 24, *PopSize* = 50, *porpas* = 80% and *Nelit* = 2%. The main controlling variable regarding to the total execution time of the genetic algorithm is in fact the population size. However, the better adoption of this parameter (without oversizing the problem) is not a trivial issue. The required population size (PopSize) depends on many factors, such as the number of variables considered in each individual, the non linearity of the objective function as well as the domain of field variables (Rexhepi *et al.*, 2013).

In the first place, the minimizing problem of the total structural weight is considered. Unless an additional restriction is imposed, the exact solution for this problem is an individual composed by the minimum admissible value in each variable, i.e. *dmin*. Then, from the sensitivity analysis explained above, three groups of variables arose and they are presented in Table 1. The relative error obtained from the evolutionary process, compared to the exact solution, is less than 1%. Also, it can be observed from Table 2 that this error is presented in elements 20 and 21, which correspond to the third group of variable with lower influence in the objective function.

Next, the geometrical solution for the minimum displacement corresponding to the middle section of the structure is analyzed. Similar to the example above, the exact solution of this problem is known and it is attained when all variables attain the maximum admissible value, i.e. *dmax*. For the case in point, four groups of variables came up applying the sensitivity analysis explained before (see Table 1). Also, the relative error obtained in this example is less than 2% and is presented in bars 2 and 4, corresponding to the fourth group of variables.

The third test involves the minimization of the Euclidean norm of internal stresses. In contrast to the precedent examples, the exact solution is unknown previously. Therefore, an intermediate solution is expected.

Following, the multiobjective minimization procedure is carried out through the combination of the objective functions fI and f2, in the Eq. (1). The impact of the weight functions on the solution shape, ω_1 and ω_2 corresponding to the minimum weight and minimum displacement of the middle section, respectively, is presented in Table 2. Also, it can be observed that the increase in the weight function ω_1 mainly affects those sensitivity variables related to the weight minimization problem introducing a bias in the minimum admissible value, dmin.

g

Function f_I (min. weight)											
Variable Group N° 1	(
Variable Group N° 2	5, 6, 7, 8, 9, 10, 11										
Variable Group N° 3 20, 21, 22											
Function f ₂ (min. displacement)											
Variable Group N° 1 7, 8, 9											
Variable Group N° 2 5, 6, 10, 11, 12, 13, 18, 19											
Variable Group N° 3	Group N° 3 14, 15, 16, 17, 21										
Variable Group N° 4 1, 2, 3, 4, 20, 22, 23, 25											
Function f ₃ (min. stress)											
Variable Group N° 1	5, 6, 7, 8, 9, 10, 11										
Variable Group N° 2	12, 13, 14, 15, 16, 17, 18, 19, 21										
Variable Group N° 3 1, 2, 3, 4, 20, 22, 23, 24											

Table 1: Variables groups obtained from sensitivity analysis for each objective function

Variables (diameters in cm)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	ngen 1	ngen 2
f1 (min. weight)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	1	1	290	551
f_2 (min. displ.)	5	4	5	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	312	619
f_3 (min. stress)	1	1	1	1	5	5	5	5	5	5	5	3	2	2	2	2	2	2	3	3	5	3	1	1	344	589
MO [1,10]	1	1	1	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	1	3	1	1	1	302	588
MO [1,5]	1	1	1	1	5	5	5	5	5	5	5	5	5	4	4	4	4	5	5	1	2	1	1	1	311	603
MO [1,1]	1	1	1	1	4	4	5	5	5	4	4	3	3	2	2	2	2	3	3	1	1	1	1	1	321	655
MO [5,1]	1	1	1	1	2	2	4	4	4	2	2	2	1	1	1	1	1	1	2	1	1	1	1	1	354	641
MO [10,1]	1	1	1	1	1	1	3	3	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	341	645
¹ number ofgene	¹ number of generations for SGA; ² number of generations for NGA;																									

Table 2: Solutions obtained with the proposed SGA for all adopted objective functions, and multiobjective analysis

Similarly, by increasing the weight function ω_2 , the obtained solution is close to the one corresponding to the displacement minimization.

Meanwhile, multiobjective analysis with weight functions $\omega_1 = \omega_2 = I$, denoted in this article as $MO[\omega_1, \omega_2]$, tends to the mean solution, which correspond to the minimization of the

Euclidean norm of the stress state (see Table 2). Figure 5 shows the representation of three multi-objective solutions corresponding to MO[1, 10], MO[1, 1] and MO[10, 1].

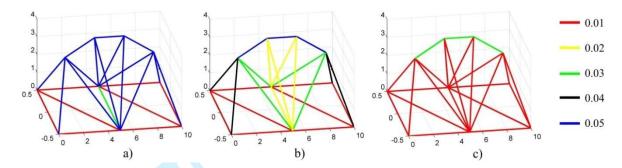


Figure 5: Multi-objective solutions considering three different combinations of weight functions: a) MO[1,10], b) MO[1,1] and c) MO[10,1]

Finally, the same sets of optimization problems are carried out without the sensitivity analysis proposed in this article. This evolutionary algorithm is known as Non-Selective Genetic Algorithm (NGA). Achieving a relative error tantamount to the one previously obtained with SGA would entail too high a computational cost as it was clearly settled down in the last two columns of Table 2 showing the required iterations of the SGA and the NGA to fulfill the convergence criterion formerly explained.

3.2 Large problem.

The same sets of analysis developed in previous section were carried out for a large problem of 3D structural optimization. Hence, this second analysis is concerning to a 3D steel tower optimization. The material properties of the steel bars are: E = 210 MPa, v = 0.27 and $\delta = 7850$ Kg/m³. The geometry, boundary conditions and the label of each variable assigned are presented in Figure 6. In this case, four horizontal external forces were applied on the top nodes of the truss structure, P=10KN.

Controlling variables adopted are: *numvars* = 80, *PopSize* = 200, *porpas* = 70% and *Nelit* = 2%.

From the sensitivity analysis the incidence of each variable is classified in four groups. Figure 7 shows the spatial distribution of each group regarding to the adopted objective function.

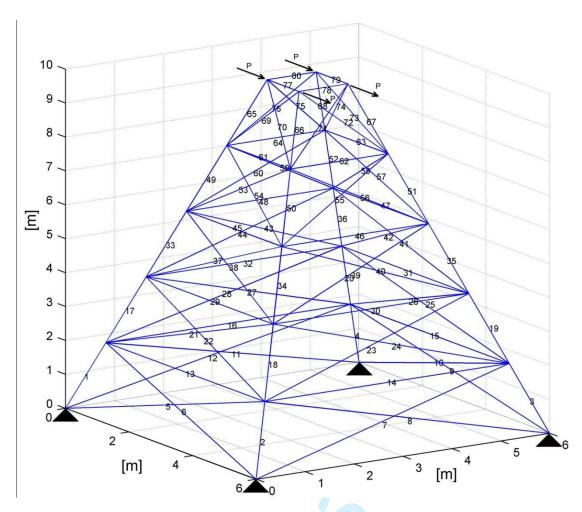


Figure 6: Geometry, boundary conditions and bars identification of the large structural optimization problem

In first place, the minimizing problem of the total structural weight and the horizontal displacement of the tower top are considered. Whether non additional restrictions are imposed the closed solution of this problems are known. Similar to the previous example, the relative error is very small and is due to the variables corresponding to the fourth group.

In Table 3 the relative errors as well as the required number of iterations for both, SGA and NGA proposed in this article, are presented.

Finally, in Figure 8 the population evolution of the SGA for the displacement objective function is plotted. Additionally, it can be identified the instant wherein a partial renewal of the population take place as well as the jump in the objective function due to the incorporation of a new group of variables (sensitivity analysis).

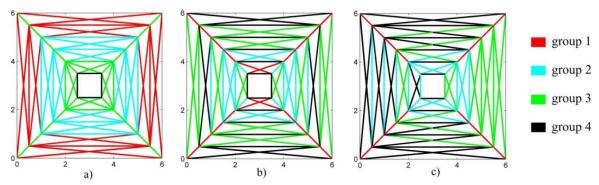


Figure 7: Top vision of the sensitivity analysis for the large structural optimization problem. The 3D structure is colored according to the assigned group obtained from the sensibility analysis and considering the following objective function: a) weight minimization; b) displacement minimization; c) stress minimization

Variables (diameters in cm)

Relative error	<i>ngen</i> for SGA	<i>ngen</i> for NGA
2.2	654	1457
1.9	691	1354
2.5	684	1524
	2.2 1.9	2.2 654 1.9 691

Table 3: Relative error obtained with the proposed SGA and NGA for all adopted objective functions.

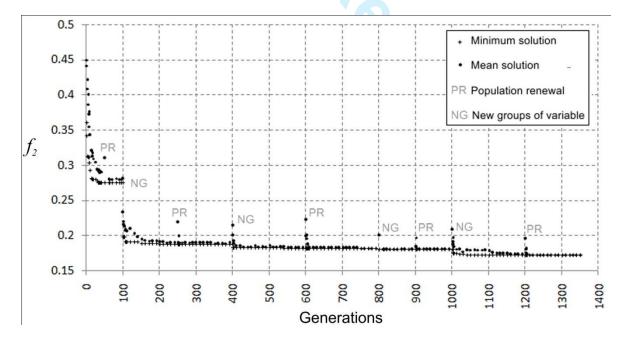


Figure 8: Population evolution of the SGA

4 CONCLUSIONS

A computational tool, based on generational elitist genetic algorithms, for multi-objective optimization of 3D steel structures is presented. The adopted objective functions consist in minimizing the total structural weight, displacement (on a particular section) and internal stress distribution. The variables sensitivity analysis put forward in this approach introduces a significant improvement in the convergence rate of the algorithm. The presented numerical results show the versatility and robustness of the proposed genetic algorithm. Its application to the 3D metallic structures design could represent a good alternative at preliminary project stages.

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