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Simultaneous optimization of production planning and inventory management of polyurethane foam plant

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Abstract In this work, the management of a polyurethane foam plant is tackled through a mixed integer linear programming model that simultaneously solves production and inventory planning problems. The production process considers the foaming stage where large polyurethane blocks are produced as well as the curing step where the blocks are dried. The proposed formulation takes into account several tradeoffs involved in the overall production process. The daily production planning is tightly related to production requirements, available space for the curing and stored elements. Moreover, the required time to dry blocks introduces a delay that must be appropriately considered in order to allow an adequate operation of downstream operations. Thus, an integrated approach where all these problems are jointly addressed is proposed using a mathematical programming model. Several study cases provided by a local company are tested to demonstrate the model performance.

Keywords Production planning · Inventory management · Polyurethane foam · MILP

List of symbols

Indices

i Block widths
j Block densities
k Block lengths

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h Rows in the curing area
g Groups of blocks

Sets

Blocks Set of possible foam blocks of width i, density j, and length k
Cart Set of long blocks that can be cured on special carts since they have a low density j
Long Set of long blocks
Orders Set of special orders to produce blocks of width i, density j, and length k that are made to order
Groups Set of blocks of width i, density j, and length k belonging to groups g

Positive variables

dif_{ijk} The difference between the final stock and the minimal stock for the block of width i, density j and length k
sf_{ijk} Final stock for the block of width i, density j and length k
sm_{ijk} Intermediate stock for the block of width i, density j and length k

Binary variables

u_{ijk} Indicates if there is no unsatisfied demand for the block of width i, density j and length k
w_{jh} Indicates if density j is produced and assigned to be cured in row h
x_{ijk} Indicates if external order blocks of width i, density j and length k are produced
y_i Indicates if the width i is selected to be produced
z_{jkh} Indicates if any block of density j and length k is placed on row h

Integer variables

n_{ijkh} Number of blocks produced of width i, density j, and length k, placed in row h
$n1_{ijkh}$ Number of long blocks of width i and density j cured on the special carts (note that the purpose of keeping index h in this variable is given by Eq. (7) but it has no physical meaning)
$n2_{ijkh}$ Number of long blocks of width i and density j placed on row h of the floor

Parameters

BM_o "Big M" parameter, where $o = 0, 1, 2,\ldots,7$
cw_i Width of the curing area when width i is selected
cl_i Length of the curing area when width i is selected
d_{ijk} Demand for blocks of width i, density j, and length k
fl_{ijk} Length of block of width i, density j, and length k occupied on the floor in the curing stage
bw_i Width of blocks from set i
fs Minimal space that must be left between blocks to allow air flow in the curing area

l_{ijk}	Length of block of width i, density j, and length k
l_min	Minimal length to produce for each density
ml_i	Minimal length to be produced for width i
np	Number of places available in the carts for the curing stage of long blocks
$Rows_i$	Number of available rows in the curing area when width i is selected
s_{ijk}	Number of blocks of width i, density j, and length k in stock at the beginning of the day
sc_g	Stock capacity of group g
$smax_{ijk}$	Maximal stock level (stock capacity) for block of width i, density j, and length k
$smin_{ijk}$	Minimal stock level for block of width i, density j, and length k
soq_{ijk}	Number of special blocks ordered of width i, density j, and length k

1 Introduction

The mattress industry has had a sustained growth in recent years motivated by market conditions and new consumption trends. Consumer habits have changed and mattresses are renewed more frequently. Purchase decisions are now driven mainly by wellbeing and self-care rather than price. Thus, mattress firms must cope with new requirements and market needs, producing a huge variety of products and foams with a high level of service. Optimal production and inventory planning is crucial to achieving these objectives.

One of the main characteristics of this problem is that several requirements must be simultaneously considered. The polyurethane foam is produced in a machine where setup and changeover costs and times are important. A detailed plan is required. Furthermore, once foam blocks are produced, they must be cured during about a day. Taking into account the volume of blocks, an appropriate space must be available. Meanwhile, the downstream mattress production process must be satisfied, and cured blocks are provided from the inventory. This process is driven by a consumption characterized by an irregular pattern. Therefore, an approach that integrates all these perspectives is required.

The integration of process and logistic decisions has been proposed by many authors and applied in several industries. A new perspective to cope with production systems is presented by Grossmann (2012). Although he analyzes a wider perspective than this work, the advantages of the integrated formulations are overwhelming. Also, Maravelias and Sung (2009) review the challenges and advantages of the integration of medium-term planning and short-term scheduling.

Different types of decision can be integrated. In Jain and Grossmann (2000) the manufacturing and packing continuous stages of a plant are integrated using an intermediate storage. They propose an MILP scheduling model, but given the combinatorial nature, there are limitations on this model as the number of orders increases. A heuristic based on partial preordering is considered that solves industrial sized problems very quickly. Lin and Floudas (2003) investigate the long-term planning problem for integrated gas field development where the key decisions involve both design of the production and transportation network structure and operation of the gas fields over time. A novel continuous-time modeling and optimization approach is proposed. A two-level formulation and solution framework is developed to take into account complicated economic calculations and results in mixed-integer nonlinear programming (MINLP) problems. Computational results show reduced computational efforts solving problems that are intractable for the discrete-time model. Verlinden et al. (2009) develop an integrated production planning methodology for sheet metal. The shop is configured as a two-stage flow shop with laser cutting and air bending. The authors argue that although in the literature, theoretical production planning models can be found for the individual processes, they focus on one single production step while in reality the planning decisions taken at the cutting stage affect the production plan for the air bending stage. An integrated production planning methodology is proposed to overcome this problem by taking into account relevant bending information at the cutting stage. Rodriguez and Vecchietti (2010) integrate the inventory and delivery optimization problems under seasonal demand in the supply chain. They propose a detailed mathematical formulation where the purchased amount is distributed among several deliveries giving rise to a nonlinear nonconvex problem that is transformed to find a global solution. On the other hand, Fumero et al. (2011) propose an MILP model to simultaneously solve planning and scheduling problems. In particular, they address a multiproduct batch plant that operates with mixed production campaigns. The same authors (Fumero et al. 2013) present a mathematical model where decisions about supply chain and batch plant design are simultaneously considered. They emphasize that significant benefits can be obtained if the interactions among different decision levels are appropriately addressed and jointly solved. Two alternative MILP models are proposed by Relvas et al. (2013) for integrated scheduling and inventory management of an oil-product distribution system. The approaches aim to attain a set of planning objectives such as fulfilling customer demands while minimizing the medium flow rate and avoiding excessively low final inventory levels. In a similar industry, Marchetti et al. (2014) propose a multi-period MILP model for the simultaneous production and distribution of industrial gas supply-chains. Process decisions include production modes and rates that determine power consumption while the distribution plan involves the selection of routes, quantity to deliver and time of each truck delivery. Cafaro and Cerdá (2016) synthesize two innovative optimization tools for the short-term planning of oil product pipelines. They integrate planning the injection, transportation and delivery of batches moving into pipelines with many operational constraints.

With the purpose of evaluating the value of integrating tactical warehouse and inventory decisions, a global warehouse and inventory model is presented by Strack and Pochet (2010). They develop two solution methodologies that offer different levels of integration of warehouse and inventory decisions. They conclude that the total cost of the inventory and warehouse systems can be drastically reduced by taking into account the warehouse capacity restrictions in the inventory planning decisions, in an aggregated way.

Finally, the simultaneous scheduling and control problems are solved for sequential batch processes in Chu and You (2014). A moving-horizon approach is developed to achieve computational efficiency and rescheduling stability. A reduced integrated problem is formulated; it can be solved efficiently online. When dynamic situations are presented in systems integrating production-inventory problems, control theory offers suitable tools for handling the time-varying phenomena (Ortega and Lin 2004). In these cases, dynamic constraints must be taken into account to address the inventory management problem (Subramanian et al. 2014). Although there are more works addressing the integrated formulation of production problems, this is not the more usual approach. As a consequence, the assessment of different tradeoffs is lost.

Some other works have been developed for the industry addressed in this article. Most literature on this subject studies the technical properties of the products or processes of the industry (Lanoë et al. 2013; Engels et al. 2013; Hopmann and Latz 2014; Bernardini et al. 2014).

Few articles deal with the production planning problem in this specific industry. Lin et al. (2013) propose a flow shop scheduling model inspired by a real production line of polyurethane (PU) foam at a plant in Taiwan. Due to different chemical compositions, various types of foams can be produced by mixing different materials on a foaming machine. Mogaji (2014) presents a decision support system for process planning and control of PU foam production. The system is developed to enhance production efficiency and presents seven modules that work together to support the decision-making task. It also includes a simple linear programming (LP) model to minimize the production costs considering limited raw materials in each period.

In the foam production process, the foam blocks have to be placed in a limited area during the curing stage. This type of decision can be related to the strip packing problem (Trespalacios and Grossmann 2016), where several small pieces must be accommodated in a larger area. A review of the literature on this topic is presented by Wäscher et al. (2007).

In the present work, an MILP model for solving planning and inventory problems in a PU foam production plant is proposed. The literature on the problem addressed in this article is scarce, and to the best of our knowledge this particular problem has not been studied before. One important aspect of this article is that a real industrial application is considered. Different challenges appear when real cases are addressed. The importance of this issue is pointed out by Harjunkoski et al. (2014), where the authors observe that there is potential for improvement, especially in transferring academic results into industry. Moreover, the proposed approach can be extended to other industries where there is a single machine producing large pieces of products of different characteristics, which must be arranged into a limited area and where the storage management with an optimal use of the available space are key issues for the process development. Some examples are sawmill processing, corrugated board box production, and paper roll production.

Different decision levels are involved, jointly formulated and solved in the proposed approach. The production process considers not only the foaming stage, but also the curing step which involves strip packing type decisions. The proposed arrangement of blocks in the limited curing area allows an effective representation of the problem, avoiding a complex formulation. Detailed inventory management decisions are also embedded in the proposed approach, achieving a more realistic representation of the problem. In this way, the presented formulation integrates several decisions and, therefore, various tradeoffs are simultaneously considered and evaluated. Despite its complexity, the proposed formulation allows an efficient solution.

The manuscript is organized as follows: in Sect. 2 the problem is explained. Section 3 presents the model formulation while Sect. 4 shows the results obtained. Final conclusions and a discussion are given in Sect. 5.

2 Problem statement

The proposed approach is motivated by production planning and stock management problems of polyurethane (PU) foam blocks for a mattress manufacturing plant. The subject considered in this work involves three basic stages: foam production line, curing of foam blocks and stock of foam blocks.

The production is carried out in a single foaming machine where various densities of PU foam can be produced. There is a set of possible foam widths but only one width can be selected per day for production given that a long setup is required between width changes. The foam is cut in different lengths and densities, forming the blocks according to characteristics required by the production procedures in downstream stages. There is a specific density sequence to be produced, from lighter to denser materials, taking into account production requirements. Also, to reduce scraps, when a density is produced, a minimal length must be planned.

The second stage is the curing process where the blocks need to cool, dry and obtain the appropriate structural properties. For this purpose, the blocks are arranged in a limited area for about 20 h. They are moved from the foam machine to the assigned place by a cart. The blocks are introduced to the curing area in the same order they are processed in the foaming machine. They are placed lengthwise, forming rows as shown in Fig. 1. The first row is located in the back of the area where blocks are placed from left to right. A fixed space between two consecutive blocks is left to help the curing process. All the blocks can be cured lying on the floor of the curing area, but some blocks of low density and short length can be also cured in a standing position, using a smaller surface.

Taking into account the produced foam width i, a maximum number of rows can be located in the curing area. This number represents a model parameter and it is calculated by dividing the curing area length (cl_i) by the block width (bw_i). Equation (1) is a general expression to determine the parameter corresponding to the number of available rows of the curing space for each width i taking into account the sizes involved and the fixed space between two consecutive blocks (fs) (Fig. 1):

$$Rows_i \leq \frac{cl_i}{(bw_i + fs)} \quad \forall i \tag{1}$$

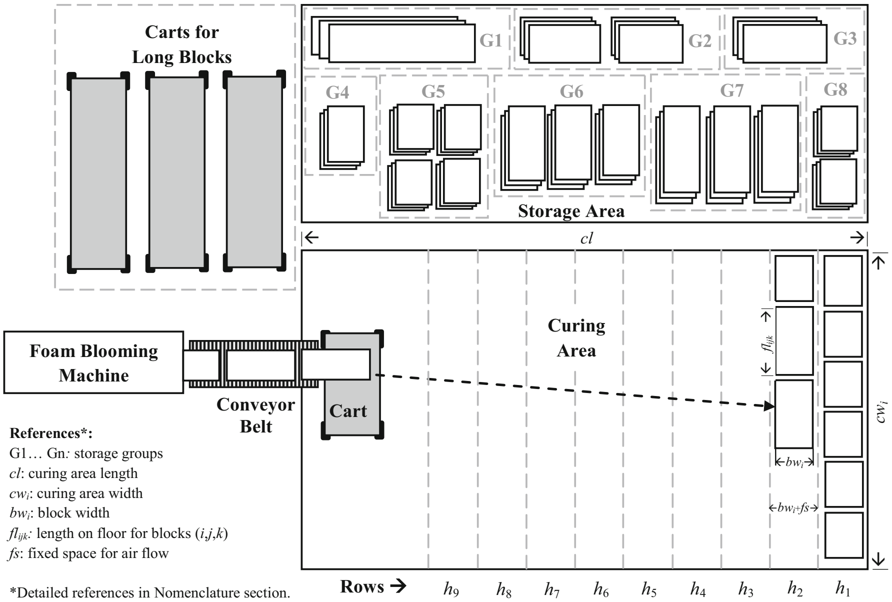

Fig. 1 Production, curing and storage of foam blocks of a mattress manufacturing plant

Since the length and width of the curing area are known, the row number depends on the width selected to be produced. Note that the curing area length (cl_i) also depends on the width i since different curing area arrangements are possible based on the orientation of the blocks in the curing area.

For some widths and densities, longer blocks are foamed. The length of these blocks is approximately four times longer than the others, and some of them can be cured on carts depending on their density, as shown in Fig. 1. Thus, the firm attains a greater curing area with a higher flexibility.

After the curing process, the blocks are moved from the curing area to the storage. In the inventory area, the blocks can be piled up, forming groups, which have an assigned limited capacity at the storage area. Each group is formed of different block types. Each block is classified according to its width, length and density, and it belongs to a unique inventory group. The maximum capacity for each group in the stock area has been introduced taking into account that there is not enough space to dedicate a specific place for each block type. As a special case, groups with only one block type could also be considered in another context. Figure 1 shows the stages of this work.

To clarify the plant floor operation, we describe the different activities. At the beginning of the working day, the blocks in the curing area foamed the previous day are moved to the storage, so when the foaming machine starts producing blocks, the curing area is empty. Every day, taking into account the production requirements, a known quantity of blocks is taken from the storage area and cut to assemble mattresses. In this stage, the planner controls the availability of the required blocks in stock. Let i be the block width, j the foam density and k the block length such that a block is designated by these three elements. If the demand (d_{ijk}) of a certain block (i, j, k) required in the downstream process is greater than the available stock (s_{ijk}), i. e. $s_{ijk} < d_{ijk}$, two options are considered by the planner before executing the model. In the first case, the demand for this block is satisfied in the stock amount and another block is used to satisfy the rest of the order. The block that replaces the required one must be larger or of higher density, i.e. (i, j, k) is replaced by (i', j', k') with i' greater than or equal to i, and k' greater than or equal to k, and density j' greater than or equal to j. In the second case, the demand for this block is satisfied in the stock amount and the rest of this order is postponed until there is availability of this block in stock. The quantity of blocks to be consumed is determined the previous evening and, consequently, it is impossible to work with a longer time horizon. The required stored blocks are taken for the downstream mattress manufacturing process, freeing up space for the next foam production. Considering the curing and stock limitations, and the demand level of the company, the foaming stage takes about 3 h per day. Therefore, a plan on a daily basis is suitable for this process, i.e. planning horizon is one day and the model is executed every day.

As previously mentioned, when the required blocks are not available in stock, the production order might be postponed or another block must be used instead of the one required in that order. In the second case, this means a greater loss of material if a larger block is used or more expensive products if a block of higher density replaces the one missing. Therefore, the main objective of the foam manufacturing area is to keep a sufficiently large and balanced stock of blocks to avoid this type of situation taking into account the available space in the storage area.

Thus, it was detected that the main goal of the firm for this area is to avoid stock-outs. We tested different performance measures in order to find a suitable target, and our proposed objective is to produce as many foam blocks as possible, to ensure an appropriate stock of each block. Therefore, the fulfillment of a suitable inventory level is also considered. If the final stock of a block is less than the required minimal level determined by the company, a penalty is included in the objective function. Then, the proposed formulation maximizes the production length minus the total length of stock-outs given by the missing blocks in storage.

Therefore, the problem for the simultaneous optimal production planning, curing and stocking of foam blocks is as follows:

Given:

- The set of widths, lengths and densities of the blocks.
- The order of densities.
- The minimum length per density.
- The minimum length to be foamed in the planning horizon.
- The curing area size (width and length).
- The number of carts and the set of blocks that can be cured in carts.
- The set of blocks that can be cured in a standing position.
- The stock level for each group at the beginning of the day.
- The minimum stock level for each type of block.
- The demand of each type of block required for the downstream process.

- The storage groups and the blocks that compose each group.
- The capacity of each storage group (number of blocks of each type).

Determine:

- The width to be foamed.
- The lengths and densities for the selected width.
- The number of blocks of each type to be foamed.
- The sequence in which the blocks are foamed.
- The block arrangement in the curing area.
- The total stock of blocks at the end of the working day.

The objective is to maximize the total foamed length minus the total length of inventory shortage, fulfilling the minimum stock requirement for the blocks of the foamed width.

3 Model formulation

In this section we present the mathematical model for the optimal production planning and stock management of foam blocks for a mattress manufacturing plant. The constraints are combined into three groups: planning constraints, stock constraints, and objective function. The result of this model is a daily production plan considering process and storage requirements as well as business policies from the company.

3.1 Planning constraints

Let y_i be a binary variable equal to one if width i is selected, then due to the set-up policies, only one foaming width can be selected:

$$\sum_i y_i \leq 1 \tag{2}$$

As previously mentioned, the blocks form rows in the curing area (Fig. 1). Let n_{ijkh} be a variable equal to the number of produced blocks of width i, density j, and length k, placed in row h, then, $n_{ijkh} = 0$ for $h > Rows_i$, that is, no blocks of width i are placed in row h for h greater than the number of available rows in the curing area for each width i:

$$n_{ijkh} \leq 0 \quad \forall i, \forall j, \forall k, \forall h > Rows_i \tag{3}$$

Let l_{ijk} be the parameter corresponding to the block length. Then, if blocks of width i are produced, i.e. $y_i = 1$, a firm production policy based on high setup costs is that a minimum length must be manufactured. Therefore, the total produced length has a lower bound, which is given by the parameter ml_i, defined taking into account setup costs:

$$\sum_{\substack{j,k,h: \\ (i,j,k) \in Blocks}} n_{ijkh} \cdot l_{ijk} \geq ml_i \cdot y_i \quad \forall i \tag{4}$$

where *Blocks* is the set of possible foam blocks defined from production requirements.

A minimum length for each produced density j has to be fulfilled. This bound is defined taking into account changeover costs. Let w_{jh} be the binary variable equal to one if density j is produced and assigned to be cured in row h, and l_min the parameter for the minimum length. Then, Eq. (5) determines that the number of blocks of density j multiplied by their corresponding length must be greater than or equal to l_min if density j is used in any row h'.

$$\sum_{\substack{i,k,h: \\ (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkh} \cdot l_{ijk} \geq l\_min \cdot w_{jh'} \quad \forall j, \forall h' \tag{5}$$

If a given width i is not selected, then no block of this width i can be produced for any density j, length k, placed in any row h as shown in Eq. (6):

$$n_{ijkh} \leq BM_0 \cdot y_i \quad \forall (i,j,k) \in Blocks, \forall (i,j,k) \in Blocks \tag{6}$$

where BM_0 is a sufficiently large parameter, which can be the maximum stock capacity for this type of block ($smax_{ijk}$).

As mentioned above, blocks must be cured after they are cut in the foaming machine. For this purpose, they are placed in the floor forming rows as shown in Fig. 1 where they must stay for 20 h. All the blocks can be cured on the floor and some of them, with density $j \in Cart$ and length $k \in Long$, can be also cured in carts. These sets consider blocks with special characteristics; *Cart* defines the set of densities that can be cured on carts, and *Long* is the set of sizes that can be allocated to them.

In order to assign foam blocks to the different curing options, new variables are introduced. While variable n_{ijkh} is used for the number of blocks of normal lengths, variables $n1_{ijkh}$ and $n2_{ijkh}$ define the number of long blocks placed on carts and on the floor in row h, respectively for blocks included in sets *Cart* and *Long*. Equation (7) ensures that the total number of produced long blocks, n_{ijkh} $k \in Long$, is equal to the number of long blocks cured on the floor, $n2_{ijkh}$, plus the number of long blocks cured on the carts, $n1_{ijkh}$ for the blocks included in this special set.

$$n1_{ijkh} + n2_{ijkh} = n_{ijkh} \quad \forall (i,j,k) \in Blocks, \; h \leq Rows_i, \forall k \in Long \tag{7}$$

Thus, the different curing options are related and variables $n1_{ijkh}$, $n2_{ijkh}$, and n_{ijkh} for long blocks are associated.

The following constraint ensures that the produced blocks fit in the curing area where parameter fl_{ijk} indicates the block distance occupied on the floor. Depending on the characteristics of the block, it can be cured lying or standing. If the block is placed lying on the floor, then fl_{ijk} represents the block length. On the other hand, if the block is placed standing on the floor, fl_{ijk} is equal to the block height. Therefore, the total width used for each row is given by the number of blocks assigned to the row multiplied by the sum of the block distance occupied on the floor fl_{ijk} and the space left between blocks fs. The total covered width in a row must be less than or equal to the curing area width cw_i, as shown in Fig. 1. This constraint is presented in Eq. (8):

$$\sum_{\substack{j,k:\\(i,j,k)\in Blocks\\k\notin Long}} n_{ijkh}\cdot\left(fl_{ijk}+fs\right)+\sum_{\substack{j,k:\\(i,j,k)\in Blocks\\k\in Long}} n2_{ijkh}\cdot\left(fl_{ijk}+fs\right)\leq cw_i \quad \forall i,\forall h\leq Rows_i \tag{8}$$

In the same way, there is a limited number of carts with certain curing capacity. It is assumed that only one long block can be assigned to a cart, and that the number of available carts is np. Therefore, the total number of blocks cured on carts can be at most np:

$$\sum_{\substack{i,j,k,h:\\(i,j,k)\in Blocks\\k\in Long\\j\in Cart\\h\leq Rows_i}} n1_{ijkh}\leq np \tag{9}$$

In contrast to the previous constraint, Eq. (10) establishes that no long block can be cured on the carts if they are of densities j with $j\notin Cart$.

$$\sum_{\substack{i,j,k,h:\\(i,j,k)\in Blocks\\k\in Long\\j\notin Cart\\h\leq Rows_i}} n1_{ijkh}\leq 0 \tag{10}$$

Some blocks of width i, density j and length k are produced to satisfy external orders. These orders satisfy special requests that are directly delivered to the clients. Therefore, they are not stocked after curing. The binary variable x_{ijk} determines if the total demand of blocks of width i, density j and length k corresponding to external orders is going to be satisfied $(x_{ijk}=1)$ or not $(x_{ijk}=0)$. If a given block type $((i,j,k)\in Orders)$ is selected to be produced, then the exact quantity of this external order, soq_{ijk}, must be produced. This constraint is shown in Eqs. (11) and (12):

$$\sum_{h\leq Rows_i} n_{ijkh}\leq soq_{ijk}\cdot x_{ijk} \quad \forall(i,j,k)\in Blocks,\forall(i,j,k)\in Orders \tag{11}$$

$$\sum_{h\leq Rows_i} n_{ijkh}\geq soq_{ijk}\cdot x_{ijk} \quad \forall(i,j,k)\in Blocks,\forall(i,j,k)\in Orders \tag{12}$$

Blocks are produced considering an increasing density order taking into account the foaming machine operation. They are placed in the curing area in the same sequence they are obtained from the foaming machine in order to make an easier block location. Equation (12) establishes that if a given density j' is placed on row h' of the curing sector (meaning that $w_{j'h'} = 1$), then no blocks of density j greater than j' can be placed on any row h lower than h':

$$\sum_{\substack{i,k,h: \\ (i,j,k)\in Blocks \\ k\notin Long \\ h<h'}} n_{ijkh} + \sum_{\substack{i,k,h: \\ (i,j,k)\in Blocks \\ k\in Long \\ h<h'}} n_{ijkh} \leq BM_1\left(1 - w_{j'h'}\right) \qquad \forall j > j', \forall h' \tag{13}$$

where BM_1 is a sufficiently large number that makes the constraint redundant when the given density j' is not placed on row h. BM_1 can be the maximum number of blocks that can be placed in the curing area.

Blocks are cut in the foaming machine in decreasing length for a given density j and they are located in the curing area in the same order they are obtained from the foaming machine. Let z_{jkh} be a binary variable equal to one if any block of density j and length k is placed on row h. Then, Eq. (14) establishes that if a given length k' of density j is placed on row h' of the curing sector (meaning that $z_{jk'h'} = 1$), then no blocks of this density j and length k less than k' can be placed on any row h' lower than h:

$$\sum_{\substack{i,k,h: \\ (i,j,k)\in Blocks \\ k\notin Long, k<k' \\ h<h'}} n_{ijkh} + \sum_{\substack{i,k,h: \\ (i,j,k)\in Blocks \\ k\in Long, k<k' \\ h<h'}} n2_{ijkh} \leq BM_2\left(1 - z_{jk'h'}\right) \qquad \forall j, \forall h', \forall k' \tag{14}$$

where BM_2 is a sufficiently large number that relaxes this constraint when the length k' of density j is not placed on row h of the curing sector. As for BM_1, BM_2 can be the maximum number of blocks that can be placed in the curing area.

If density j is not assigned to row h of the curing area (i.e., $w_{jh} = 0$), then no blocks of this density can be placed on h. Similarly, if length k and density j are not assigned to row h (i.e., $z_{jkh} = 0$), the number of blocks of those length and density is zero in that row. These restrictions are stated using Big-M type expressions, as shown in Eqs. (15) to (17):

$$\sum_{\substack{i,k: \\ (i,j,k)\in Blocks \\ k\notin Long}} n_{ijkh} + \sum_{\substack{i,k: \\ (i,j,k)\in Blocks \\ k\in Long}} n2_{ijkh} \leq BM_3 \cdot w_{jh} \qquad \forall j, \forall h \tag{15}$$

$$\sum_{\substack{i: \\ (i,j,k)\in Blocks}} n_{ijkh} \leq BM_4 \cdot z_{jkh} \qquad \forall j, \forall k \notin Long, \forall h \tag{16}$$

$$\sum_{\substack{i: \\ (i,j,k)\in Blocks}} n2_{ijkh} \leq BM_5 \cdot z_{jkh} \qquad \forall j, \forall k \in Long, \forall h \tag{17}$$

BM_3, BM_4 and BM_5 can be calculated as the maximum number of blocks that can be placed in a row.

On the other hand, if the number of blocks of density j and length k assigned to row h is zero, then the binary variable z_{jkh} that assigns blocks of density j and length k to row h must be also zero (Eqs. (18), (19)), and, if a given density j is assigned to row h (i.e., $w_{jh} = 1$), then some blocks of this density must be located in this row as shown in Eq. (20):

$$\sum_{\substack{i:\\(i,j,k)\in Blocks}} n_{ijkh} \geq z_{jkh} \qquad \forall j, k \notin Long, \forall h \tag{18}$$

$$\sum_{\substack{i:\\(i,j,k)\in Blocks}} n2_{ijkh} \geq z_{jkh} \qquad \forall j, k \in Long, \forall h \tag{19}$$

$$\sum_{\substack{i,k:\\(i,j,k)\in Blocks\\k\notin Long}} n_{ijkh} + \sum_{\substack{i,k:\\(i,j,k)\in Blocks\\k\in Long}} n2_{ijkh} \geq w_{jh} \qquad \forall j, \forall h \tag{20}$$

Logical relations can be defined among the binary variables y_i, x_{ijk}, z_{jkh} and w_{jh}:

$$\sum_{k} z_{jkh} \geq w_{jh} \qquad \forall j, \forall h \tag{21}$$

Equation (21) establishes that if density j is assigned to row h, at least one length k of density j must be assigned to row h.

$$z_{jkh} \leq w_{jh} \qquad \forall j, \forall k, \forall h \tag{22}$$

Equation (22) states that if density j is not assigned to row h, no length k of density j can be assigned to row h.

$$\sum_{\substack{j':\\j'\leq j}} w_{j'h'} \geq w_{jh} \qquad \forall j, \forall (h, h')|h' < h \tag{23}$$

If a density j' is assigned to row h', then any density j must be assigned to all rows h previously located to h'. The purpose of Eq. (23) is to avoid alternative solutions when the limitation of the number of produced blocks is given by the storage capacity and there is an excess of space in the curing area.

$$x_{ijk} \leq y_i \qquad \forall (i, j, k) \in Orders \tag{24}$$

Equation (24) admits the production of blocks of width i required for external orders only when this width has been selected.

3.2 Stock equations

After curing, the foam blocks are stored. The warehouse policy determines a set of groups of blocks which have a specific assigned space in the storage area, as shown in Fig. 1. These groups are related a priori to specific blocks of different width, density and length by the set $Groups$ ($(i,j,k,g) \in Groups$). A block of a given width, density and length belongs to a unique group.

The stock capacity for each group (sc_g), as well as the minimum ($smin_{ijk}$) and maximal stock ($smax_{ijk}$) and the initial stock (s_{ijk}) for each block type, all expressed in number of blocks, are model parameters. As previously described, a daily production planning is modeled. Three moments are considered for the stock, the initial moment where the stock is equal to the final stock of the previous day (including the pieces produced the previous day), the intermediate stock sm_{ijk}, after the demand of the current day is taken for downstream processes, and the final stock sf_{ijkt} which is the inventory level that considers the blocks produced and cured in the current day,$\sum_h n_{ijkh}$.

At the beginning of the production day, the demand (d_{ijk}) is taken from the stock area. This demand is satisfied by the initial stock, i.e. the available blocks at the end of the previous working day. In the intermediate moment, if the demand of blocks d_{ijk} is greater than the initial stock s_{ijk}, an unsatisfied demand occurs, and the model reports the unsatisfied blocks. In this case, the intermediate stock is $sm_{ijk} = 0$. On the other hand, if the initial stock is enough to satisfy the demand, the intermediate stock is the difference between the initial stock and the demand, as in the following constraints:

$$sm_{ijk} \le s_{ijk} - d_{ijk} + BM_6\left(1 - u_{ijk}\right) \qquad \forall (i,j,k) \in Blocks \tag{25}$$

$$sm_{ijk} \ge s_{ijk} - d_{ijk} - BM_6\left(1 - u_{ijk}\right) \qquad \forall (i,j,k) \in Blocks \tag{26}$$

$$sm_{ijk} \le BM_6 u_{ijk} \qquad \forall (i,j,k) \in Blocks \tag{27}$$

where u_{ijk} is a binary variable equal to 1 if no unsatisfied demand occurs for block of width i, density j and length k, and 0 otherwise. In this way, if the initial stock is enough to satisfy the current demand, $u_{ijk} = 1$ and Eqs. (25) and (26) state that the stock after taking the demand is $sm_{ijk} = s_{ijk} - d_{ijk}$, while Eq. (27) is redundant. Otherwise, $u_{ijk} = 0$, Eq. (27) indicates that $sm_{ijk} = 0$, and Eqs. (25) and (26) are redundant. BM_6 can be the maximum stock capacity for block (i, j, k), $smax_{ijk}$.

Finally, the stock at the end of the day is given by the intermediate stock plus the blocks produced during the day:

$$sf_{ijk} = sm_{ijk} + \sum_h n_{ijkh} \qquad \forall (i,j,k) \in Blocks \tag{28}$$

The following equation determines that the total number of blocks of a given group g in the stock cannot be greater than the stock capacity of that group:

$$\sum_{\substack{i,j,k,h: \\ (i,j,k) \in Blocks \\ (i,j,k) \notin Orders \\ (i,j,k,g) \in Groups}} sf_{ijk} \le sc_g \qquad \forall g \tag{29}$$

This last expression excludes blocks that belong to external orders (*Orders*) that are not stored.

Similarly to the previous equation, Eq. (30) establishes that the number of blocks in stock at the end of the planning day must be less than or equal to the block stock capacity parameter, for each block of width i, density j and length k. Blocks that belong to external orders (*Orders*) are not stored, and thus they are not included in this equation.

$$sf_{ijk} \leq smax_{ijk} \qquad \forall(i,j,k) \in Blocks,\ \forall(i,j,k) \notin Orders \tag{30}$$

If width i is selected to be produced, then the stock of all blocks of this width must fulfill the minimal stock as shown in Eq. (31), where BM_7 is a sufficiently large constant to make this constraint redundant when width i is not produced. A good selection for BM_7 is $BM_7 = \max_{(i,j,k) \in Blocks}\{smin_{ijk}\}$. Blocks corresponding to external orders (*Orders*) are not considered.

$$sf_{ijk} - smin_{ijk} \geq BM_7(y_i - 1) \qquad \forall(i,j,k) \in Blocks,\ \forall(i,j,k) \notin Orders \tag{31}$$

3.3 Objective function

The objective function seeks simultaneously the maximization of the length of the produced foam and the minimization of the length of blocks needed in the inventory to satisfy the required minimum stock. As previously mentioned, the foam production section aims to assure the provision of the requested blocks to the following production stage. It is important to mention that if the maximum block production is considered as the sole performance measure of the model, the solution would only take into account the available curing space as well as the empty places in stock. Therefore, the formulation would lack criteria to evaluate whether or not the proposed production plan is balanced. In addition, the production planners have defined a minimal stock of blocks according to their expertise regarding the historical consumption of blocks in the downstream process. To take into account this information and provide a solution of better quality, a second term is included in the objective function that penalizes the unsatisfied minimum stock requirement. The proposed objective function represents a good compromise among production, storage and downstream demand fulfillment.

As stated in Eq. (31), if the width i is selected, all final stock of blocks of this width must be greater than or equal to the lowest admitted stock $smin_{ijk}$. For the other widths, the difference between the final stock and the minimal stock is not modified because no blocks are produced, then it can be greater than, less than or equal to zero. In order to represent this difference, the variable dif_{ijk} is defined. Then, Eq. (32) states that the amount of missing blocks in stock, of width i, density j and length k, is equal to the difference between the minimal stock level ($smin_{ijk}$.) and the final stock level). Thus, if the minimal stock of block of width i, density j and length k is satisfied, dif_{ijk} takes value zero. Otherwise, dif_{ijk} is equal to the number of blocks of width i, density j and length k required to complete the minimal stock.

$$smin_{ijk} - sf_{ijk} \leq dif_{ijk} \qquad \forall (i,j,k) \in Blocks, \; \forall (i,j,k) \notin Orders \tag{32}$$

$$dif_{ijk} \geq 0 \qquad \forall (i,j,k) \in Blocks, \; \forall (i,j,k) \notin Orders \tag{33}$$

It is worth noting that for the width produced in the current day, $dif_{ijk} = 0$ due to Eq. (31).

Therefore, the objective function to be maximized is:

$$C = \sum_{\substack{i,j,k,h: \\ (i,j,k) \in Blocks \\ h \leq Rows_i}} n_{ijkh} \cdot l_{ijk} - \sum_{\substack{i,j,k: \\ (i,j,k) \in Blocks}} dif_{ijk} \cdot l_{ijk}. \tag{34}$$

In summary, the proposed mathematical model for the simultaneous production and inventory planning involves maximizing the objective function of Eq. (34) subject to constraints (2)–(33).

4 Results

Different production scenarios can be obtained according to the problem input data. In this section, three study cases are presented to show various planning schemes and highlight the several tradeoffs that are simultaneously evaluated in the proposed approach. Also, the objective function is tested and validated through an example. The examples have been adjusted to an appropriate size for an article.

In the first example, the stock capacity is completed for the width selected by the model, and therefore no more blocks of this width can be produced. This means that the stock capacity is the limiting resource while the curing area is not filled. In the second case, no more blocks can be placed in the curing area, and therefore the maximum stock of blocks of the selected width is not fulfilled. Finally, in the last example, the model is solved maximizing the foaming length in the objective function, i.e. without penalizing the negative difference between the inventory level and the minimal stock parameter. In this way, the objective function and the corresponding optimal solution is tested and compared with the previous example. In all cases, the model was implemented and solved in GAMS version 24.1.3 with a 2.8 GHz Intel Core i7 processor. The CPLEX 12.5.1 solver was employed for solving the MILP problems. The computational performance of the three examples is presented in Table 1.

It is worth highlighting that the good computational performance of the model is because the constraints are strongly dependent on the width selection and the binary variables are closely linked through specific constraints. As only one width is produced, when the solver selects the width, many variables take the value zero, and therefore, many decisions are made. Moreover, the logical constraints help the convergence since they reduce the search space in the feasible region. In summary, the model structure is suitable for fast convergence despite the large number of discrete decisions.

Table 1 Performance of the three examples

Equations	Continuous variables	Discrete variables	CPUs* example 1	CPUs* example 2	CPUs * example 3
150,108	15,179	14,907	66.08	10	52.3

* 0% gap (in all cases)

To show the power of the model, for the examples, 3 possible widths, 16 different densities, and 12 lengths are considered. Table 5 from the Appendix shows, for each possible foam block belonging to the set *Blocks* grouped according to the stock groups (*Groups*), the stock capacity of each block ($smax_{ijk}$) and group (sc_g), and the minimum stock for each block ($smin_{ijk}$). Table 6 from the Appendix displays the initial stock (s_{ijk}) and the demand (d_{ijk}) for the instances.

The curing area size is determined by the parameters cw_i and cl_i. In these examples, for i equal to width 190 and 200 cm the curing area width is fixed to 2040 cm, while the curing area length is 4500 cm. For width 214 cm, some long blocks are produced, and therefore more blocks can be accommodated in the curing area if this area is transposed. Then, for i equal to 214, $cw_i = 4500$ and $cl_i = 2040$ cm. The parameter cl_i is used to determine the value of $Rows_i$ i.e. the number of admitted rows in the curing area, according to Eq. (1). It is worth mentioning that the planners can modify these parameters to improve the block arrangement in this sector.

The minimum density length is 1500 cm (l_min) and the considered value for the minimum total length is 15,000 cm (ml). There are 5 available places in carts ($np = 5$) for curing long blocks ($1200 \in Long$) of densities $j \in Carts$, i.e. blocks of 1200 cm in length of certain densities. In addition, 10 blocks of width 214, density BS28, and length 225 are required as a special order ($soq_{214,BS28,225} = 10$, where $(214, BS28, 225) \in Orders$). Therefore, if the width 214 is selected to be produced, the total special order could be manufactured.

Example 1 Besides the parameters previously presented, the initial stock and the demand presented in the first and third columns of Table 6 from the Appendix, respectively, must be considered.

The optimal solution selects the width 190 cm and 152 blocks of this width are foamed. The total foamed length is equal to 392.62 m. In Table 2, the detailed foaming program is shown. The order in which blocks for each density are produced is provided by the company. From this table, the total foamed length for each density can be calculated.

Figure 2 displays the optimal planning in the curing area. Note that the total available rows (21) are used, but some of them are not completely occupied. For example, in row h3 there are two blocks of density CO28 and length 2 m, which are lying down on the floor occupying 4.42 m of this row. Therefore, nearly 16 m of this row is empty. Note that the width used in this row is 4.42 m because an additional fixed space must be considered to allow the air flow in the curing process. A similar

Table 2 Example 1. Detailed program of foam blocks

Density (j)	Length (k) (cm)	No. of blocks ($\Sigma_h n_{ijkh}$)	Total length ($\Sigma_h n_{ijkh} * l_{ijk}$) (m)
CO28	200	**13**	26.78
CO28	250	**1**	2.58
CO28	270	**1**	2.79
CO28	280	**4**	11.56
GR28	240	**8**	19.76
GR28	260	**3**	8.04
GR28	280	**13**	37.7
RS26	280	**8**	23.04
AZ24	240	**4**	9.88
AZ24	280	**2**	5.76
AM20	200	**4**	8.4
AM20	240	**21**	52.5
AM20	260	**4**	10.8
AM20	280	**9**	26.01
LI18	240	**6**	15.12
LI18	280	**4**	11.6
N18L	200	**4**	8.4
N18L	240	**9**	22.5
N18L	280	**11**	31.9
BCOL	240	**23**	57.5
	Total	**152**	392.62

situation can be observed in rows h7, h8, and h20. In h20 for example, the blocks are standing, therefore the length occupied in the floor is calculated according to the block height which is 1.18 m, i.e., the occupied length is 6.65 m that corresponds to 1.18 m [block height] by 5 [no. of blocks] plus 0.15 m [space between blocks] by 5 [no. of blocks]. In this case, alternative solutions for the block arrangement can be obtained because the available space in the curing area is greater than the required surface.

Figure 3 shows the stock management for each group. Blocks of width 190 cm are stored in groups G5 to G11, G13, G18 and G19. Groups G14, G16 and G22 include blocks of width 190 cm as well as blocks of width 200 cm. Some of these groups do not achieve their maximum stock capacity, even when there is free space for more blocks in the curing area. However, the free space in stock corresponds to the blocks of width 200 cm which are not produced. The maximum foamed length is reached with blocks of width 190. The maximum stock capacity for this width is completed except for group G13 due to the minimum density length requirement. Therefore, the bottleneck in this case is the stock capacity.

Example 2 Considering the data in the second and fourth columns of Table 6 from the Appendix, the optimal solution selects to foam blocks of width 214 cm. The

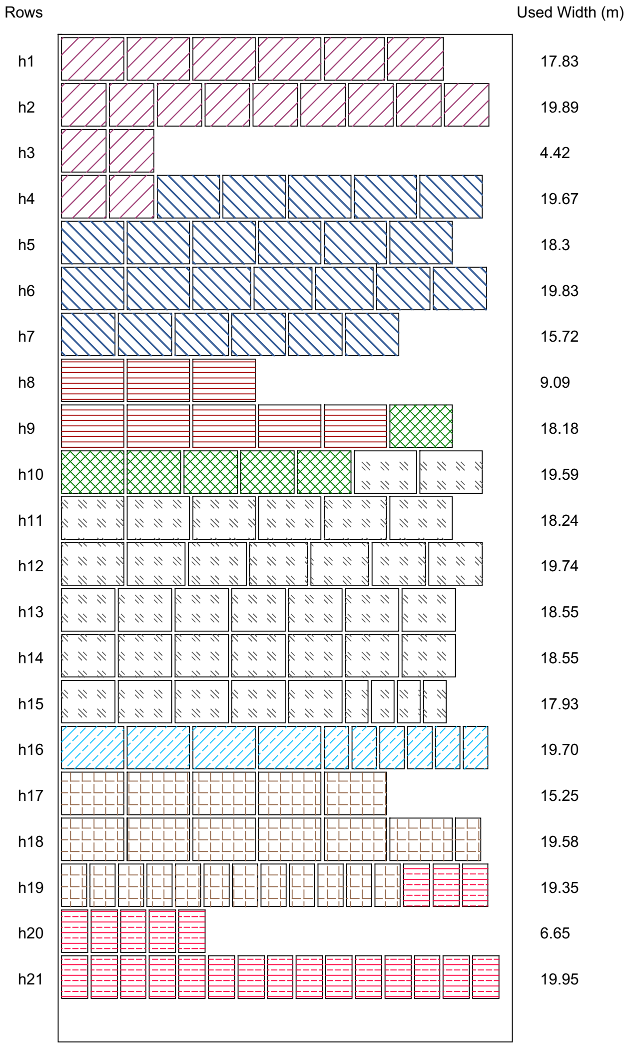

Fig. 2 Example 1. Optimal block arrangement in curing area

number of produced blocks is 38 and the total length is 358.50 m, while the total length of missing blocks in stock is 49.43 m. The detailed foaming program is shown in Table 3. As can be observed, the special order of 10 blocks of density BS28 and length 225 cm is carried out.

Figure 4 shows the block arrangement in the curing area. Note that the number of rows in the curing area is less than in the previous case since when width 214 cm is selected, the area dimensions are transposed. In this case, all the available rows are employed and 23 long blocks are cured on the floor. Although some rows are not totally covered, no more long blocks can be accommodated in the rows of this area.

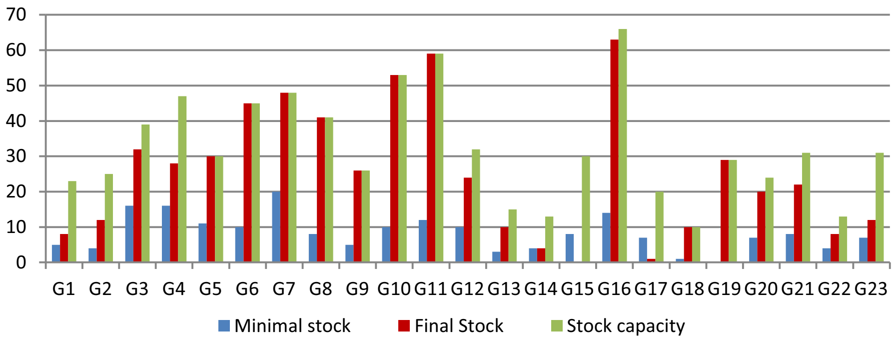

Fig. 3 Example 1. Stock management of groups

Table 3 Example 2. Detailed program of foam blocks

Density (j)	Length (k) (cm)	No. of blocks ($\Sigma_h n_{ijkh}$)	Total length ($\Sigma_h n_{ijkh}$* l_{ijk}) (m)
BS28	225	**10**	22.50
BS28	1200	**2**	24
FR28	1200	**6**	72
GS26	1200	**6**	72
BB20	1200	**5**	60
BB12	1200	**9**	108
	Total	**38**	358.50

Therefore, the total available space in carts (5 blocks) is used for curing long blocks of low density.

The stock management of these blocks is shown in Fig. 5. The long blocks are grouped in a last cluster, G23. Note that the blocks of the special order are not stored. There are 3 places unoccupied in G23. This means that, in this case, the most limited resource is the curing area. All the required shorter blocks are foamed, while no more long blocks are produced because they cannot be accommodated in the curing area. For the remaining groups no blocks are produced, therefore the final stock is equal to the initial stock minus the demand.

Example 3 The objective function is changed in this example and the term that calculates the lacked blocks according to the minimum stock is removed. Therefore, Eq. (34) is replaced by:

$$C = \sum_{\substack{i,j,k,h: \\ (i,j,k)\in Blocks \\ h \le Rows_i}} n_{ijkh} \cdot l_{ijk} \tag{35}$$

and all the parameters are as for Example 2. Now, the optimal solution selects width 190 cm instead of 214 cm. The detailed planning program is shown in Table 4.

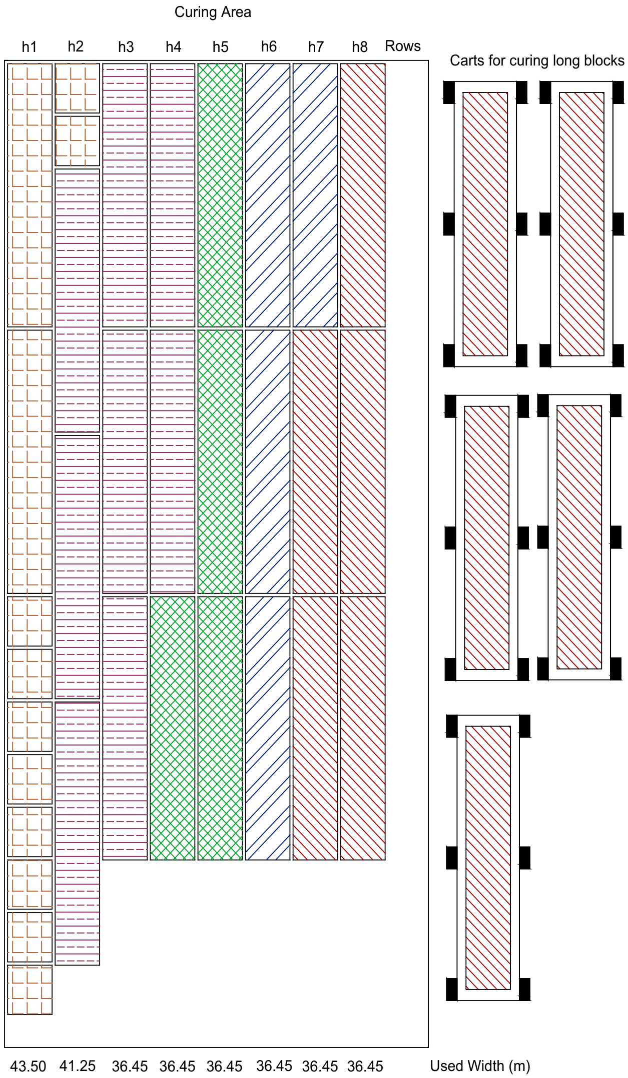

Fig. 4 Example 2. Optimal block arrangement in the curing area

The total foamed length in the previous example was 358.5 m, while in this case, the total foamed length is 361.84 m. Taking into account the problem data, in both examples the shortage of blocks when width 190 cm is produced is 133.43 m, while when width 214 cm is produced it is 49.43 m. Therefore, when the length of missing blocks is subtracted from the length of foam blocks, the best solution is to produce blocks of width 214 cm, while when shortage is not taken into account the best solution is to produce blocks of width 190 cm.

This shows that the purpose of maintaining a more balanced stock level can be achieved when the objective function is given by Eq. (34).

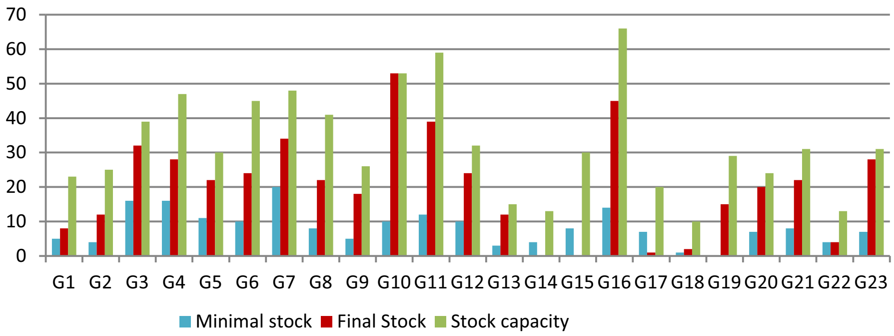

Fig. 5 Example 2. Stock management of groups

Table 4 Example 3. Detailed program of foam blocks

Density (*j*)	Length (*k*) (cm)	No. of blocks ($\Sigma_h n_{ijkh}$)	Total length ($\Sigma_h n_{ijkh} * l_{ijk}$) (meters)
CO28	200	**11**	22.66
CO28	250	**3**	7.74
CO28	270	**1**	2.79
CO28	280	**3**	8.67
GR28	240	**8**	19.76
GR28	260	**3**	8.04
GR28	280	**16**	46.4
RS26	280	**8**	23.04
AZ24	240	**4**	9.88
AZ24	280	**4**	11.52
AM20	200	**4**	8.4
AM20	240	**21**	52.5
AM20	260	**2**	5.4
AM20	280	**12**	34.68
LI18	240	**8**	20.16
LI18	280	**6**	17.4
N18L	200	**4**	8.4
N18L	240	**9**	22.5
N18L	280	**11**	31.9
	Tota	**138**	361.84

5 Conclusion

In this article, an MILP formulation for the simultaneous optimization of PU foam production planning and inventory management was presented. Planning decisions include two main features of the production process. In the foaming step, only one foam width can be produced per day and the number and type of foam blocks must be decided. In the curing process, the blocks must be placed according to the available curing space. Given that only one width can be produced per day, storage planning is critical in the plant. Besides, mattress production requirements cannot be known in advance, so an appropriate inventory must be maintained to overcome manufacturing fluctuations. The lack of blocks in stock with respect to a minimal level is then incorporated in the objective function, allowing a homogeneous distribution of blocks in storage. Therefore, the model allows obtaining the largest foaming production considering the lack of blocks and satisfying production, curing and stocking policies.

Several tradeoffs are simultaneously considered. First, the model achieves an appropriate planning of the foaming sector with adequate links with production through inventory management. Previous efforts with disarticulated proposals failed to attain an efficient operation. Block production not only is limited by foaming requirements but also by curing area and stock space. The concurrent consideration of these perspectives allows tighter resource utilization. Thus, mathematical programming is an effective tool to improve the production and inventory management.

Although the model is a sophisticated approach from the mathematical point of view, with an important number of discrete variables, it is a simple tool from the managers' perspective. Several runs can be compared and the impact of daily mattress production plans can be anticipated.

The inventory management through block groups with limited capacity has enabled a stock policy avoiding shortages. If additional space was available in the inventory area, a dedicated stock for each block could be adopted. However, space limitations necessitated a different approach. Block groups are sufficiently flexible to maintain an appropriate block stock in the available area.

Three study cases were presented to demonstrate the model performance. Through the examples, the approach capabilities were highlighted and several tradeoffs among production and curing processes and stock management were analyzed. For the company, the approach represents a useful tool for deciding the daily production planning. The structure of the model, where several logical relations among variables are included, allows solution in a reasonable computational time, providing a powerful tool for helping the firm with decision making.

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Appendix

See Tables 5, 6.

Table 5 Stock capacity data

Group	Block (width.density. length)	Stock capacity (number of blocks)	Minimum stock (number of blocks)	Group stock capacity (number of blocks)
G1	200.VE22.200	8	2	23
	200.VE22.240	2	0	
	200.VE22.280	8	2	
	200.VE22.300	5	1	
G2	200.LI18.200	10	3	25
	200.LI18.240	3	0	
	200.LI18.280	6	1	
	200.LI18.300	6	0	
G3	200.GR28.160	22	8	39
	200.GR28.180	8	2	
	200.GR28.210	9	6	
G4	200.AZ24.160	26	10	47
	200.AZ24.210	15	5	
	200.AZ24.300	6	1	
G5	190.AZ24.240	11	1	30
	190.AZ24.280	19	10	
G6	190.AM20.240	45	10	45
G7	190.AM20.260	4	0	48
	190.AM20.280	44	20	
G8	190.GR28.260	6	0	41
	190.GR28.280	35	8	
G9	190.GR28.240	26	5	26
G10	190.BCOL.240	53	10	53
G11	190.N18L.240	34	10	59
	190.N18L.280	25	2	
G12	200.B14P.270	11	2	32
	200.B14P.300	21	8	
G13	190.BC35.240	2	0	15
	190.BC35.280	13	3	
G14	200.AM20.180	5	2	13
	190.AM20.200	4	0	
	200.AM20.200	4	2	
G15	200.AM20.160	30	8	30
G16	200.CO28.160	11	0	66
	190.CO28.200	27	10	
	200.CO28.210	14	0	

Table 5 continued

Group	Block (width.density. length)	Stock capacity (number of blocks)	Minimum stock (number of blocks)	Group stock capacity (number of blocks)
	190.CO28.250	5	2	
	190.CO28.270	4	1	
	190.CO28.280	5	1	
G17	200.RS26.160	12	5	20
	200.RS26.210	8	2	
G18	190.RS26.280	10	1	10
G19	190.LI18.240	15	0	29
	190.LI18.280	14	0	
G20	200.BC35.160	11	5	24
	200.BC35.210	13	2	
G21	200.N18L.160	31	8	31
G22	200.N18L.180	5	2	13
	190.N18L.200	4	0	
	200.N18L.200	4	2	
G23	214.BS28.1200	2	0	31
	214.FR28.1200	8	2	
	214.GS26.1200	6	1	
	214.BB12.1200	10	2	
	214.BB20.1200	5	2	

Table 6 Initial stock and demands

Block (width.density.length)	Initial stock (number of blocks)		Demand (number of blocks)	
	Example 1	Examples 2 and 3	Example 1	Examples 2 and 3
200.VE22.200	5	5	0	2
200.VE22.240	0	2	0	0
200.VE22.280	3	5	3	2
200.VE22.300	3	5	0	2
200.LI18.200	7	10	0	2
200.LI18.240	2	3	0	2
200.LI18.280	0	0	0	0
200.LI18.300	3	2	0	2
200.GR28.160	20	11	1	10
200.GR28.180	6	6	0	6
200.GR28.210	7	7	0	7
200.AZ24.160	23	13	1	13
200.AZ24.210	5	3	4	3
200.AZ24.300	5	3	0	3
190.AZ24.240	7	7	0	0

Table 6 continued

Block (width.density.length)	Initial stock (number of blocks)		Demand (number of blocks)	
	Example 1	Examples 2 and 3	Example 1	Examples 2 and 3
190.AZ24.280	18	10	1	1
190.AM20.240	40	35	16	16
190.AM20.260	2	2	2	2
190.AM20.280	40	37	5	5
190.GR28.260	3	3	0	0
190.GR28.280	35	32	13	13
190.GR28.240	23	23	5	5
190.BCOL.240	30	53	0	0
190.N18L.240	29	29	4	4
190.N18L.280	20	20	6	6
200.B14P.270	9	5	1	4
200.B14P.300	19	10	3	9
190.BC35.240	0	0	0	0
190.BC35.280	12	0	2	2
214.BS28.1200	0	0	0	0
214.FR28.1200	5	5	3	3
214.GS26.1200	4	4	0	0
214.BB12.1200	2	0	0	0
214.BB20.1200	4	2	0	0
200.AM20.180	0	0	0	0
190.AM20.200	4	4	4	4
200.AM20.200	0	0	0	0
200.AM20.160	0	0	0	0
200.CO28.160	11	11	2	11
190.CO28.200	21	21	7	7
200.CO28.210	13	14	0	13
190.CO28.250	5	1	1	1
190.CO28.270	3	1	0	0
190.CO28.280	2	2	1	1
200.RS26.160	4	4	4	4
200.RS26.210	3	4	2	3
190.RS26.280	4	4	2	2
190.LI18.240	10	8	1	1
190.LI18.280	10	8	0	0
200.BC35.160	11	5	0	5
200.BC35.210	10	13	1	10
200.N18L.160	23	13	1	13
200.N18L.180	2	3	0	2
190.N18L.200	2	2	2	2
200.N18L.200	3	3	1	3

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