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An MILP model for planning of batch plants operating in a campaign-mode

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Abstract A mixed integer linear programming (MILP) for the detailed production planning of multiproduct batch plants is presented in this work. New timing decisions are incorporated to the model taking into account that an operation mode based in campaigns is adopted. This operation mode assures a more efficient production management adjusted to the specific context conditions of the considered time horizon. In addition, special considerations as sequence-dependent changeover times and different unit sizes for parallel units in each stage are taken into account. The problem consists of determining the amount of each product to be produced, stored and sold over the given time horizon, the composition of the production campaign (number of batches and their sizes), the assignment, sequencing and timing of batches, and the number of repetitions of the campaign, for a given plant with known product recipes. The objective is to maximize the net profit fulfilling the minimum and maximum product demands. The proposed model provides a useful tool for solving the optimal campaign planning of installed facilities in reasonable computation time, taking different decisions about the operations management.

Keywords Multiproduct batch plants · Sequence-dependent changeovers · Production campaign · Planning · Scheduling · MILP model

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Abbreviation

Indices

b, b'	Batch
i, i'	Product
j	Stage
k	Unit
l, l'	Slot
m	Index utilized for the representation in base 2 for the number of repetition of the campaign
r	Raw material

Sets

I	Products
IB_i	Admitted batches of product i in the campaign composition
K_j	Nonidentical parallel batch unit that operate out-of-phase in stage j

Parameters

B_i^{max}	Maximum feasible batch size for product i
B_i^{min}	Minimum feasible batch size for product i
$c_{ii'k}$	Sequence-dependent changeover time between products i and i' at unit k
co_i	Operating cost coefficient of product i
DE_i^L	Minimum demand of product i
DE_i^U	Maximum demand of product i
F_{ri}	Raw material conversion factor
H	Planning horizon
$IMinit_r$	Inventory of raw material r at the beginning of planning horizon
$IPinit_i$	Inventory of product i at the beginning of planning horizon
L	Number of slots postulated for all units of each stage
M_b	Big-M constant parameters for $b = 1, 2, 3, 4$
np_i	Selling price of product i
NBC_i^{UP}	Maximum number of batches of product i in the campaign
NC^{UP}	Maximum number of times that the campaign can be repeated over the planning horizon
Q_i^L	Lower bound for production level of product i
Q_i^U	Upper bound for production level of product i
SF_{ij}	Size factor of product i in stage j
t_{ik}	Processing time of product i in batch unit k
V_k	Size of unit k
α_{ik}	Equipment utilization minimum rate for product i at unit k
β_r	Inventory cost coefficient for raw material r
δ_i	Inventory cost coefficient for product i
κ_r	Price of raw material r
λ	Weighting factor for the variable CT in the objective function

Binary variables

x_m Variable utilized for the representation in base 2 for the number of campaign repetition
X_{kl} Indicates if slot $l$of unit k is employed
Y_{bkl} Indicates if batch b is assigned to slot $l$of unit k
z_{ib} Indicates if batch b of product i is selected

Continuous variables

B_{ib} Size of batch b of product i
CR_r Amount of raw material r purchased during the planning horizon
CT Campaign cycle time
IM_r Inventory of raw material r at the end of the time horizon
IP_i Inventory of final product i at the end of the time horizon
NB_{it} Total number of batches of product i processed in the time horizon
NBC_i Number of batches of product i included in the campaign
NC Number of times that the campaign is cyclically repeated over the time horizon
NP Net profit
Q_i Amount of product i to be produced in the planning horizon
QS_i Amount of product i sold at the end of the planning horizon
RM_r Amount of raw material r used for producing of all products during the time horizon
RM_{ri} Amount of raw material r used to make product i
TF_{kl} Final processing time of slot l in unit k for the first campaign cycle
TI_{kl} Initial processing time of slot l in unit k for the first campaign cycle
w_{ibm} Variable that represents the product $B_{ib}\quad x_m$
ww_m Variable that represents the product CTx_m
$YY_{blb'l'k}$ Continuous variable on interval [0, 1] that indicates if batch b is assigned to slot l of unit k and batch b' to slot l' of the same unit

1 Introduction

Multiproduct batch plants are characterized by their flexibility to manufacture multiple products using the same equipment. These plants consist of a collection of processing units where batches of various products are processed through a set of operations. These operations are characterized by a processing time and they do not involve both simultaneous feed and removal of products from the unit during this processing time. Units that perform the same operation are grouped in a production stage, and they can operate in parallel mode (in phase or out-of-phase). In this type of batch plants, where the process structure is sequential, each batch follows the same sequence through all the processing stages (Voudouris and Grossmann 1992).

At the plant floor level production management decisions have to be coordinated and integrated in order to obtain optimal production targets. Although production planning and scheduling decisions are closely related since the result of planning problem is the production target of scheduling problem, traditional approaches resort to hierarchical methodologies for solving them. At a first level, the planning problem is solved to define the production targets and then the detailed scheduling program is determined in order to meet these targets.

Mathematical programming has become one of the most widely explored methods for production process optimization, and in particular for obtaining detailed production planning schemes. The reviews of Shah (1998), Grossmann et al. (2002), Kallrath (2002) and Maravelias and Sung (2009) summarize the formulations and strategies proposed for scheduling and planning problems in the last two decades, presenting the optimization challenges and opportunities. These authors coincide that the complexity associated to the large size of the integrated model is an important drawback and constitutes a limitation. To overcome the above difficulty, most of the works presented in the literature aims at decreasing the problem scale through different types of problem simplification and developing efficient solution strategies.

Li and Ierapetritou (2009) presented a general solution framework based on decomposition to solve the integrated production planning and scheduling problem using a bilevel optimization formulation. They also incorporated a penalty term into the objective function of the scheduling model to penalize the unsatisfied production targets. Erdirik-Dogan and Grossmann (2006) proposed a slot-based MILP model for the simultaneous planning and scheduling of single-stage single-unit multiproduct continuous plants in a multiperiod context. A bilevel decomposition algorithm in which the original problem is decomposed into an upper level planning and a lower level scheduling problem was also developed in order to deal with complex problems. Later, Erdirik-Dogan and Grossmann (2008) extended their work incorporating parallel units to the formulation, and subsequently the same problem was addressed by Liu et al. (2010). These authors propose a MILP model and a rolling-horizon algorithm that improve the computational performance of the analyzed examples compared with the approach introduced by Erdirik-Dogan and Grossmann (2008). It is worth mentioning that some authors addressed the integrated problems through hybrid modeling approaches (Bilgen and Çelebi 2013; Coban and Hooker 2013; Chu et al. 2015) as well as using heuristic methods such as simulated annealing (Reklaitis 2000) and genetic algorithms (Berning et al. 2004; Yan and Zhang 2007; Shao et al. 2009).

Assuming a given plant, i.e., its configuration and the unit sizes are known, different production problems can be posed depending of the contemplated scenario. In particular, when products demands can be accurately forecasted during a relatively long time horizon due to a stable context, more efficient management and control of the production resources can be attained if the operating condition is based on campaign of cyclic schedule. In this case, the campaign consists of several batches of different products that are going to be manufactured and the same pattern is repeated at a constant frequency over a time horizon. This campaign-based operation mode, namely mixed product campaign (MPC), has several advantages, for example, more standardized production during certain periods of time, easier and profitable operations decisions, more efficient operation control, and adequate inventory levels without generating excessive costs and minimizing the possibility of stock-outs.

The scheduling problem using MPCs was barely addressed in the literature. Birewar and Grossmann (1989) developed slot-based MILP formulations for scheduling of multiproduct batch plants using production campaigns, considering different transfer policies (unlimited intermediate storage, UIS, and zero wait, ZW) and where the number and size of batches are data problem. They determined the optimal campaign cycle time, for simple plants including only one unit per processing stage. In Fumero et al. (2011) two MILP models for the simultaneous design and scheduling of a multistage batch plant are proposed. The parallel units are considered identical and no changeover times are taken into account. The same assumptions were taken into account in the simultaneous planning and scheduling approach presented by Fumero et al. (2012a).

Under this context, a detailed planning model is presented in this work. This proposal considers that production planning is simultaneously solved with the cyclic scheduling based on MPCs. This type of detailed planning is used for product manufacturing with relatively constant demand during the planning horizon. This leads to a more regular production mode which is more appropriate for a make-to-stock production policy. From the computational point of view, the cyclic scheduling allows reducing the size of the overall scheduling problem, which is often intractable.

The multistage multiproduct batch plant under consideration has nonidentical parallel units, operates under ZW transfer policy, and sequence-dependent changeover times are taken into account. Given the plant configuration and unit sizes, the product recipes, and the minimum and maximum demands of each product over a know planning time horizon, this approach simultaneously determines: production targets of each product, selection and sizing of batches of the campaign to be carried out, assignment of batches to process units, sequencing of batches on units, timing of batches, and number of campaign repetitions along the time horizon. The objective is to maximize the total net benefit, given by the income for sales minus the total costs. Nevertheless, similar models can be developed taking advantage of this proposal, considering different policies and scenarios. Several tradeoffs can be evaluated as raw material procurement and consumption, units operation, and product inventory and sale.

From the mathematical and computational points of view, with the aim of reducing the combinatorial complexity associated to the scheduling decisions, additional constraints are considered in order to eliminate equivalent symmetric solutions. Then, the detailed planning approach through MPCs considering sequence-dependent changeover times for multistage batch plants with nonidentical parallel units is efficiently solved.

The rest of this paper is organized as follows. The detailed planning problem is first introduced in Sect. 2 and its mathematical formulation is then presented in Sect. 3. In Sect. 4, the examples are depicted in order to validate the proposed approach. Finally, in Sect. 5 the conclusions and some remarks are presented.

2 Problem definition

In this work, the integrated planning and scheduling problem is addressed for a multistage multiproduct batch plant that operates in a campaign mode. In this plant, each processing stage $j \in \{1, 2, \ldots, J\}$ has a set K_j of parallel batch units that operate out-of-phase and where different unit sizes are admitted. Since the parallel units in each stage may be not identical, the numerical sequence 1, 2, …, K is used to denote all plant units, and V_k represents the size of unit k. A set I of products must be manufactured in the plant following the same sequence of stages and using R raw materials. Taking into account that the plant operates through campaigns of cyclic (periodic) execution, the detailed planning problem consists of merely solving the campaign batching and scheduling and executing it repeatedly. Although the batching and scheduling decisions are critical when several products are involved and units with different sizes are available in each stage, the cyclic approach reduces the complexity of the operations and avoids lead to large-scale and computationally intractable models.

The production recipes, i.e. the processing time of each product i in unit k, t_{ik}, the conversion factor F_{ri} that represents the required amount of raw material r to elaborate one mass unit of product i, and the size factor SF_{ij} that denotes the required capacity of units in stage j to produce one mass unit of final product i, are problem data. Additionally, upper and lower bounds, on the demands of each product i in the planning horizon H, DE_i^L and DE_i^U, are known. Then, the production level of each product in the campaign is a model variable, and this amount can be fulfilled with one or more batches of different sizes. Therefore, an index b is introduced to denote the bth batch required in the campaign to meet the production of the corresponding product.

Given that the number and size of batches of each product i in the campaign are decision variables in the model, the campaign composition and its cycle time are not known a priori. Only upper limits are imposed on the number of batches of each product i in the campaign, and therefore a set of generic batches associated to that product, IB_i, is proposed, where $|IB_i| = NBC_i^{UP}$.

For each raw material, the initial inventory level, the unit cost, and its availability are known parameters. Also, prices of final products and their required final stock levels, are problem data. For each unit k, an equipment utilization minimum rate is considered for each product i, which is denoted by α_{ik}. This value is associated to physical and operative conditions of use of the plant equipment, and it is defined by the operations manager of the facility.

Intermediate storage tanks are not allowed. In addition, taking into account the sequential structure of the process, no batch splitting or mixing is considered, i.e. each batch is treated as a discrete entity throughout the whole process. It is assumed that a batch cannot wait in a unit after finishing its processing. Therefore, the ZW transfer policy between stages is adopted, i.e., after being processed in stage j, a batch b is immediately transferred to the next stage $j + 1$. Besides, batch transfer times between units are assumed very small compared to process operation times and, consequently, they are included in the processing times.

Sequence-dependent changeover times, $c_{ii'k}$, are considered between consecutive batches processed in the same unit k, even of the same product. This transition time corresponds to the preparation or cleaning of the equipment to perform the processing of the following batch. It is necessary for various reasons: ensure products quality, maintain the equipment, safety reasons, etc.

For campaign scheduling decisions, an asynchronous slot-based continuous-time representation has been used (Fumero et al. 2012b). Unlike the most of scheduling approaches presented in the literature, the number and sizes of batches to be scheduled are optimization variables. Although there are some attempts to link decisions of batching and scheduling (Prasad and Maravelias 2008; Marchetti et al. 2010, 2012; Sundaramoorthy and Maravelias 2008a, b; Sundaramoorthy et al. 2009; Fumero et al. 2014), in all these works the product demands of each order are problem data. In addition, taking into account that the parallel unit sizes are different, the number of slots that must be postulated for each unit of stage j is not a trivial decision. This parameter can be approximated considering the admitted maximum number of batches of each product at the campaign. Then, the number of slots postulated for all units of each stage is the same and it is given by:

$$L = \sum_{i \in I} NBC_i^{UP}$$

Taking into account the assumed periodic scheduling strategy, the optimal campaign is cyclically repeated along the planning horizon. The number of repetitions of the campaign, denoted by NC, is a discrete variable of the model, and an upper bound for this variable can be estimated from the length of the planning horizon and the processing times of all products elaborated in the period.

The decisions involved in the production planning consist of determining for each product, the quantity to be produced, the total sales, and the inventory level at the end of the horizon.

The amounts of raw material purchased and used in the process are also determined. From the point of view of scheduling decisions, the model allows determining, the number and size of batches of each product in the optimal campaign, the assignment of batches to equipment items in each stage and its sequencing, the initial and final processing times for batches processed in each unit, the campaign cycle time, and the number of campaign repetitions over the planning horizon.

The considered objective function is the maximization of the net profit over the planning horizon, given by incomes (product sales) minus costs (raw materials, operation and inventory).

3 Mathematical formulation

3.1 Production planning constraints

The following constraints manage the purchase of raw materials, the production, and the raw materials and products inventories, in order to meet the demands limits and ensure the required product and raw material storage at the final of the planning horizon.

The proposed model assumes that the production of each final product i, Q_i, requires $r = 1, 2, \ldots, R$ raw materials. The mass balance [Eq. (1)] determines the amount consumed of raw material r to make product i, RM_{ri}, where parameter F_{ri} denotes the conversion of raw material r for product i.

$$RM_{ri} = F_{ri} Q_i \quad \forall r, i \tag{1}$$

Constraint (2) specifies the total raw material consumption.

$$RM_r = \sum_i RM_{ri} \quad \forall r \tag{2}$$

Thus, the stock of raw material r at the end of the planning horizon, IM_r, depends on the amount stored at the beginning of time horizon, $IMinit_r$; the purchases during this period, CR_r; and the amount consumed for production, RM_r:

$$IM_r = IMinit_r + CR_r - RM_r \quad \forall r \tag{3}$$

Furthermore, purchases of raw materials cannot exceed the respective availabilities:

$$CR_r \leq QM_r \quad \forall r \tag{4}$$

In a similar way, Eq. (5) sets the level of final product i stored at the end of planning horizon, IP_i, which is equal to the amount stored at the start of time horizon, $IPinit_i$, plus the production during this time period, Q_i, less the sold amount QS_i, where the sold amount is bounded by the minimum and maximum demands:

$$IP_i = IPinit_i + Q_i - QS_i \quad \forall i \tag{5}$$

$$DE_i^L \leq QS_i \leq DE_i^U \quad \forall i \tag{6}$$

3.2 Batches selection and sizing constraints

As already mentioned, the number and size of batches of product i that must be manufactured at the campaign, as well as the number of repetitions of the campaign are model variables. Then, a binary variable z_{ib} is introduced, which takes value 1 if batch b of product i, from the set proposed IB_i, is selected to satisfy the production requirements of that product and 0 otherwise.

Let B_{ib} be the size of batch b of product i and Q_i the amount of product i elaborated during the planning period. Then, taking into account that the campaign will be cyclically repeated NC times over the planning horizon:

$$Q_i = \sum_{b \in IB_i} B_{ib} NC \quad \forall i \tag{7}$$

$$B_i^{min} z_{ib} \leq B_{ib} \leq B_i^{max} z_{ib} \quad \forall i, b \in IB_i \tag{8}$$

where $B_i^{min} = \underset{j \in J}{max} \left\{ \underset{k \in K_j}{min} \left\{ \alpha_{ik} \frac{V_k}{SF_{ij}} \right\} \right\}$, $B_i^{max} = \underset{j \in J}{min} \left\{ \underset{k \in K_j}{max} \left\{ \frac{V_k}{SF_{ij}} \right\} \right\}$ are the minimum and maximum feasible batch sizes, respectively, for product i. That is, the minimum and maximum capacities of batch units are bounds on the amount of processed material in each batch.

Due to B_{ib} and NC are optimization variables, Eq. (7) is reformulated to avoid non linearities. Discrete variable NC is expressed using a base-2 representation and therefore it can be treated as a continuous variable:

$$NC = \sum_{m=0}^{M} x_m 2^m \tag{9}$$

Parameter M is taken as $M = ceil \left(\log_2 (NC^{UP} + 1) - 1 \right)$, where ceil is a function that rounds the argument to the next integer, NC^{UP} is the maximum number of times that the campaign can be cyclically repeated over the time horizon, which is estimated by the planner taking into account different parameters of the model (processing times, minimum batch sizes, maximum demands, planning horizon, etc), and x_m are binary variables used for the representation.

In particular, if the total production at the period is null because the current inventory allows reaching the requirements to achieve the maximum benefit, all binary variables x_m take value zero:

$$\sum_i \sum_{b \in IB_i} z_{ib} \geq x_m, \quad \forall m \tag{10}$$

Then, replacing Eq. (9) into Eq. (7), the following constraint is hold:

$$Q_i = \sum_{b \in IB_i} \sum_{m=0}^{M} x_m B_{ib} 2^m \quad \forall i \tag{11}$$

Bilinear terms in Eq. (11) are eliminated using Glover's procedure (Glover 1975). Thus, a non negative continuous variable, w_{ibm}, is defined, which is equal to B_{ib} if x_m takes value 1, and 0 otherwise. So Eq. (11) is represented by:

$$Q_i = \sum_{b \in IB_i} \sum_{m=0}^{M} 2^m w_{ibm} \quad \forall i \tag{12}$$

Besides, the following constraints are imposed, where M_1 is a sufficiently large number that makes the constraint redundant when x_m takes value 0:

$$w_{ibm} - B_{ib} \geq M_1 (x_m - 1) \quad \forall i, b \in IB_i, m \tag{13}$$

$$w_{ibm} \leq B_{ib} \quad \forall i, b \in IB_i, m \tag{14}$$

$$w_{ibm} \leq B_i^{max} x_m \quad \forall i, b \in IB_i, m \tag{15}$$

Taking into account that the size of unit k denoted by V_k and the size factor SF_{ij} are model parameters, if batch b of product i is processed in unit k of stage j the following inequalities limit the size B_{ib} of batch b between the minimum and maximum processing capacities of unit k:

$$\alpha_{ik}\frac{V_k}{SF_{ij}} \leq B_{ib} \leq \frac{V_k}{SF_{ij}}$$
$$\forall\ i, b \in IB_i, k \in \{\text{units of stage } j \text{ used to process batch } b\} \tag{16}$$

where α_{ik} is the minimum filled rate required to process product i in unit k. Due to the units selected to process the batches of each product are optimization variables and their sizes are different, Eq. (16) must be expressed through a variable that indicates this selection, as it will be seen later.

Besides, without loss the generality and in order to reduce the number of alternative solutions, the selection of batches of a same product as well as the sizes assigned to them are made in ascending and descending numerical order, respectively, that is:

$$z_{ib+1} \leq z_{ib} \quad \forall i, b \in IB_i, b+1 \in IB_i \tag{17}$$

$$B_{ib+1} \leq B_{ib} \quad \forall i, b \in IB_i, b+1 \in IB_i \tag{18}$$

3.3 Assignment, sequencing and timing constraints

For these decisions, an asynchronous slot-based continuous-time representation has been used. The assignment, sequencing and timing constraints for the batches that compose the campaign are largely inspired from detailed model (DP) proposed by Fumero et al. (2014) for the problem of scheduling of multiproduct batch plants operating in campaign-mode. A detailed description of assumptions about units and slots utilization at each plant stage, which allow reducing the search space as well as eliminate alternative solutions, can be found in the previous article. However, in order to facilitate the readability of the model, the assignment and timing main constraints are presented in this manuscript.

Each selected batch must be assigned, in each stage, to a specific slot of a unit for its processing. Then, the binary variable Y_{bkl} is introduced, which takes value 1 if batch b is assigned to slot l in unit k and 0 otherwise. Although this variable is enough for formulating the scheduling decisions, the binary variable X_{kl}, which specifies the slots set utilized in unit k for processing batches, is also used in order to reduce the search space and, therefore, to improve the computational performance.

Logical relations can be defined among binary variables z_{ib}, X_{kl} and Y_{bkl}. In fact, if slot l of unit k is not utilized, then none of the proposed batches is processed in it. Moreover, if slot l of unit k is utilized, then only one of the proposed batches is processed in it. Then, the following constraint is imposed:

$$\sum_{i\in I}\sum_{b\in IB_i} Y_{bkl} = X_{kl} \quad \forall j, k \in K_j, 1 \leq l \leq L \tag{19}$$

On the other hand, if batch b of product i is selected (i.e. $z_{ib} = 1$), then this batch is processed, in each stage j, in only one slot of some of the available units at the stage. This condition is guaranteed by:

$$\sum_{k\in K_j}\sum_{1\leq l\leq L} Y_{bkl} = z_{ib} \quad \forall j, i, b \in IB_i \tag{20}$$

Without loss of generality and in order to reduce the search space, it is assumed that slots of each unit are consecutively used in ascending numerical order. Hence, slots of zero length take place at the end of each unit. Eq. (21) establishes that for each unit k, slot $l+1$ is only used if slot l has been already allocated:

$$X_{kl} \geq X_{kl+1}, \quad \forall j, k \in K_j, 1 \leq l \leq L \tag{21}$$

Finally, variable Y_{bkl} allows correctly expressing the inequalities posed in (16) as:

$$\alpha_{ik} \frac{V_k}{SF_{ij}} Y_{bkl} \leq B_{ib} \quad \forall i, b \in IB_i, 1 \leq l \leq L \tag{22}$$

$$B_{ib} \leq \frac{V_k}{SF_{ij}} + M_2 \left(1 - \sum_{1 \leq l \leq L} Y_{bkl}\right) \forall i, b \in IB_i \tag{23}$$

where scalar M_2 is a sufficiently large number that makes the constraint redundant when batch b is not assigned to any slot of unit k.

Taking into account the cyclic character of the used scheduling strategy, the timing decisions for the set of proposed slots in each unit are posed for the first cycle.

Nonnegative continuous variables, TI_{kl} and TF_{kl}, are used to represent the initial and final processing times, respectively, of the proposed slots in each unit k. When slot l is not the last slot used in unit k of stage j for processing one batch, that is, if $Y_{b'kl+1}$ takes value 1 for some batch b',final processing time TF_{kl} of slot l in unit k is constrained by:

$$TF_{kl} = TI_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'kl+1} \quad \forall j, k \in K_j, 1 \leq l < L \tag{24}$$

On the other hand, when the sequence of slots used in unit k is 1, 2, …l, i.e. slot $l$is the last slot used at unit k of stage j to process some batch, taking into account that the campaign is cyclically repeated over the planning horizon, the final processing time TF_{kl} is calculated considering the changeover time required for processing the batch assigned to slot 1 in unit k of stage j. Then, the following constraint, analogous to (24), must be satisfied:

$$TF_{kl} = TI_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) Y_{bkl} Y_{b'k1} \quad \forall j, k \in K_j, 1 \leq l \leq L \tag{25}$$

A nonnegative variable $YY_{blb'l'k}$ is defined to eliminate the bilinear products in Eqs. (24) and (25), which takes value 1 if $Y_{bkl} = 1$ and $Y_{b'kl'} = 1$, and 0 otherwise. However, taking into account that the slots of each unit are consecutively used in ascending order, it is only necessary to link the assignments variables Y_{bkl} and $Y_{b'kl+1}$, i.e., those relative to consecutive slots on the same unit k, as well as Y_{bkl} and $Y_{b'k1}$, for all slot l, in order to represent the previous constraints. Therefore, Eqs. (24) and (25) are represented using the following Big-M expressions, respectively:

$$TF_{kl} - TI_{kl} - \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k} \geq M_3 (X_{kl+1} - 1)$$

$$\forall j, k \in K_j, 1 \leq l < L \tag{26a}$$

$$-TF_{kl} + TI_{kl} + \sum_{i \in I} \sum_{i' \in I} \sum_{b \in IB_i} \sum_{\substack{b' \in IB_{i'} \\ b \neq b'}} (t_{ik} + c_{ii'k}) YY_{blb'l+1k} \geq M_3 (X_{kl+1} - 1)$$

$$\forall j, k \in K_j, 1 \leq l < L \tag{26b}$$

$$TF_{kl} - TI_{kl} - \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'1k} \geq -M_3 X_{kl+1}$$
$$\forall j, k \in K_j, 1 \leq l \leq L \tag{27a}$$
$$-TF_{kl} + TI_{kl} + \sum_{i\in I}\sum_{i'\in I}\sum_{b\in IB_i}\sum_{\substack{b'\in IB_{i'}\\ b\neq b'}} (t_{ik} + c_{ii'k})YY_{blb'1k} \geq -M_3 X_{kl+1}$$
$$\forall j, k \in K_j, 1 \leq l \leq L \tag{27b}$$

where M_3 is a sufficiently large number.

Constraints to avoid the overlapping between the processing times of different slots in a unit as well as to match the initial times of empty slots with the final time of the last previously used slot are added to the formulation.

$$TF_{kl} \leq TI_{kl+1} \qquad \forall j, k \in K_j, 1 \leq l < L \tag{28}$$
$$TF_{kl} - TI_{kl+1} \geq -M_4 X_{kl+1} \qquad \forall j, k \in K_j, 1 \leq l < L \tag{29}$$

where M_4 is a sufficiently large number that make the constraint redundant when slot $l+1$ is used.

In order to assure ZW transfer policy, constraints of Big-M type are included, depending if slot l is or is not the last slot used at unit k of stage j for processing one batch. Thus, the following constraint set is added to the formulation in order to consider both cases:

$$TF_{kl} - \sum_{i'\in I}\sum_{b'\in IB_{i'}} c_{ii'k}YY_{blb'1k} - TI_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} - 2 - \sum_{\tilde{l}>l} X_{k\tilde{l}})$$
$$\forall j, j+1, k \in K_j, k' \in K_{j+1}, \quad 1 \leq l \leq L_{kj}, 1 \leq l' \leq L_{k'j+1}, i \in I, b \in IB_i \tag{30a}$$
$$-TF_{kl} + \sum_{i'\in I}\sum_{b'\in IB_{i'}} c_{ii'k}YY_{blb'1k} + TI_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} - 2 - \sum_{\tilde{l}>l} X_{k\tilde{l}})$$
$$\forall j, j+1, k \in K_j, k' \in K_{j+1}, \quad 1 \leq l \leq L_{kj}, 1 \leq l' \leq L_{k'j+1}, i \in I, b \in IB_i \tag{30b}$$

$$TF_{kl} - \sum_{i'\in I}\sum_{\substack{b'\in IB_{i'}\\ b'\neq b}} c_{ii'k}YY_{blb'l+1k} - TI_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} + X_{kl+1} - 3)$$
$$\forall j, j+1, k \in K_j, k' \in K_{j+1}, \quad 1 \leq l < L_{kj}, 1 \leq l' \leq L_{k'j+1}, i \in I, b \in IB_i \tag{31a}$$
$$-TF_{kl} + \sum_{i'\in I}\sum_{\substack{b'\in IB_{i'}\\ b'\neq b}} c_{ii'k}YY_{blb'l+1k} + TI_{k'l'} \geq M_3(Y_{bkl} + Y_{bk'l'} + X_{kl+1} - 3)$$
$$\forall j, j+1, k \in K_j, k' \in K_{j+1}, \quad 1 \leq l < L_{kj}, 1 \leq l' \leq L_{k'j+1}, i \in I, b \in IB_i \tag{31b}$$

The expression for the cycle time of the campaign, *CT*, is obtained from the initial and final times of the first and last slot proposed for processing batches in each unit, respectively, and it is given by:

$$CT \geq TF_{kL} - TI_{k1}, \quad \forall j, k \in K_j \tag{32}$$

Significant reductions in resolution times are achieved by establishing the following lower limit on the campaign cycle time (Fumero et al. 2012b). Assuming that the idle time in each unit during the processing of the campaign is zero, then:

$$CT \geq \sum_{i \in I} \sum_{b \in IB_i} \sum_{l} t_{ik} Y_{bkl} \quad \forall j, k \in K_j \tag{33}$$

Taking into account that the campaign is executed *NC* times with period *CT*, the total time required to produce all batches cannot exceed the length of planning horizon, H:

$$CTNC \leq H \tag{34}$$

The representation of variable *NC* [Eq. (9)] allows rewriting Eq. (34) as:

$$\sum_{m=0}^{M} 2^m x_m CT \leq H \tag{35}$$

In order to avoid the nonlinearity of this expression given for the product of binary and continuous variables, a new continuous variable ww_m is defined, which is equal to *CT* if x_m = 1; otherwise the value is 0. Then,

$$\sum_{m=0}^{M} 2^m ww_m \leq H \tag{36}$$

$$ww_m - CT \geq M_4(x_m - 1) \quad \forall m \tag{37}$$

$$ww_m \leq CT \quad \forall m \tag{38}$$

$$ww_m \leq CT^U x_m \quad \forall m \tag{39}$$

where M_4 is a sufficiently large parameter and CT^U is an upper bound for the campaign cycle time, which is approximated by the planner taking into account the processing times and the maximum number of batches for all products in the campaign.

3.4 Objective function

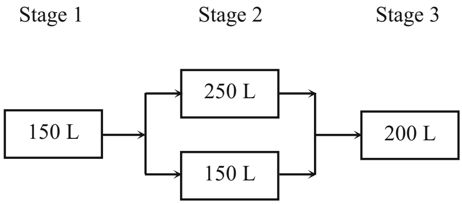
The problem goal is to maximize the net profit (*NP*) given by the difference between the incomes due to products sales and the total costs that include purchases of raw materials, inventories and operation costs.

$$\begin{aligned} NP = & \sum_i np_i QS_i - \sum_r \kappa_r CR_r \\ & - \left[\sum_r \beta_r \left(\frac{IMinit_r + IM_r}{2} \right) H + \sum_i \delta_i \left(\frac{IPinit_i + IP_i}{2} \right) H \right] - \sum_i co_i Q_i \end{aligned} \tag{40}$$

To determine the incomes, the product price, np_i, is multiplied by the amount of sold product i during time period. Parameter α_r denotes the price of raw material r used to manufacture products, while β_r and δ_i are the inventory costs per unit of raw material and final product, respectively. Finally, parameter co_i denotes the operating cost coefficient.

Taking into account that different production campaigns can be obtained for a same optimal planning, a penalty term that involves the cycle time of the campaign is included in the objective function. Through this heuristic, the computational performance is improved and the number of alternative solutions is reduced. This new term is the product of the variable *CT* with a weighting factor λ, which is appropriately selected taking into account the involved

Fig. 1 Plant structure and unit sizes



parameters. In this way, the approach reaches the more profitable solution with minimum campaign cycle time. Therefore, the following objective function is proposed:

$$f = NP - \lambda CT \tag{41}$$

Finally, all the constraints for the detailed planning of a multiproduct multistage batch plant with nonidentical parallel units, sequence-dependent changeovers, and a campaign-based operation, are linear in order to generate a MILP model that can be solved to global optimality.

4 Examples

In this section two illustrative examples are presented in order to evaluate the proposed formulation for the optimal production planning for multiproduct batch plants operating through campaigns of cyclic execution. In both examples, coefficient α_{ik} representing the minimum filled rate required to process product i in unit k, is assumed to be 0.50 for all products and equipment items. That is, this parameter accounts for 50 % of total available capacity of units. All the examples has been solved with a 0 % optimality gap in GAMS (Brooke et al. 2012), using CPLEX 12.5 solver, on an Intel Core i7, 3.4 GHz with 4 GB RAM.

4.1 Example 1

Four products (A, B, C, D) are produced through three stage from two raw materials (R1, R2). The plant structure and unit sizes are illustrated in Fig. 1. Units at each stage are denoted by the sets: $K_1 = \{1\}$, $K_2 = \{2, 3\}$, and $K_3 = \{4\}$, respectively.

A planning horizon time of 1 week ($H = 144$ h) is assumed. Data on processing times and size factors for each product, and conversion factors of raw material into product are shown in Table 1, while the sequence-dependent changeover times are given in Table 2.

The raw materials availability (kg) for the purchase are 5000 for R1 and 4000 for R2, and their costs ($/kg) are 0.4 and 0.6 for respectively. The final products selling prices ($/kg) are 3.2, 3.0, 2.8, 2.5, for products A, B, C and D. The inventory cost coefficients ($/ ton h) for both raw materials and final products are 0.1 and 0.2, respectively. The maximum product demands are 780, 1290, 1980 and 2400, respectively, while a minimum of 50 % of those demands must be satisfied in the planning horizon.

For this case, the initial inventories of both raw materials and products are null. The production of product D will be suspended by a week, and therefore it is required to maintain a stock of 1000 kg of this product at the end of the given planning horizon.

Considering the nonidentical parallel unit sizes at each stage and the size factors for each product in each stage, and assuming that the equipment utilization minimum rate is 0.50

Table 1 Example 1—model parameters

Products	Processing time, t_{ik}(h)				Size factor, SF_{ij} (L/kg)			Conversion factor, F_{ri}	
	J1	J2		J3	J1	J2	J3	R1	R2
	1	2	3	4					
A	6.0	14.0	12.0	6.0	0.6	0.9	0.8	0.75	0.5
B	2.0	4.0	2.0	2.0	0.7	1.2	0.85	1.0	0.25
C	5.0	11.0	9.0	2.0	0.75	1.0	0.7	0.5	1.25
D	2.0	1.5	1.5	3.0	0.3	0.4	0.45	0.8	0.2

Table 2 Example 1—sequence-dependent changeover times

i	Sequence-dependent changeover time: $c_{ii'k}$ (h)											
	J1 $k = 1$				J2 $k = 2, 3$				J3 $k = 4$			
	A	B	C	D	A	B	C	D	A	B	C	D
A	0.0	0.5	0.3	0.0	0.0	0.3	0.4	0.5	0.25	0.3	0.0	0.6
B	0.8	0.0	0.6	0.8	1.2	0.25	0.6	0.6	1.2	0.25	0.8	0.7
C	1.0	0.5	0.0	1.0	0.0	0.5	0.25	0.5	1.5	0.5	0.0	0.5
D	0.8	0.0	0.6	0.0	0.9	0.25	0.7	0.25	1.25	0.5	1.25	1.25

for all products and equipment items, the feasible batch sizes for products A, B and C are bounded by: $125 \leq B_A \leq 250$, $118 \leq B_B \leq 208.3$, $143 \leq B_C \leq 200$, $250 \leq B_D \leq 444$.

Taking into account that the total production for all products and the amount of each product elaborated in the campaign are optimization variables, the number of batches that must be postulated for each product in the campaign as well as the its number of repetitions are not trivial decisions. Model parameters concerning to maximum demands, possible batch sizes, cycle time of products, length of planning horizon, etc. allow obtaining a better estimation of this values and a reduction in the model size. In this example, a maximum of two batches in the campaign composition is assumed for products A, B and D, and three batches for product C; and an upper limit for the number of repetitions of the campaign equal to 6. Then, the sets of batches proposed for each product are: $IB_A = \{b_1, b_2\}$, $IB_B = \{b_3, b_4\}$, $IB_C = \{b_5, b_6, b_7\}$, $IB_D = \{b_8, b_9\}$, and three binary variables have been defined for the base-2 representation of variable NC. In order to avoid undesirable combinations for the value of NC, the following constraint is added to the formulation: $x_2 + x_1 + x_0 \leq 2$.

Table 3 lists the objective function values for the linear programming (LP) relaxation and the MILP model, as well as the model size and computational statistics for the proposed formulation.

The amounts of final products produced and sold, amounts of raw materials purchased and used for producing all products, and the inventories levels of both raw materials and products, are summarized in Table 4.

The optimal campaign consists of one batch of product A (b_1), one of B (b_3), two of C (b_5, b_6) and one of D (b_8). The optimal production sequence obtained in each batch unit for the different stages, considering sequence-dependent changeover times, is illustrated in the Gantt chart of Fig. 2. As it can be noted from Fig. 2, in order to decrease the campaign cycle

Table 3 Model sizes and computational statistics

Model	LP relaxation	Objective function	Constraints	Variables		Nodes	CPU time (s)
				Binary	Continuous		
Example 1	13,150.50	11,059.54	26,321	352	5059	17,756	124.66
Example 2	149,427.40	148,532.40	46,959	519	8453	47,407	481.96

Table 4 Example 1—optimal production planning variables

Product	Production (kg) Q_i	Sales (kg) QS_i	Inventory (kg) IP_i	Raw material	Purchases (kg) CR_r	Material used (kg) RM_r	Inventory (kg) IM_r
A	780.0	780.0	0.0	R1	4956.2	4956.2	0.0
B	1250.0	1250.0	0.0	R2	3710.3	3710.3	0.0
C	1980.0	1980.0	0.0				
D	2664.0	1664.0	1000.0				

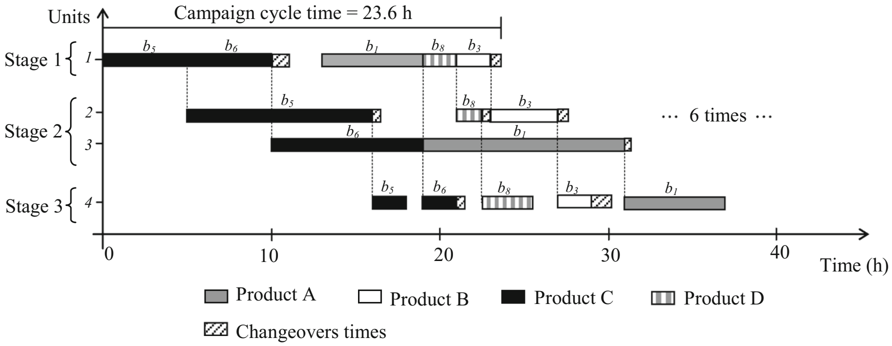


Fig. 2 Example 1—optimal production schedule for the campaign

time and improve the use of the equipment items, the processing sequence of the batches is not the same on stages 1 and 3. The campaign cycle time is equal to 23.6 h and it is repeated 6 times.

The size for each batch that composes the campaign is equal to 130 kg for product A, 208.3 kg for product B, 187 and 143 kg for product C, and 444 kg for product D. The capacities used in each unit of the different stages for processing the selected batches are reported in Table 5. The batches that reach the minimum and maximum capacities are highlighted in italics and bold, respectively. Batch b_1 of product A is processed in units 1, 3 and 4 using 52, 78 and 52 % of their capacities, respectively. Batch b_3 of product B is processed in units 1, 2 and 4 and its size is the maximum possible. Two batches of product C are processed for meeting its demand. Batch b_6 size is larger than b_5 and additionally the size of b_5 is the required minimum. Finally, for product D, batch b_8 is processed in units 1, 2, and 4 and its size is the maximum possible,

As it can be observed from Table 4, the optimal campaign allows satisfying the maximum demands of products A, and C. However, for products B and D, although the batch size used

Table 5 Example 1—capacities used in each unit of each stage (L)

Product	Batch	Stage 1	Stage 2		Stage 3
		$k = 1$	$k = 2$	$k = 3$	$k = 4$
A	b_1	78.00		117.00	104.00
B	b_3	145.83	**250.00**		117.08
C	b_5	140.36	187.14		131.00
	b_6	107.14		142.86	*100.00*
D	b_8	133.2	177.6		**200.00**

Table 6 Example 1—economic evaluation results ($)

Description	Optimal value
Sales income	15,950.00
Raw material cost	4,208.66
Raw material inventory cost	0.00
Product inventory cost	14.40
Operating costs	667.40
Total	11,059.54

in each case is the maximum possible, the total produced over the given time horizon does not meet the maximum demand for those products. Although the availability of raw materials is not zero, these quantities as well as the length time horizon are not sufficient to meet the maximum demands for products B and D.

Finally, a detailed analysis of the economic results is summarized in Table 6.

4.2 Example 2

The batch facility considered in this example consists of three processing stages with two nonidentical units operating in parallel on stages 1 and 2, and one unit on the remaining stage, as is shown in Fig. 3. The unit sizes are also depicted in the figure.

The plant can produce four products A, B, C and D from two raw materials.

As in Example 1, the objective is to determine the detailed production plan for a time horizon equal to 1 week ($H = 144$ h).

Data on processing times and size factors for each product, and conversion factors of raw material into product are shown in Table 7, while the sequence-dependent changeover times are given in Table 8.

The raw material availability during the planning horizon is equal to 55,000 kg for raw material R1 and 70,000 kg for raw material R2, and the costs ($/kg) are 0.5 for R1 and 1.4 for R2. The final products prices ($/kg) are 2.0, 3.0, 2.5, and 2.0 for products A, B, C and D, respectively. The inventory cost coefficients ($/ ton h) of both final raw materials and products are 0.1 and 0.2, respectively. The maximum demands (kg) are 25,000, 40,000, 30,000 and 20000, for products A–D respectively; and a minimum of 50 % of those demands must be satisfied in the planning horizon.

The initial inventories (kg) of raw materials are 10,000 and 20,000, respectively; while the available stock for products A and D at the beginning of the planning horizon are

Fig. 3 Example 2—plant structure and unit sizes

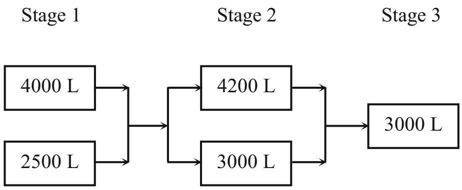


Table 7 Example 2—model parameters

Products	Processing time, t_{ik}(h)					Size factor, SF_{ij}(L/kg)			Conversion factor, F_{ri}	
	J1		J2		J3	J1	J2	J3	R1	R2
	1	2	3	4	5					
A	10	7	18	14	5	0.70	0.60	0.50	0.5	0.7
B	11	7	13	13	4	0.60	0.70	0.45	1.0	0.7
C	8	6	11	8	3	0.70	0.65	0.55	0.4	1.0
D	6	5	3	3	1	0.60	0.80	0.40	0.3	1.3

Table 8 Example 2—sequence-dependent changeover times

i	Sequence-dependent changeover time: $c_{ii'k}$ (h)											
	J1				J2				J3			
	$k = 1, 2$				$k = 3, 4$				$k = 5$			
	A	B	C	D	A	B	C	D	A	B	C	D
A	0.0	0.5	0.3	0.0	0.25	0.3	0.0	0.5	0.0	0.6	0.6	0.6
B	0.8	0.0	0.6	0.8	1.2	0.25	0.8	0.4	0.8	0.0	0.8	0.5
C	1.0	0.5	0.8	1.0	0.5	1.5	0.55	0.5	2.0	1.5	0.0	1.0
D	0.4	0.5	1.0	0.0	0.25	0.5	0.5	0.25	0.5	0.5	0.5	0.0

$IPinit_A$ =1000 kg and $IPinit_D$ = 10,000 kg. It is required to maintain a stock of 500 kg of product A and 2500 kg of product C at the end of the given planning horizon.

For this example, the minimum feasible batch sizes for products A, B, C, and D are 3000 , 3333 , 2727 and 3750 kg, respectively, while the maximum feasible batch sizes are 5714 kg for product A, 6000 kg for B, 5454 kg for C and 5250 kg for D. Based on the above values and taking into account the different parameters of the model, the maximum number of batches in the campaign composition is assumed to be three for products A and C, four batches for product B, and two batches for product D; and the upper limit for the number of repetitions of the campaign equal to 7. Then, the sets of batches proposed for each product are: $IB_A = \{b_1, b_2, b_3\}$, $IB_B = \{b_4, b_5, b_6, b_7\}$, $IB_C = \{b_8, b_9, b_{10}\}$, $IB_D = \{b_{11}, b_{12}\}$, and three binary variables have been defined for the base-2 representation of variable NC.

Objective function values for LP relaxation and MILP model, as well as the model size and computational statistics for the proposed formulation are listed in Table 3.

The optimal amounts of final products produced and sold, amounts of raw materials purchased and used for producing all products, and the inventories levels of both raw materials and products, are summarized in Table 9.

Table 9 Example 2—optimal production planning variables

Product	Production (kg) Q_i	Sales (kg) QS_i	Inventory (kg) IP_i	Raw material	Purchases (kg) CR_r	Material used (kg) RM_r	Inventory (kg) IM_r
A	20,000.0	20,500.0	500.0	R1	43,000.0	53,000.0	0.0
B	40,000.0	40,000.0	0.0	R2	54,500.0	74,500.0	0.0
C	32,500.0	30,000.0	2500.0				
D	0.0	10,000.0	0.0				

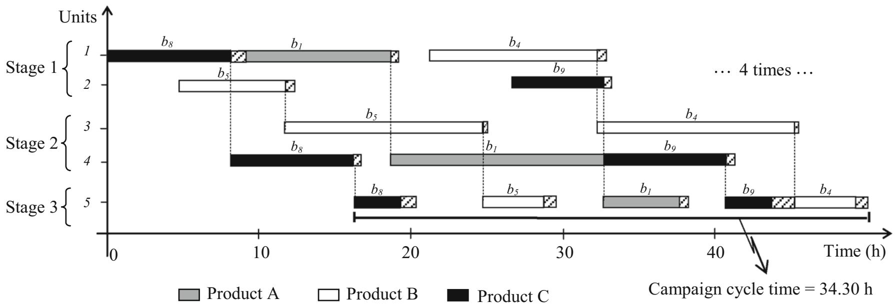


Fig. 4 Example 2—optimal production schedule for the campaign

The optimal campaign consists of one batch of product A (b_1), two of B (b_4, b_5), two of C (b_8, b_9) and zero of D. The optimal production sequence obtained in each batch unit for the different stages, considering sequence-dependent changeover times, is illustrated in the Gantt chart of Fig. 4. The campaign cycle time is reached at unit 5 and it is equal to 34.3 h, and the campaign is repeated 4 times over the planning horizon.

The size for each selected batch that composes the campaign and the capacities used in each unit of the different stages for processing those batches are depicted in Table 10. The batches that reach the maximum capacities are highlighted in bold.

Productions of B and C satisfy the maximum demand limits because they are the most profitable products. The total production of B is sold, while a portion of the total production of C is used to satisfy the required stock level for this product at the end of the planning horizon and the rest is sold. Batch b_4 of product B is processed in units 1, 3 and 5 and its size is the maximum possible. Then, batch b_5 fulfills the required amount of that product occupying approximately 96, 67 and 60 % of the capacities of units 2, 3 and 5, respectively. On the other hand, batch b_8 of product C is processed in units 1, 4 and 5 and its size is the maximum possible to be processed in unit 4 of stage 2. Then, batch b_9 fulfills the required amount of that product occupying approximately 98, 76 and 64 % of capacities, respectively.

Analyzing results, it is found that the total of raw materials R1 and R2 consumed to produce B and C are 53,000 and 74,500 kg, respectively. Therefore, 12,000 and 15,500 kg of raw materials R1 and R2, respectively, are available for the production of A and D. From these last two products, A is the most profitable. A total of 20,000 kg of A is produced and considering the time horizon available for reaching the optimal plan, only one batch of product A (b_1) is processed in units 1, 4 and 5 taking the maximum possible size in unit 4 of stage 2. Then, 2000 kg of R1 and 1500 kg of R2 are available for production of D. For this product, the initial inventory level allows satisfying the minimum demand limit (10,000 kg)

Table 10 Example 2—selected batch sizes (kg) and capacities used in each unit of each stage (L)

Product	Batch/size	Stage 1		Stage 2		Stage 3
		$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
A	b_1 (5000)	3500.00			**3000.00**	2500.00
B	b_4 (6000)	3600.00		**4200.00**		2700.00
	b_5 (4000)		2400.00	2800.00		1800.00
C	b_8 (4615.4)	3230.77			**3000.00**	2538.46
	b_9 (3509.6)		2456.73		2281.25	1930.29

Table 11 Example 2—economic evaluation results ($)

Description	Optimal value
Sales income	256,000.00
Raw material cost	97,800.00
Raw material inventory cost	216.00
Product inventory cost	201.60
Operating costs	9,250.00
Total	148,532.40

but no more batches of this product can be processed in the campaign because the amounts of raw materials and the planning horizon time are not sufficient.

Finally, a detailed analysis of the economic results is summarized in Table 11.

5 Conclusions

In this work, the detailed production planning of multistage multiproduct batch plants that operate in campaign-mode is addressed. Planning and scheduling decisions are modeled according to this operation mode. In addition, sequence-dependent changeover times and nonidentical parallel unit sizes are taken into account. The scheduling constraints are based on a continuous-time representation using the concept of slots. Various equations of the integrated model have been reformulated in order to attain a MILP model and assure the global optimality of the solution.

The planning and scheduling decisions are integrated via production, inventory and batching constraints and the different trade-offs between both decisions levels can be adequately evaluated. From the planning perspective, the raw materials purchase, production, and inventory levels are obtained. From the scheduling perspective, the selection and sizing of the batches that compose the campaign, the assignment of batches to units and its sequencing, and the initial and final operation times has been achieved as well as the number of campaign cycles. The proposed model selects the best campaign configuration with minimum cycle time that can be executed during the given planning horizon in order to satisfy the demand limits and maximize the total net profit.

The proposed model can be used by the planner as a tool for the decision–making about the operations management and different options can be assessed. In a stable context, a make to stock policy can be effectively carried out assuring the appropriate employment of the plant resources. Through the examples the capabilities of the proposed formulation are shown.

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