

# Unit Commitment Scheduling Including Transmission Constraints: a MILP Formulation

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## Abstract

The benefits of good scheduling of electrical units are widely known (De la Torre et al, 2008). In this paper, a new approach to account for the Security-Constrained Unit Commitment (SCUC) is presented. A model is developed as a deterministic optimization problem, giving rise to a MILP formulation. Demand, reserve, and unit constraints are taken from a previous paper (Marcovecchio et al, 2014). Transmission constraints including buses balance, lower and upper bound for line power flows, and bus voltage angle constraints are included in this paper. Identification of loops matrix is not necessary as it is the case in several formulations (Stagg et al, 1968). Scheduling was solved for a 6-bus 3-generator and 11 transmission line problem and a 31-bus 16-generator and 43-transmission line problem. Computational times are very low, being 0.189 and 118.160 CPU sec. respectively. Relationship between capacity usage and occupied time for all generators is analyzed. In a similar way, power flow in each line related to the power flow in output and input bus connected to each line is addressed. This information is depicted by simple graphs.

**Keywords:** Optimization, Power System Scheduling, Security-Constrained Unit Commitment (SCUC), Transmission Constraints.

## 1. Introduction

Planning and operation of power systems is faced for complex and changing challenges. As a consequence, it is always a subject of scientific interest. The Unit commitment (UC) is one of the most important functions of system operators in the electricity market. The basic goal of a UC problem is to determine the optimal schedule of generating units in a power system that satisfies a given load demand and specific unit constraints while minimizing operational costs. Various operating constraints and non linear costs are considered according to unit characteristics. Many approaches have been applied by many authors to settle the problem more conveniently. The first studies appeared over 40 years ago (Guy, 1971).

Unit Commitment Problem has been usually solved without paying much attention to network security constraints (Kazarlis et al, 1996; Ostrowski et al, 2012; Quan et al, 2015). If network constraints are not considered, the provided solution could cause overload for transmission lines, and solution would be infeasible. When transmission

constraints are included, the problem is known as Security-Constrained Unit Commitment (SCUC) problem. This furnishes an economically feasible schedule. In such cases, obtaining feasible solutions becomes a more difficult task, since both UC and SCUC problems are NP-hard mixed integer problems.

Adding Transmission constraints to Unit Commitment problems have been a challenge faced by researchers for too many years. One of the first and most widely cited papers which deal with transmission constraints appeared in 1995. It was presented by J. Shaw: DC power flow model for unit SCUC formulation was presented to solve network constraints applying Lagrangian relaxation. This approach is considered a starting point and it was often cited along the following years by researchers working in that field. Since then, many methods have been proposed to resolve transmission constrained UC (Padhy, 2004; Fu et al, 2005; Guan et al, 2005; Senthil Kumar et al, 2010; Reza Norouzi, 2014). Transmission limits are modeled by voltage bus angle differences or line power flow constraints (Tseng et al, 1998). There is extensive literature that presents iterative and simulation methods to solve these problems, with no guarantee for a global optimal solution (Zhu, 2009).

The present work is based on a previously developed UC formulation in which the number of discrete decisions is greatly reduced, giving rise to smaller MILP problems as compared to previous approaches. Transmission constraints are included by means of transmission lines power limits. When transmission lines are active, voltage angles differences between buses are linked to the real power transmitted by lines. All lines are assumed to be active along the considered time horizon. By implementing the proposed model, problems of power systems with network restrictions can be solved in efficient computational times.

Two problems are resolved to prove effectiveness. We compared different solvers, objective function values, and CPU times. Further information derived from the optimal scheduling is also presented. The states of lines and buses are discussed. This analysis allows lines and buses to be ordered according to their relative contribution to the distribution network.

## **2. SCUC Formulation**

SCUC problem is composed by UC problem plus network constraints. This problem consists in considering  $I$  thermal units on  $T$  time periods which are the scheduling horizon. Marcovecchio et al. have formulated the problem in an effective way, reducing the number of decision variables. The objective consists of minimizing the operating cost for power supply which satisfies the demand. Several constraints were imposed to start and shut down generator times. Operating costs are convex linear and nonlinear terms. An MILP (Mixed Integer Lineal Problem) is obtained by linear approximation of the convex part of the objective function.

### 3. Transmission Line Method applied to Solving SCUC Problem

Power balance in each bus of the system is included:

$$\sum_{i=1}^I p_{i,bu,t} + \sum_{l=1}^L \sum_{bu_i}^{BU} p_{-l,bu_i,bu_o=bu,t} - \sum_{l=1}^L \sum_{bu_o}^{BU} p_{-l,bu_i=bu,bu_o,t} = \sum_{j=1}^J bkt_{j,bu,t} \quad (1)$$

For all “bu” and “t”. Being: I = Total number of units; J= Total number of loads; L = total number of transmission lines; BU = Total number of buses; p = power produced by generator unit “i” [MW]; p<sub>-l</sub> = real power of line “l” that connects bus i “from” to bus o “to” [MW], bkt = power load [MW]. Sets: bu = bus; bu<sub>i</sub> = “from” bus; bu<sub>o</sub> = “to” bus;

Sign convention: power entering the bus: positive. Powers leaving the bus: negative.

Real power transmitted by each line is proportional to the voltage angle difference between the terminal buses of the line. In high-voltage power systems, the angle difference  $\theta_{bu_i bu_o}$  is very small. Furthermore, the line resistances are negligible compared to the line reactances. According to these considerations, we obtain

$$p_{-l,bu_i bu_o} = -b_{bu_i bu_o} \theta_{bu_i bu_o} = \frac{\theta_{bu_i} - \theta_{bu_o}}{x_{bu_i bu_o}} = \frac{\theta_{bu_i} - \theta_{bu_o}}{x_l} \quad (2)$$

“ $\theta_{bu_i bu_o}$ ” being the difference in voltage angle between two connected buses. “x” is the constant line reactance (p.u.). Then, by reformulating equation (2) with (1), the real power balance across buses is:

$$\sum_{i=1}^I p_{i,bu,t} + \sum_{l=1}^L \sum_{bu_i}^{BU} \left( \frac{\theta_{bu_i=bu,t} - \theta_{bu_o=bu,t}}{x_l} \right) - \sum_{l=1}^L \sum_{bu_o}^{BU} \left( \frac{\theta_{bu_o=bu,t} - \theta_{bu_i=bu,t}}{x_l} \right) = \sum_{j=1}^J bkt_{bu,t} \quad (3)$$

### 4. Numerical Test

Models were developed in *General Algebraic Modeling System (GAMS)* and solved using different solvers for each test. Simulations were performed in an Intel i5, 4 GB Ram computer.

#### 4.1. 3 generator – 6 bus – 11 line problem:

The data for this problem are provided in the paper by Grey (2008). Two cases of this system are tested for proving the performance of the proposed model.

Case 4.1.1: The first case involves a UC problem without network constraints. In the paper by Grey, the Objective Function is: \$ 78,322. We obtain \$ 73,721 for the total

cost and we used three solvers for the MILP formulation: GUROBI (0.083 sec. of CPU time); CPLEX (0.078 sec.).

Case 4.1.2: For the second case, network constraints were included. Thus, SCUC is addressed. We introduce a modification to the original problem: The upper and lower power flow limit per line is set in 80 and -80 [MW] respectively. These modifications are introduced for better exhibiting the effects of network constraints in the objective function. The total cost in this case is \$ 83,768. Again, we use again three solvers; and the CPU times are the following: GUROBI 0.189 and CPLEX 0.250 seconds. Both cases were solved up to zero GAP between relaxed and feasible solution.

#### 4.2. 16 generator - 31 bus – 43 line problem:

All the data used for this system are presented by Guo (2012). This problem is a modified version of the already known IEE original problem presented by Shaw in 1995.

Case 4.2.1: As in the previous case, we consider a UC problem without network constraints. Comparison with the objective function used by another author could not be performed due to the lack of this information. Total cost is \$ 1,103,044. CPU times were 51.380 sec. with GUROBI and 62.400 sec. using CPLEX.

Case 4.2.2: Now we consider the UC problem with network constraints. The Total Cost is \$ 1,105,217. CPU times: 118.160 sec. (GUROBI) and 126.439 sec (CPLEX). Both cases were solved up to zero GAP.

The relationship between generators power supply and commitment time is depicted in Figure 1: In an ordinate axis: the power produced by each unit in active time divided by the maximum power that can be produced by each unit in active time. In the abscissa axis: the sum of hours in active time for each unit divided by 24 hours (programming horizon). As it can be seen, 5 generators are occupied for more than 80% of the time horizon whereas these generators are loaded with more than 30% of their capacity. On the other hand, 4 generators are occupied for less than 30% of the time horizon and lasted with less than 5% of their capacity.

$$\frac{\sum_{t=1}^{t=24} p_{i,t} \left[ \frac{\text{MW.H}}{\text{MW.H}} \right]}{\sum_{t=1}^{t=24} p_{i,t}^{\text{UP}} \left[ \frac{\text{MW.H}}{\text{MW.H}} \right]} \in [0,1] \quad \text{vs.} \quad \frac{\sum_{t=1}^{t=24} a_{i,t} \cdot t_{i,t} \left[ \frac{\text{H}}{\text{H}} \right]}{24 \text{ hs}} \in [0,1] \quad (4) \text{ and } (5)$$

We analyse the behaviour for system lines. Power in line “l” at time t is divided by the maximum power that can be supported by line “l” in the same time for each hour in the programming horizon. We have six lines whose factor is 1.0 at least one time (lines: 6,7,8,33,37 and 38). We only show the first three lines in Figure 2 for clarifying. At the optimal solution, three different line load states can be identified: lines with high, medium, and low power loads.

$$\frac{p_{l,t}}{p_{l,\max,t}} \left[ \frac{\text{MW.H}}{\text{MW.H}} \right] \in [0,1] \quad (6)$$

$$t \in [1,24] \quad (7)$$

Figure 1. Power capacity utilization against time utilization for generators

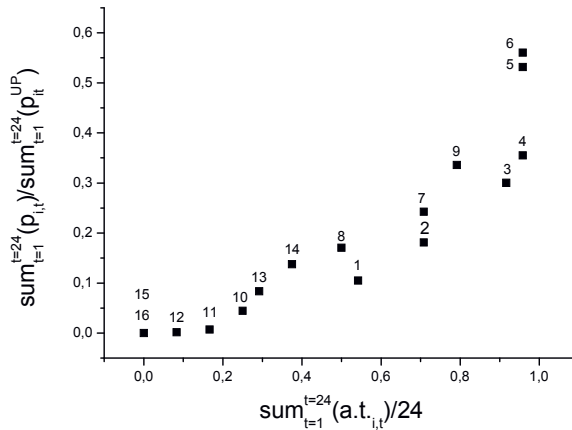
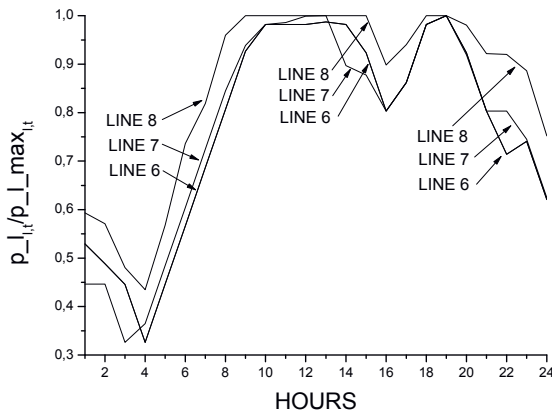


Figure 2. Using the first 3 lines with factor 1.0 through 24 hours



## 5. Conclusions

An efficient approach for treating transmission limits in Security Constrained Unit Commitment problems was presented. Two different size problems were tested.

Using the proposed MILP formulation ensures a global optimal solution. In fact, UC solution for case 4.1.1 reaches a global minimum which becomes \$ 4,601 cheaper than that proposed in the paper by Grey. A larger problem for case 4.2.2 involving 16 generators was solved in 118 seconds.

New parameters: Power Capacity Utilization and Time Utilization for generators and lines were presented. Simple graphs were developed on the basis of these parameters,

providing helpful information that helps to characterize the system. Thus, critical lines and generators can be identified and classified.

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