Journal of Sound and Vibration (2000) **235**(3), 530–538
doi:10.1006/jsvi.2000.2904, available online at http://www.idealibrary.com on IDEAL®

TRANSVERSE VIBRATIONS OF AN ORTHOTROPIC RECTANGULAR PLATE OF LINEARLY VARYING THICKNESS AND WITH A FREE EDGE

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(*Received* 25 *January* 2000)

1. INTRODUCTION

Transverse vibrations of an orthotropic plate of non-uniform thickness and with a free edge (Figure 1) are of importance in several technological applications mainly in civil engineering situations where they are used as structural elements in buildings, bridges, etc., the design of certain printed circuit boards, etc. In spite of the importance of the title problem, apparently no studies have been published on the subject matter [1, 2]. The present paper deals with the determination of the fundamental frequency of the plate or slab shown in Figure 1 when the edge $x = a$ is free and the other three are subjected to all possible supported combinations (clamped or simply supported).

The eigenvalues are determined the following two different approaches:
(1) The optimized Rayleigh–Ritz method using the first terms of a pseudo-Fourier expansion [3, 4], (2) the finite element method [5]. The edges are ordered in a counter clock-wise fashion starting from the edge $x = 0$. For instance, a SS-C-F-SS situation means that the edge $x = 0$ is simply supported, $y = 0$ is clamped, the edge $x = a$ is free and $y = b$ is simply supported (Figure 2(c)). All the different combinations of edge conditions are shown in Figure 2.

2. APPROXIMATE SOLUTION BY MEANS OF THE OPTIMIZED RAYLEIGH–RITZ METHOD

Following Lekhnitskii's standard notation and in the case of normal modes the amplitude of vibration $W(x, y)$ must satisfy the governing energy functional [6]

$$\mathbf{J}(W) = \frac{1}{2}\int_0^b\int_0^a \left[D_1(x)\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_2(x)\left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\mu_2 D_1(x)\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4D_k(x)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] \mathrm{d}x\,\mathrm{d}y - \frac{\rho}{2}\omega^2 \int_0^b\int_0^a h(x)\, W^2\, \mathrm{d}x\,\mathrm{d}y \tag{1}$$

and appropriate boundary conditions.

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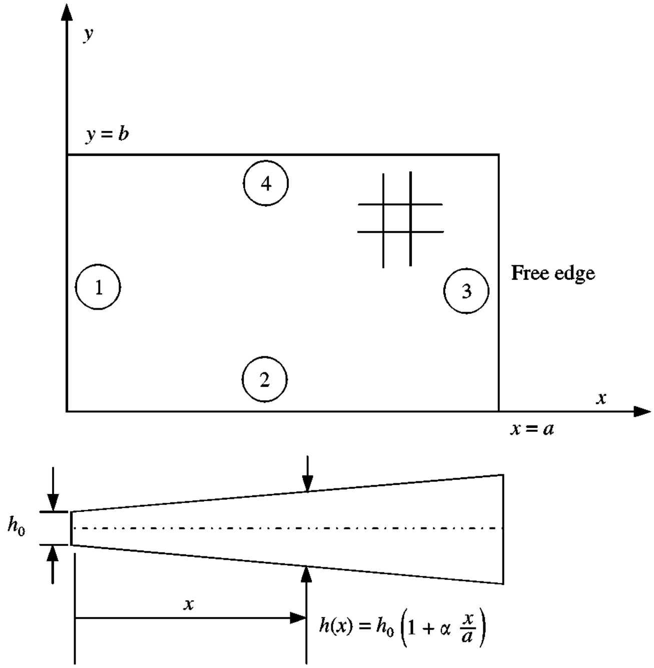

Figure 1. Orthotropic plate of varying thickness.

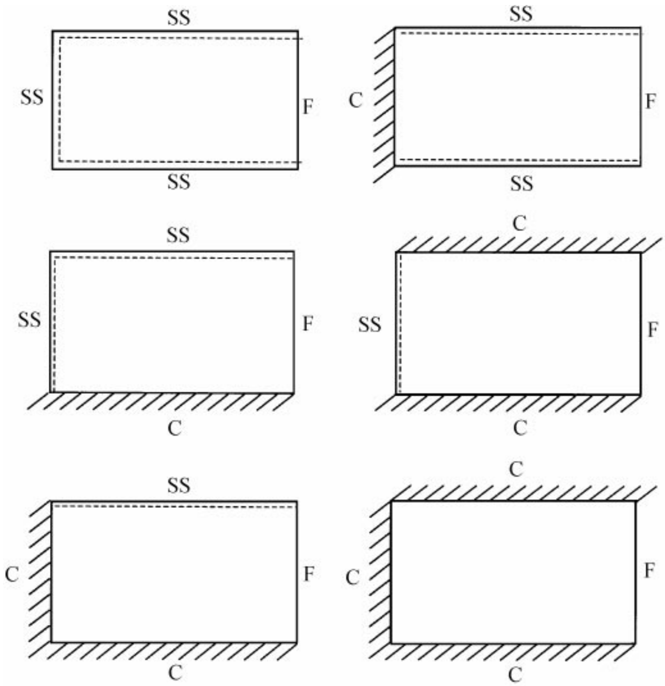

Figure 2. Different combinations of boundary conditions for the system under study.

In equation (1) one has

$$D_1(x) = \frac{E_1 h(x)^3}{12(1-\mu_1\mu_2)}, \quad D_2(x) = \frac{E_2 h(x)^3}{12(1-\mu_1\mu_2)}, \quad D_k(x) = \frac{G h(x)^3}{12},$$

where $h(x) = h_0(1 + \alpha\, x/a)$.

The following approximations for $W(x,y)$ have been used (see Figure 2 [4, 7]:

$$W_a = A_1 \sin\frac{\pi x}{\gamma_1 a}\sin\frac{\pi y}{b} + A_2 \sin\frac{\pi x}{\gamma_2 a}\sin\frac{\pi y}{b}, \quad \text{(SS-SS-F-SS)}, \tag{2a}$$

$$W_a = A_1\left(\sin\frac{\pi x}{\gamma_1 a}\right)^2 \sin\frac{\pi y}{b} + A_2\left(\sin\frac{\pi x}{\gamma_2 a}\right)^2 \sin\frac{\pi y}{b}, \quad \text{(C-SS-F-SS)}, \tag{2b}$$

$$W_a = \left(A_1 \sin\frac{\pi x}{\gamma_1 a} + A_2 \sin\frac{\pi x}{\gamma_2 a}\right)\left[\left(\frac{y}{b}\right)^2 - \frac{5}{3}\left(\frac{y}{b}\right)^3 + \frac{2}{3}\left(\frac{y}{b}\right)^4\right], \quad \text{(SS-C-F-SS)}, \tag{2c}$$

$$W_a = \left(A_1 \sin\frac{\pi x}{\gamma_1 a} + A_2 \sin\frac{\pi x}{\gamma_2 a}\right)\left[\left(\frac{y}{b}\right)^2 - 2\left(\frac{y}{b}\right)^3 + \left(\frac{y}{b}\right)^4\right], \quad \text{(SS-C-F-C)}, \tag{2d}$$

$$W_a = \left[A_1\left(\sin\frac{\pi x}{\gamma_1 a}\right)^2 + A_2\left(\sin\frac{\pi x}{\gamma_2 a}\right)^2\right]\left[\left(\frac{y}{b}\right)^2 - \frac{5}{3}\left(\frac{y}{b}\right)^3 + \frac{2}{3}\left(\frac{y}{b}\right)^4\right], \quad \text{(C-C-F-SS)}, \tag{2e}$$

$$W_a = \left[A_1\left(\sin\frac{\pi x}{\gamma_1 a}\right)^2 + A_2\left(\sin\frac{\pi x}{\gamma_2 a}\right)^2\right]\left[\left(\frac{y}{b}\right)^2 - 2\left(\frac{y}{b}\right)^3 + \left(\frac{y}{b}\right)^4\right], \quad \text{(C-C-F-C)}, \tag{2f}$$

where $\gamma_1 (> 1)$ and γ_2 are Rayleigh's optimization parameters. Each one of equations (2) satisfies the essential boundary conditions and substituting equations (2) into equation (1) and applying the well-known Ritz minimization conditions yields, finally, a determinantal equation whose lowest root is the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h_0/D_{10}}\,\omega_1 a^2$ where $D_{10} = E_1 h_0/12(1-\mu_1\mu_2)$. Since

$$\Omega_1 = \Omega_1(\gamma_1, \gamma_2), \tag{3}$$

by requiring

$$\frac{\partial \Omega_1}{\partial \gamma_1} = \frac{\partial \Omega_1}{\partial \gamma_2} = 0, \tag{4}$$

one is able to optimize the fundamental eigenvalue.

3. FINITE ELEMENT SOLUTION

An independent finite element solution has been obtained using the algorithm developed in reference [5]. The finite element mesh used is a function of $\lambda = b/a$ and the resulting number of equations is indicated in Tables 1 and 2.

Each plate element is considered to be of uniform thickness equal to the average thickness of the corresponding plate element.

TABLE 1

Fundamental frequency coefficient of an isotropic SS–SS–F–SS rectangular plate of linearly varying thickness: $h(x) = h_0(1 + \alpha x/a)$

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Number of equations (FEM)	Remarks
2/5	2·6290	2·7490	2·8764	3·0106	3·1505	3·2959	3·4432		(a) Analytical
	2·6276	2·7457	2·8732	3·0081	3·1488	3·2939	3·4425	2·050	(b) FE
				3·008					(c) Reference [2]
2/3	5·1480	5·4642	5·7836	6·1049	6·4328	6·7659	7·1028		(a) Analytical
	5·1487	5·4533	5·7693	6·0938	6·4245	6·7598	7·0985	1·230	(b) FE
				6·0933					(c) Reference [2]
1	9·5917	10·3062	11·0183	11·7195	12·4227	13·1282	13·8350		(a) Analytical
	9·5780	10·2734	10·9766	11·6845	12·3951	13·1071	13·8196	4·040	(b) FE
				11·680					(c) Reference [2]
3/2	19·2156	20·879	22·5305	24·1111	25·6666	37·2043	28·7513		(a) Analytical
	19·1756	20·8019	22·4139	24·0101	25·5903	27·1545	28·7030	1·220	(b) FE
				24·0075					(c) Reference [2]
5/2	49·4152	54·3019	59·0477	63·6015	67·8880	71·9657	75·9977		(a) Analytical
	49·2559	54·0779	58·7584	63·2867	67·6539	71·8538	75·8850	2·020	(b) FE
				63·3125					(c) Reference [2]

TABLE 2

Fundamental frequency coefficient of an orthotropic SS–SS–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Number of equations (FEM)	Remarks
2/5	2·4270	2·5246	2·6295	2·7343	2·8556	2·9758	3·0995		(a) Analytical
	2·4255	2·5211	2·6256	2·7369	2·8535	2·9744	3·0987	2·050	(b) FE
2/3	4·4706	4·7020	4·9387	5·1036	5·4209	5·6697	5·9223		(a) Analytical
	4·4638	4·6874	4·9213	5·1629	5·4102	5·6618	5·9164	1·230	(b) FE
1	7·8083	8·8157	8·8223	9·2976	9·8102	10·3094	10·8109		(a) Analytical
	7·7850	8·2720	8·76718	9·26773	9·7718	10·2782	10·7860	4·040	(b) FE
3/2	14·7753	15·9358	17·0907	18·1714	19·2665	20·3467	21·4221		(a) Analytical
	14·7027	15·8194	16·9325	18·0408	19·1435	20·2405	21·3316	1·220	(b) FE
5/2	36·3602	39·7178	43·0434	46·1603	49·1789	52·1301	55·0207		(a) Analytical
	36·1056	39·3744	42·5750	45·7088	48·7773	51·7826	54·7269	2·020	(b) FE

4. NUMERICAL RESULTS

It was considered of basic interest to also obtain frequency values for the isotropic plate configuration. The corresponding Poisson's ratio was taken equal to 0·30. Accordingly in

TABLE 3

Fundamental frequency coefficient of an isotropic C–SS–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0{\cdot}3$	$\alpha = -0{\cdot}2$	$\alpha = -0{\cdot}1$	$\alpha = 0$	$\alpha = 0{\cdot}1$	$\alpha = 0{\cdot}2$	$\alpha = 0{\cdot}3$	Remarks
2/5	4·6415	4·7178	4·8088	4·9119	5·0251	5·1471	5·2769	(a) Analytical
	4·6374	4·7124	4·8010	4·9004	5·0088	5·1246	5·2466	(b) FE
2/3	6·5361	6·8435	7·1658	7·5011	7·8436	8·1950	8·5532	(a) Analytical
	6·5283	6·8321	7·1493	7·4766	7·8116	8·1524	8·4976	(b) FE
1	10·4768	11·2189	11·9711	12·7300	13·4954	14·2644	15·0370	(a) Analytical
	10·4588	11·1948	11·9388	12·6873	13·4385	14·1907	14·94305	(b) FE
3/2	19·6745	21·3798	23·0774	24·773	26·4628	28·1428	29·8205	(a) Analytical
	19·6375	21·3338	23·0199	24·6943	26·3561	28·0050	29·6411	(b) FE
5/2	49·5508	54·4455	59·2258	63·897	68·4723	72·9660	77·392	(a) Analytical
	49·3721	54·2579	59·0287	63·6799	68·2086	72·6123	76·8912	(b) FE

TABLE 4

Fundamental frequency coefficient of an orthotropic C–SS–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0{\cdot}3$	$\alpha = -0{\cdot}2$	$\alpha = -0{\cdot}1$	$\alpha = 0$	$\alpha = 0{\cdot}1$	$\alpha = 0{\cdot}2$	$\alpha = 0{\cdot}3$	Remarks
2/5	4·5242	4·5814	4·6523	4·7343	4·8260	4·9260	5·0333	(a) Analytical
	4·5201	4·5759	4·6444	4·7230	4·8098	4·9035	5·0029	(b) FE
2/3	6·0146	6·2435	6·4860	6·7391	7·0012	7·2708	7·5455	(a) Analytical
	6·0061	6·2314	6·4685	6·7146	6·9674	7·2253	7·4871	(b) FE
1	8·9018	9·4351	9·9799	10·5349	11·0971	11·6682	12·2051	(a) Analytical
	8·8799	9·40585	9·9391	10·4769	11·0174	11·5592	12·1017	(b) FE
3/2	15·4338	16·6268	17·8204	19·0151	20·3323	21·4100	22·6115	(a) Analytical
	15·3857	16·5672	17·7454	18·9187	20·0864	21·2481	22·4036	(b) FE
5/2	36·5960	39·9596	43·2383	46·5297	49·7522	52·9438	56·1115	(a) Analytical
	36·3708	39·7143	43·0020	46·2357	49·4171	52·5482	55·6312	(b) FE

the mechanical parameters of equation (1), one has

$$\mu_1 = \mu_2 = \mu = 0{\cdot}30, \quad D_2/D_1 = 1, \quad D_k/D_1 = 0{\cdot}35. \qquad \text{(5a–c)}$$

For the orthotropic configurations the following values of orthotropic parameters were chosen:

$$\mu_2 = 0{\cdot}30, \quad D_2/D_1 = 1/2, \quad D_k/D_1 = 1/3. \qquad \text{(6a–c)}$$

Tables 1, 3, 5, 7, 9 and 11 depict values for elastic isotropic plates for the different combinations of edge restraints previously described, while Tables 2, 4, 6, 8, 10 and 12 deal with the orthotropic structural configurations. In all cases the results have been determined for $\lambda = b/a = 2/5$, 2/3, 1, 3/2 and 5/2; $\alpha = -0{\cdot}30$, $-0{\cdot}20$, $-0{\cdot}10$, 0, 0·10, 0·20 and 0·30.

TABLE 5

Fundamental frequency coefficient of an isotropic SS–C–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Remarks
2/5	3·1510	3·3255	3·5075	3·6872	3·8932	4·0935	4·2949	(a) Analytical
	3·1347	3·3073	3·4900	3·6805	3·8770	4·0781	4·2828	(b) FE
2/3	6·7610	7·2466	7·7427	8·2385	8·7408	9·2453	9·7596	(a) Analytical
	6·5283	7·2211	7·7106	8·2080	8·7112	9·2184	9·7283	(b) FE
1	13·4868	14·6564	15·7613	16·8699	17·9767	19·0738	20·1813	(a) Analytical
	13·4438	14·5601	15·6772	16·7924	17·9037	19·0103	20·1114	(b) FE
3/2	28·3419	31·0110	33·6519	36·1575	38·6507	41·0326	43·3860	(a) Analytical
	28·2258	30·8698	33·4614	35·9974	38·4763	40·8978	43·2626	(b) FE
5/2	74·9709	82·9806	90·6909	97·9879	104·6160	110·6260	116·0670	(a) Analytical
	74·6109	82·5429	90·1823	97·4327	104·1946	110·3616	115·9254	(b) FE

TABLE 6

Fundamental frequency coefficient of an orthotropic SS–C–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Remarks
2/5	2·7868	2·9180	3·5075	3·6872	3·8932	4·0935	4·2949	(a) Analytical
	2·7567	3·3073	3·4900	3·6805	3·8770	4·0781	4·2828	(b) FE
2/3	5·5288	5·8744	6·2285	6·5818	6·9416	7·3065	7·6753	(a) Analytical
	5·5011	5·8378	6·1855	6·5410	6·9022	7·2675	7·6358	(b) FE
1	10·4030	11·1955	11·9960	12·7691	13·5457	14·3236	15·1024	(a) Analytical
	10·3519	11·1220	11·8982	12·6774	13·4580	14·2387	15·0188	(b) FE
3/2	21·0194	22·8674	24·7190	26·4700	28·1981	29·9058	31·5956	(a) Analytical
	20·8918	22·6952	24·4784	26·2403	27·9809	29·7005	31·3999	(b) FE
5/2	54·3618	59·8114	65·1242	70·1792	74·9261	79·4661	83·8188	(a) Analytical
	53·9458	59·3059	64·5040	69·5249	74·3543	78·9814	83·4015	(b) FE

TABLE 7

Fundamental frequency coefficient of an isotropic SS–C–F–C rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Remarks
2/5	3·7953	4·0331	4·3024	4·5696	4·8415	5·1173	5·3968	(a) Analytical
	3·7863	4·0321	4·2889	4·5541	4·8256	5·1019	5·3818	(b) FE
2/3	8·8488	9·5632	10·2921	11·0133	11·7462	12·1708	13·7890	(a) Analytical
	8·8224	9·5285	10·2448	10·9675	11·6942	12·4230	13·1527	(b) FE
1	18·4899	20·1743	21·8574	23·5021	25·1150	26·6813	28·2347	(a) Analytical
	18·4101	20·0800	21·7349	23·3716	24·9880	26·5833	28·1574	(b) FE
3/2	39·8158	43·8190	47·7837	51·4350	55·1105	58·4590	61·6176	(a) Analytical
	39·6128	43·6177	47·5040	51·2499	54·8558	58·2486	61·4845	(b) FE
5/2	106·6482	118·7780	130·4600	141·3460	150·5822	158·2631	164·9081	(a) Analytical
	106·0593	118·0783	129·6664	140·5094	149·9869	157·6712	164·0340	(b) FE

TABLE 8

Fundamental frequency coefficient of an orthotropic SS–C–F–C rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Remarks
2/5	3·1873	3·3657	3·5540	3·7481	3·9487	4·1545	4·3642	(a) Analytical
	3·1677	3·3440	3·5303	3·7242	3·9239	4·1280	4·3356	(b) FE
2/3	6·8786	7·3789	7·8925	8·4020	8·9185	9·4397	9·9645	(a) Analytical
	6·8501	7·3390	7·8390	8·3467	8·8599	9·3768	9·8963	(b) FE
1	13·7730	14·9403	16·1249	17·2643	18·4026	19·5372	20·6668	(a) Analytical
	13·6960	14·8404	15·9848	17·1363	18·2632	19·3946	20·5200	(b) FE
3/2	29·0098	31·7657	34·4903	37·1034	39·6413	42·1114	44·5280	(a) Analytical
	28·8060	31·220	34·1827	36·7854	39·3284	41·8112	44·2344	(b) FE
5/2	76·8420	85·1087	93·0695	100·6000	107·4090	113·6220	119·3410	(a) Analytical
	76·2296	84·3839	92·2474	99·7247	106·6934	113·0457	118·7584	(b) FE

TABLE 9

Fundamental frequency coefficient of an isotropic C–C–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0.3$	$\alpha = -0.2$	$\alpha = -0.1$	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	Remarks
2/5	4·9632	5·0917	5·2365	5·3948	5·5644	5·7430	5·9312	(a) Analytical
	4·9431	5·0691	5·2104	5·2104	5·5275	5·69993	5·8779	(b) FE
2/3	7·8496	8·3397	8·8454	9·3631	9·8901	10·4246	10·9651	(a) Analytical
	7·8252	8·3091	8·8081	9·3168	9·8328	10·3541	10·8792	(b) FE
1	14·0669	15·2470	16·4307	17·6232	18·8137	20·0044	21·1948	(a) Analytical
	14·0214	15·1913	16·3645	17·5375	18·7083	19·8757	21·0387	(b) FE
3/2	28·5528	31·2860	33·9853	36·6535	39·2935	41·9035	44·4850	(a) Analytical
	28·4455	30·1611	33·8397	36·4792	39·0780	41·6370	44·1555	(b) FE
5/2	74·9805	83·0373	90·7598	98·1784	105·2850	112·0866	118·5386	(a) Analytical
	74·6331	82·5961	90·3045	97·7019	104·7254	111·3208	117·4654	(b) FE

Table 1 contains results available in reference [2] for the isotropic plate and $\alpha = 0$. The agreement with the finite element results is excellent and the differences with the analytically determined eigenvalues are less than 1%.

In general, the difference between analytical and finite element results is always less than 1% for all values of λ and α, with the exception of Tables 7 and 8 where the maximum disagreements occur for $\alpha = 0{\cdot}30$ and are of the order 1%.

It should be pointed out that the optimization procedure with respect of γ_1 and γ_2, essentially a non-linear optimization procedure, has been performed numerically.

One may state that the optimized Rayleigh–Ritz method coupled with the use of a pseudo-Fourier expansion yields good accuracy for a rather complex eigenvalue problem.

TABLE 10

Fundamental frequency coefficient of an orthotropic C–C–F–SS rectangular plate of linearly varying thickness

λ	$\alpha = -0{\cdot}3$	$\alpha = -0{\cdot}2$	$\alpha = -0{\cdot}1$	$\alpha = 0$	$\alpha = 0{\cdot}1$	$\alpha = 0{\cdot}2$	$\alpha = 0{\cdot}3$	Remarks
2/5	4·7391	4·8292	4·9341	5·0510	5·1788	5·3152	5·4593	(a) Analytical
	4·7091	4·7961	4·8968	5·0086	5·1293	5·2575	5·3918	(b) FE
2/3	6·8420	7·1891	7·5472	7·9170	8·2996	8·6878	9·0824	(a) Analytical
	6·8061	7·1416	7·4980	7·8600	8·2289	8·2289	8·9815	(b) FE
1	11·2191	12·0453	12·8803	13·6620	14·5672	15·4163	16·2679	(a) Analytical
	11·1641	11·9788	12·7996	13·6235	14·4486	15·2734	16·0973	(b) FE
3/2	21·4187	23·3119	25·1963	27·0160	28·9366	30·7943	32·6434	(a) Analytical
	21·3015	23·1756	25·0349	26·8781	28·7048	30·5148	32·3087	(b) FE
5/2	54·9270	59·8114	65·2392	70·4168	75·4728	80·4245	85·2718	(a) Analytical
	54·0398	59·4573	64·7406	69·8828	74·8777	79·7211	83·4015	(b) FE

TABLE 11

Fundamental frequency coefficient of an isotropic C–C–F–C rectangular plate of linearly varying thickness

λ	$\alpha = -0{\cdot}3$	$\alpha = -0{\cdot}2$	$\alpha = -0{\cdot}1$	$\alpha = 0$	$\alpha = 0{\cdot}1$	$\alpha = 0{\cdot}2$	$\alpha = 0{\cdot}3$	Remarks
2/5	5·3840	5·5849	5·8045	6·0387	6·2858	6·5430	6·8090	(a) Analytical
	5·3685	5·5662	5·7812	6·0100	6·2499	6·4988	6·7551	(b) FE
2/3	9·6526	10·3874	11·1365	11·8965	12·6657	13·4381	14·2163	(a) Analytical
	9·6220	10·3476	11·0863	11·8339	12·5874	13·3446	14·1040	(b) FE
1	18·8199	20·5712	22·3190	24·0588	25·8006	27·5353	29·2687	(a) Analytical
	18·7441	20·4780	22·2051	23·9718	25·6259	27·3162	28·9920	(b) FE
3/2	39·8868	43·9754	47·9765	51·8876	55·7144	59·4668	63·1562	(a) Analytical
	39·6872	43·7467	47·7188	51·5926	54·8558	59·0122	62·5475	(b) FE
5/2	106·656	118·774	130·488	141·477	151·592	160·568	169·203	(a) Analytical
	106·061	118·086	129·706	140·691	150·655	159·273	166·687	(b) FE

TABLE 12

Fundamental frequency coefficient of an orthotropic C–C–F–C rectangular plate of linearly varying thickness

λ	$\alpha = -0{\cdot}3$	$\alpha = -0{\cdot}2$	$\alpha = -0{\cdot}1$	$\alpha = 0$	$\alpha = 0{\cdot}1$	$\alpha = 0{\cdot}2$	$\alpha = 0{\cdot}3$	Remarks
2/5	4·9822	5·1149	5·2642	5·4271	5·6013	5·7853	5·9778	(a) Analytical
	4·9569	5·0862	5·2307	5·3874	5·5541	5·7289	5·9105	(b) FE
2/3	7·9438	8·4486	8·9696	9·5025	10·0447	10·5939	11·1496	(a) Analytical
	7·9061	8·4021	8·9118	9·4314	9·9581	10·4898	11·0253	(b) FE
1	14·3307	15·5470	16·7688	17·9930	19·2175	20·4489	21·6671	(a) Analytical
	14·2511	15·4489	16·6493	17·8490	19·0460	20·2390	21·4273	(b) FE
3/2	29·2086	32·0244	34·8032	37·5480	40·2647	42·9463	45·6003	(a) Analytical
	29·0121	31·7964	34·5420	37·2465	39·9088	42·5287	45·1064	(b) FE
5/2	76·8500	85·1731	93·1279	100·773	108·065	115·043	121·656	(a) Analytical
	76·2491	84·4313	92·3583	99·9705	107·199	113·981	120·286	(b) FE

ACKNOWLEDGMENTS

The present study has been sponsored by CONICET Research and Development Program, by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur and by FONCyT.

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