Optimal Shutdown Policy for Maintenance of Cracking Furnaces in Ethylene Plants

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This work addresses the cyclic scheduling of cracking furnace shutdowns in ethylene plants within a short-term production planning model, based on a discrete time representation. Cracking furnaces are continuous parallel reactors that show decaying performance during their operation due to coke deposition on coil walls. For that reason, they must be periodically shutdown and cleaned. This behavior is modeled through binary variables and coil internal roughness, a variable whose increase has a linear dependence on operation time. After cleanup, roughness is at its lowest value and starts increasing again during operation. The cyclic scheduling model includes not only furnaces models but an entire plant mathematical model at each time interval to carry out production planning for meeting varying demands, as well as to determine main plant operating variable profiles and to predict an ethane recycle stream, which is an important feed to cracking furnaces and constitutes a key variable for the optimal shutdown schedule. The model includes nonlinear mathematical functions for each cracking furnace production as a function of main process variables, simplified models for distillation columns in the separation train, and raw material and product storage equations. Additional binary variables are included to force null values for production in shutdown furnaces. The resulting mixed-integer nonlinear programming (MINLP) model is solved in GAMS with DICOPT++.

1. Introduction

Planning and scheduling of continuous processes have been increasingly addressed through mathematical programming techniques. Since earlier work by Sahinidis et al.[1] on the optimization of a long-range planning model through the formulation of a multiperiod MILP (mixed-integer linear programming) model, several authors addressed the capacity-planning problem in process industries. They include successive extensions and improvement of solution techniques by Sahinidis and Grossmann[2], Liu and Sahinidis,[3] and Iyer and Grossmann.[4] The last two papers include multiple scenarios to account for demands and price uncertainty and different solution strategies. A few papers have been presented that include uncertainty in the continuous process-planning model through the application of two-stage and multistage stochastic approaches.[5−7] Regarding short-term planning for large-scale processes, Bok et al.[8] propose an MILP and a bilevel decomposition strategy to efficiently solve it. Jackson and Grossmann[9] propose Lagrangean decomposition (spatial and temporal) techniques for the solution of a nonlinear programming (NLP) problem that models multisite planning of production, transportation, and sales in a chemical company. Neiro and Pinto[10] formulate a planning model of a refinery complex as an MINLP (mixed-integer nonlinear programming) problem, including process unit models. Schulz et al.[11] propose an MINLP model for short-term planning of a petrochemical complex, considering nonlinear production models that include process operating conditions, inventories, and demand satisfaction. They solve the problem with available solvers and show the advantages of including detailed process models.

Regarding the scheduling of entire ethylene plants, Tjoa et al.[12] propose an MINLP model that takes into account different naphtha feedstocks, plant operating conditions, production inventories, and demands, where the main complexity arises in the feedstock management system, including vessels arrivals, storage tanks, and blending.

However, an important problem in ethylene plants, not addressed in the preceding papers, is the scheduling of furnace shutdown periods. Cracking furnaces typically operate in parallel and exhibit decaying performance due to coke deposition inside coils. This decay results in negative effects on ethylene yield. Consequently, they have to be periodically shutdown for cleanup. The problem of cyclic scheduling of continuous parallel units with decaying performance has been addressed by Jain and Grossmann,[13] focusing on the particular case of cracking furnaces in naphtha-fed ethylene plants. The process model, included within the objective function, is represented as one of exponential decay in performance. Multiple feeds and unlimited storage are considered, and inventory costs are neglected. In this way, these authors formulate a special kind of MINLP, with continuous time representation, and propose an NLP-based branch-and-bound method that allows the solution to global optimality. On the basis of this model, Schulz et al.[14] propose an extension for ethane-fed ethylene plants, which have an ethane recycle as part of the furnace feed. They include the estimation of the recycle mean value through the addition of a nonlinear plant model. More recently, the problem of processes with decaying performance has been addressed by Houze et al.[15] They formulate a multiperiod model for optimal catalyst replacement, incorporating an empirical correlation to model the deactivation of a catalyst over time.

In this work, the optimal scheduling of cracking furnace shutdowns integrated to short-term production planning is addressed through the formulation of a multiperiod MINLP model, with a discrete time representation. The entire plant mathematical model is included at each time interval to predict an ethane recycle stream, which is an important feed to the cracking furnaces. The recycle stream flow rate significantly influences the optimal shutdown schedule.The model solution also provides main optimization and operating variable profiles. The decay in furnace performance along operating time has been modeled through binary variables and coil roughness, an empirical variable whose dependence on time has been determined through rigorous furnace simulations. To reflect the cyclic nature of the furnace-shutdown scheduling within a fixed-length

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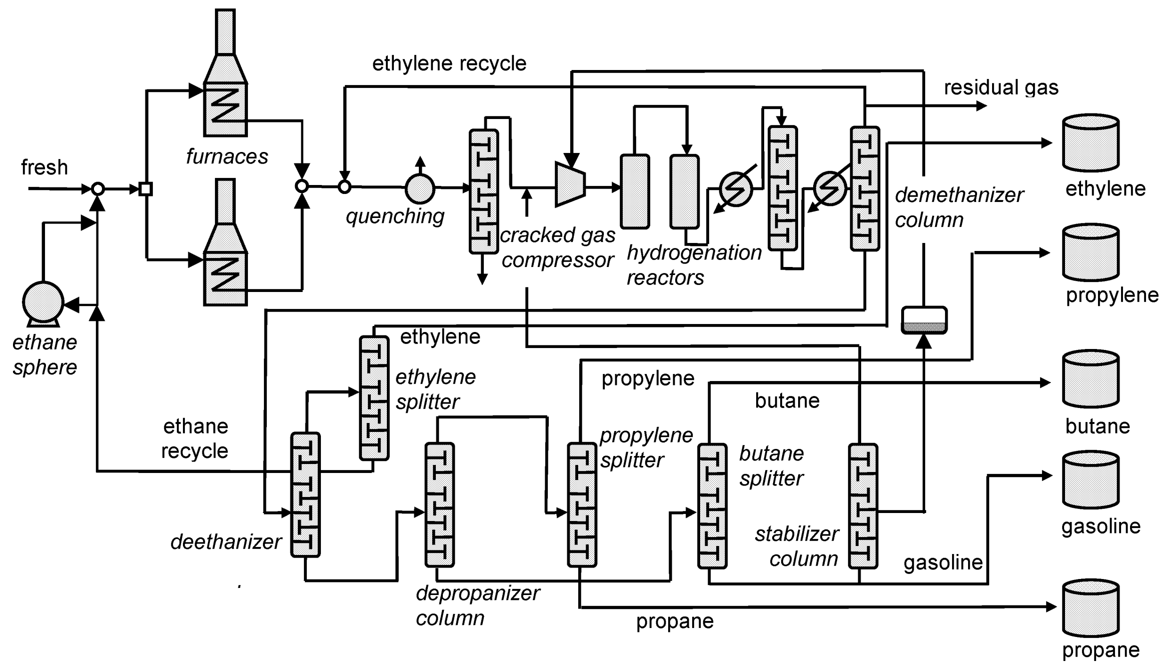

Figure 1. Ethylene plant flowsheet.

horizon, additional conditions have been imposed. At least one complete cycle is allowed for each furnace along the time horizon, enforcing that the maximum attained roughness for each furnace be the same at the cleanup times (not necessarily equal to maximum roughness for the remaining furnaces) and greater than initial and final roughness in the time horizon. To represent a realistic situation, the model considers that the state of the furnaces is different at the beginning of the time horizon, which is represented by a different value of roughness for each one. The model also takes into account storage capacity bounds for both raw material and products, as well as inventory costs and sales for varying product demands throughout the time horizon. Inventory level targets are also imposed for ethane and ethylene tanks. The objective is to maximize profit, calculated as the difference between income (revenue from sales of ethylene, propylene, propane, butane, gasoline, and residue gas) and cost (raw material, natural gas for boilers and furnaces, inventory costs, and cleanup costs for furnaces). The multiperiod MINLP problem has been solved in GAMS[16] using DICOPT++.[17]

2. Ethylene Plant Description and Furnace Maintenance

A typical ethane-fed ethylene plant has several parallel cracking furnaces, cracked gas compressor, heat recovery network, separation system, refrigeration system, and steam plant, as is schematically shown in Figure 1. The plant fresh feed, which has high ethane content, is mixed with two additional ethane streams, one from storage and the ethane recycle stream, and it is then diluted with process steam. Thermal cracking in the furnaces produces ethylene and subproducts. The outlet gas is cooled and compressed from ~0.2 to 30 kg/cm^2 and is further cooled to −100 °C. Compressed gas at cryogenic conditions is fractionated in the separation train, consisting of several columns (demethanizer, deethanizer, depropanizer, and debutanizer) and two additional splitters to further separate ethane from ethylene and propane from propylene. There are two main process recycles in the plant: (a) ethane recycle, which is the bottom stream from the ethane−ethylene splitter and is sent to the cracking furnaces after mixing with the ethane fresh feed, and (b) ethylene recycle, which is recycled from the gasoline stabilization column to the gas compressor inlet.

Coke is produced as a byproduct in the thermal cracking of ethane. It deposits on the coil internal surface, resulting in a decrease of ethylene yield due to both higher heat transfer resistance to the reacting mixture (a strong negative effect as the reaction is highly endothermic) and the decrease of the coil's cross-sectional area. This also results in a decrease in the residence time inside the coils. Constant conversion operation can be achieved by increasing coil temperature with time and, eventually, by reducing the feed flowrate, thus reducing production and increasing utility costs. Increasing coil temperature above a certain limit reduces furnace life. On the other hand, keeping feed flowrate and coil temperature constant, thus decreasing conversion with time, also leads to lower profit and increased formation of byproducts. Consequently, cracking furnaces must be periodically shut down and cleaned, with the associated cost of cleanup and loss of production. There is a tradeoff between operating the furnaces for long periods of time with low cleaning cost and higher cleanup costs with better performance. In the plant under study, both conversion and fresh feed flowrate are kept within narrow bounds, with the consequent increase in fuel gas consumption in each furnace as operation time increases.

A model for the scheduling of furnace shutdown periods and the determination of each furnace load, optimal conversion, and main operating variables, together with entire plant variables, heat loads, and steam requirement, provides a useful tool in plant operation for maximizing profits along a given time horizon.

3. Mathematical Model

The scheduling of parallel cracking furnace shutdowns in an ethylene plant has been formulated as a multiperiod MINLP with discrete time representation. On the basis of a previously developed steady-state rigorous model for the plant,[18] nonlinear correlations have been included for the entire process at each time interval, mainly comprising furnace operation and separation train.

To model the furnaces' decaying performance along the time horizon, their output products are calculated as nonlinear func-

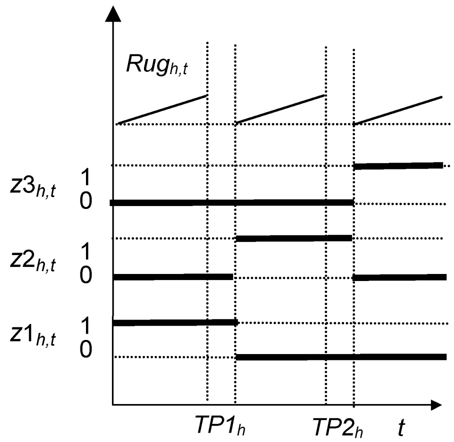

Figure 2. Roughness and binary variables behavior.

Table 1. Roughness Coefficients

furnace	$C1$	$C2$ (1/week)
1	6.420E−04	4.970E−04
2	2.112E−03	4.900E−04
3	3.053E−03	4.821E−04
4	2.570E−03	4.821E−04
5	3.456E−03	4.690E−04
6	3.876E−03	4.620E−04
7	1.097E−03	4.550E−04
8	2.434E−03	4.480E−04

tions of dilution ratio (RD_t), outlet furnace pressure ($\mathrm{Pf}_{h,t}^{\mathrm{out}}$), furnace conversion ($\mathrm{CONV}_{h,t}$), total furnace inlet flowrate ($\mathrm{Ff}_{h,t}^{\mathrm{in}}$), and furnace coil roughness ($\mathrm{Rug}_{h,t}$). The first three are typical optimization variables for this kind of process. Dilution ratio is defined as the process steam-to-hydrocarbon feed ratio; it constitutes an important optimization variable because steam is added to ethane feed to minimize coke deposition on the coil walls.

Coil roughness (Rug) has been defined to account for coke deposition on the internal coil surface; it is an empirical continuous variable during the furnace operation whose value linearly increases as a function of operation time. After the furnace shutdown for cleanup, the coil roughness gets its lowest value and starts increasing again, as is shown in Figure 2. Coil roughness coefficients have been determined with rigorous furnace simulations. The model has been verified against plant data. Parallel furnaces have a different performance, which is associated with their roughness slope. The current results are presented assuming a formulation using eight furnaces, whose coil roughness intercept and slope values are shown in Table 1. The intercept is the same for clean furnaces, but the initial intercept is different for each furnace, as they are at different dirtiness states. Each shutdown period is estimated to be 1 week.

To represent coil roughness cyclic behavior along the time horizon, as well as heat load required by each furnace, two sets of binary variables have been defined: $z_{i,h,t}$ and $y_{m,h,t}$. Variables $z_{i,h,t}$ are defined by eqs 1−14, where $\mathrm{TP}_{k,h}$ is the cleanup period number k for furnace h in the time horizon. Figure 2 shows these variables' behavior as related to roughness. If time period t is before or equal to $\mathrm{TP}_{1,h}$, then $z_{1,h,t}$ is equal to 1; otherwise, it is equal to 0:

$$t \leq \mathrm{TP}_{1,h} + \mathrm{BM1}(1 - z_{1,h,t}) \qquad \forall h,t \tag{1}$$

$$t \geq TP_{1,h} + 1 - \mathrm{BM1}\cdot z_{1,h,t} \qquad \forall h,t \tag{2}$$

For time intervals between the first and the second shutdown period for furnace h, i.e., $(\mathrm{TP}_{1,h} + 1) \leq t \leq \mathrm{TP}_{2,h}$, variable $z_{2,h,t}$ is equal to 1; otherwise, it is equal to 0:

$$t \geq \mathrm{TP}_{1,h} + 1 - \mathrm{BM1}^*(1 - z_{2,h,t}) \qquad \forall h,t \tag{3}$$

$$t \leq \mathrm{TP}_{1,h} + \mathrm{BM1}\cdot a_{h,t} \qquad \forall h,t \tag{4}$$

$$t \geq \mathrm{TP}_{1,h} + 1 - \mathrm{BM1}(1 - a_{h,t}) \qquad \forall h,t \tag{5}$$

$$t \leq \mathrm{TP}_{2,h} + \mathrm{BM1}(1 - z_{2,h,t}) \qquad \forall h,t \tag{6}$$

$$t \leq \mathrm{TP}_{2,h} + \mathrm{BM1}(1 - b_{h,t}) \qquad \forall h,t \tag{7}$$

$$t \geq \mathrm{TP}_{2,h} + 1 - \mathrm{BM1}\cdot b_{h,t}, \qquad \forall h,t \tag{8}$$

where $a_{h,t}$ and $b_{h,t}$ are binary variables that define $z_{2,h,t}$ as their product. It has been modeled as follows. If $a_{h,t} = 1$ and $b_{h,t} = 1$, then $z_{2,h,t} = 1$. Moreover, if $a_{h,t} = 0$ or $b_{h,t} = 0$, then $z_{2,h,t} = 0$. Following a systematic procedure proposed[19,20] to obtain a mathematical representation for logic propositions, these conditions have been respectively written as

$$1 - a_{h,t} - b_{h,t} + z_{2,h,t} \geq 0 \qquad \forall h,t \tag{9}$$

$$a_{h,t} - z_{2,h,t} \geq 0 \qquad \forall h,t \tag{10}$$

$$b_{h,t} - z_{2,h,t} \geq 0 \qquad \forall h,t \tag{11}$$

For time intervals after the last shutdown (it is the second one in the present model, but it can be generalized for more shutdowns), i.e., $t \geq \mathrm{TP}_{2,h} + 1$, variable $z_{3,h,t}$ is equal to 1; otherwise, it is equal to 0:

$$t \leq \mathrm{TP}_{2,h} + \mathrm{BM1}\cdot z_{3,h,t} \qquad \forall h,t \tag{12}$$

$$t \geq \mathrm{TP}_{2,h} + 1 - \mathrm{BM1}(1 - z_{3,h,t}) \qquad \forall h,t \tag{13}$$

At time interval t, each furnace h can be only in one state i (first, second, or third cycle):

$$\sum_{i=1}^{3} z_{i,h,t} = 1 \qquad \forall h,t \tag{14}$$

Coil roughness for each furnace is represented by parallel linear displaced functions in $(\mathrm{TP}_{1,h} + 1)$ and $(\mathrm{TP}_{2,h} + 1)$, as is shown in Figure 2. This behavior is modeled by inequalities 15−20:

$$\mathrm{Rug}_{h,t} \geq C1_h + C2_h\cdot t - \mathrm{BM2}(1 - z_{1,h,t}) \qquad \forall h,t \tag{15}$$

$$\mathrm{Rug}_{h,t} \leq C1_h + C2_h\cdot t + \mathrm{BM2}(1 - z_{1,h,t}) \qquad \forall h,t \tag{16}$$

$$\mathrm{Rug}_{h,t} \geq C1\mathrm{clean}_h + C2_h\lfloor t - (\mathrm{TP}_{1,h} + 1)\rfloor - \mathrm{BM2}(1 - z_{2,h,t}) \qquad \forall h,t \tag{17}$$

$$\mathrm{Rug}_{h,t} \leq C1\mathrm{clean}_h + C2_h\lfloor t - (\mathrm{TP}_{1,h} + 1)\rfloor + \mathrm{BM2}(1 - z_{2,h,t}) \qquad \forall h,t \tag{18}$$

$$\mathrm{Rug}_{h,t} \geq C1\mathrm{clean}_h + C2_h\lfloor t - (\mathrm{TP}_{2,h} + 1)\rfloor - \mathrm{BM2}(1 - z_{3,h,t}) \qquad \forall h,t \tag{19}$$

$$\mathrm{Rug}_{h,t} \leq C1\mathrm{clean}_h + C2_h\lfloor t - (\mathrm{TP}_{2,h} + 1)\rfloor + \mathrm{BM2}(1 - z_{3,h,t}) \qquad \forall h,t \tag{20}$$

Each furnace complete cycle length is determined by imposing that the furnace is always cleaned when its coils reach the same value of roughness, also determined by the model; i.e., $\mathrm{Rug}_{h,\mathrm{TP}_{1,h}} = \mathrm{Rug}_{h,\mathrm{TP}_{2,h}}$, which has been written with a tolerance of 0.000 15 as eqs 21 and 22:

$$C1_h - C1\text{clean}_h + C2_h(1 + 2\text{TP}_{1,h} - \text{TP}_{2,h}) \le 0.000\ 15 \quad \forall h,t \quad (21)$$

$$C1_h - C1\text{clean}_h + C2_h(1 + 2\text{TP}_{1,h} - \text{TP}_{2,h}) \ge -0.000\ 15 \quad \forall h,t \quad (22)$$

At least one complete cycle is required (between $\text{TP}_{1,h}$ and $\text{TP}_{2,h}$), so the following constraints must hold, imposing that coil roughness be lower than its maximum attained value for each furnace both at the beginning and at the end of the time horizon (eqs 23 and 24).

$$\text{Rug}_{h,1} \le C1_h + C2_h{\cdot}\text{TP}_{1,h} \qquad \forall h,t \qquad (23)$$

$$\text{Rug}_{h,16} \le C1_h + C2_h{\cdot}\text{TP}_{1,h} \qquad \forall h,t \qquad (24)$$

The second set of binary variables, $y_{m,h,t}$, is used to define $\text{TP}_{k,h}$ and to make continuous variables associated to each furnace equal to zero during the shutdown period. When furnace h is shut down in time period t, $y_{m,h,t}$ is equal to 1; otherwise, it is equal to 0. Index m corresponds to the number of shutdown (=1,2); if $m = 3$, the corresponding binary variable represents a combination of both $y_{1,h,t}$ and $y_{2,h,t}$, as will be later described.

$$\text{TP}_{k,h} = \sum t y_{m,h,t} \qquad \forall h,t,m = 1, 2 \qquad (25)$$

The model considers scenarios where there must be, at least, one shutdown over the time horizon and the shutdown period is 1 week long for all the furnaces. For operating reasons, at most two furnaces can be simultaneously cleaned at the same time period. This behavior is modeled by eqs 26−29.

$$\sum_t y_{1,h,t} = 1 \qquad \forall h \qquad (26)$$

$$\sum_t y_{2,h,t} \le 1 \qquad \forall h \qquad (27)$$

$$\sum_{k=1}^{2} y_{k,h,t} \le 1 \qquad \forall h,t \qquad (28)$$

$$\sum_h y_{k,h,t} \le 2 \qquad \forall t,k = 1, 2 \qquad (29)$$

Variable $y_{3,h,t}$ is equal to 1 if either $y_{1,h,t}$ or $y_{2,h,t}$ is equal to 1; $y_{3,h,t}$ is equal to 0 if both $y_{1,h,t}$ and $y_{2,h,t}$ are equal to 0:

$$y_{3,h,t} - y_{1,h,t} \ge 0 \qquad \forall h,t \qquad (30)$$

$$y_{3,h,t} - y_{2,h,t} \ge 0 \qquad \forall h,t \qquad (31)$$

$$y_{1,h,t} + y_{2,h,t} - y_{3,h,t} \ge 0 \qquad \forall h,t \qquad (32)$$

The heat load to furnace h, $\text{Qf}_{h,t}$, is written as a linear function of its operation time and feed flowrate, $\text{Ff}_{h,t}^{\text{in}}$. Additionally, it must be zero during the shutdown period. Furnace heat load is modeled by eqs 33−38:

$$\text{Qf}_{h,t} \ge [C3_h + C4_h{\cdot}t](1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} - \text{BM3}(1 - z_{1,h,t}) \quad \forall h,t \quad (33)$$

$$\text{Qf}_{h,t} \le [C3_h + C4_h{\cdot}t](1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} + \text{BM3}(1 - z_{1,h,t}) \quad \forall h,t \quad (34)$$

$$\text{Qf}_{h,t} \ge \{C3_h + C4_h[t - (\text{TP}_{1,h} + 1)]\}(1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} - \text{BM3}(1 - z_{2,h,t}) \quad \forall h,t \quad (35)$$

$$\text{Qf}_{h,t} \le \{C3_h + C4_h[t - (\text{TP}_{1,h} + 1)]\}(1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} + \text{BM3}(1 - z_{2,h,t}) \quad \forall h,t \quad (36)$$

$$\text{Qf}_{h,t} \ge \{C3_h + C4_h[t - (\text{TP}_{2,h} + 1)]\}(1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} - \text{BM3}(1 - z_{3,h,t}) \quad \forall h,t \quad (37)$$

$$\text{Qf}_{h,t} \le \{C3_h + C4_h[t - (\text{TP}_{2,h} + 1)]\}(1 - y_{3,h,t}) + C5_h{\cdot}\text{Ff}_{h,t}^{\text{in}} + \text{BM3}(1 - z_{3,h,t}) \quad \forall h,t \quad (38)$$

Total heat load is calculated as the summation over all the furnaces:

$$\text{Qft}_t = \sum_h \text{Qf}_{h,t} \qquad \forall h,t \qquad (39)$$

The following big-M inequalities ensure that the feed of component j for furnace h ($\text{ff}_{h,j,t}^{\text{in}}$) is zero when it is being cleaned. Analogous equations have been used to model conversion and furnace outlet pressure behavior.

$$\text{ff}_{h,j,t}^{\text{in}} \le M_j(1 - y_{3,h,t}) \qquad \forall h,t \qquad (40)$$

$$\text{ff}_{h,j,t}^{\text{in}} \ge m_j(1 - y_{3,h,t}) \qquad \forall h,t \qquad (41)$$

Equation 42 computes furnace h inlet pressure at time interval t ($\text{Pf}_{h,t}^{\text{in}}$) as a nonlinear function of feed flowrate to the furnace, coil roughness, dilution ratio with steam, demethanizer pressure (or furnace outlet pressure + Δp), and ethane conversion, with the last three being optimization variables.

$$\text{Pf}_{h,t}^{\text{in}} = f_1(\text{Ff}_{h,t}^{\text{in}},\text{Rug}_{h,t},\text{RD}_t,\text{Pf}_{h,t}^{\text{out}},\text{CONV}_{h,t}) \qquad \forall h,t \quad (42)$$

Furnace h production of component j at time interval t ($\text{ff}_{h,j,t}^{\text{out}}$) is calculated by inequalities 43−46, which ensure that it is zero when furnace h is being cleaned.[15] It is a nonlinear function of component and total feed flowrate to the furnace, dilution ratio, furnace inlet pressure, and conversion. The dependence on coil roughness is given through furnace inlet pressure.

$$\text{ff}_{h,j,t}^{\text{out}} \ge f_2(\text{Ff}_{h,t}^{\text{in}},\text{ff}_{h,j,t}^{\text{in}},\text{RD}_t,\text{Pf}_{h,t}^{\text{in}},\text{CONV}_{h,t}) - \text{BM5}_j{\cdot}y_{3,h,t} \quad \forall h,j,t \quad (43)$$

$$\text{ff}_{h,j,t}^{\text{out}} \le f_2(\text{Ff}_{h,t}^{\text{in}},\text{ff}_{h,j,t}^{\text{in}},\text{RD}_t,\text{Pf}_{h,t}^{\text{in}},\text{CONV}_{h,t}) + \text{BM5}_j{\cdot}y_{3,h,t} \quad \forall h,j,t \quad (44)$$

$$\text{ff}_{h,j,t}^{\text{out}} \ge 0 - \text{BM5}_j(1 - y_{3,h,t}) \qquad \forall h,j,t \qquad (45)$$

$$\text{ff}_{h,j,t}^{\text{out}} \le 0 + \text{BM5}_j(1 - y_{3,h,t}) \qquad \forall h,j,t \qquad (46)$$

Downstream from the furnaces, correlations and mass balances are included to describe the separation train. There are no zero flowrates in this section. The product streams of the different units that constitute the separation train are represented by eq 47 as a function of total furnaces production of individual products ($\text{ffout}_{j,t}$), demethanizer pressure (Pdem_t), ethylene-to-ethane ratio at the entrance of the separation train (Rl_t), and each unit's separation factors ($\text{sf}_{u,j,t}$).

$$\text{ff}_{u,j,t}^{\text{st}} = f3(\text{fftot}_{j,t},\text{Pdem}_t,\text{Rl}_t,\text{sf}_{u,j,t}) \qquad \forall u,j,t \qquad (47)$$

The following equations represent the mixer at the entrance of the plant (Figure 3), where ethane recycled from the ethylene

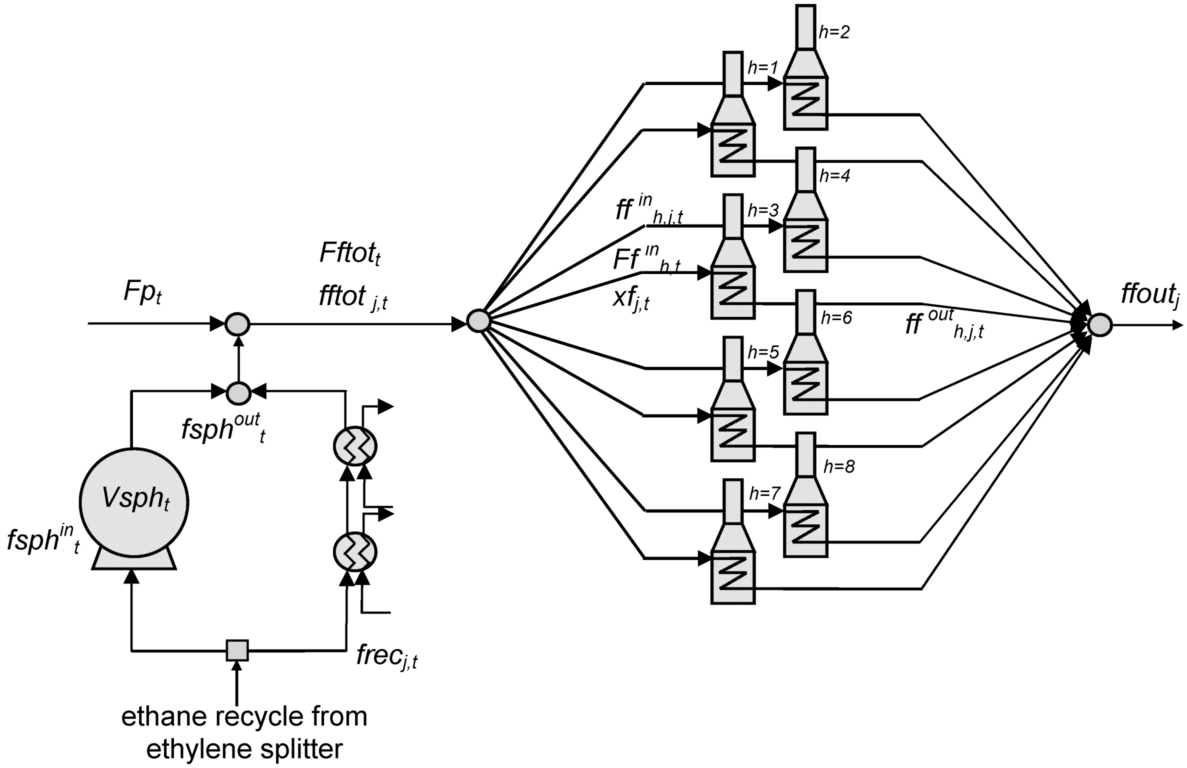

Figure 3. Detail of feed header and furnaces.

Table 2. Fresh Feed Composition

component	mole fraction
CH_4	0.003 820 7
C_2H_6	0.994 139 0
C_3H_8	0.002 040 3

splitter bottoms ($frec_{j,t}$) and from the storage tank ($fsph_t^{out}$) joins fresh feed, which has a given constant composition (xp_j, see Table 2) throughout the time horizon and a variable flowrate (Fp_t). In this way, plant downstream behavior is taken into account in furnace feed.

$$fftot_{j,t} = xp_j \cdot Fp_t + frec_{j,t} + fsph_t^{out} \qquad \forall j,t \qquad (48)$$

As is shown in Figure 3, there is a stream splitter at the entrance of the furnaces and a mixer after them, which have been modeled with eqs 49−54:

$$Ff_{h,t}^{in} = \sum_j ff_{h,j,t}^{in} \qquad \forall h,t \qquad (49)$$

$$Fftot_t = \sum_j fftot_{j,t} \qquad \forall t \qquad (50)$$

$$\sum_j xf_{j,t} = 1 \qquad \forall t \qquad (51)$$

$$fftot_{j,t} = \sum_h ff_{h,j,t}^{in} \qquad \forall j,t \qquad (52)$$

$$ff_{h,j,t}^{in} = xf_{j,t} \cdot Ff_{h,t}^{in} \qquad \forall h,j,t \qquad (53a)$$

$$fftot_{j,t} = xf_{j,t} \cdot Fftot_t \qquad \forall h,j,t \qquad (53b)$$

$$\sum_h ff_{h,j,t}^{out} = fftot_{j,t} \qquad \forall j,t \qquad (54)$$

Equation 55 represents mass balances in ethane storage tanks (raw material) at each time period t, and eqs 56 and 57 calculate an economic penalty when the stored ethane is above or below a security level ITsph. There are upper and lower bounds on the tanks' capacity. Inventory costs affect the inlet stream to the tank to avoid sending ethane to the furnaces through the storage tank instead of directly recycling through the recycle stream.

$$Vsph_t = Vsph_0 + \sum_1^t (fsph_t^{in} - fsph_t^{out}) \qquad \forall t \qquad (55)$$

$$pensph_t \geq ITsph - Vsph_t \qquad \forall t \qquad (56)$$

$$-pensph_t \leq ITsph - Vsph_t \qquad \forall t \qquad (57)$$

Equation 58 represents the molar balance in the final product storage tanks (ethylene, propane, propylene, butane, and gasoline). There are also inequalities analogous to eqs 56 and 57 for ethylene storage; i.e., a target inventory level has been imposed.

$$Vp_{p,t} = Vp_{p,0} + \sum_1^t (fp_{p,t}^{in} - sp_{p,t}^{out}) \qquad \forall p,t \qquad (58)$$

Sales ($sp_{p,t}^{out}$) cannot exceed the demand forecast ($demp_{p,t}$), and the unmet demand ($deltap_{p,t}$) is penalized in the objective function.

$$sp_{p,t}^{out} \leq demp_{p,t} \qquad \forall p,t \qquad (59)$$

$$sp_{p,t}^{out} + deltap_{p,t} = demp_{p,t} \qquad \forall p,t \qquad (60)$$

The objective function is the maximization of net profit, defined as the difference between the sales revenue and the costs (heating, cleaning, inventory, and raw material costs) and the penalties related to the unmet demand and the unmet security level in storage tanks. Table 3 shows raw material costs and product prices.

4. Numerical Results

In the present analysis, a plant with eight parallel cracking furnaces and a time horizon of 16 weeks has been considered.

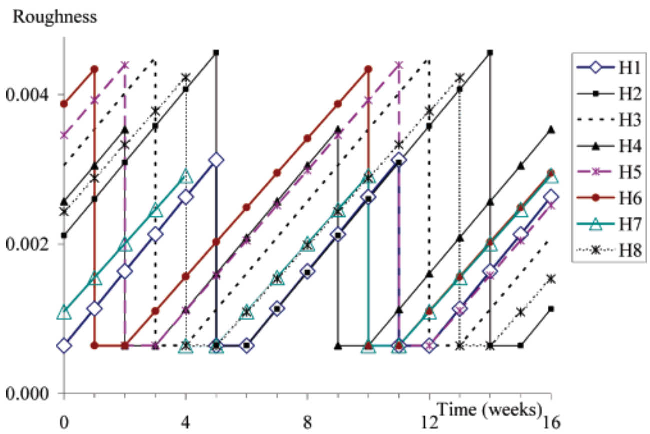

Figure 4. Roughness profiles for the eight parallel furnaces (Case 1).

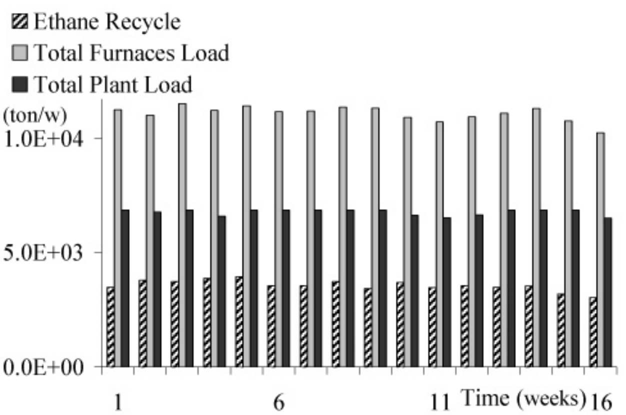

Figure 5. Fresh feed, ethane recycle, and furnaces total charge profiles (Case 1).

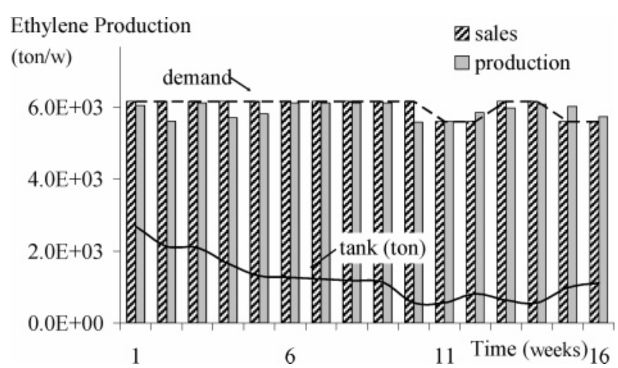

Figure 6. Ethylene sales, production, demand, and storage profiles (Case 1).

The solution of the MINLP model determines the cyclic scheduling of furnace shutdowns and cycle length within a short-term production-planning problem, as well as operational profiles for key process variables on a time-varying product-demand scenario. Two case studies, with different product demands, have been addressed. The problem comprises 12 347 constraints, 5 841 continuous variables, and 1 024 binary variables. The entire model has been formulated in GAMS[16] and solved with DICOPT++,[17] with CONOPT3[21] and CPLEX,[22] in a 3GHz Pentium IV PC, with 512 MB of RAM.

In the first case, an ethylene demand profile has been imposed, with small decreases in periods 11, 12, 15, and 16 (see Figure 6). Figure 4 shows coil roughness behavior for all the furnaces along the scheduling horizon, as well as shutdown periods. Each furnace is at a different part of its cycle at the beginning of the time horizon; for example, furnace H1 is clean, which is represented by setting its roughness at the lowest initial value. Roughness linear coefficients for each furnace are shown in Table 1, where the first column corresponds to the intercept values for the start period ($C1_h$) and the second one presents the slope. The intercept value for just-cleaned furnaces ($C1\mathrm{clean}_h$) is the same for all furnaces and is equal to 6.42×10^{-4}. Optimal values for the shutdown periods for this case are detailed in Table 4, in the second and third columns. It can be seen that the model determines a cycle length, between $\mathrm{TP}_{1,h}$ and $\mathrm{TP}_{2,h}$, that can be considered as representative of each furnace. In the given time horizon, each furnace starts at any part of its cycle, completes one entire cycle, and starts a third cycle up to the end of the time horizon. Two complete cycles have been determined by the model for furnaces H1, H4, and H7, with cycle lengths of 5, 6, and 5 weeks and maximum attained roughnesses of 0.003 13, 0.003 535, and 0.002 917, respectively. The roughness upper bound is stated as 0.01, but the maximum attained value is 0.004 56 in furnace H2, with a length cycle of 8 weeks (Table 4).

Table 3. Raw Material and Product Prices

component	price ($US/kg)
C_2H_4	0.340 00
C_2H_6	0.085 00
C_3H_6	0.248 15
C_3H_8	0.248 15
C_4H_{10}	0.248 15
C_5H_{12}	0.138 51

Table 4. Optimal Shutdown Periods for Cases 1 and 2

furnace	case 1		case 2	
h	$\mathrm{TP}_{1,h}$ (week)	$\mathrm{TP}_{2,h}$ (week)	$\mathrm{TP}_{1,h}$ (week)	$\mathrm{TP}_{2,h}$ (week)
1	5	11	6	13
2	5	14	3	10
3	3	12	3	12
4	2	9	4	13
5	2	11	1	9
6	1	10	1	10
7	4	10	5	12
8	4	13	2	9

Figure 5 shows the plant fresh feed (FP_t) profile, ethane recycle stream, and total charge to furnaces (Fftot_t). The mass balance in the mixer is completed with an ethane stream from storage. It can be seen that fresh feed remains practically constant at its upper bound throughout the entire time horizon; there is a slight decrease during the periods of lower ethylene demand. However, furnace total charge has smooth variations that include the effect of the ethane recycle stream and an additional ethane stream from storage.

Figure 6 shows ethylene production and sales profiles, as well as weekly demands (constant at 6 160 ton/week and decreased to 5 600 ton/week in periods 11, 12, 15, and 16). As expected, the lowest ethylene production occurs in periods 2, 4, 5, 10, and 11, when two furnaces are simultaneously shutdown. However, sales satisfy demands throughout the entire time horizon, and the ethylene tank level remains around its target value of 1 120 tons. Figure 7 shows propane production, sales, and constant demand, which is satisfied at any time period though the time horizon. As inventory costs have been included and no target inventory level has been imposed on this product, storage tank level decreases to lower values to the end of the horizon. Figure 8 shows the main remaining products from the furnaces, i.e, hydrogen, methane, propylene, butane, and gasoline. On the other hand, the ethane storage tank level fluctuates around its target value, as is shown in Figure 9; the dashed line indicates the storage level target and the continuous one indicates the tank content. In this tank, inventory costs are applied not to the tank content but to what is submitted to storage

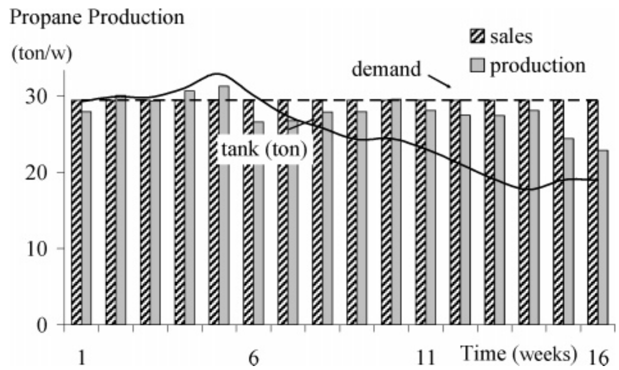

Figure 7. Propane sales, production, demand, and storage profiles (Case 1).

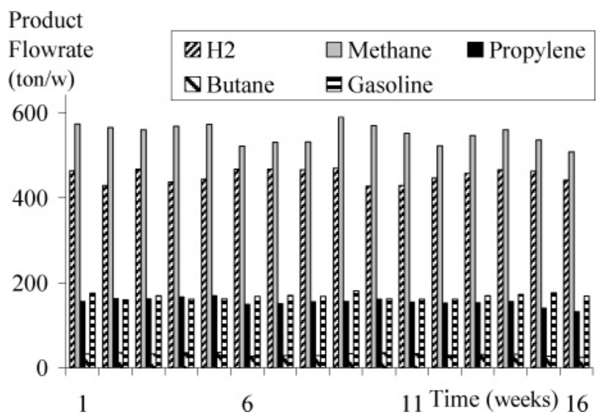

Figure 8. Total furnaces production of hydrogen, methane, propylene, butane, and gasoline (Case 1).

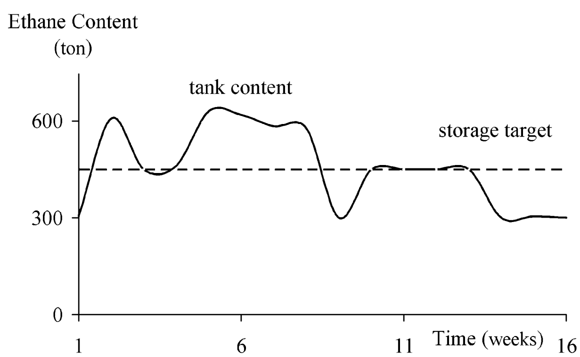

Figure 9. Ethane storage profile and target inventory level (Case 1).

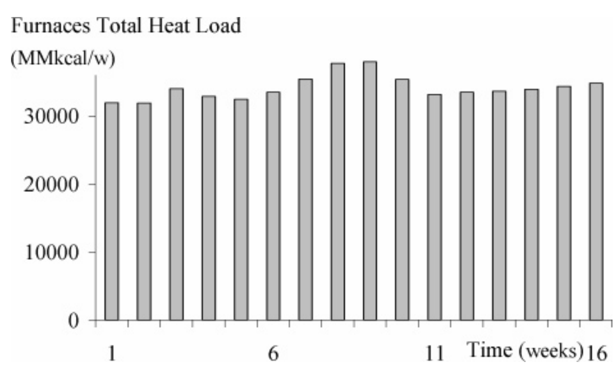

Figure 10. Total heat load to furnaces (Case 1).

from the ethane−ethylene splitter, to avoid recycling ethane to the furnaces through this storage tank.

Heat load profiles are shown in Figures 10 and 11. Total heat load to furnaces (Figure 10), which depends on both each furnace's operation time and the feed flowrate, is fairly constant

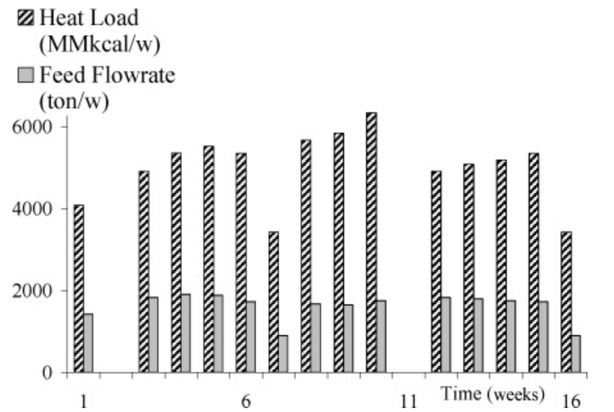

Figure 11. Feed flowrate and heat load requirements for furnace H5 (Case 1).

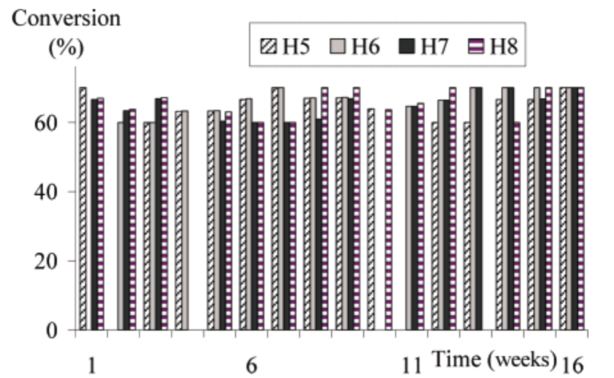

Figure 12. Ethane conversion profiles for furnaces H5−H8 (Case 1).

throughout the entire time horizon, having its maximum value in periods 7, 8, and 9. Periods 7 and 8 correspond to operation with all eight furnaces (none is shut down for cleanup), and roughness, as well as feed flowrate, is high for the furnaces during time period 9 (see Figure 5). As an example of the behavior of heat load requirements to each individual furnace, Figure 11 shows both feed flowrate and heat load to furnace H5. It can be clearly noted that the required heat load to the furnace depends on both feed and operating time (eqs 33−38); for constant charge, heat load increases with time as coil roughness does. Time periods 3−10, between the first and second shutdowns, show this behavior, except at time periods 6 and 7, when inlet flowrate decreases. The same applies to the third cycle, when heat load linearly increases during periods 12−15, but it decreases at the last time period due to an important decrease in feed flowrate to the furnace. It can also be seen that both heat load and feed flowrate are zero during the shutdown periods (2 and 11).

Conversion profiles for furnaces H5−H8 are given in Figure 12; these are optimization variables for the ethylene process, and they remain between narrow bounds throughout the entire time horizon. Lower conversion is required for high charge, and higher conversion is associated to lower charges (see Figure 5, feed flowrate to furnace H5), as is determined in real-time plant process optimization.

The problem has been solved in four major iterations of the MINLP algorithm, requiring 4 472 CPU seconds. The objective function, net profit for the entire horizon of 16 weeks, is 23 686 667.66 $US. The model has also been run for a longer time horizon, 18 weeks, with similar operating results, but with a 1% decrease in the objective function, on a weekly basis (1 464 545 $US/w against 1 480 416 $US/w). For this reason, a time horizon of 16 weeks has been considered as an acceptable one.

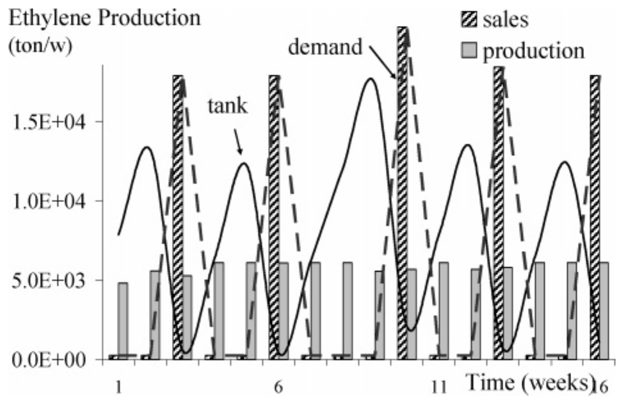

Figure 13. Ethylene sales, production, demand, and storage profiles (Case 2).

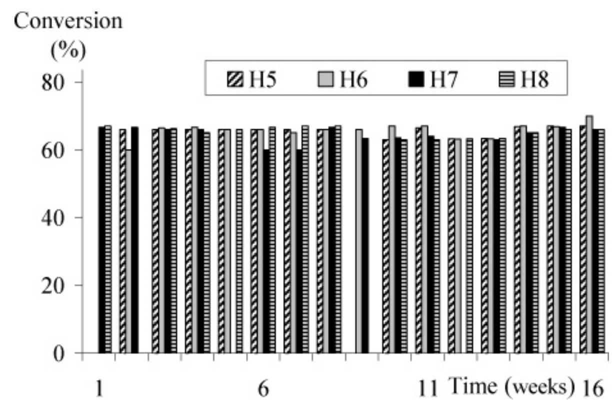

Figure 14. Ethane conversion profiles for furnaces H5−H8 (Case 2).

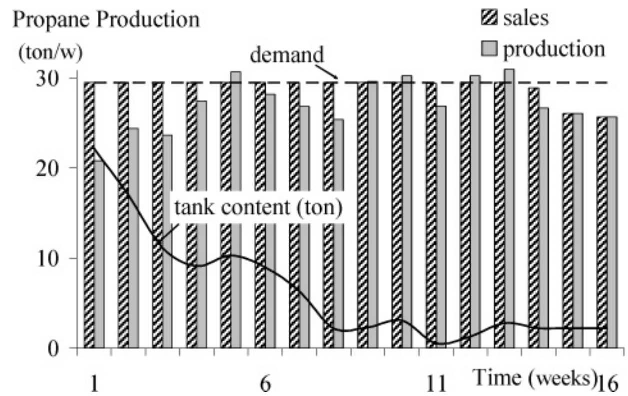

Figure 15. Propane sales, production, demand, and storage profiles (Case 2).

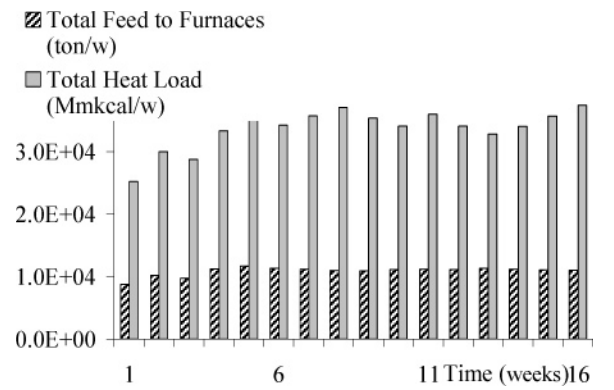

Figure 16. Total heat load and total charge to furnaces (Case 2).

The second case study addressed in this work constitutes a more realistic scenario. This is a situation of scheduled vessel arrivals that is represented in our model by low ethylene weekly demands, 280 ton/week, and sharp increases during time periods corresponding to the ship arrivals and loading. These increases correspond to 17 920 ton/week at time periods 3, 6, 13, and 16, and 21 000 ton/week at the tenth time period. This is the only difference with Case 1, and the overall ethylene demand along the entire time horizon is the same. As can be seen in Figure 13, the imposed high demands are always satisfied (sales equal to the corresponding demand value at each time period), with the consequent oscillating profile in ethylene storage tank level and increased associated inventory costs. While starting from the same initial storage tank level as in Case 1 (2 800 ton), during the first time period ethylene production is completely stored, except for the demanded 280 ton. For this reason, the ethylene tank has 7 800 ton at the end of the first time period. To meet the new demand profile, the model determines different shutdown periods and cycles length for the furnaces, which are shown in the fourth and fifth columns of Table 4. Furnaces are at the same initial state as in Case 1, represented by the same roughness coefficients shown in Table 1. In this case, furnace initial shutdowns are concentrated at the beginning of the time horizon (there are five shutdowns during periods 1−3), as the first high demand at the third week can be partly satisfied with initially stored ethylene. Optimal cycles lengths have been modified to meet the sharp demand profile, rendering the maximum attained roughness value at 0.0045 for furnaces H3 and H4. Conversion profiles have also been changed to satisfy the new ethylene requirements, as is shown in Figure 14 for furnaces H5−H8. Taking into account that higher conversion implies lower selectivity to ethylene formation, one can see an overall decrease in ethane conversion that results in higher selectivity and, consequently, lower production of byproducts (for example, propane and propylene). This fact can be seen in Figure 15, which shows propane production, sales, and demand, with this last being the same as in Case 1. It can be noted that propane demand cannot be satisfied during the last three time periods in this new scenario, even though it was satisfied in Case 1. This is because the total propane production along the time horizon has decreased from 447 ton (Case 1) to 433 ton (Case 2). The same behavior is shown by propylene production, which decreases from 1 890 to 1 716 ton in this last case.

The problem solution, which required three major iterations of the MINLP algorithm, has been obtained in 5 756 CPU seconds. In this case, net profit is 0.6% lower than that corresponding to Case 1, mainly because of higher inventory costs associated to the ethylene tank (>120 000 ton stored along the entire time horizon, against 20 000 ton in Case 1).

In Case 2, the required total heat load varies more significantly along the time horizon, as is shown in Figure 16. It has lower values during the first three time periods, associated to low total charge to furnaces (in the same figure) and concentration of furnace shutdowns during those periods; five furnaces are cleaned up during the first three weeks. Heat load increases up to its maximum value in the middle of the time horizon, when all furnaces are in operation. During the last time periods, it linearly increases, even though feed flowrate is kept constant, due to furnaces coking (note that no more cleanups are performed after period 13).

Two different case studies have been analyzed, but the proposed model is general enough to explore different plant scenarios including varying demands and product prices, raw material availability and costs, tighter limitations on storage, and process units' capacities.

5. Conclusions

An MINLP multiperiod model has been formulated to represent the problem of optimal shutdowns scheduling for furnace maintenance in ethylene plants with ethane recycle, within a short-term production-planning problem, on a discrete time representation. The model is able to realistically capture an important feature of plant operation, which is the decay in furnace performance throughout operating time. This has been modeled by the definition of coil internal roughness, which is an empirical variable whose linear dependence on time has been determined through rigorous simulations. Two sets of binary variables have been introduced to complete the model of decaying performance and select optimal shutdown periods. As compared to the case of catalyst replacement in chemical processes,[15] each process unit is used after cleanup. When starting operation again, the roughness returns to zero and starts increasing with time.

The proposed mathematical model allows for production planning and scheduling of furnace shutdowns, taking into account unit operating variables, main plant optimization variables, time-varying product demands, and limitations on equipment and storage capacity, including nonlinear models for furnace production and further product separation at each time interval. In this way, the model constitutes a powerful tool to increase overall plant net profit. The model has been developed at an academic level. However, cycle lengths are in agreement with historical plant values, which vary between 40 and 60 days in this type of furnace. Operating variables are in good agreement with current ones, and the trend of plant operation is closely represented. Any change in furnace coils could be easily included in the model through the tuning of roughness coefficients and/or time dependence. We conclude that the proposed model is general enough to include current plant data to represent changing situations.

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Note Added after ASAP Publication: The version of this paper that was published on the Web 3/11/2006 had two equations labeled "(53)". The numbering error was corrected, and the corrected version of the paper was published on the Web 3/27/2006.

Nomenclature

Indexes

h = furnaces (1 to 8)

j = components (hydrogen, methane, ethane, ethylene, acetylene, propane, propylene, butane, butylene, and pentane)

p = final products (ethylene, propane, propylene, butane, and gasoline)

t = time period, weeks (1 to 16)

u = units in the separation train (demethanizer, deethanizer, debutanizer, ethane–ethylene splitter, and propane–propylene splitter)

m = number of shutdown if equal to 1 or 2; any shutdown if equal to 3

k = number of shutdown for each furnace (1 to 2)

Parameters

BM1, BM2, BM3, BM4, BM5$_j$ = big-M parameters for conditional constraints

M_j, m_j = big-M parameters that represent upper and lower bounds on component individual flowrates

$C1_h$, $C1\mathrm{clean}_h$, $C2_h$, $C3_h$, $C4_h$, $C5_h$ = parameters for coil roughness and heat correlations

$\mathrm{demp}_{p,t}$ = forecast demand of product p on week t (kmol)

ITsph = inventory target or security level for ethane storage tank (kmol)

$\mathrm{sf}_{u,j,t}$ = separation factor for component j in unit u at time period t

$\mathrm{Vp}_{p,0}$ = initial stock level of product p (kmol)

Vsph_0 = initial stock level of ethane in storage plant (kmol)

xp_j = fresh feed composition

Binary Variables

$y_{m,h,t}$ = 1 when furnace h is being cleaned up; = 0, otherwise

$z_{i,h,t}$ = 1 during ith cycle of furnace h; = 0, otherwise

Continuous Variables

$\mathrm{CONV}_{h,t}$ = ethane conversion in furnace h at period t

$\mathrm{deltap}_{p,t}$ = difference between the demand prediction and the sales of product p in period t (kmol/w)

$\mathrm{Ff}_{h,t}^{\mathrm{in}}$ = total inlet flowrate to furnace h in period t (kmol/w)

$\mathrm{ff}_{h,j,t}^{\mathrm{in}}$ = inlet flowrate of component j to furnace h in period t (kmol/w)

$\mathrm{ff}_{h,j,t}^{\mathrm{out}}$ = outlet flowrate of component j to furnace h in period t (kmol/w)

Fftot_t = total inlet flowrate to furnaces in period t (kmol/w)

$\mathrm{fftot}_{j,t}$ = total inlet flowrate of component j to furnaces in period t (kmol/w)

$\mathrm{frec}_{j,t}$ = molar flowrate of component j in ethane recycle stream in period t (kmol/w)

$\mathrm{ff}_{u,j,t}^{\mathrm{st}}$ = molar flowrate of component j as top product in unit u in the separation train (kmol/w)

$\mathrm{ff}_{u,j,t}^{\mathrm{sb}}$ = molar flowrate of component j as bottom product in unit u in the separation train (kmol/w)

Fp_t = fresh feed flowrate in period t (kmol/w)

$\mathrm{fp}_t^{\mathrm{in}}$ = inlet flowrate of product p to its storage tank in period t (kmol/w)

$\mathrm{fsph}_t^{\mathrm{in}}$ = inlet flowrate of ethane to storage tank in period t (kmol/w)

$\mathrm{fsph}_t^{\mathrm{out}}$ = outlet flowrate of ethane from storage tank in period t (kmol/w)

$\mathrm{Pf}_{h,t}^{\mathrm{in}}$ = furnace h inlet pressure in period t (bar)

$\mathrm{Pf}_{h,t}^{\mathrm{out}}$ = furnace h outlet pressure in period t (bar)

Pdem_t = demethanizer column pressure in period t (bar)

pensph_t = difference between the inventory target and the actual inventory in ethane storage tank in period t (kmol/w)

$\mathrm{Qf}_{h,t}$ = heat load to furnace h in period t (MMkcal/w)

Qtot_t = total heat load to furnaces in period t (MMkcal/w)

RD_t = dilution ratio in period t

Rl_t = ethylene/ethane ratio at the entrance of the separation train

$\mathrm{Rug}_{h,t}$ = internal coil roughness for furnace h in period t

$\mathrm{sp}_t^{\mathrm{out}}$ = sales of product p and outlet flowrate of product p from its storage tank in period t (kmol/w)

$\mathrm{TP}_{k,h}$ = shutdown k of furnace h (week)

$\mathrm{Vp}_{p,t}$ = inventory of product p in period t (kmol)

Vsph_t = ethane inventory level in ethane storage tank in period t (kmol)

$\mathrm{xf}_{j,t}$ = furnaces feed composition

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