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Novel achromatic single reflection quarter-wave retarder: Design and measurement

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In this work, we present an achromatic quarter-wave retarder whose design is based upon the reflection properties of an isotropic-anisotropic interface. In theory, it is possible to obtain a π/2 phase shift by means of a total internal reflection at an isotropic-isotropic interface. However, in order to achieve such a phase shift, it is necessary to use a medium with a particularly high refractive index. We have previously shown that these phase shifts can be achieved by means of a total internal reflection in an isotropic-uniaxial interface, which allows the use of smaller refractive index media. By means of this property, we designed, built, and characterized a novel quarter-wave retarder that makes it possible to obtain circularly polarized light from a linear polarization state. We developed some guidelines that allowed us to obtain a device of competitive performance, low cost, and manageable manufacture.

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I. INTRODUCTION

Nowadays, precise control of polarization is necessary to obtain an optimal performance from innumerable optical components and systems. In order to handle the state of polarization, it is necessary to attain polarized light. Thus, polarization control is achieved by the introduction of phase differences between privileged components of the electric field. This task is usually performed by retarders that can contain either isotropic-anisotropic or isotropic-isotropic interfaces. Although there is a wide range of different kinds of retarders, the retardance or phase difference can be introduced mainly by two distinct methods:

1. One way is by making the waves associated to the components of the electric field travel different optical paths. In this case, the phase difference is introduced progressively as the waves propagate through the media. The classic example are the wave plates. They are the most widely used retarders and basically consist of a plane parallel plate made of a birefringent material. There are zero order wave plates (true or compound) and multiple order wave plates. A true zero order wave plate is made of a single birefringent material and its thickness can be so small as to limit its manipulation. However, this inconvenience can be overcome by the use of low birefringence polymers yielding thicker and more robust wave plates.1 A compound zero order wave plate is constructed combining multiple order rotated wave plates. Due to the mechanism of introducing the phase difference, they can be very sensitive to the wavelength or to the angle of incidence.2–7

2. The other way corresponds to the introduction of the phase shift by means of total reflections in interfaces between different media. The classical example is the Fresnel Rhomb and its different variations (e.g., double rhombs).8–11 Classically, the Fresnel Rhomb is a device that makes it possible to obtain circularly polarized light (CPL) since it introduces a π/2 phase shift between s and p components of the electric field. It consists of a rhomb-type isotropic prism that introduces the desired phase shift by means of two total internal reflections (TIR). This mechanism of introducing the phase shift causes a strong achromatic behavior but is rather sensitive to the angle of incidence.10 Moreover, this kind of retarders can be generally considered as true zero order. It is theoretically possible to obtain a π/2 phase shift by means of a single TIR in an isotropic-isotropic interface, but since a high refractive index (n > 2.4) is needed (e.g., diamond), it is practically unsuitable. Instead, as we showed in a previous work,12 it is possible to obtain such phase shift by means of only one TIR in a non-absorbing isotropic-uniaxial interface13 without the requirement of such elevated refractive indices. Our goal was to design, fabricate, and characterize a novel, simple, inexpensive prototype of quarter-wave retarder based on a single TIR.

We begin by a brief theoretical development in order to find the optimum angle of incidence that permits obtaining CPL, and then we explain the practical characteristics of the prototype. Second, we evaluate experimentally the performance of the device as a λ/4 retarder. We show the experimental results we obtain by means of varying the angle of incidence around the optimum value for two different wavelengths.

II. DESIGN OF A QUARTER-WAVE TIR RETARDER

The device is designed to obtain CPL from linearly polarized light (LPL) by means of a total internal reflection. The interface where the phase jump occurs is composed by
an isotropic medium (internal medium) of refractive index \( n_i \) and a uniaxial medium (external medium) of principal indices \( n_o \) and \( n_e \).

The reflected field \( \vec{E}^* \) can be obtained from the incident field \( \vec{E} \) by means of a four coefficient matrix

\[
\begin{bmatrix}
\vec{E}^* \cdot \hat{s} \\
\vec{E}^* \cdot \hat{p}
\end{bmatrix} =
\begin{bmatrix}
R_{ss} & R_{sp} \\
R_{ps} & R_{pp}
\end{bmatrix}
\begin{bmatrix}
\vec{E} \cdot \hat{s} \\
\vec{E} \cdot \hat{p}
\end{bmatrix}.
\]

(1)

In order to obtain \( s - p \) modes separation, the optical axis should be either contained in the plane of incidence or perpendicular to it. This way, the perpendicular \( s \) and parallel \( p \) modes are separated and both \( R_{sp} \) and \( R_{ps} \) are zero.\(^{12} \)

The choice of the optical axis perpendicular to the plane of incidence is more difficult to fulfill since it may require an arduous grinding on the crystal in order to get the optical axis parallel to the contact face (\( x - cut \)). Instead, if the choice corresponds to the optical axis parallel to the plane of incidence, there is no need to change the bulk direction of the optical axis and it is possible to use one of the crystallographic structural planes (i.e., natural cleavage). This way, if we consider the case of optical axis contained in the plane of incidence, then the optical axis forms an angle, \( \theta \), with the interface (Fig. 1).

If the index of refraction of the isotropic medium is greater than both principal refractive indices, there would be two angles of total reflection. One of them, \( \alpha_o T \), corresponds to the perpendicular mode \( (s) \). It is associated to the extinction of the refracted ordinary waves and can be found as in isotropic media

\[
\sin \alpha_o T = \frac{n_e}{n}.
\]

(2)

The other one, \( \alpha_e T \), corresponds to the parallel mode \( (p) \). It is associated with the extinction of the extraordinary refracted waves and can be determined by means of the generalized Snell’s law\(^{14} \)

\[
\sin \alpha_e T = \left( \frac{n_o^2 \cos^2 \theta + n_e^2 \sin^2 \theta}{n} \right)^{\frac{1}{2}}.
\]

(3)

The reflection coefficient \( R_{ss} \) has the same characteristics as the perpendicular mode reflection coefficient in an isotropic-isotropic interface (with indices \( n \) and \( n_o \)). It may be written as a function of the ordinary total reflection angle

\[
R_{ss} = \frac{\cos \alpha - (\sin^2 \alpha_o T - \sin^2 \alpha)^{\frac{1}{2}}}{\cos \alpha + (\sin^2 \alpha_o T - \sin^2 \alpha)^{\frac{1}{2}}}.
\]

(4)

where \( \alpha \) is the angle of incidence on the isotropic-uniaxial interface (Fig. 1). On the other hand, \( R_{pp} \) is associated to the extraordinary electric fields and does not coincide with the parallel mode reflection coefficient in an isotropic-isotropic interface, since it depends on the value of both principal indices \( (n_o \) and \( n_e) \) and the direction of the optical axis,\(^{15} \)

\[
R_{pp} = \frac{-n_o n_e \cos \alpha - n^2(\sin^2 \alpha_o T - \sin^2 \alpha)^{\frac{1}{2}}}{n_o n_e \cos \alpha + n^2(\sin^2 \alpha_o T - \sin^2 \alpha)^{\frac{1}{2}}}.
\]

(5)

If we consider a negative birefringent material \( (n_o > n_e) \) with its optical axis contained in the plane of incidence, then \( \alpha_o T > \alpha_e T \). It means that, as we increment the angle of incidence, we will first find the occurrence of the total reflection of the parallel mode (extraordinary) and second the total reflection of the perpendicular mode (ordinary). Moreover, for angles of incidence which are equal or higher to \( \alpha_o T \), we get proper TIR, since there are no refracted traveling waves.

The prototype proposed consists of an isotropic glass prism (refractive index \( n \)) in optical contact with a negative uniaxial crystal slab of refractive indices \( n_o \) and \( n_e \) with its optical axis \( z_3 \) at an angle \( \theta \) from the isotropic-uniaxial interface. The plane of incidence contains the optical axis and the surface normals to faces of the prism (Fig. 1).

The electric field at the output of the device \( \vec{E}_{out} \) can be calculated as a function of the incident field \( \vec{E}_{inp} \) (neglecting multiple reflections)

\[
\begin{bmatrix}
\vec{E}_{out} \cdot \hat{s} \\
\vec{E}_{out} \cdot \hat{p}
\end{bmatrix} =
\begin{bmatrix}
T_{ss}R_{ss}T_{s1} & 0 \\
0 & T_{pp}R_{pp}T_{p1}
\end{bmatrix}
\begin{bmatrix}
\vec{E}_{inp} \cdot \hat{s} \\
\vec{E}_{inp} \cdot \hat{p}
\end{bmatrix}.
\]

(6)

where \( T_{s1} \) and \( T_{p1} \), and \( T_{s2} \) and \( T_{p2} \), are the transmission coefficients corresponding to the first and second air-glass and glass-air interfaces.\(^{16,17} \) In order to obtain CPL, the quotient between the output components of the electric field \( \vec{E}_{out} \cdot \hat{p} \) and \( \vec{E}_{out} \cdot \hat{s} \) (Eq. (6)) must hold

\[
\frac{T_{p2}R_{pp}T_{p1}}{T_{s2}R_{ss}T_{s1}} \vec{E}_{inp} \cdot \hat{p} = T_{p2}R_{pp}T_{p1} \frac{1}{\tan \gamma} = \exp \pm j \frac{\pi}{2}.
\]

(7)

where \( \gamma \) is the angle between the plane of polarization of the incoming light beam and the plane of incidence. Thus, CPL is obtained when the phase difference between \( R_{pp} \) and \( R_{ss} \) is \( \pm \pi/2 \) (Eq. (7)). To achieve this, at least one of the modes should undergo total reflection. When both modes reach total reflection, both reflection coefficients become unitary. However, due to the isotropic-isotropic interfaces transmission properties, the amplitudes might need to be compensated by the angle of the polarizer \( \gamma \). For angles of incidence \( \alpha > \alpha_e T \), the coefficient of reflection \( R_{pp} \) is\(^{12} \)

\[
\frac{T_{p2}R_{pp}T_{p1}}{T_{s2}R_{ss}T_{s1}} \vec{E}_{inp} \cdot \hat{p} = T_{p2}R_{pp}T_{p1} \frac{1}{\tan \gamma} = \exp \pm j \frac{\pi}{2}.
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(7)

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\[
\frac{T_{p2}R_{pp}T_{p1}}{T_{s2}R_{ss}T_{s1}} \vec{E}_{inp} \cdot \hat{p} = T_{p2}R_{pp}T_{p1} \frac{1}{\tan \gamma} = \exp \pm j \frac{\pi}{2}.
\]
FIG. 2. Phases $\phi_{pp}$ and $\phi_{ss}$ of the reflection coefficients for a glass-calcite interface (glass refractive index $n_g$, calcite principal refractive indices $n_o = 1.6558$ and $n_e = 1.4852$, and $\theta = 45.5^\circ$). $\phi_{pp}$ (solid red curve), $\phi_{ss}$ (solid blue curve), and $\phi_{pp} - \phi_{ss}$ (dashed green curve). (a) $n = 1.6900$, (b) $n = 1.7900$, (c) $n = 1.8900$.

$$R_{pp} = \exp j\phi_{pp} = \exp \left\{ -2 \arctan \left( \frac{n^2 \sin^2 \alpha - n_e^2 \sin^2 \alpha_T}{n_o n_e \cos \alpha} \right)^{\frac{1}{2}} \right\},$$

and for angles of incidence $\alpha > \alpha_{OT}$, the coefficient of reflection $R_{ss}$ is

$$R_{ss} = \exp j\phi_{ss} = \exp \left\{ -2 \arctan \left( \frac{(\sin^2 \alpha - \sin^2 \alpha_T)^{\frac{1}{2}}}{\cos \alpha} \right) \right\}.$$  

(8)

(9)

Since we are dealing with negative uniaxial crystals, the extraordinary total reflection occurs for angles of incidence that are smaller than the minimum required for ordinary total reflection and $\phi_{pp} - \phi_{ss} \leq 0$. This way, the maximum absolute value of the phase shift between $s$ and $p$ components corresponds to $\alpha = \alpha_{OT}$ (Fig. 2). A conditional relation between the refractive indices and the angle $\theta$ arises from the requirement of existence of at least one angle of incidence that makes it possible to reach a $-\pi/2$ retardance between components $p$ and $s$ of the field. By means of Eqs. (3) and (8), we obtain the following inequation for negative uniaxial crystals:

$$n^4 \sin^2 \theta \left( \frac{n_o^2}{n_e^2} n^2 \right) - n^2 \left( \frac{1}{n_o^2} - \frac{1}{n_e^2} \right) + \frac{n_o^2}{n_e^2} \geq 0.$$  

(10)

As shown in Fig. 2(b), if equality holds, there is only one angle of incidence that allows for a $-\pi/2$ phase shift. For the rest of the cases that satisfy the inequation, there are two angles of incidence that allow for this phase shift (Fig. 2(a)).

By means of Eq. (10), if the refractive indices and the direction of the optical axis of the negative uniaxial crystals remain fixed, it is possible to obtain the conditions that the index of the isotropic medium $n$ should satisfy. The roots of this bi-quadratic,

$$n^2 = n_o^2 n_e \pm \left[ n_o^2 - 4(n_e^2 - n_o^2) \sin^2 \theta \right] \left( \frac{1}{n_o^2} - \frac{1}{n_e^2} \right) \sin^2 \theta,$$

(11)

give two possible values which are obtained under the imposition of real positive roots. Since $n_e^2 - 4(n_e^2 - n_o^2) \sin^2 \theta < n_o^2$, they are

FIG. 3. Refractive indices $n_1$ and $n_2$ associated to the roots of Eq. (10) versus $\theta$. The interface considered is a glass-calcite one where the refractive index of the isotropic medium is $n$. Colored regions indicate the values of $n$, which make it possible to obtain a $\pi/2$ phase shift.

FIG. 4. $\phi_{pp} - \phi_{ss}$ for different values of the refractive index $n$.

FIG. 5. Scheme of the experiment. Top view: (BC) Beam combiner, (P) polarizer for adjusting the input polarization plane, (RS) rotatory stage, (RA) rotatory analyzer, and (PD) photodiode.
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where \( n_1 \) and \( n_2 \) are the two roots that meet the criteria. Finally, for a given negative uniaxial crystal (i.e., for a given set of \( n_o, n_e, \) and \( \theta \)), the properties of the polynomial of Eq. (10) restrict the refractive index of the isotropic medium to \( n < n_1 \) or \( n > n_2 \). In Fig. 3, we present the variation of the refractive index of the isotropic medium \( n \) for the case of calcite as a function of \( \theta \) (Eqs. (10) and (11)).

For a given value of \( \theta \), there are two values of \( n \) (\( n_1 \) and \( n_2 \)) that limit the possibility of reaching a ~\( \pi/2 \) phase shift. For negative uniaxial crystals we obtain two intervals, one corresponding to \( (n_o, n_e, \theta) \) and the other one with a lower bound given by \( n_2 \). In the case of calcite (Fig. 3), the values associated to the second root \( n_2 \) are too high and nonsuitable for every value of \( \theta \). This way, a suitable value of refractive index is confined to \( (n_o, n_1) \).

We decided to use a 4 cm × 6 cm × 1.4 cm calcite slab with natural cleavage (around \( \theta = 45.5^\circ \)) for a central wavelength \( \lambda_c = 632.8 \text{ nm} \) (\( n_o = 1.6558 \) and \( n_e = 1.4852 \)). Thus, according to Eqs. (10) and (12), the maximum value of refractive index that would allow us to obtain a phase shift of \( -\pi/2 \) is \( n = 1.79 \). Figure 4 shows the phase shift as a function of the angle of incidence for three different interfaces composed by an isotropic media (\( n = 1.69, 1.79, \) and 1.89) and the selected crystal slab. While for smaller refractive indices it is possible to reach an absolute value of the phase shift even higher than \( \pi/2 \), the angle of incidence becomes more grazing as \( n \) decreases.

As an isotropic medium, we used an equilateral prism that was available in our laboratory (face: 7 cm, length: 3 cm). We measured the refractive index of the prism for \( \lambda_v = 632.8 \text{ nm} \) by the minimum deviation method. The resulting value of the refractive index, \( n = 1.796 \pm 0.003 \), is slightly higher than the calculated upper bound, and a lower one would have been desirable (\( n < 1.79 \)). However, the available prism and calcite slab allow us to attain a phase retardance which is very close to the quarter wave (about \( -89^\circ \)). In order to properly obtain a \( -\pi/2 \) phase shift with this prism, the calcite contact surface should have been carefully ground and polished till \( \theta = 46.45^\circ \).

Since the phase shift introduced is closest to \( -\pi/2 \) for an angle of incidence of \( \alpha = \alpha_{\text{opt}} = 67.4^\circ \), the incidence at the prism entrance face must be at an angle \( i_1 = 13^\circ \) and the input polarization should be adjusted in order to obtain \( \gamma = 46.1^\circ \) (Eq. (7)). Thus, there is an angle of \( 34^\circ \) between the incident and the emergent beams. However, if working with normal incidence and an isosceles prism, the angle of deviation will correspond to the vertex angle.

### III. Experimental Analysis

The prism and the calcite slab are fixed together over a rotatory stage making sure that light travels around a plane parallel to the optical table and perpendicular to the axis of rotation (Fig. 5). Special care is taken regarding the
parallelism between the plane of incidence, the optical axis, and the surface normals along the different interfaces.

The angle of incidence $i_1$ is controlled by the rotatory stage with a resolution of 0.1°, whereas the angle of polarization $\gamma$ of the incident LPL is controlled by a rotatable linear polarizer. Two different He–Ne lasers were employed: $\lambda_{v1} = 632.8\text{ nm}$, corresponding to the calculations made in Sec. II, and $\lambda_{v2} = 543.5\text{ nm}$. The latter was employed in order to test the behavior of the device at an additional wavelength. After reflection, the outgoing beam passes through a rotating analyzer (mounted on a rotator) and finally reaches a photodetector PD (photodiode, amplifier, and DAQ).

Figure 6 shows the measurements performed for angles of incidence $i_1 = 12^\circ$, $13^\circ$, and $14^\circ$. If the light that reaches the analyzer is circularly polarized, the intensity detected by PD should not change while rotating the analyzer. Measurements were performed separately for the two different wavelengths and for three different angles of incidence. It is possible to appreciate both a good agreement between measurements and theoretical predictions, and a rather satisfactory performance of the device as a $\lambda/4$ retarder. No significant differences were found between the results obtained for the two wavelengths employed.

The best performance as a quarter-wave retarder is obtained for an angle of incidence $i_1 = 13^\circ$, where the light is practically circularly polarized. The measured ellipticity was 0.95 for $\lambda_{v1}$ and 0.92 for $\lambda_{v2}$. Ideally, an optimum shape of the isotropic medium should enable us to work with normal incidence on the isotropic-isotropic interfaces (i.e., $i_1 = 0^\circ$). This will allow us to reduce the chromatic dependence due to the prism dispersion in these interfaces. Nevertheless, the behavior of the prototype evidences acceptable achromatism. For both wavelengths, the angle of incidence that allows us to obtain circularly polarized light is the same (Fig. 6(b)). On the other hand, Figs. 6(a) and 6(c) show that the device is rather dependent on the angle of incidence. The ellipticity dramatically decreases when the angle of incidence varies just 1°, reaching values of approximately 0.7 (Fig. 6(a)).

In order to compare our prototype with commercial retarders, we also evaluate the performance of a Fresnel Rhomb (quarter-wave, single prism) and a wave plate (quarter-wave, mica, true zero order) under similar experimental conditions. In Figs. 7 and 8, we present the results obtained with the same experimental configuration (Fig. 5). The Fresnel Rhomb (Fig. 7) shows an acceptable achromatism. However, it is rather sensitive to the angle of incidence. On the other hand, the wave plate (Fig. 8) shows a differential behavior for the two wavelengths considered, whereas it shows high stability against the angle of incidence (angles of incidence up to 10° were evaluated with no significant variations).

IV. CONCLUSIONS

The experimental results obtained are comparable to those obtained with devices such as Fresnel Rhombs and wave plates. Some of these devices show achromaticity (e.g., the Fresnel Rhomb and the prototype presented here) and others show certain independency regarding the angle of incidence (e.g., wave plates). Other aspects might be considered depending on the case of application such as beam displacement. In the case of the Fresnel Rhomb, there is a lateral shift, whereas in the case of wave plates it is negligible. In the case of our prototype, there is a beam deviation between the incident and the emergent beams mainly due to the reflection at the isotropic-uniaxial interface.

A uniaxial crystal is necessary for the construction of both a plate retarder and a single reflection quarter-wave retarder as the one presented here. Whereas the wave plate requires a high grade crystal without volumetric imperfections and two polished plane parallel surfaces, the device presented here does not demand the use of a high grade uniaxial crystal, since there is no transmission of traveling waves through it. The requirements on the calcite slab only apply on its surface quality, accuracy, and homogeneity, whereas its volume features can be kept unattended.

On the other hand, a single Fresnel Rhomb requires an isotropic prism. Since circularly polarized light is produced by means of two TIR, in addition to the general volumetric features, there is also a requirement of parallelism and surface quality regarding four interfaces. The prism required for our device also plays an important role and dispersion effects could modify its achromaticity. In fact, its achromaticity is related to both calcite and prism dispersion. In order to reduce the size of the prototype, a Dove prism could be used instead of a triangular prism. Moreover, different shapes of prisms can be implemented in order to work with normal incidence on the isotropic-isotropic interfaces. This can be achieved either by means of plane, spherically, or cylindrically shaped interfaces.
Some improvements and variations can be inferred by means of the study presented here. For a more detailed study, the dependence of the phase shift with the wavelength should be included.\textsuperscript{13} In addition to uniaxial crystals, there is a wide range of materials that can be used, such as liquid crystals, and electro-optic crystals. These ones can change their birefringence by means of external electric fields and thus provide the ability of wavelength tuning.

The dimensions of the device can be significantly reduced since there is only one TIR. Due to its simplicity, we also believe that this kind of retarder can be miniaturized and integrated as an optoelectronic-VLSI. The study presented states some rules of design that would be useful for the manufacturing process of this novel kind of retarder.

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\begin{thebibliography}{18}
\bibitem{17} M. Born and E. Wolf, \textit{Principles of Optics}, 7th ed. (The Press Syndicate of the University of Cambridge, Cambridge, United Kingdom, 1999).
\end{thebibliography}

\begin{quotation}
In order to achieve optical contact between the prism and the calcite slab, we used an optical liquid with an intermediate refractive index (Methylene iodide: $n_{MI} = 1.74 \pm 0.01$).
\end{quotation}