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Analysis of the nonlinear relationship between commodity prices in the last two decades

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Abstract We propose the construction of a network to study the correlation among price indices of different commodities, by using the *Multifractal Cross-Correlation* method proposed by Podobnik and Stanley. This estimator, based on the method Multifractal Detrended Fluctuation Analysis, is effective for self-similar signals with characteristics such as those we here analyze. We construct different networks for time periods between 1991 and 2012. Each node represents a commodity group and the links are the cross-correlation between nodes. We study the evolution of these networks from January 1991 to April 2012. The results show that after 2007, high connectivity arises between the nodes of the matrix. We conjecture that this is a consequence of the cash flow from equities and real estate markets to the commodity market due to the subprime mortage crisis.

Keywords Econophisycs · Complex networks · Cross-correlation · Commodities · Multifractality

1 Introduction

Commodities are valuable goods produced in large quantities and with a very low level of differentiation or specialization. Broadly speaking, we can say that commodities are equivalent to raw materials, although this is not always true, especially in recent years: for example, aspirin, which is a pharmaceutical product, is so undifferentiated, that today it is considered as a commodity. One can also speak of financial commodities such as 10 years bonds, or

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currencies like the dollar or the euro. One of the characteristics of a commodity is that its price is determined by a market. Well-established physical commodities have actively traded spot and derivative markets. The economies of developing countries are often highly dependent on them. We finally remark that, especially in times of crisis, commodities can be seen as a refuge investment. On the other hand, most of these products are necessary for subsistence and often require storage and transport. For this reason, these products have a slower response to changes in demand. This last trait has consequences on the associated dynamics. Matias et al. Matia et al. (2003) have conjectured that this feature is the main cause of its multifractal behavior because they respond more slowly than common stock shares and their changes usually present a higher degree of correlation.

There are many articles which study cross-correlation matrices of multivariate signals. The idea behind this work is that there is a complex system whose internal structure is manifested in the cross-correlations between their constituents. These interactions cause collective modes and reveal the underlying dynamics (Gopikrishnan et al. (2001)). Taking into account this idea together with the assumed multifractal nature of stock prices series, we here build cross-correlation multifractal matrices, using a procedure proposed by Podobnik and Stanley (2008).

We apply this methodology to the study of the evolution of daily price of 24 commodity considered “classic” , i.e. raw products, organized in six groups, as defined by the Down Jones: industrial metals, precious metals, livestock, grains, softs and energy.

2 Mathematical methods

2.1 The multifractal fluctuation analysis

We adopt the *Multifractal Detrended Fluctuation Analysis* (MFDFA) method, a generalization of the *Detrended Fluctuation Analysis* (DFA) method, that has been proved to be a particularly flexible method, specially to deal with non-stationary series, (Kantelhardt et al. 2002; Serrano and Figliola 2009; LeiteSiqueira et al. 2010).

The MFDFA multifractal spectrum estimation of a one dimensional series $\{x(i), i = 1, \cdots, N\}$, is based on the construction and analysis of the *fluctuation function*, that is defined in terms of the so called *profile* of the series: $Y(k) = \sum_{i=1}^{k}[x(i) - \langle x\rangle]$, where $\langle x\rangle$ is the mean value of the series $\{x(i)\}$. The profile is then cut into $N_s = N/s$ non overlapping segments of equal length s. The detrended time series for segment ν, denoted by $Y_\nu(i)$, is calculated as the difference between the original time series and a polynomial $p_\nu(i)$ that fits the series in the ν-th segment.

$$Y_\nu(i) = Y(i) - p_\nu(i) \tag{1}$$

The *fluctuation function* is defined as:

$$F_s^2(\nu) = \frac{1}{s}\sum_{i=1}^{s}\{Y[(\nu-1)s+i] - p_\nu(i)\}^2. \tag{2}$$

For simplicity we will use a polynomial fit of order 1, so that if we were to use the usual notation our algorithm would strictly be the 1-MFDFA. In this paper we will instead use the simpler notation MFDFA, though we should keep in mind that different degrees in the polynomial would imply different elimination of trends in the data. For each of the N_s segments, the variance of the detrended time series $Y_\nu(i)$ is evaluated by averaging over all the data points i in the ν-th segment. Then, averaging over all segments, it is possible to obtain the q-th fluctuation function:

$$F_q(s) \;=\; \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \left[F_s^2(\nu)\right]^{q/2} \right\}^{1/q} \,, \tag{3}$$

where, in general, the index q can take any real value, and q works as a *mathematical microscope* that amplifies different behaviors of the data series, as we will show. The scaling behavior of the fluctuation function is determined by analyzing log–log plots $F_q(s)$ versus s for each value of q. If the series $x(i)$ is long-range power-law correlated $F_q(s)$ increases, for large values of s, as a power-law:

$$F_q(s) \;\sim\; s^{h(q)} \,. \tag{4}$$

For more details see Kantelhardt et al. (2002).

Following from Eqs. (3) and (4) and assuming that the length N of the series is an integer multiple of the scale s,

$$\sum_{\nu=1}^{N/s} \mid Y(\nu s) \;-\; Y((\nu-1)s) \mid^q \;\sim\; s^{qh(q)-1} \,. \tag{5}$$

Kantelhardt and co-workers argued that this multifractal formalism corresponds to the standard box counting theory and they relate both formalisms, it is obvious that the term $\mid Y(\nu s) - Y((\nu-1)s) \mid$ is identical to the sum of the numbers $x(i)$ whitin each segment ν of size s. This sum is the box probability $p_s(\nu)$ in the standard formalism for normalized series $x(i)$.

The scaling function $\eta(q)$ is usually defined from last equation:

$$\eta(q) \;=\; q\, h(q) \;-\; 1 \tag{6}$$

where q is a real parameter. The Hölder exponent α and the multifractal spectrum $f(\alpha)$ are related to $\eta(q)$ via a Legendre transform, in the case that $\eta(q)$ is concave:

$$\alpha \;=\; \eta'(q) \tag{7}$$

and

$$f(\alpha) \;=\; q\, h \;-\; \eta(q) \,. \tag{8}$$

In this way, the MFDFA can be framed into a multifractal formalism. In multifractal systems, the strength of multifractality can be described by the width of the spectrum $\Delta\alpha$. It is easy to show that: $\alpha_{max} = h(-\infty)$ and $\alpha_{min} = h(+\infty)$. So, to estimate α_{max} and α_{min} we can use the function $h(q)$ with $\mid q \mid >> 1$.

For a stationary series like the fractional Gaussian noise (fGn), the profile is like the fractional Brownian motion (fBm). For theses processes, $0 < h(q=2) < 1$ and $h(q=2)$ is the Hurst exponent, H. In the case of monofractal signals with compact support, $h(q)$ is independent of q, because the scaling behavior of $F_q(s)$ is the same for all segments. Only if regions with large and small fluctuations scale differently in s, the function $h(q)$ will depend significantly on q.

2.2 The detrended cross-correlations analysis

We here follow Podobnik and Stanley (2008) proposal to evaluate the *Cross-Correlation Function*. Let two series have the same length and sampled frequency ($\{s_1(i), i = 1, \cdots, N\}$

and $\{s_2(i), i = 1, \cdots, N\}$). We evaluate a Multi-Fractal Detrended Cross-Correlation (MDCC) function as follows Podobnik and Stanley (2008):

$$f_{MDCC}^2(\nu) = \frac{1}{r}\sum_{i=1}^{r}\{(Y_{1r}[(\nu-1)r+i])(Y_{2r}[(\nu-1)r+i])\} \tag{9}$$

where

$$Y_{1,2}(k) = \sum_{i=1}^{k}[s_{1,2}(i) - < s_{1,2} >]. \tag{10}$$

The MDCC estimator is the q-norm of $f_{MDCC}^2(\nu)$:

$$F_{MDCC}(q,r) = \left\{\frac{1}{2N_r}\sum_{\nu=1}^{2N_r}\left[f_{MDCC}^2(\nu)\right]^{q/2}\right\}^{1/q}. \tag{11}$$

When the series are non-linearly cross correlated, they present a relation like:

$$F_{MDCC}(q,r) \propto r^{h_{MDCC}(q)}. \tag{12}$$

In a similar way to Eq. (4), the exponent $h_{MDCC}(q)$ can be obtained from the slope of the log–log graph of $F_{MDCC}(q,r)$ versus r.

In the case $q = 2$ the cross correlation estimator $h_{MDCC}(q = 2) = h_{DCC}$ is known as Detrended Cross-Correlation. When $i = j$ the fluctuation function $F_{MDCC}(q,r)$ becomes the function $F_s(q)$, $h_{MDCC}(q)$ the standard *generalized Hurst exponent*, $h(q)$, and h_{DCC} the Hurst exponent H.

The notions of persistence and anti-persistence are relevant to analyse the behavior of markets. These are used by market analysts to estimate future behavior and are related to the Hurst exponent: H. When $0.5 < H < 1$ the behavior is persistent: there is a higher probability that a positive trend (rise) will follow a rise and a negative (low), another low. In contrast, if $0 < H < 0.5$, the behavior is anti-persistent: a rise will most likely be followed by a decrease and vice-versa. When $H = 0.5$ future and past are uncorrelated: it is not possible to anticipate any trend.

The concepts of persistence and anti-persistence can also be applied to the case of the cross-correlations between two series: When $0 < h_{DCC} < 0.5$ or $0.5 < h_{DCC} < 1$ it is also true that the past behaviour of the cross-correlation series can be used to estimate the future trend. On the other hand if $h_{CCD} = 0.5$ this is not possible because the past behaviour of one series does not influence the future trend of the other.

3 Data

Daily price data were used in the series of 24 commodities. The data were obtained from the website: www.djindexes.com and cover the period: from January 2, 1991 to April 23, 2012. On the price series $\{x(i), i = 1, \cdots, N\}$ the returns are defined as: $\{r(i) = log\frac{x(i)}{x(i+1)}, i = 1, \cdots, N-1\}$. In this analysis we use the returns of 6 commodities indices defined by Dow Jones as described in Table 1. Each one of them is built as a weighted average of the different goods include in the table. Furthermore, as we wish to study the evolution of these groups, the total series was divided into 8 overlapping periods. Table 2 shows the start and end of each period.

Table 1 Commodities grouped according to the classification given by the site's Dow Jones

G1 Industrial metal	G2 Precious metal	G3 Grain
Aluminum	Gold	Corn
Copper	Silver	Soybeans
Lead	Platinum	Wheat
Nickel		
Tin		
Zinc		
G4 Energy	**G5 Softs**	**G6 Livestock**
Brent Crude	Cocoa	Lean Hogs
Heating Oil	Coffee	Live Cattle
Natural Gas	Cotton	
Unleaded Gas	Sugar	
WTI Crude Oil		

Table 2 The eight periods of 1186 data

Period	1	2	3	4
From	1/2/1991	5/17/1993	9/27/1995	2/11/1998
To	9/26/1995	2/10/1998	6/20/2000	11/7/2002
Period	**5**	**6**	**7**	**8**
From	6/21/2000	11/8/2002	3 /30/2005	8/9/2007
To	3/29/2005	8/8/2007	12/14/2009	4/23/2012

4 Methodology

The aim of this work is to study the evolution of the cross-correlation matrix of the six above defined commodities indices. To this end, cross correlation matrices V^p, $\{p = 1, \cdots, 6\}$ corresponding to each period p were built.

We first calculated the cross correlation estimator $h_{MDCC}(q = 2) = h_{DCC}$ between all pairs i, j of the commodities for each period, and named them $H_{i,j}$. In this way, we obtained eight 6×6 matrices. In the diagonal are the Hurst coefficients of each group $(H(i,i))$.

In principle, we have an array with values in the range between 0 and 1. As we explained in the previous section, if a cross-correlation is persistent, the value of $H(i, j)$ lies between 0.5 and 1, if anti- persistent, between 0 and 0.5 while when the value is equal to 0.5, the two series i and j move independently in time. Then, in order to study the system as an unweighted network, we considered that two nodes g and h are connected if $|H_{g,h} - 0.5| > u$ where we choose the threshold u to be $u = 0.04$. We chose this threshold, experimentally.

With this new matrix, we then studied the evolution of the network M of commodities, in 8 periods. The results are present in the Fig. 1.

5 Results and conclusions

As we can see in Fig. 1, the nodes of the network are in general weekly connected between periods 1 to 6, that correspond from January 2, 1991 to August 8, 2007. In the first period some cross-correlations appears between industrial metals and grains and grains and energy. In the second period, there is correlation between grain and livestock, while softs and precious met-

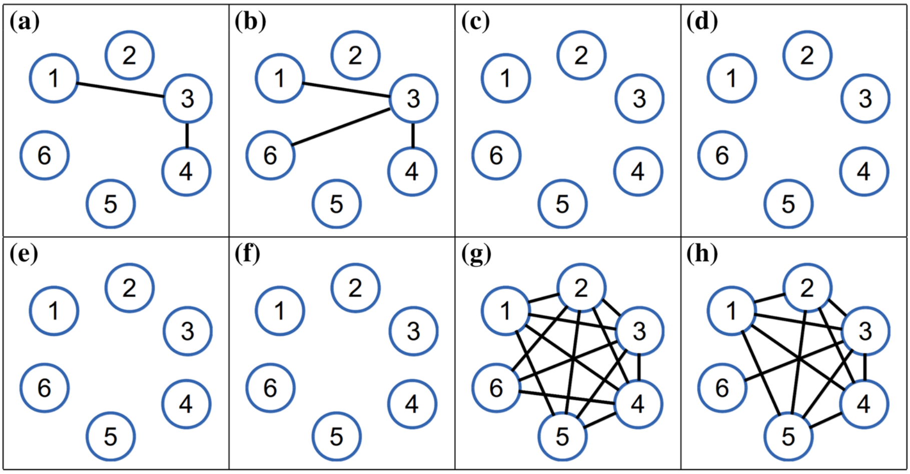

Fig. 1 Network for the six groups of commodities. (**a**) Correspond to period 1, (**b**) period 2, (**c**) period 3, (**d**) period 4, (**e**) period 5, (**f**) period 6, (**g**) period 7 and (**h**) period 8

als are still uncorrelated. These two periods cover the 90's. (it ends in 1997 approximately). The subsequent periods 3 to 6 show total uncorrelation among the six indices. Finally periods 7 and 8 show a significant and remarkable large correlation between all nodes, the maximum being reached reached in the 7 node.

These last two periods coincide with the international mortgage crisis. It is known that at that time money escaped from equities and real estate markets to be invested in other funds, particularly commodities.

These results suggest that the nonlinear correlation that we observed was a consequence of an external shock and not an emergent consequence of a complex dynamic. It is worth remarking that this is similar to what happens in the ARFIMA examples considered in Podobnik and Stanley (2008), two series both of them showing persistent behaviour will have a cross-correlation series with h_{DCC} =.5 if the external driving noise acting in each of them are statistically independent.

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