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TEACHING OF MATHEMATICS AND PHYSICS IN SECONDARY SCHOOL THROUGH RESEARCH AND STUDY PATHS

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ABSTRACT

In this paper we present a proposal for teaching based on the Research and Study Paths (RSP) involving Mathematics and Physics together at the Secondary School. We adopt the ideas of the Anthropological Theory of didactic (ATD). The Research and Study Path (RSP) and the Praxeological Model of Reference (PMR) are described and analyzed. We carried out an implementation with N=68 students of 5th year at the secondary school. Some results obtained in this experience are described.

Keywords: Mathematics and physics teaching, Anthropological theory of didactics (ATD), Study and research paths (RSP), Praxeological model of reference (PMR).

Contribution/ Originality

This study is one of very few studies which have investigated the implementation of Research and Study Paths (RSP) in usual courses of secondary school. This teaching by research requires of the joint study of physical and mathematical notions, in the math classes, and in this sense also this is one contribution.

1. INTRODUCTION

The present work is part of a larger investigation, whose objective is to promote a teaching by research in secondary school. Didactic devices called Research and Study Paths (RSP) are designed, implemented and analyzed, leading to the study of mathematics in a functional way and in conjunction with other disciplines.

The Path is introduced by a question Q_0: *Why did the "Movediza" stone fall down?* In order to answer Q_0 it is necessary to formulate other possible derivative questions, even in a discipline such as Physics. The search for possible answers requires the joint study of pertinent Mathematics and Physics, which is unusual in this institution.

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This study describes what is called Praxeological Model of Reference (PMR) around Q_0 and some derivative questions, carrying out a brief analysis of the possible knowledge in Mathematics and Physics likely to be studied by means of the proposed path and of some partial results.

2. RESEARCH AND STUDY PATHS

The theoretical framework adopted is Yves Chevallard´s Anthropological Theory of Didactics (ATD) (Chevallard, 1999), (Chevallard, 2009), (Chevallard, 2013). ATD proposes a didactic device known as Research and Study Path (RSP). The RSPs consist in the study of a question *Q*, called *generative question*, and the construction of possible answers *R* performed by the students. *Q* should necessarily be a sharp question and generate a wide number of derivative questions which, in turn, when the answer is obtained they will lead to the construction of knowledge in diverse disciplines. Therefore, the study process is determined by the succession of pairs of questions and answers $P=(Q_i;R_i)$ which originate the different potential paths arising from the study of Q_0 (Otero *et al.*, 2013).

The RSP starts from the generative question Q_0: *Why did the Movediza stone in Tandil fall down?* This question is fully connected with the context in which it will be implemented that is the secondary school in Tandil, a city in the Argentine Republic. The *Movediza* or swinging stone was a big 248-tons block of granite which remained in a "curious" oscillating balance for several decades on the top part of a 300-metre-high hill (Peralta *et al.*, 2008). On 12 February, 1912 the stone fell down the cliff becoming worldwide recognized for its singularity and producing an important number of legends and sayings for over a hundred years. Moreover, diverse geological and physical explanations of its fall still remain open. Different hypotheses involving the historical, social and economic contexts, as well as myths and legends are known at present. From the analysis of these hypotheses, numerous questions whose answers contain a wide range of current contents in the secondary school curricula can be generated.

3. PRAXEOLOGICAL MODEL OF REFERENCE

A Praxeological Model of Reference (PMR) (Chevallard, 2013) is developed in order to analyze the specific and didactic knowledge taking part in all the possibilities of the RSPs. It is a tool suggested by the ATD which allows considering exhaustively the feasible ways to perform the study of Q_0, which involves an *a priori* didactic analysis.

In the elaboration of the PMR four hypotheses in connection with the fall were considered: H_1: *the blowing-up by means of explosives on a quarrymen strike*; H_2: *as a consequence of erosion;* H_3: *popular myths and legends;* H_4: *asa consequence of mechanic resonance*. Any of these hypotheses would produce an ample set of questions deriving from Q_0. The proposed work refers to H_4 which is the hypothesis connected to Mathematics and Physics.

Thus, being the research-leading question: W*hich aspects of the oscillating systems are relevant and need to be considered within the study so as to answer* Q_0? A set of questions is therefore generated in relation to the physics and mathematics-associated models which allow the study of a functional mathematics in Secondary Education and beyond it. The oscillating phenomena can be approached by means of the spring and pendulum physical model, to which

different conditions can be set. The related mathematical models are differential equations, functions (trigonometric, exponential and logarithmic) and trigonometry, among others. In *Figure1* a PMR diagram is presented which summarizes all the possible paths and the physical and mathematical notions likely to be studied and re studied.

The oscillating movement is a periodic movement around a stable balanced point. The mechanically balanced points are, in general, those in which the net force which acts on the particle is zero. If this equilibrium is stable, little displacements will bring about the existence of a restoring force, eventually leading the particle towards the balance point once more. In the Simple Harmonic Oscillator (SHM), the recovering force will depend on the position, as it represents an important physical model in Physics.

Depending on the conditions and the energy variation in the system, the oscillating movement can be classified into: *simple harmonic*, *damped and driven*, having different characteristics. In the secondary school Physics curriculum, the oscillating movement is often reduced to the SHM and the spring model, as the ideal pendulum is approached separately, in both cases the Mathematics is left aside, considering it as "conceptual" and advantageous. On the other hand, the oscillating movement is used in Mathematics in order to introduce harmonic functions, which are the answers for differential functions and leaving aside both essential mathematical aspects and the physical model. The attempt to teach science in a functional manner presupposes that none of the models can be avoided, and that it is carried out within a research framework. In the proposed RSP it is necessary to study the different movements to establish a suitable physical and mathematical model.

In case it is not possible to access to the MO of the differential equations so as to approach the SHM, the MO relative to the *trigonometric functions* and the relation of the frequency of movement and the phase with the parameters of these functions will be studied. When the movement is suspended or is not by the internal friction and when energy is released in the system by means of an external force, it is possible to study as well the *exponential functions* which show the decay in the energy of the oscillator. It is also possible to study the *complex numbers* in order to characterize them.

In summary, the complexity of the RSP proposed is evident, given that it allows the study of an important variety of physical and mathematical notions involved in the study of different oscillating models, with different parameters and conditions whose solutions and solving techniques correspond to the same mathematical model.

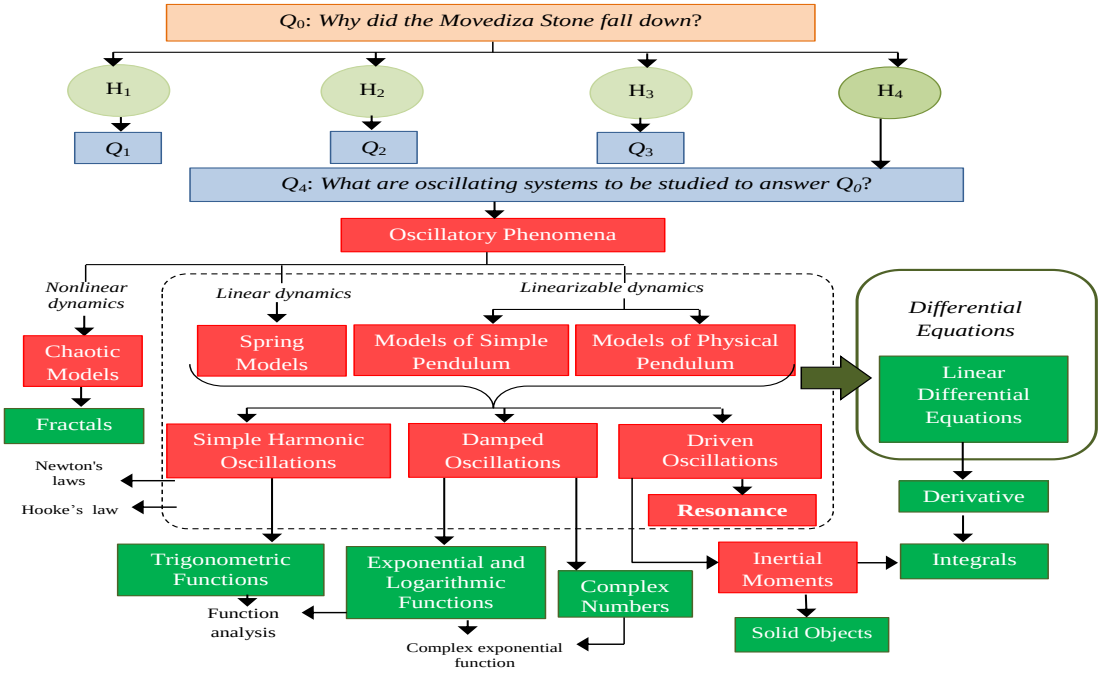

Figure-1. Praxeological Model of Reference (PMR) related to H_4.

4. BRIEF COMMENTS ON THE RESULTS OBTAINED

Two exploratory implementations were performed, in parallel, in 5th year courses of secondary schools in the city of Tandil, Argentina, including in total N=69 students aged 16-17 years old. The oscillating phenomena were studied and mathematically described and characterized, given that by the implementation in the Mathematics lessons of the questions: *How can it be mathematically described? How can it be modelized?*they were present along the whole path. The simple harmonic oscillations, the suspended oscillations and the forced oscillations for the spring and pendulum models were studied and were characterized by means of harmonic trigonometric functions. A partial answer to Q_0 linked to the resonance phenomenon was obtained.

The physical pendulum issue remains unsolved in the secondary school, for which the differential equation will be approached by means of software, attempting the calculation of its inertia moment and the limit oscillation angle at which the fall might have occurred by using the stone dimensions, which is believed to have been 5 degrees according to tradition, though never confirmed.

The results obtained allow highlighting the scopes of the proposed RSP in order to study Mathematics and Physics in a joint and functional way and, in consequence, the didactic potential of the RSP as a tool which produces a substantial change in the teaching of Science in Secondary Education is justified. The RPM analysis allows understanding the scopes of the different organizations which take part in the study programs and the variety of underlying physical and mathematical notions which can be meaningfully studied from the generative question that triggers the whole study. The outcome obtained at present has shown that the teaching by research through

the RSP in ordinary secondary school courses in Argentina has been feasible and positive, although further results are still expected to be produced in future implementations.

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