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Sensitivity analysis of markup equilibria in complementary markets

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ABSTRACT

We study the competitive structure of a market in which firms compete to provide various products within a bundle. Firms adopt price functions proportional to their per-unit costs by selecting markups. We consider two measures reflecting, respectively, the intensity of direct competition and the impact of complementarity on each producer's markup. We characterize the sensitivity of these terms to various changes in the market structure and relate this to changes in producer profits and the social efficiency of the market.

1. Introduction

We consider markets where products are encoded by links in a series–parallel (SP) network. Customers purchase product bundles, given by paths of the network, where parallel links represent substitutes, while series links represent complements. The resulting market conditions are captured by the model of markup equilibrium discussed by Correa et al. [5] (hereafter CFLS). In this paper, we extend their analysis to study the sensitivity of prices, profits and welfare to changes in market conditions and competitive structure. This enables comparisons of competition across a broad set of alternative market configurations in which both complements and substitutes are present.

The CFLS model considers producers that face linear marginal costs and compete to provide all or some portion of the bundle to customers, who in turn choose a set of producers offering the lowest combined price (path in the network). The model relies on a form of supply function equilibria [9], in which producers set prices by choosing a markup to apply to production costs. This is an attractive modeling choice in the bundled setting, as scheduled quantity-dependent price adjustments remove the ambiguity around revenue-splitting that would result in a Cournot-type model of complementary producers. Supply function models yield a structure where bundle-level purchase quantities (i.e., path flows) uniquely determine both the producer-level purchase quantities (link flows) and the market price of each producer's output. In contrast, a pure quantity-commitment model lacks a mechanism for setting individual prices.

Practically, bundling is important to many industries. In freight shipping, point to point routes often involve multiple carriers, each servicing a distinct geography and/or mode of transport. The model also applies to decentralized assembly supply chains, where a manufacturer contracts separately to purchase components from various suppliers. Such outsourcing typically requires a modular product structure that is amenable to an SP representation. Taking the assembler as a monopsonistic buyer, one could employ our model to understand the market around individual component suppliers (e.g., producers of processors, hard disks, and displays in a computer system supply chain). The SP structure provides a general framework to study markets where customers have a need of a set of elements that compose the final product. The study of markets arising from a network structure has drawn attention from the business strategy community [17,3] and in the operations literature surrounding transportation networks [10,12,15,16], telecommunication and computing services [1,4,13] and decentralized assembly supply chains [7,14,8,11].

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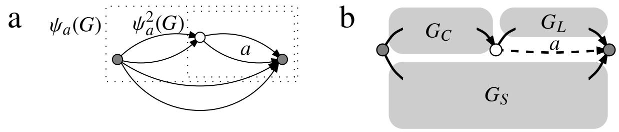


Fig. 1. (a) A series–parallel market with 6 producers. Boxes represent submarkets $\psi_a(G)$ and $\psi_a^2(G)$. (b) Competition for producer a at depth 3: SP networks G_C, G_S, and G_L are the complement, local and substitute markets of a, respectively.

We use the equilibrium characterization provided by [5] to derive a number of important structural insights within this framework. Among our findings are that:

(i) an increase in any producer's cost of production increases the markups of *all competitors* in equilibrium.
(ii) an increase in a producer's own costs *can increase* that producer's equilibrium profits.
(iii) an increase in the costs of production for complementary items *decreases bundle share* for efficient producers, but their less-efficient competitors may actually *gain bundle share*.
(iv) mergers that consolidate market power locally may in fact *improve social efficiency* in the full market for bundles.

At an intuitive level, the relationships we observe depend on whether certain producers interact more as competitors or as complementors. As we will show, both elements are present in most inter-producer relationships.

2. The SP markup equilibrium model of competition

This section provides a quick overview of the markup equilibrium model, as defined in CFLS; we refer the reader to [5] for details. This model is a special form of supply function equilibrium [9] where producers specify price functions by committing to a fixed percentage markup over per-unit production costs. The market is encoded by an SP network $G = (V_G, A_G)$, where each link $a \in A_G$ represents a producer. SP graphs are created by sequentially joining smaller SP graphs in series or in parallel, and capture well the modularity of constructing bundles, which are given by origin-to-destination paths. Indeed, the SP framework accommodates complements, substitutes, and multiple layers thereof. Let $\mathcal{B} := \{B_1 \dots B_m\}$ represent a set of bundles, all equivalent in the eyes of our customers. We say that $a \in B_i$ if producer a contributes to B_i. Producer a may contribute to multiple bundles so that its total production x_a equals $\sum_{B_i \ni a} f_i$ where the vector $f \in \mathbb{R}^m$ describes the allocation of consumption across bundles. A basic example is a computer system, with the option to purchase CPU, keyboard and monitor individually or in integrated bundles. A structure of this type is shown in Fig. 1(a).

Each producer faces a marginally-increasing cost curve and commits to an upward-sloping price function. For the analysis below, we assume that costs are linear and equal to $c_a x_a$ per unit. This leads to the price function $p_a(x_a) = \alpha_a c_a x_a$ per unit for the firm's chosen markup of $\alpha_a \geq 1$. Demand for otherwise-identical products will split into proportions that equalize price among active producers. Complementarity arises from the decentralized production of component products within some demanded bundle of goods.

Notation is needed to describe the recursive structure of SP networks. A *submarket* g refers to an SP subnetwork nested within G. We denote the join of a collection $\mathcal{g}$ of submarkets using the operators $P(\mathcal{g})$ and $S(\mathcal{g})$, respectively. Inversely, the mapping $\psi(g)$ returns the set of submarkets comprising g. A submarket is labeled either *series* or *parallel*, as indicated by the type of join applied last in its construction. We require when g is a series submarket that all elements of $\psi(g)$ be parallel submarkets, and vice versa, so that $\psi(g)$ represents the largest (by cardinality) set of submarkets from which g can be formed in a single composition. For submarkets $g' \subseteq g$, the restricted mapping $\psi_{g'}(g)$ selects the submarket of g that contains g'. We let $\nu_g := (G, \psi_g(G), \psi_g^2(G), \dots, \psi_g^{h_g}(G) = g)$ denote the unique sequence of submarkets starting with G within which g is nested, where h_g is the depth at which g is nested. For example, the sequence of submarkets ν_a in Fig. 1(a) is $(G, \psi_a(G), \psi_a^2(G), a)$. Finally, let $\nu_{g,P} = (g_1, g_2, \dots)$ (alternatively, $\nu_{g,S}$) be the subsequence of odd or even elements of ν_g obtained when restricting to only parallel (series) submarkets. The sequence $\nu_{g,P}$ provides the increasingly specific decisions that a customer must make before purchasing from g. Lastly, given $g \subseteq g'$, we use $g' \setminus g$ to denote the market in g' with producers from g removed and $\vec{\alpha}_{-g}$ to denote the markups vector of producers in $G \setminus g$.

The game has two phases: all producers choose markups simultaneously, followed by the allocation of an inelastic unit demand across the bundles. CFLS show that a unique markup equilibrium $\vec{\alpha}$, a unique production vector $\vec{x}$, and an aggregate bundle price p_G exist if and only if the network G is 3-edge-connected. We henceforth assume that G satisfies this property.

Several results discussed in CFLS will be useful as preliminaries. Fixing a markup vector $\vec{\alpha}$, one can construct a *price multiplier* $R_g(\vec{\alpha})$, used to compute the resulting price p_g as $d_g R_g(\vec{\alpha})$, where d_g is the demand for g. Price multipliers are constructed recursively according to $R_{S(\mathcal{g})}(\vec{\alpha}) = \sum_{g \in \mathcal{g}} R_g(\vec{\alpha})$ for a series market, $R_{P(\mathcal{g})}(\vec{\alpha}) = (\sum_{g \in \mathcal{g}} 1/R_g(\vec{\alpha}))^{-1}$ for a parallel submarket, and $R_a(\alpha_a) = \alpha_a c_a$ for a producer. Furthermore, the network can be pivoted around any submarket g to produce a *substitute network* $G \ominus g$ that encodes the local view of competition from g. When the full market is clear from the context we will omit it for brevity and just write $\ominus g$. The optimal markups for producers in g depend on producers outside g only through the aggregate multiplier $R_{\ominus g}$. Using this, a best-response markup of producer a to its competitors' markups $\vec{\alpha}_{-a}$ is $2 + R_{\ominus a}(\vec{\alpha}_{-a})/c_a$. This formula provides a system of equations that is used by CFLS to characterize equilibria.

While pivot $\ominus a$ redefines the network so that all paths act as substitutes for a, an additional scaling factor is needed to adjust the size of the relevant market to reflect the nature of complementarity introduced by producer a's competitors. The demand of a turns out to be $x_a = \mu_a R_{\ominus a}/(R_{\ominus a} + \alpha_a c_a)$, where the scaling factor is given by $\mu_a := \prod_{g \in \nu_{a,S}} R_{\ominus g}/\big(R_{\ominus g} + R_{g \setminus \psi_a(g)}\big)$.

3. Sensitivity analysis of producer outcomes

We now study the effects of changing market parameters on the outcome experienced by a producer a in equilibrium. The impact of any perturbation manifests itself as a combination of its effects on the multipliers $R_{\ominus a}$ and μ_a. We analyze these effects for a perturbation of a producer's own efficiency parameter, as well as for changes in the structure of its competition. In the latter case, we distinguish between those competitors whose markups decrease μ_a (net complements of a) and those whose markups increase μ_a (net substitutes of a).

3.1. Sensitivity of markups

In this section we formalize the impact of a fixed submarket g_F on the competition in another submarket g. We make the distinction between the full game and a *local game* on $\ominus g_F$. In the latter case, markups for producers in g_F are fixed – and aggregated together by R_{g_F} – while the game is played only among producers outside it. Hence, the demand becomes elastic: a small R_{g_F} indicates the existence of attractive options inside g_F. As R_{g_F} shrinks, the competition in $\ominus g_F$ becomes more intense. We let the *submarket response function* $\phi_{g|g_F}(R_{g_F})$ capture the value corresponding to R_g that arises from the equilibrium of the local game and possibly the value of R_{g_F}. For brevity, when we write $\phi_g(\cdot)$, omitting the local submarket, we refer to $\phi_{g|\ominus g}(\cdot)$. The following technical lemma will be useful in the sequel.

Lemma 1. *Let g, g_F be submarkets of G. If $g \subseteq \ominus g_F$ or $g_F \subseteq g$, then $\phi'_{g|g_F}(R_{g_F}) < \phi_{g|g_F}(R_{g_F})/R_{g_F}$. Consequently, $\phi_{g|g_F}(R_{g_F})/R_{g_F}$ is decreasing in R_{g_F}.*

The perturbations we consider can be expressed generally as a shift in the response function for either a single producer or some subnetwork of producers. By a shift, we mean that $\phi_g(R_{\ominus g})$ is replaced by a function $\hat{\phi}_g(R_{\ominus g})$ such that (for an upward shift) $\hat{\phi}_g(R) \geq \phi_g(R)$ for all R in the domain. For a single producer, a shift in $\phi_a(\cdot)$ can result only from a change in c_a, but for a submarket g it can be the result of any number of structural or parametric changes within g. For example, Section 4 shows that a merger of direct competitors results in an upward shift, while a merger of complements results in a downward shift.

The next observation concerns the sensitivity of price multipliers. As price multipliers of any producers are strategic complements, we are able to show that an upward (downward) shift in $\phi_g(\cdot)$ induces an increase (decrease) in the equilibrium price multipliers of *all* competitors in $\ominus g$. The conditions placed on g' in the result exclude only those submarkets that overlap with g partially, as their behavior is dependent on the nature of the structural change that causes the shift.

Proposition 1. *Fix $g \subseteq G$. For any submarket $g' \subseteq G$ satisfying $g \subseteq g'$ or $g' \subseteq \ominus g$, an upward shift in $\phi_g(\cdot)$ leads to an increase in the equilibrium multiplier $R_{g'}$.*

Note that this result also implies (see the proof) that the shift increases $R_g/R_{g'}$, so the multiplier increase is greatest on R_g. Below we expand the analysis to include producer outcomes (i.e., demand, price, and profits) that depend on complementarity, measured by μ_a, besides the overall magnitude of markups.

3.2. Own-cost perturbation

Consider an increase in c_a, and the corresponding upward shift in $\phi_a(\cdot)$. In Proposition 1, we show that this drives up R_b for all producers b. For $b \neq a$, this implies an increase in markup α_b. On the other hand, producer a's own markup can be shown to decrease with the perturbation. Among direct competitors, this implies the intuitive result that more efficient producers apply larger markups. Proposition 2, below, summarizes the impact of own-cost perturbations on equilibrium markups, market share, and price.

While the markup for producer a decreases, note that the size of the producer's absolute profit margin increases. Increasing c_a by Δ makes R_a increase by $\Delta_a = 2\Delta + \Delta_{\ominus a}$, where $\Delta_{\ominus a}$ is the resulting change in $R_{\ominus a}$. Since $\phi_{\ominus a}(\cdot)$ is unchanged, $R_{\ominus a} = \phi_{\ominus a}(R_a)$ increases so that $\Delta_a > 2\Delta$. So, for any fixed production quantity x, the profit margin, $(R_a - c_a)x$, increases. On the other hand, the market share x_a will decrease following the shift. This result follows from the repeated application of Lemma 1, which indicates that each subproduct containing a applies a proportionally larger increase in its multiplier than do its competitors.

Turning to prices, the effect of a cost perturbation on the price p_a is dampened by the subsequent reduction of producer a's markup, as well as the reallocation of demand toward competing producers. However, some portion of the cost increase is ultimately reflected in the new equilibrium price. Indeed, regardless of the overall market structure, the price p_a can only increase with c_a.

Proposition 2. *For any producer a, an increase in c_a leads to a decrease in the equilibrium markup α_a, a decrease in the equilibrium market share x_a, and an increase in the equilibrium price p_a.*

Although markups, market share and prices are monotone, profits $\pi_a := (R_a - c_a)x_a^2$ can shift in either direction, depending on the relative size of the effects on markups and market share. (The numerical example in Table 3 illustrates both cases.) This in itself is an intriguing phenomenon, as it suggests that it may in some cases be to a producer's advantage to be *less* efficient. We note that this effect is not unique to models with complementarity. The critical observation is that for sufficiently inelastic demand, the overall level of profits may be higher in a market where the aggregate cost is larger. Decreasing efficiency weakens a producer's competitive position, but may also increase the overall profits, and this increase may dominate the individual effects. The same effect can occur in a Cournot model, although the cost shock must affect at least 3 producers [6]. In fact, the presence of complementary markets makes a profit decrease more likely. As c_a increases, the corresponding increase in markups from producers of complementary items contributes to a further decrease in producer a's market share.

3.3. Competitor perturbation

Also of interest is the effect of the producer's position in relation to a competitor who may alter its efficiency. Here, the impact on profits will depend on the extent to which the producer provides a substitute or a complement to the perturbed producer. There is a substitutive aspect to each relationship, with the exact nature captured by the producer's substitute network structure. (As an exception, when two full markets compete serially for a fixed demand, producers in one market have no effect on demand for producers in the other.) Any increase in a competitor's cost structure will relax competition from substitutes in the sense that $R_{\ominus a}$ increases. This increases $(R_a - c_a)$ as a result, and because $\phi'_a(R_{\ominus a}) < R_a/R_{\ominus a}$ (see Lemma 1), the 'unscaled' market share, x_a/μ_a, also increases with any competitor's shift upward. The combined result is that profits π_a must increase if the perturbation increases μ_a (this follows since the ratio π_a/μ_a^2 always increases).

Despite these effects, profits π_a may move in either direction, with the determining factor being the direction of the change in scaling factor μ_a and its size in relation to substitution-based effects. With the exception of a producer who spans the full market, all producers have some bearing on the complementarity faced by each other producer. Hence, the effect of the perturbation on μ_a can be either positive or negative. For more insight, we specialize to producer a at depth 3 in the submarket tree. The general market of this type is illustrated in Fig. 1(b). The producer faces some direct competition in the local market, depicted as G_L. In addition, the competition outside of the localized market is divided into a complementary section and a substitutive section (depicted by the submarkets G_C and G_S, respectively). We allow generality in that G_L, G_C, and G_S can be arbitrary submarkets, but limit the depth of a to 3 for simplicity.

An upward shift in $\phi_{G_S}(\cdot)$, positioned as a substitute to bundles containing a, leads to an increase in R_{G_S} that is proportionally larger than the increase in R_{G_C}. This implies an increase in μ_a and thus in π_a.

Proposition 3. *Using the notation of Fig. 1(b), any upward shift in $\phi_{G_S}(\cdot)$ increases the equilibrium market share x_a, prices p_a, and profits π_a for the producer a.*

Similarly, an upward shift in $\phi_{G_C}(\cdot)$, positioned in complement to producer a, leads to a decrease in μ_a. Here, the conclusions in terms of market share and profits for the producer a are not clear *a priori* and will depend on the structure of the individual submarkets. For instance, an efficient producer will lose market share when the cost of complementary items increases

Table 1
Sensitivity analysis of model's outputs for producer a caused by perturbations in a submarket. The rows refer to an increase in c_a and in the complementary and substitute markets, respectively. Arrows point to the possible directions of change of each of the outputs.

Submarket	Cost (c)	Price mult. (R)	Markup (α)	Demand (x)	Price (p)	Profit (π)
a	↑	↑	↓	↓	↑	↕
G_C	–	↑	↑	↕	↕	↕
G_S	–	↑	↑	↑	↑	↑

(see Proposition 4, below). However, we observe that the most inefficient producers of a component may stand to gain market share, as this relaxes local competition. Furthermore, in a market for small-enough components, all producers lose market share when the cost of complements increases, as effects on μ_a tend to dominate.

Proposition 4. *Using the notation of Fig. 1(b), when c_a is small enough, an upward shift in $\phi_{G_C}(\cdot)$ decreases the equilibrium market share x_a.*

Table 1 summarizes this section's sensitivity analysis by pointing out the direction of change of each output of the model for producer a. Rows a, G_C, and G_S indicate the location of the perturbation. To interpret with the example of Section 1, the last entry for G_C indicates that a CPU manufacturer may gain or lose profits from additional inefficiencies in the market for monitors. Indeed, both outcomes may occur under specific conditions. The last row shows that a cost increase for a producer of integrated computers allows a CPU manufacturer, modeled by a, to increase markups while gaining demand, and so increase its prices and profits. In the case of a producer's own price, this quantity moves in the same direction as the cost perturbation, although market share and in some cases profits move in the opposite direction. Note that the directional changes for perturbations within G_C or G_S apply for any type of shift.

4. Effect of mergers on market outcomes

Here we will focus on the effect of changes to a network structure that leave the overall production capacity fixed. Thus, we see how the network structure dictates the intensity of competition, as measured by the effect on total producer profits (i.e., industry markup) and the social cost of production. We start in Section 4.1 by looking at the overall markup applied to a bundle. Recalling Proposition 1, structural changes that either intensify or relax competition locally within a particular submarket have the corresponding effect on the overall bundle price, and so we analyze these local effects. In Section 4.2, we observe that the connection between local and global effects does not extend to the social cost criterion. In this case, the full market structure should be considered to assess the impact of a structural change.

For this analysis, recall that R_G is the market price in equilibrium for a given network, and note that $C^{\text{OPT}} = R_G|_{\vec{\alpha}=1}$ is the total cost of satisfying demand in a socially optimal manner. A comparison of these terms gives a measure of the extent to which bundles have been marked up. In particular, $R_G/(R_G|_{\vec{\alpha}=1})$ measures the 'average' markup, and $R_G - R_G|_{\vec{\alpha}=1}$ is equal to the total producer profit. In terms of social cost, let $C(\vec{\alpha})$ represent the total production cost resulting from an equilibrium markup vector $\vec{\alpha}$, and the ratio $C(\vec{\alpha})/(R_G|_{\vec{\alpha}=1})$ determines the inefficiency of that vector. As such, we are particularly interested in 'mergers', i.e., changes to the market structure for which $R_G|_{\vec{\alpha}=1}$ remain constant. In this case, the effects on profits and efficiency can be observed through R_G and $C(\vec{\alpha})$ alone.

We define a *merger* as a change in the network structure where multiple producers are combined in a way that preserves the aggregate cost structure. The cost of the new link should match the cost of using the subnetwork that it replaces, assuming that flow is allocated optimally within the original subnetwork. We denote the optimally aggregated cost of a submarket g by c_g. The procedure for aggregating costs optimally is identical to that for aggregating price multipliers: $c_{S(\mathcal{g})} = \sum_{g\in\mathcal{g}} c_g$, and $c_{P(\mathcal{g})} = (\sum_{g\in\mathcal{g}} 1/c_g)^{-1}$. Any changes in market outcomes result entirely from changes in the way the producers interact.

4.1. Industry markups

Proposition 1 illustrates a consistency between the local competitiveness of subproducts and the markups of the market as a whole. A change in the network structure that causes an upward shift in a local response function will increase R_G as well. In the case of a merger, this implies an increase in the overall industry markups. In this section, we highlight the local effect of both *parallel mergers* between direct competitors and *integrated production* of complementary components.

We first look at parallel mergers, where two parallel links, a_1 and a_2 are combined to form a_P. We denote the cost of the merged producer by c. Letting $\theta = c_{a_2}/(c_{a_1} + c_{a_2})$, we have $c_{a_1} = c/\theta$ and $c_{a_2} = c/(1-\theta)$. Any parallel merger can be described in this way for some $\theta \in (0, 1)$. We show that any such merger results in an upward shifted response function, $\phi_{a_P}(\cdot)$ relative to the aggregated response function $\phi_{g_P}(\cdot)$, where $g_P = P(a_1, a_2)$. This is consistent with intuition since a merger reduces the competition in the market.

Theorem 1. *Parallel mergers increase the price of a bundle.*

With respect to any fixed $R_{\ominus g_P}$, the two producers prior to merging behave equivalently to the elastic duopoly studied in [2]. There, the equilibrium for the duopoly is derived in a closed-form, from which it can be shown that both $\phi_{g_P}(R_{\ominus g_P})$ and $\phi'_{g_P}(R_{\ominus g_P})$ increase as θ gets further from 1/2. In a series–parallel setting, we can apply Proposition 1 to extend this parametric relationship to the market price R_G. We conclude that not only does any parallel merger increase the equilibrium price, but the size of the price increase is increasing in the symmetry of the merging firms. That is, for two parallel links with a given aggregate cost, c, the configuration leading to the lowest prices (equivalently, the largest upward shift in ϕ_{g_P} upon merging) is that of $c_1 = c_2 = 2c$. Prices increase as the distribution becomes less symmetric, with the highest prices coming from complete asymmetry in which one link is eliminated altogether.

We next discuss the case of mergers involving complementary producers. Two producers in series are an unstable configuration because the network would not be 3-connected, so we will not analyze mergers originating from this structure. Rather, we look at the case of a single producer in series with a set of parallel producers who compete with each other directly, i.e., the simplest configuration involving complementarity. We consider the effect of consolidating all of these producers to a single one. We interpret this as a scenario where the production being offered by the direct competitors is carried out in-house by the producer occupying a single link. In this way we study the effect of *integrated production*.

Let c be the cost of the integrated producer a_S. We consider parallel links a_1 and a_2, forming g_P. The submarket g_P is connected in series with a third producer a_M to form $g_S = S(a_M, g_P)$. We require $c_{g_S} = c$, and in particular, for $\theta_P, \theta_M \in (0, 1)$, let $c_{a_1} = (1-\theta_M)c/\theta_P$, $c_{a_2} = (1-\theta_M)c/(1-\theta_P)$, and $c_{a_M} = \theta_M c$. For any choice of θ_P and θ_M, $\phi_{a_S}(\cdot)$ is a downward shift of $\phi_{g_S}(\cdot)$ so that integrated production results in a lower price than the subcontracting setup.

Theorem 2. *Integrated production decreases the equilibrium price of a bundle. The size of the effect is decreasing in θ_M.*

Table 2
Social cost comparison between the social optimum (top) and equilibria with symmetric (medium) and asymmetric (bottom) costs in g.

Social optimum	Pr. 1	Pr. 2	g	Pr. 5	Pr. 6	Market (G)
Efficiency (c)	1	1	4.5	2	2	0.833
Market share (x)	0.083	0.083	0.167	0.417	0.417	1
Cost (cx^2)	0.007	0.007	0.126	0.347	0.347	0.833

Symmetric producers	Pr. 1	Pr. 2	Pr. 3	Pr. 4	Pr. 5	Pr. 6	Market (G)
Efficiency (c)	1	1	9	9	2	2	0.833
Markup (α)	6.96	6.96	2.70	2.70	5.08	5.08	4.60
Market share (x)	0.123	0.123	0.123	0.123	0.377	0.377	1
Cost (cx^2)	0.015	0.015	0.135	0.135	0.285	0.285	0.870

Merged producer	Pr. 1	Pr. 2	g	Pr. 5	Pr. 6	Market (G)
Efficiency (c)	1	1	4.5	2	2	0.833
Markup (α)	8.12	8.12	4.21	5.90	5.90	5.64
Market share (x)	0.102	0.102	0.204	0.398	0.398	1
Cost (cx^2)	0.010	0.010	0.187	0.317	0.317	0.842

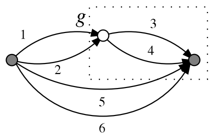


Fig. 2. Total production cost in this market is smaller when producers 3 and 4 are merged.

The theorem establishes that the markup of an integrated producer a_S provides a lower bound on $\phi_{g_S}(R_{\ominus g_S})$. In the limit as θ_M nears 1, the behavior of g_S resembles that of the integrated producer. The response function then shifts up monotonically as θ_V is decreased. Looking at the competitive portion of the network, g_P, the competition among these producers is most intense when this local market is relatively small. As g_P grows larger relative to its substitutes, the sensitivity $\phi'_{g_P}(R_{\ominus g_P})$ to competitor markups, notably those of the monopolist a_M, increases so that the dynamic of competition within g_S begins to resemble more closely that of serial monopolies. The implication is then that integrated production produces the largest decrease in a bundle price when the local monopolist, a_M, incurs a small portion of the production costs in the market g_S.

4.2. Social cost of production

In contrast to industry markups, the social cost of production may decrease following a parallel merger that consolidates market power locally. Relative to total profit, social cost depends on the symmetry, rather than the size of markups. So, when a bundle consisting of product g is inherently more expensive to produce than substitute bundles, likely leading to high markups on those substitutes, what is perceived locally as an inefficiency in the market for g may be a force that drives markups on g closer to those on substitutes. In effect, this reduces the degree of distortion in the overall market. We proceed with an example to demonstrate this possibility.

Consider the market G in Fig. 2, where the submarket for product $g = \psi_4^2(G)$ is a duopoly. For a fixed multiplier $R_{\ominus g}$, the two producers face an elastic combined demand. Considering this market for g alone, the most efficient configuration would appear to be the symmetric one (indeed, this is shown for an elastic duopoly model in [2]). For comparison, we consider the efficiency that results in G when producers 3 and 4 are merged into a single producer with efficiency parameter $c_g = [1/c_3 + 1/c_4]^{-1}$ so that the aggregate cost structure is maintained.

We start with the symmetric scenario where costs in g are $c_3 = c_4 = 9$. According to Table 2, in an optimal allocation, 83.3% of customers purchase products 5 and 6, and the average bundle cost under the optimum is 0.83. Regarding the equilibrium allocation, producers 5 and 6, each being more efficient than the other purchase combinations, apply relatively large markups of $\alpha_5 = \alpha_6 = 5.08$ to their products. In comparison, the price of a combination purchase from the other producers is only 3.1 times larger than the cost (of $10x$ for x units). This distortion encourages a larger proportion of costly combination purchases, and the average cost of a bundle in equilibrium is 0.87.

Now, we merge producers 3 and 4 into a single producer with combined cost $c_g = 4.5$. Studying g in isolation would suggest that this arrangement is inefficient. Yet, the merged producer applies a larger markup that raises the price of a combination purchase to 4.9 times the cost. This shifts some demand back to producers 5 and 6 so that the average cost of a bundle falls to 0.84, despite the market power of the merged producer. Although the difference in social cost between these two scenarios is rather small, the direction of change is surprising as it goes contrary to what a local model of market g suggests.

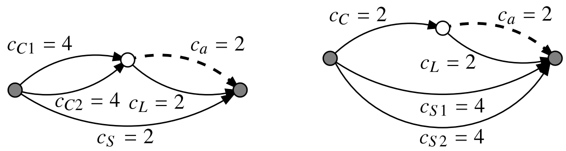
5. Effect of ignoring complementary markets

We close by looking at the impact of model misspecification that ignores complementarity. For a producer a competing in a parallel submarket g of G (that is, $g = P(\mathcal{g})$ with $a \in \mathcal{g}$), a likely alternative to modeling G fully is to estimate only the parameters μ_g and $R_{\ominus g}$. This is equivalent to estimating the demand function for submarket g, while treating g as a market of direct competitors subject to an elastic demand. To illustrate the misspecification, consider a CPU manufacturer evaluating a decision to invest in more efficient capacity. This investment will likely trigger smaller markups from producers of other components (e.g. monitors) and integrated computer models. If, however, the producer restricts analysis to the CPU submarket, then these markups are implicitly assumed to remain at their pre-investment levels, neglecting the competitors' responses. We show that a localized approach of this type can yield misleading conclusions about the sensitivity of producer profits to changes in model primitives.

Consider a perturbation of producer a's own efficiency parameter by Δ, as described in Section 3.2, for a network as in Fig. 1(b). By altering $\phi'_{G_C}(\cdot)$ and $\phi'_{G_S}(\cdot)$ we can demonstrate estimation errors in both directions that arise from a localized view (the localized model treats both of these sensitivities as zero). Fig. 3 provides two specific examples: a bundle with a sensitive substitute market in (a) and with a sensitive complement market in (b). In both, we fix $c_{G_S} = c_{G_C} = 2$. However, in (a), G_S contains a single link, while G_C is a symmetric pair (note that a single link is more sensitive than any parallel pair), and in (b) we model the reverse. Table 3 summarizes the original equilibrium in each market, as well as the equilibrium when c_a is increased by $\Delta = 1$. Lastly, we look at a localized

Table 3
Summary of perturbed equilibria.

	c_a	R_a	$R_{\ominus a}$	R_V	R_H	μ_a	x_a	π_a
	Market (a)							
Initial	2	13.1	9.1	9.6	20.1	0.678	0.278	0.854
Perturbed	3	16.0	10.0	9.9	21.5	0.685	0.263	0.898
Localized pert.	3	15.6	9.6	–	–	0.678	0.258	0.842
	Market (b)							
Initial	2	13.1	9.1	20.1	9.6	0.322	0.132	0.193
Perturbed	3	16.0	10.0	21.5	9.9	0.315	0.121	0.190
Localized pert.	3	15.6	9.6	–	–	0.322	0.123	0.191



(a) A sensitive substitute market.

(b) A sensitive complement market.

Fig. 3. A localized model underestimates profits in (a) and overestimates profits in (b).

model of competition in the market $L_+ = P(G_L, a)$, wherein $R_{\ominus G_{L+}}$ remains fixed to $R_{G_C} + R_{G_S}$ as computed with the original costs. The table includes equilibrium values for this localized model when c_a is perturbed.

We observe that the profits estimated by the localized model are too low when the substitute market is sensitive and too high when the complement market is sensitive. In both cases, the localized model ignores a shift in μ_a, and the difference in outcomes reflects a difference in the true direction of this effect. In (a), the localized model ignores a positive shift from $\mu_a = 0.678$ to 0.685. In (b), the localized model ignores a negative shift from $\mu_a = 0.322$ to 0.315. Furthermore, the example in (a) demonstrates that even the *direction* of the change in profits may differ in the localized model. These flaws are problematic for a producer evaluating a decision to invest in more efficient technology (i.e., decreasing the efficiency parameter). As such, they provide producers with motivation to explicitly model their complementary markets.

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Appendix. Proof of results

Proof of Lemma 1. We must first provide a definition of the local game underlying the function $\phi_{g|g_F}(R_{g_F})$. As $\vec{\alpha}$ captures the decisions of all producers in the first-stage game, we let $\vec{\alpha}_{\ominus g_F|g_F}(R_{g_F})$ represent the decisions of producers in $G \setminus g_F$ in a markup game played with R_{g_F} held fixed. For a submarket $g \subseteq \ominus g_F$, $R_{g|g_F}(\vec{\alpha}_g, R_{g_F})$ is defined analogously to $R_g(\vec{\alpha})$ and computed in the same inductive manner. For producer $a \in G \setminus g_F$, the best-response function in the local market game becomes $\Gamma_{a|g_F}(\vec{\alpha}_{\{\ominus g_F \setminus a\}|g_F}, R_{g_F}) = 2 + R_{\ominus a|g_F}(\vec{\alpha}_{\{\ominus g_F \setminus a\}|g_F}, R_{g_F})/c_a$. The first-stage decisions, $\vec{\alpha}_{\ominus g_F|g_F}(R_{g_F})$, then satisfy

$$\alpha_a = \Gamma_{a|g_F}(\vec{\alpha}_{\{\ominus g_F \setminus a\}|g_F}, R_{g_F}) \quad \text{for all } a \in G \setminus g_F.$$

Now, let g be composed of two component submarkets g_I and g_O, and hold R_{g_I} fixed to induce $R_{g_O|g_I} = \phi_{g_O|g_I}(R_{g_I})$ and $R_{\ominus g|g_I} = \phi_{\ominus g|g_I}(R_{g_I})$. It was shown in Theorem 6.3 of CFLS that $\phi'_{g'}(R_{\ominus g'}) < \phi_{g'}(R_{\ominus g'})/R_{\ominus g'}$ and furthermore, $\phi'_{g_O|g_I}(R_{g_I}) < R_{g_O|g_I}/R_{g_I}$. As, $\phi'_{g_I|g_I}(R_{g_I}) = 1$, it follows that $\phi'_{g|g_I}(R_{g_I}) < \phi_{g|g_I}(R_{g_I})/(R_{g_I})$. For g_F nested deeper within g, $\phi'_{g|g_F}(R_{g_F}) = \phi'_{g|\psi_{g_F}(g)}(R_{\psi_{g_F}(g)|g_F})$ $\phi'_{\psi_{g_F}(g)|\psi^2_{g_F}(g)}(R_{\psi^2_{g_F}(g)|g_F}) \cdots \phi'_{\psi^{-1}(g_F)|g_F}(R_{g_F}) < (R_{g|g_F}/R_{\psi_{g_F}(g)|g_F}) \cdots (R_{\psi^{-1}(g_F)|g_F}/R_{g_F}) = \phi_{g|g_F}(R_{g_F})/(R_{g_F})$. Then, for $g \subseteq \ominus g_F$, disjoint from g_F, $\phi'_{g|g_F}(R_{g_F}) = \phi'_g(R_{\ominus g|g_F})\phi'_{\ominus g|g_F}(R_{g_F}) < (R_{g|g_F}/R_{\ominus g|g_F})(R_{\ominus g|g_F}/R_{g_F}) = \phi_{g|g_F}(R_{g_F})/(R_{g_F})$. □

Proof of Proposition 1. Let $\bar{g} = \psi_g^{-1}(g)$ be a composition of the submarkets g and g''. We will show that an upward shift in $\phi_g(R_{\ominus g})$ implies upward shifts in $\phi_{\bar{g}}(R_{\ominus \bar{g}})$ and in $\phi_{g''|\ominus \bar{g}}(R_{\ominus \bar{g}})$. The result then follows by induction up the submarket tree. The response $\phi_{\bar{g}}(R_{\ominus \bar{g}})$ is an increasing function of $R_{g|\ominus \bar{g}}$ and $R_{g''|\ominus \bar{g}}$. If $\bar{g}$ is series then $R_{g''|\ominus \bar{g}} = \phi_{g''|\ominus \bar{g}}(R_{\ominus \bar{g}}) = \phi_{g''}(R_{\ominus \bar{g}} + R_{g|\ominus \bar{g}})$, and $R_{g|\ominus \bar{g}}$ is the unique fixed point of $h_{g|\ominus \bar{g}} : h_{g|\ominus \bar{g}}(R) \rightarrow \phi_g(R_{\ominus \bar{g}} + \phi_{g''}(R_{\ominus \bar{g}} + R))$. If $\bar{g}$ is parallel then $R_{g''|\ominus \bar{g}} = \phi_{g''|\ominus \bar{g}}(R_{\ominus \bar{g}}) = \phi_{g''}([1/R_{\ominus \bar{g}} + 1/R_{g|\ominus \bar{g}}]^{-1})$, and $R_{g|\ominus \bar{g}}$ is the unique fixed point of $h_{g|\ominus \bar{g}} : h_{g|\ominus \bar{g}}(R) \rightarrow \phi_g([1/R_{\ominus \bar{g}} + 1/\phi_{g''}([1/R_{\ominus \bar{g}} + 1/R]^{-1})]^{-1})$. In either case, $R_{g''|\ominus \bar{g}}$ is increasing in $R_{g|\ominus \bar{g}}$. Furthermore, the inner function of the composition $h_{g|\ominus \bar{g}}$ is unaffected by the shift in $\phi_g(\cdot)$, while the outer function is shifted upward. Thus, $h_{g|\ominus \bar{g}}(R)$ is shifted upward. The function $h_{g|\ominus \bar{g}}(R)$ is continuous and $h_{g|\ominus \bar{g}}(0) > 0$ so that $h_{g|\ominus \bar{g}}(R)$ intersects with the 45° line once and from above. The upward shift pushes this point to a larger value of R, thus increasing $R_{g|\ominus \bar{g}}$ for any fixed value of $R_{\ominus \bar{g}}$. As a result, $R_{g''|\ominus \bar{g}}$ and $\phi_{\bar{g}}(R_{\ominus \bar{g}})$ increase as well. □

Proof of Proposition 2. In equilibrium, R_a is a fixed point of $h_a : h_a(R_a) \rightarrow \phi_a(\phi_{\ominus a}(R_a))$. Perturbing c_a by a small Δ increases R_a by $\Delta_R = 2\Delta/(1 - h'_a(R_a)) = 2\Delta/(1 - \phi'_{\ominus a}(R_{\ominus a}))$. By Lemma 1, $\phi'_{\ominus a}(R_{\ominus a}) < R_{-a}/R_a = 1 - 2/\alpha_a$, and so $\Delta_R < \alpha_a \Delta$. The effect on α_a, denoted by Δ_α, satisfies $\Delta_R = c_a \Delta_\alpha + \alpha_a \Delta$ and so $\Delta_\alpha < 0$. For market shares, recall that x_a can be written as $\prod_{g \in \nu_P(a)} R_g/R_{\psi_a(g)}$, where $\nu_P(a) = (G_1, G_2, \ldots, G_d)$ is the sequence of parallel submarkets within which producer a is nested. Each term $R_g/R_{\psi_a(g)}$ in this product is equivalently written as $R_{g \setminus \psi_a(g)}/(R_{\psi_a(g)} + R_{g \setminus \psi_a(g)})$. For any $g \in \nu_P(a)$, $R_{\psi_a(g)}$ increases with c_a by Proposition 1. We note that $\phi_{g \setminus \psi_a(g)}(\cdot)$ remains unchanged, and applying Lemma 1, $\phi'_{\{g \setminus \psi_a(g)\}|\psi_a(g)}(R_{\psi_a(g)}) < R_{g \setminus \psi_a(g)}/R_{\psi_a(g)}$, so that $R_{g \setminus \psi_a(g)}/R_{\psi_a(g)}$ decreases. This is true for each $g \in \nu_P(a)$, causing $R_g/R_{\psi_a(g)}$ to decrease, and so x_a decreases. For prices, rewrite the above product as $x_a = R_{G_1}\left[\prod_{l \in \{2 \ldots d\}} R_{G_l}/R_{\psi_a^{-1}(G_l)}\right] R^{-1}_{\psi_a(G_d)}$. R_{G_1} increases with c_a by Proposition 1. Applying Lemma 1 and Proposition 1 for each term in the brackets ensures that this product is increasing as well, leaving only $R^{-1}_{\psi_a(G_d)}$ to decrease with c_a. Now looking at $p_a = R_a x_a$, we need only show that $R_a/R_{\psi_a(G_d)}$ is nondecreasing with c_a. If a has direct competition, i.e., $a \in \psi(G_d)$, then this term cancels out of p_a. Otherwise, $R_a/R_{\psi_a(G_d)} = R_a/R_{\psi_a^{-1}(a)}$ is increasing in c_a by Lemma 1. □

Proof of Proposition 3. By Proposition 1, R_{G_S} increases with the upward shift in $\phi_{G_S}(\cdot)$. Considering that $G_S \subseteq R_{\ominus_a}$ and $\mu_a = R_{G_S}/(R_{G_C} + R_{G_S})$, Lemma 1 implies an increase in both μ_a and

$x_a/\mu_a = R_{\ominus a}/(R_{\ominus a} + R_a)$ so that x_a increases. Proposition 1 also implies an increase in R_a and $R_a - c_a$ so that p_a and π_a increase as well. □

Proof of Proposition 4. Define a sequence $\{c_a^j\}$ of efficiency parameters for producer a, and the corresponding sequence $\{\vec{\alpha}^j\}$ of equilibria. To specify from among the sequence $\{\vec{\alpha}^j\}$ of equilibria, we reintroduce the explicit dependence of $R_g(\vec{\alpha}_g)$ on $\vec{\alpha}_g$. Let $\{c_a^j\}$ be such that $c_a^j \to 0$ as $j \to \infty$. We note that the response functions $\phi_{G_C}(\cdot)$, $\phi_{G_S}(\cdot)$, and $\phi_{G_L}(\cdot)$ are unaffected by c_a^j so that the equilibrium $\vec{\alpha}^j$ is dictated entirely by the function $\phi_a^j(R_{\ominus a}) = 2c_a^j + R_{\ominus a}$. Furthermore, the smoothness of response functions and of $\phi_a^j(R_{\ominus a})$ with respect to c_a^j ensures that $\{\vec{\alpha}^j\} \to \vec{\alpha}$ as $j \to \infty$, where $\vec{\alpha}$ is the equilibrium corresponding to $\phi_a(R_{\ominus a}) := R_{\ominus a}$, and that $\phi_g'^j(R_g(\vec{\alpha}_g^j)) \to \phi_g'(R_g(\vec{\alpha}_g))$ for $g \subseteq G$. As a result, letting Δ_x^j represent the change in $x_a(\vec{\alpha}^j)$ resulting from a particular upward shift in $\phi_{G_C}(\cdot)$, we see that $\Delta_x^j \to \Delta_x$, where Δ_x is the change in x_a when $c_a = 0$. When $c_a = 0$, producer a's market share is given by $x_a(\vec{\alpha}) = \mu_a(\vec{\alpha})R_{\ominus a}(\vec{\alpha})/(R_{\ominus a}(\vec{\alpha}) + \alpha_a c_a) = \mu_a(\vec{\alpha})/2$. Applying Lemma 1 to $\mu_a(\vec{\alpha}) = R_{G_S}(\vec{\alpha})/(R_{G_S}(\vec{\alpha}) + R_{G_C}(\vec{\alpha}))$, we see that x_a decreases with a shift in $\phi_{G_C}(\cdot)$ and that $\Delta_x < 0$. So, for c_a near enough to zero, producer a loses market share. □

Proof of Theorem 1. We first look at the pre-merger response function for g_P. Here, $\phi_{g_P}(R_{\ominus g_P}) = [1/R_{a_1|\ominus g_P} + 1/R_{a_2|\ominus g_P}]^{-1}$, where $R_{a_1|\ominus g_P} = \phi_{a_1}(R_{\ominus a_1|\ominus g_P}) = 2c/\theta + [1/R_{\ominus g_P} + 1/R_{a_2|\ominus g_P}]^{-1}$ and $R_{a_2|\ominus g_P} = \phi_{a_2}(R_{\ominus a_2|\ominus g_P}) = 2c/(1-\theta) + [1/R_{\ominus g_P} + 1/R_{a_1|\ominus g_P}]^{-1}$. We observe that $R_{a_1|\ominus g_P} < (2c + R_{\ominus g_P})/\theta$ and $R_{a_2|\ominus g_P} < (2c + R_{\ominus g_P})/(1-\theta)$. Combining gives $\phi_{g_P}(R_{\ominus g_P}) < 2c + R_{\ominus g_P}$ for all $R_{\ominus g_P}$. Of course, when a_P is replaced with a single link with cost c, the response function is $\phi_{a_P}(R_{\ominus a_P}) = 2c + R_{\ominus a_P}$. □

Proof of Theorem 2. As always, $\phi_{a_S}(R_{\ominus a_S}) = 2c + R_{\ominus a_S}$. We show that $\phi_{g_S}(R_{\ominus g_S}) \geq \phi_{a_S}(R_{\ominus a_S})$ for fixed $R_{\ominus g_S} = R_{\ominus a_S}$. Because markups are bounded below by 2, $R_{g_P|\ominus g_S} \geq 2(1-\theta_V)c$. We know that $R_{a_V|\ominus g_S}$ is equal to $2\theta_V c + R_{\ominus a_S} + R_{g_P|\ominus g_S}$. Thus, $\phi_{g_S}(R_{\ominus g_S}) = R_{g_P|\ominus g_S} + R_{a_V|\ominus g_S} \geq 2c + R_{\ominus a_S} + R_{g_P|\ominus g_S} \geq \phi_{a_S}(R_{\ominus a_S})$. For the second statement, see that as θ_V decreases, $\phi_{g_P}(\cdot)$ is shifted upward, while $\phi_{a_V}(\cdot)$ shifts downward, leading to a decrease in $R_{a_V|\ominus g_S}$, and so in $R_{\ominus g_P|\ominus g_S}$. Similarly, the sensitivity $\phi'_{g_P}(R_{\ominus g_P|\ominus g_S})$ increases (this effect is derived from closed-form expressions of [2] for equilibria in an elastic duopoly). For a fixed value of $R_{\ominus g_S}$, $\phi'_{g_S}(R_{\ominus g_S}) = (1 + 3\phi'_{g_P}(R_{\ominus g_P|\ominus g_S}))/(1 - \phi'_{g_P}(R_{\ominus g_P|\ominus g_S}))$ and so is also increasing. Integrating $\phi'_{g_S}(R)$ over $[0, R_{\ominus g_S}]$ shows that $\phi_{g_S}(R_{\ominus g_S})$ increase with a decrease in θ_V. □

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