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Adoption of innovations with contrarian agents and repentance

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HIGHLIGHTS

- Data of adoption of innovations provide evidences of a complex behavior.
- A key factor of the model is the idiosyncratic resistance to change.
- Social influence is included by favoring the adoption or acting against it.
- The inclusion of repentance generates a rich landscape including cycles.

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ABSTRACT

The dynamics of adoption of innovations is an important subject in many fields and areas, like technological development, industrial processes, social behavior, fashion or marketing. The number of adopters of a new technology generally increases following a kind of logistic function. However, empirical data provide evidences that this behavior may be more complex, as many factors influence the decision to adopt an innovation. On the one hand, although some individuals are inclined to adopt an innovation if many people do the same, there are others who act in the opposite direction, trying to differentiate from the "herd". People who prefer to behave like the others are called mimetic, whereas individuals who resist adopting new products, the stronger the greater the number of adopters, are named contrarians. Besides, in the real world new adopters may have second thoughts and change their decisions accordingly. In this contribution we include this possibility by means of repentance, a feature which was absent in previous models. The model of adoption of an innovation has all the ingredients of a previous version, in which the agents decision to adopt depends on the appeal of the novelty, the inertia or resistance to adopt it, and the social interactions with other agents, but now agents can repent and turn back to the old technology. We present analytic calculations and numerical simulations to determine the conditions for the establishment of the new technology. The inclusion of repentance can modify the balance between the global incentive to adopt and the number of contrarians who prevent full adoption, generating a rich landscape of temporal evolution that includes cycles of adoption.

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1. Introduction

Innovation is at the core of the changing in living conditions all along the human history. It is also one of the main driving forces of sustainable economical development in modern societies. However, even when innovations may represent a clear improvement over existing technologies, its adoption is not guaranteed because it depends on other factors that can restrain the adoption process, like the individual resistance or a high price. Besides, the adoption may be boosted by means of advertising or interpersonal influence. Rogers [1] was the first one to address theoretically the problem of innovation adoption, as early as 1962. In his qualitatively description, he claims that adoption curves are S-shaped (logistic) as a function of time: there are few early adopters, and only when their number becomes larger than a threshold, adoption develops up to a saturation point.

Systems of heterogeneous interacting individuals are complex systems whose properties have been studied in the contexts of economics (see [2–4] and references therein), criminality [5], game theory [6], and in many other social and biological systems. It has been shown that when the individuals are mimetic, i.e. they choose to imitate the behavior of the others (also called herding or congregator behavior), the possible equilibria have well known properties [7,8]. If some individuals do not exhibit a mimetic behavior, adoption dynamics is more involved, but also more interesting. Such individuals, called *contrarians*, have been described in different contexts in the literature. Galam [9] introduced contrarian agents in voter models, in such a way that they adopt opinions that are systematically opposite to the one of the majority of their neighbors. Subsequent extensions of this 2-state model explain the global balance between the two competing opinions observed in some real situations, in cases where the number of contrarians exceeds a threshold value [10,11]. Other possibilities have been proposed more recently by Crokidakis et al. [12], where agents can have either positive (mimetic) or negative (contrarian) interaction with a given probability, and by Masuda [13], who considered different models in which the decision of each contrarian depends on its neighborhood (made of contrarians and/or mimetics).

With or without contrarians, the time evolution of the fraction of adopters is a Markov chain: $n(t+1) = \mathcal{F}(n(t))$. The fixed points attractors of the dynamics satisfy $n = \mathcal{F}(n)$. However, as demonstrated by Goles et al. [14], systems with interacting binary agents evolve toward fixed points only when the interactions are *symmetric* and positive. Negative symmetric interactions may lead either to fixed points or to cycles of length 2, depending on details of the dynamics and on the initial state. These results rely on the existence of an energy function that is a decreasing (more rigorously, non-increasing) function of time under the system's dynamics. However, a system with contrarians does not necessarily have an underlying energy function, because the interactions between mimetic agents and contrarian agents are not symmetric. Being an individual property, contrarians have negative interactions with all other agents. Thus, interactions between contrarians are symmetric – both being negative – but interactions between a contrarian and a mimetic agent are anti-symmetric. Consequently, the existence of fixed points is not guaranteed.

The dynamics of adoption in models with only mimetic individuals has been studied a long time ago by Bass [7], in which was probably the first mathematical, however phenomenological, formulation of the problem. Later, Phan et al. [15] studied the problem in different types of networks, showing that the fraction of adopters increases with time through avalanches that depend on the underlying network structure. Moreover, the fraction of adopters at equilibrium in the absence of contrarians has been obtained analytically in [16] for a uniform distribution of the resistance to adopt and small values of the interaction weights. Numerical results have been obtained when contrarians are included: in the context of innovation the most important consequence of the inclusion of contrarians is the non-trivial restraining effect on the adoption curves, i.e, a small fraction of contrarians produce a large reduction on the final fraction of adopters [16].

All these models are suitable for situations where users cannot change their decisions, such as the case of expensive technologies. But there are other situations, as for example the choice of an operating system, a software, or an internet supplier, where the decision can be revised periodically. In such cases, adopters may change their minds and abandon the innovation. In this article we investigate that possibility on a society where individuals exhibit a mimetic or contrarian behavior (kept fixed during the whole adoption process), but they can repent for their decisions, going back to a non-adopter state. We study this model using analytical and numerical approaches, and considering different distributions of the idiosyncratic resistance to adopt.

We explore the parameters space by comparing the results of simulations with a mean field analytic approach, analyzing the phase diagram of the system for different proportions of mimetics and contrarians. The paper is organized as follows: In Section 1 we present the model, in Section 2 we consider a uniform distribution of the resistance to adopt (analytically and numerically), and in Section 3 we analyze the case of a logistic distribution. Conclusions are presented in Section 4.

2. The model of adoption with social interactions

A "microscopic" model of adoption dynamics has been proposed recently by some of us [16]. The model considers heterogeneous idiosyncratic individuals in the presence of advertising. For the sake of clearness we show below the payoff (P) function that every agent evaluates, adopting the novelty if its value is positive:

$$P = A - R_i + J_i n, \qquad (1)$$

where A represents the advertising, R_i is the individual resistance to adopt, and J_i is the agent weight interaction that multiplies n, the fraction of adopters. Mimetic individuals have a positive interaction with the adopters ($J_i > 0$), increasing their payoff function with the fraction of them. Contrarians, instead, have the opposite behavior, with a preference to adopt that decreases ($J_i < 0$) when the fraction of adopters increases. Both, mimetics and contrarians, cannot change their minds, consequently, the fraction of adopters is a non-decreasing function of time.

Similarly to the model described above, we consider here a system of N individuals that must decide whether to adopt or not a novelty. Again, the parameter $A \geq 0$ is a global incentive to adopt, the same for all agents. This incentive is proportional to the advantages introduced by the new technology, to advertising, and to eventual social values associated with the possession of the new product. On the other hand, each individual has a resistance to adopt the innovation given by a $R_i = R + r_i$ $(1 \leq i \leq N)$, where R is the population's average resistance and the r_i are (quenched) idiosyncratic deviations distributed among the population according to a probability density function $\mathcal{P}(r)$ of zero mean and standard deviation s. This resistance can be associated to suspicion against the novelty, to a certain laziness that induces to remain with the old technology or to limited resources for the acquisition of the new technology. When confronted with the decision to adopt or not the new technology, we assume that there are two kinds of individuals: a fraction f of the population is composed by contrarian agents, i.e., they resist to imitate what others do, while a fraction $1-f$ of the population is mimetic, so they tend to follow the herd. We assume these attitudes also remain fixed during the adoption process. At each time step, each agent weights the global decision of the others with a strength J_i, which represents the social influence on his own decision. As we said, mimetic individuals have positive J_i and contrarians have negative J_i, so the first ones increase, while the second ones decrease, their willingness to adopt the innovation proportionally to the fraction of adopters n.

As it stands, the model has in principle five parameters: A, R, s, J_i, and f, but we can get rid of two by using $|J_i|$ as a normalization factor for the others and defining an effective incentive d, as follows:

$$d \equiv \frac{A-R}{J}, \qquad \sigma \equiv \frac{s}{J} \quad \left(i.e., \ u_i \equiv \frac{r_i}{J} \right). \tag{2}$$

Notice that this normalization is equivalent to consider $|J_i| = J = 1$. So depending of the type of agents (mimetic or contrarian), the payoffs (1) can be written as:

$$\pi_i^M = d - u_i + n_i \quad \text{if } i \text{ is mimetic,} \tag{3a}$$
$$\pi_i^C = d - u_i - n_i \quad \text{if } i \text{ is contrarian,} \tag{3b}$$

where n_i is the fraction of adopters without counting individual i:

$$n_i = \frac{1}{N-1} \sum_{k \neq i} \omega_k, \tag{4}$$

with $\omega_k = 1$ if k is an adopter and zero otherwise.

When calculating the payoffs we can check each individual at random and immediately update his decision according to the sign of its payoff. On the other hand we can perform a synchronous updating of all agents at once. In the later case agents determine their payoffs taking into account the present value of the number of adopters, n, and this value is refreshed only after all agents have been checked. We will discuss this point in further detail in the next section.

If adopters are not allowed to change their minds, as in the original model, only the payoffs of non-adopters are important for the dynamics; however, the equilibrium properties of the present model depend both on adopters and non-adopters payoffs. In the limit of large populations with large numbers of adopters we may replace $N-1 \approx N$, drop down the constraint $k \neq i$ in the equation above and approximate n_i by the bare fraction of adopters n:

$$n_i \approx n \equiv \frac{1}{N} \sum_k \omega_k. \tag{5}$$

Individuals adopt the new technology whenever their payoffs are positive. The adoption dynamics may become quite complex upon introduction of contrarians that decide according to the majority rule [16]. Here we include the possibility of coming back from previous decisions, thus, individuals will abandon innovation if the payoff is negative.

When considering a large number of agents we can take the limit $N \to \infty$. Introducing the fraction of adopters (5) in Eqs. (3), and assuming that the idiosyncratic normalized resistance to adopt u_i are quenched random variables of probability density $\mathcal{P}(u)$, the adoption probability (the probability of positive payoff) in the limit $N \to \infty$ is

$$P(\omega = 1|M) = \int_{-\infty}^{d+n} \mathcal{P}(u) du \tag{6a}$$
$$P(\omega = 1|C) = \int_{-\infty}^{d-n} \mathcal{P}(u) du \tag{6b}$$

where M stands for mimetic and C for contrarian agents.

If the fraction of adopters at time t is $n(t)$, the adopters' dynamics is given by the following equation:

$$n(t+1) = (1-f)\int_{-\infty}^{d+n(t)} \mathcal{P}(u)du + f\int_{-\infty}^{d-n(t)} \mathcal{P}(u)du$$
$$= \int_{-\infty}^{d+n(t)} \mathcal{P}(u)du - f\int_{d-n(t)}^{d+n(t)} \mathcal{P}(u)du \tag{7}$$

In the absence of contrarians, $f = 0$, the phase diagram of the model is well known [2,3,8]. The stationary states satisfy

$$n = \int_{-\infty}^{d+n} \mathcal{P}(u)du \tag{8}$$

which is easily solved (see [8]).

In the following sections we include contrarians and the possibility of repentance, we assume payoffs given by Eqs. (3) and we include the possibility of changing decisions for both mimetic and contrarian agents. Agents adopt if the payoffs are positive and do not adopt otherwise. Moreover, those that have adopted previously may go back to no-adoption if the payoffs turn out to be negative. We analyze two particular distributions $\mathcal{P}(u)$, the uniform distribution and the logistic one, and we compare the results with the original model without repentance [16].

3. Uniform distribution

In this section, we present first results of simulations and then we discuss analytic results for a uniform distribution, i.e. $\mathcal{P}(u) = (2\,u_0)^{-1}$ in $[-u_0, u_0]$ and $\mathcal{P}(u) = 0$ elsewhere. We will compare the results of this model with the model without repentance when $u_0 = 0.5$ ($\mathcal{P}(u) = 1$) [16].

3.1. Numerical results

When performing the simulations we consider two different dynamics, corresponding to synchronous and non-synchronous updates. In the case of synchronous Parallel Dynamics (PD), the payoffs are evaluated for all agents at the same time, then the status of each agent is changed or not accordingly to their payoff and thereafter the new fraction of adopters, n, is updated. In the Monte Carlo sequential dynamics (MC), one agent is selected at random and its status is updated depending on its payoff, then the number of adopters is immediately adjusted. This process is repeated N times, which corresponds to one MC step. The reason for considering these two dynamics is that the first one (PD) is better adapted to be compared with analytical results, while MC simulations could provide a better description of the changes in real societies. The difference between the two dynamics is that in the MC method a sequential update is performed, which means that the number of adopters changes in a continuous way during each MC step, while in the PD case the number of adopters is updated at the end of each step, after evaluating the payoffs of all agents. A second difference between the two types of dynamics is that with the PD procedure all agents are visited, while in MC dynamics they may not.

In order to illustrate the dynamics let us first consider a very simple case with just two agents, $N = 2$, and the three possible combinations: two mimetics, one mimetic and one contrarian, and two contrarians. We choose parameters such that for both agents d is slightly higher than u_i, for example $d = 0.01$ and $u_1 = u_2 = 0$. The results are exhibited on Fig. 1 (black filled squares correspond to PD and red open circles to MC dynamics). In the case of two mimetic agents, both of them will adopt immediately and no further changes are observed, the system arrives at a fixed point. With one mimetic and one contrarian, both agents adopt in the first time step but then only the mimetic remains as adopter. The system gets to a fixed point with $n = 0.5$. In the cases above the description corresponds strictly to PD while some random variations are possible with MC dynamics. An interesting time evolution arises when there are two contrarians because both of them adopt when $n = 0$, but as soon as they adopt, $n_i = 1/2$ (notice that for small values of N, the approximation given by Eq. (4) is not valid), so the contrarian's payoff becomes negative and at the next evaluation both become non-adopters. Therefore, the system exhibits a strictly periodic behavior in parallel simulations. In MC simulations, however, there are no oscillations: after the first agent adopts, when the second is selected, it will not adopt because its payoff will be negative, so the evolution stops at 50% of adopters. Obviously, the $N = 2$ case does not correspond to any real system, but we include it here, despite being an over simplification, because it helps understand the results on large systems that we present below.

Let us now consider the case with an intermediate number of agents, $N = 100$; henceforth we use the approximation given by Eq. (5). In Fig. 2, it can be verified that when the fraction of contrarians is large, $f = 0.9$, the system exhibits oscillations in the parallel dynamics case, while there are no oscillations with Monte Carlo dynamics. For a lower fraction of contrarians, $f = 0.5$, oscillations are of smaller amplitude and there are no oscillation when $f \leq 0.2$.

For a much larger number of agents, $N = 10^7$, oscillations are damped in the long term. Even for a high proportion of contrarians, $f = 0.9$, and with parallel dynamics, oscillations decay after a short transient, as can be seen in Fig. 3. For a lower fraction of contrarians, oscillations are very short lived; for $f = 0.5$, for instance, no more than three cycles are seen in Fig. 3(b). The previous results, all put together, suggest that for uniform distribution of resistance to adoption – which is a raw simplification of the society representation – oscillations are possible for a large number of contrarians, but are a finite

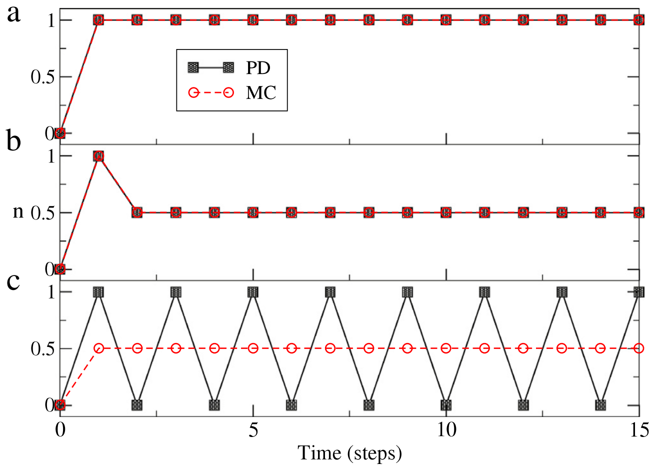

Fig. 1. Case of two agents: (a) Two mimetics, the final value of n is $n = 1$; (b) one mimetic and one contrarian, n quickly converges to $n = 0.5$; (c) two contrarians, the system exhibits oscillations. Black squares correspond to PD and open red circles to MC dynamics. In all cases, $d = 0.01$ and $u_1 = u_2 = 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

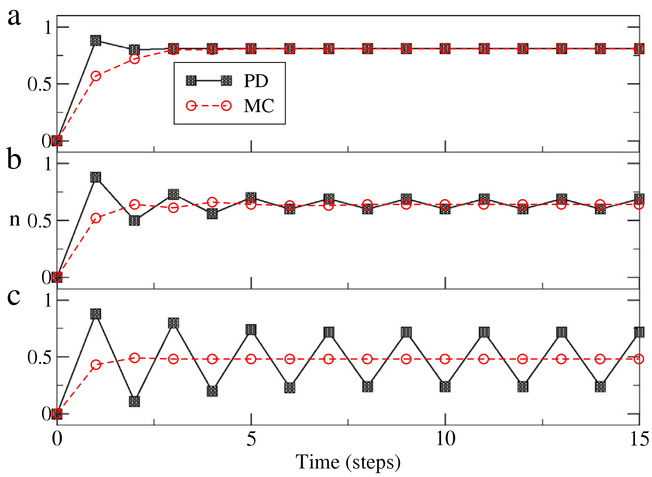

Fig. 2. Fraction of adopters as a function of time for $N = 100$ agents and different values of the fraction of contrarians f: (a) $f = 0.2$, no oscillations; (b) $f = 0.5$, small amplitude oscillations; and (c) $f = 0.9$, large amplitude sustained oscillations. Black squares correspond to PD and open red circles to MC dynamics. In all cases, $d = 0.4$ and $u_0 = 0.5$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

size and/or transient effect. We have investigated the relation between oscillations and system size, and it seems that the critical number of agents is around $N \approx 10\,000$.

It is also interesting to investigate the effect of the advertising. The values considered, $d = 0.4$ may be too high. It can be argued that a too high value of the incentive to adopt could have a role in the appearance, or not, of oscillations. And it has. In order to check this we try a lower value of the incentive, $d = -0.2$. As u goes from -0.5 to $+0.5$ that value of d implies that 30% of the agents have an idiosyncrasy below d, i.e. 30% are potential early adopters. As we are interested in possible oscillations we focus on a high concentration of contrarians, $f = 0.9$, and three system sizes, $N = 2$, $N = 100$, and $N = 10^7$. The results are shown in Fig. 4 where a clear feature can be seen: the asymptotic values for the average fraction of adopters are much lower ($n \approx 0.15$) than in the case of $d = 0.4$. This is expected because the advertising is what promotes the adoption in the first place. But on the other hand the oscillatory behavior is very similar to previous results with a bigger value of d, so our conclusions regarding the oscillatory behavior (and the existence or not of oscillations) are robust against a change of the advertising. However, if the width of the distribution is narrower, no oscillations appear for low values of d.

Note that the effect of repentance with the associated cycle dynamics is only possible if $f > 0$. If there are no contrarians, even if it is possible to repent, no agent will do it because for a mimetic the payoff cannot decrease. In other words, cycles

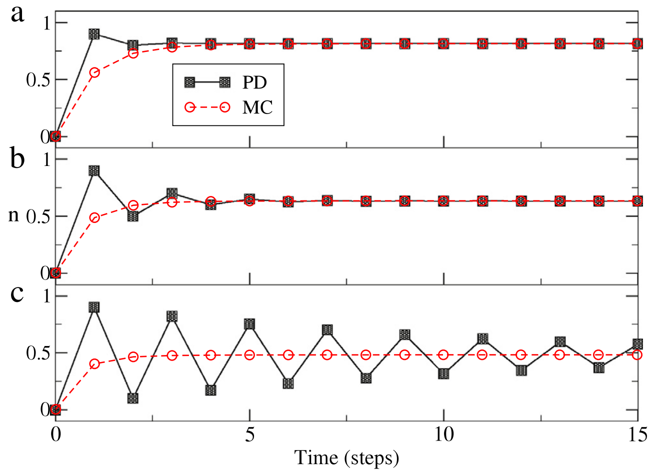

Fig. 3. Fraction of adopters as a function of time for $N = 10^7$ agents and different values of the fraction of contrarians f: (a) $f = 0.2$, no oscillations; (b) $f = 0.5$, very short lived oscillations; and (c) $f = 0.9$, transient oscillations. Black squares correspond to PD and open red circles to MC dynamics. The other parameters in all cases are $d = 0.4$ and $u_0 = 0.5$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

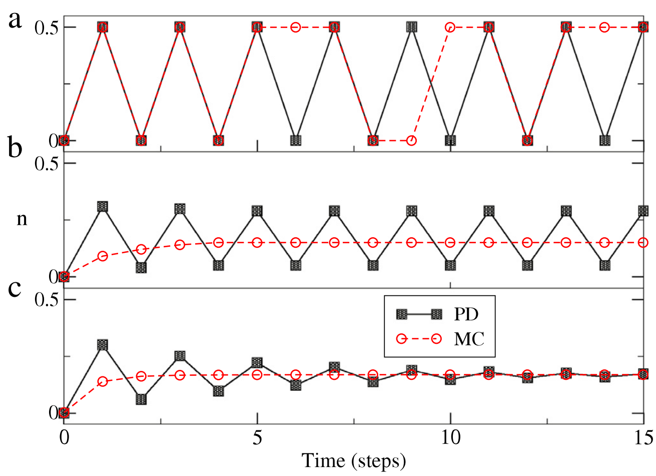

Fig. 4. Fraction of adopters as a function of time for a large fraction of contrarians ($f = 0.9$), but a low value of advertising ($d = -0.2$). Results for different system sizes: (a) $N = 2$, (b) $N = 100$, and (c) $N = 10^7$. Black squares correspond to PD and open red circles to MC dynamics. The qualitative behavior is the same as in Figs. 2 and 3 with the same value of f, but a bigger value of d. There is subtlety regarding the $N = 100$ case: depending on how evenly is the actual distribution of u_i, oscillations can or cannot be observed. The case shown in panel (b) correspond to an even realization of the uniform distribution. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

are only possible if both, contrarians and repentance, are present in the model. Moreover, the effect of contrarians modifies the number of adopters, and this change may induce mimetics to abandon the innovation. We remark that in this section we have restrained our simulations to a distribution of idiosyncratic resistance to adopt that is uniform in $[-0.5, 0.5]$, to compare to previous results without repentance [16]. If a narrower distribution is considered, stable oscillations may appear for relatively high values of the advertising. In the next section we show, as an example, that such is the case for $u_0 = 0.25$ and $\mathcal{P}(u) = 2$ (see Fig. 6). Moreover, the effect of the width of the distribution on the existence of oscillations will be discussed in detail in Section 4, as the logistic distribution is easier to work with.

3.2. Analytic results

In this section we present analytic mean field results and compare them with numerical simulations of the preceding section. As the analytic calculations implicitly assume $N \to \infty$ we will compare them with the numerical results for the biggest system, $N = 10^7$.

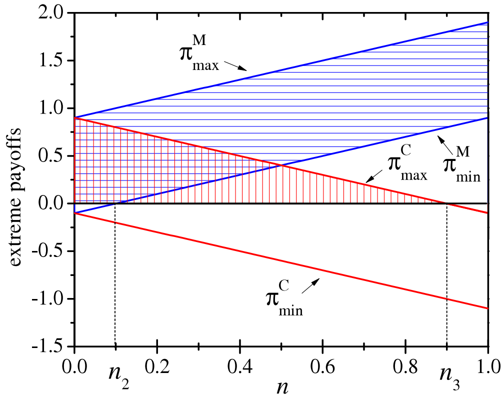

Fig. 5. Extreme payoffs as a function of the number of adopters, n, for the uniform distribution with $d = 0.4$, $u_0 = 0.5$. π lines are extreme payoffs, blue lines for mimetics and red lines for contrarians. In general they are given by Eq. (9), in this case by Eq. (12). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Due to the compact support of $\mathcal{P}$, the possible (normalized) payoffs as a function of n (see Eq. (3) and Fig. 5) are bounded by

$$\begin{aligned} \pi^M_{max}(n) &= d + u_0 + n \\ \pi^M_{min}(n) &= d - u_0 + n \\ \pi^C_{max}(n) &= d + u_0 - n \\ \pi^C_{min}(n) &= d - u_0 - n \end{aligned} \tag{9}$$

and the fixed point equations of the dynamics, $n(t+1) = n(t) = n$ are:

$$n = n^M + n^C; \tag{10a}$$

$$n^M = (1-f) \int_{\max[0,\pi^M_{min}(n)]}^{\max[0,\pi^M_{max}(n)]} \mathcal{P}(u)du \tag{10b}$$

$$n^C = f \int_{\max[0,\pi^C_{min}(n)]}^{\max[0,\pi^C_{max}(n)]} \mathcal{P}(u)du \tag{10c}$$

that must be solved for n. We call n^M the number of adopters who are mimetic and n^C those who are contrarian.

There are different regimes that have to be analyzed separately, depending on the signs of the extreme payoffs at $n = 0$ and at $n = 1$. To illustrate this point we show on Fig. 5 the extreme payoffs as a function of the number of adopters, n, for $d = 0.4$ and $u_0 = 0.5$. Blue lines are extreme payoffs for mimetics and red lines for contrarians. The area with positive payoff corresponds to the number of mimetics (blue ones) and contrarians (red ones) but one should make attention to the fact that these areas are weighted with the factors $(1-f)$ for mimetics, and f for contrarians. When $f = 0.5$ both areas have the same weight. In this case, starting with $n = 0$ the number of adopters after the first step is of the order of $n = 0.9$. But with such a high number of adopters most of the contrarians will defect the innovation and the number of adopters will fall to $n = 0.5$ in the second step. After that, the number of adopters increases again, and then decreases to finally converge to an intermediary value of $n = 0.6$. These decaying oscillations converging to $n = 0.6$ are also observed in the numerical results, on Fig. 3(b)

We consider now the points where the extreme payoffs change sign by solving Eq. (10) for the payoff equal to zero:

$$\begin{aligned} n_1 &= -d - u_0 \\ n_2 &= -d + u_0 \\ n_3 &= d + u_0 = -n_1 \\ n_4 &= d - u_0 = -n_2 \end{aligned} \tag{11}$$

Two of these points, n_2 and n_3, are also indicated on Fig. 5; $\pi^M_{max}(n)$ is always positive and $\pi^C_{min}(n)$ always negative, so n_1 and n_4 are both negative and are not solutions. In the general case we can state that $u_0 > 0$, $n_2 > n_1$ and $n_3 > n_4$, but depending on the relative values of d and u_0, n_3 may be larger or smaller than n_2. Here we would like just to analyze the two cases that we have simulated numerically: $d = 0.4$ and $d = -0.2$, both with $u_0 = 0.5$. In the first case ($d = 0.4$) one has the

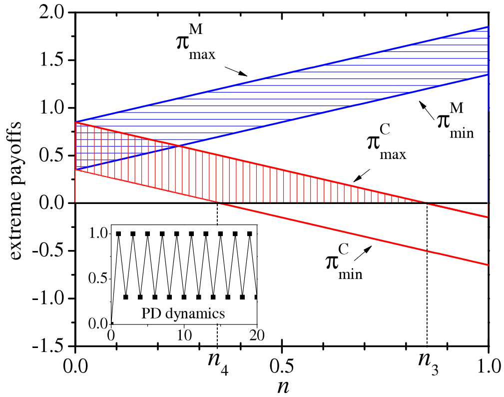

Fig. 6. Extreme payoffs as a function of the number of adopters, n, for the uniform distribution with $d = 0.6$, $u_0 = 0.25$. π lines are extreme payoffs, blue lines for mimetics and red lines for contrarians (given by Eq. (9)). Inset: Numerical results of the fraction of adopters as a function of time in the parallel update dynamics for $N = 10^7$ ($d = 0.6$, $u_0 = 0.25$, and $f = 0.7$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

following boundaries:

$$\begin{aligned}\pi^M_{\max}(n) &= 0.9 + n\\ \pi^M_{\min}(n) &= -0.1 + n\\ \pi^C_{\max}(n) &= 0.9 - n\\ \pi^C_{\min}(n) &= -0.1 - n.\end{aligned} \tag{12}$$

Those boundaries are the ones plotted on Fig. 5. The boundary $\pi^M_{\max}(n)$ is always positive, $\pi^M_{\min}(n)$ is positive for $n > 0.1$, $\pi^C_{\max}(n)$ is positive if $n < 0.9$ and $\pi^C_{\min}(n)$ is always negative, so in the corresponding contrarian integral (Eq. (10c)) the lower bound is always 0. The fixed point can be evaluated using Eqs. (10). Considering $f = 0.5$ in those equations, it is easy to verify that the equilibrium is in the region $0.1 \le n \le 0.9$ and the result is $n \approx 0.63$, that coincides very well with the asymptotic limit shown in Fig. 3(b). For $f = 0.9$ the asymptotic value is $n \approx 0.48$ that also coincides with the numerical result (Fig. 3(c)). The figure also explains the oscillations before attaining the fixed point, as described above.

In the case with $d = -0.2$, the boundaries are:

$$\begin{aligned}\pi^M_{\max}(n) &= 0.3 + n\\ \pi^M_{\min}(n) &= -0.7 + n\\ \pi^C_{\max}(n) &= 0.3 - n\\ \pi^C_{\min}(n) &= -0.7 - n.\end{aligned} \tag{13}$$

Now, the boundary $\pi^M_{\max}(n)$ is always positive, $\pi^M_{\min}(n)$ is positive for $n > 0.7$, $\pi^C_{\max}(n)$ is positive if $n < 0.3$ and $\pi^C_{\min}(n)$ is always negative. Let us examine the case $f = 0.9$. If one assumes a trial value, n_t, restricted to $n_t > 0.3$ there are no contrarians adopting and the number of adopters should be $n = (1 - f)n_t$ that is always lower than n_t, in contradiction with the hypothesis. So, n must be lower than 0.3 and the solution is $n \approx 0.16$ again in agreement with the simulations (see Fig. 4(c)).

Finally, and as we said, sustained oscillations can be obtained for a set of parameters such that $0 < n_4 \equiv d - u_0 < n_3 \equiv d + u_0 < 1$, provided that both, u_0 and d be small enough. We show in Fig. 6 the extreme payoffs obtained analytically for $d = 0.6$ and $u_0 = 0.25$. The interpretation of this figure is the same as the one done for Fig. 5. Moreover, the inset of Fig. 6 shows the numerical result for the same values of parameters and $f = 0.7$. It can be observed that the fraction of adopters as a function of time for parallel update dynamics present oscillations, in accordance with the analytical result.

4. Logistic distribution

While the uniform distribution is simpler than other distributions, the discontinuity at the borders generates some complications particularly for the analytic calculations. Also, one may imagine that the distribution of idiosyncrasies in a real society exhibit a concentration of values around the mean value and a relatively low concentration in the extremes. Taking these points into consideration one could envisage the use of a Gaussian distribution, as most of the values of the resistance to adopt will be distributed within a bell-shape of a few standard deviations width. However, the integral of the Gaussian is not analytic. So, to avoid the cumbersome complications raised by the uniform and Gaussian distributions,

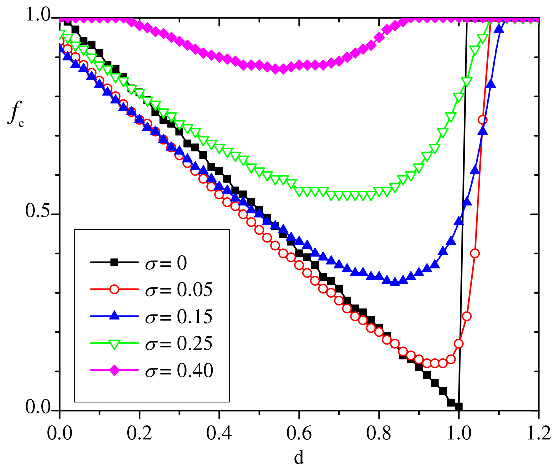

Fig. 7. Thresholds value of the fraction of contrarians above which oscillations appear. The curves correspond to different values of the standard deviation of the logistic distribution, σ, as indicated. We have represented just positive values of d as there are no oscillations for negative values. The curves go through a minimum that is lower the narrower the distribution. Notice that there are no oscillations when the advertising is slightly higher than $d = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

we consider hereafter a bell-shaped logistic distribution of the resistances to adopt, $\mathcal{P}(u_i)$, that is continuous and has infinite support. The probability density of the logistic distribution is:

$$\mathcal{P}(u) = \frac{\beta}{2\cosh^2(\beta\, u)}, \tag{14}$$

with its standard deviation given by $\sigma = \frac{\pi}{2\beta\sqrt{3}}$.

Following the scheme of the previous section, we present first the numerical results.

4.1. Numerical results

We have performed different simulations with the logistic distribution given by Eq. (14). The results are presented in Figs. 7–10. When the simulation is performed in parallel (PD), permanent oscillations may appear. This is the case for intermediate values of the advertising, d, and relatively high values of the fraction of contrarians, f. This can be verified in Fig. 7 where we have represented a threshold value of the fraction of contrarians f_c, above which oscillations appear, as a function of the advertising d. Notice that there are no oscillations for $d < 0$ or for high values of d. Oscillations are present in a region of values of d around $d = 1$, i.e. when the advertising is as strong as the social interaction; oscillations are cycles of period two and arise because contrarians adopt when the number of adopters is low, but abandon the innovation when the number of adopters is high. However some mimetics may follow the contrarian's behavior. The amplitude of the oscillations decreases when decreasing f or when d is smaller or bigger than 1 (see Fig. 7). The region where stable oscillations occur is larger the narrower the width of the distribution of idiosyncrasies, σ.

We have represented in Figs. 8 and 9 the time evolution of the number of adopters exhibiting the oscillations, when they happen, or the convergence to a fixed point when there are no oscillations. When performing Monte Carlo simulations there are no oscillations in none of the cases. Fig. 10 summarizes the numerical results for the logistic distribution. Red curves (dashed) correspond to the final number of adopters (fixed points) when performing MC simulations, while black curves correspond to Parallel Dynamics (PD). In the later case, oscillations may be observed above a critical value of the fraction of contrarians. When there are no oscillations, the results for PD and MC simulations coincide. When oscillations are present, PD results are different from MC results, and the plot shows bot the extreme amplitude of the oscillations and, in between, the average value of the number of adopters.

4.2. Analytic results

The dynamics of adoption, given by Eq. (7), is

$$n(t+1) = (1-f)\mathcal{F}(d+n(t)) + f\,\mathcal{F}(d-n(t)) \tag{15}$$

where $\mathcal{F}(u)$ is the cumulative distribution

$$\mathcal{F}(u) = \int_{-\infty}^{u} \mathcal{P}(x)dx = \frac{1}{1+e^{-2\beta u}}. \tag{16}$$

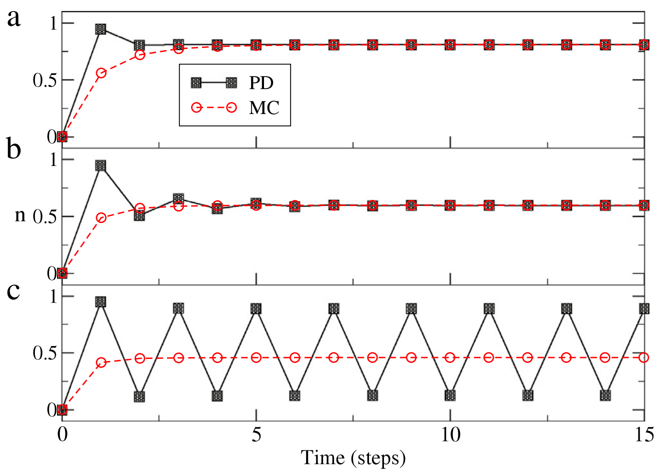

Fig. 8. Temporal behavior of the fraction of adopters for the logistic distribution with $\sigma = 0.25$, $d = 0.4$, and different values of the fraction of contrarians f: (a) $f = 0.2$, (b) $f = 0.5$, and (c) $f = 0.9$. Results are for a large number of agents, $N = 10^7$. Open red circles correspond to the MC simulations and black squares to PD simulations. In the PD case it is possible to see the oscillations in the number of adopters for a high concentration of contrarians. We have considered much longer times than those represented in the figure and the oscillations are stable. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

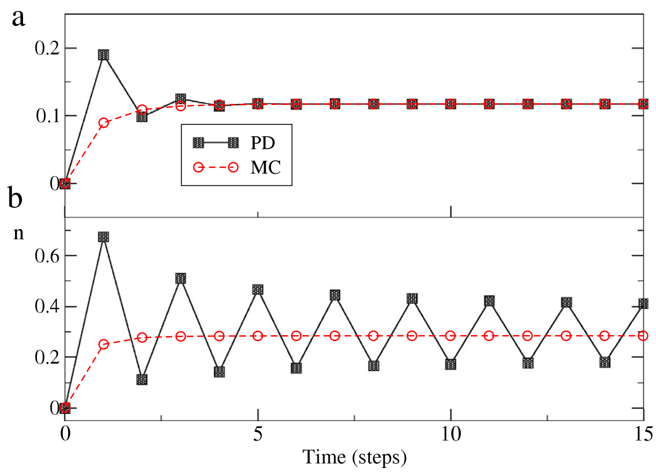

Fig. 9. Temporal behavior for the logistic distribution with $\sigma = 0.25$, $f = 0.9$ and two different values of the parameter d (the normalized effective marketing): (a) $d = -0.2$ and (b) $d = 0.1$. Oscillations in the number of adopters are obtained if $d > 0$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The fixed points may be obtained by solving the transcendental equation

$$n = y_1(n) = \frac{1-f}{1+e^{-2\beta(d+n)}} + \frac{f}{1+e^{-2\beta(d-n)}} \tag{17}$$

through the intersections of the function $y_1(n)$ with the line $y_2(n) = n$. Fig. 11 presents some examples for different parameter values.

Fig. 11 represents a plot of $y_1(n)$ and the line $y_2(n) = n$. The intersections correspond to the fixed points and are stable solutions provided that $y_1' \equiv \frac{dy_1}{dn} < 1$. However, solutions with $|y_1'| \equiv |\frac{dy_1}{dn}| > 1$ are unstable, and we are then obliged to consider a second iteration, i.e., $y_1(y_1(n))$. The solutions for this second iteration are represented on Fig. 12: if more than one intersection is present, the upper and lower intersections correspond to the extreme value of the oscillations.

The comparison between numerical and analytical solutions is discussed in detail in the caption of Figs. 11 and 12. We find a very good agreement of both solutions, then, there is no need of further discussion of this point. We will concentrate in the next section in the discussion of the results and comparison with a previous model [16].

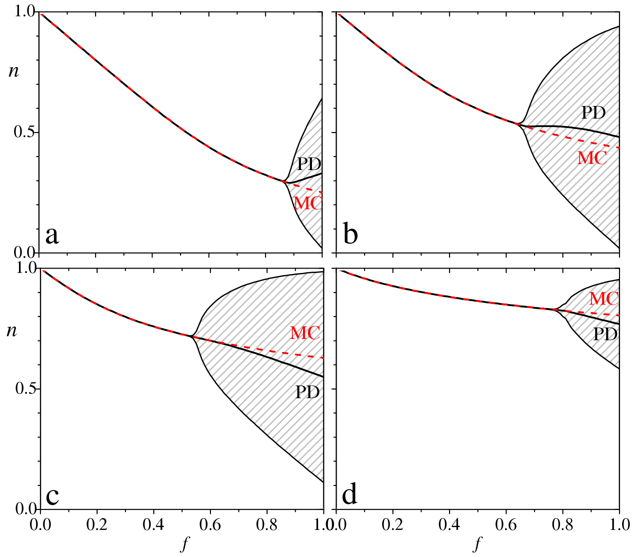

Fig. 10. This figure summarizes the numerical results for the logistic distribution of idiosyncrasies with $\sigma = 0.25$. All four panels exhibit the fraction of adopters as a function of the fraction of contrarians for four different values of d: (a) $d = 0.1$, (b) $d = 0.4$, (c) $d = 0.7$, and (d) $d = 1.0$. As expected, the number of adopters decreases when the number of contrarians increases. The red curves (dashed) correspond to Monte Carlo simulations and the black ones to a parallel dynamics. An oscillatory behavior is obtained only for parallel dynamics and the black lines correspond to the average value of the oscillations, while the shadowed areas indicates the amplitude of the oscillations. Both dynamics exhibit identical results for low and intermediate values of f, but there exists a critical value of f when the parallel dynamics exhibits period two oscillations. When increasing d the region of oscillations increases up to $d = 0.7$ and then decreases for $d = 1.0$. When $d < 0$ there are no oscillations and both dynamics produce the same results. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

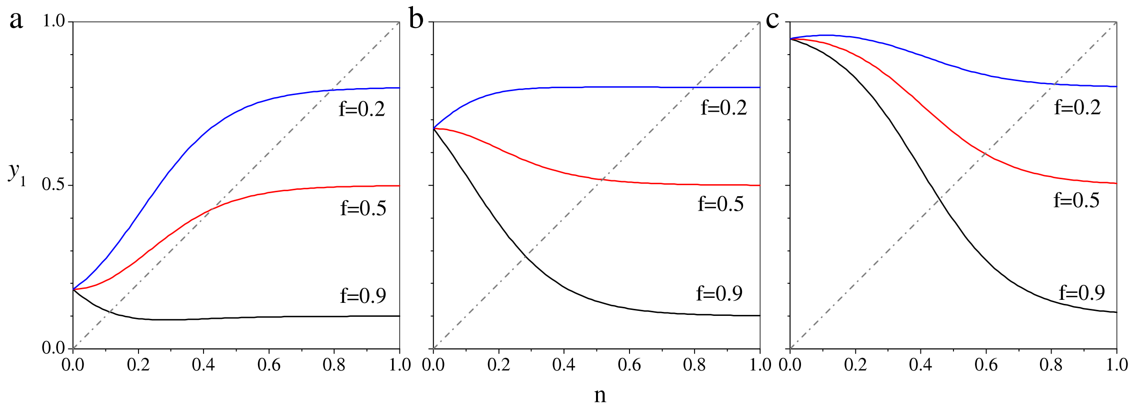

Fig. 11. Fixed points of $y_1(n)$. The fixed points correspond to the intersections of $y_1(n)$ and $y_2(n)$ (indicated by the dot-dashed gray line in the figure). When the absolute value of the derivative is lower than one, the solutions are stable and correspond to a fixed point of the dynamics. The three panels show different cases for different values of the parameter d of the logistic distribution of idiosyncrasies: (a) For $d = -0.2$ the derivatives at the intersections are always $|y_1'| < 1$, thus no oscillations are expected. (b) When $d = 0.1$ three possible stable intersections appear in each case, and the values roughly correspond to the numerical results plotted on Fig. 10(a). (c) When $d = 0.4$ and $f = 0.2$ the stable solution correspond to $n \approx 0.8$ that coincides with the numerical solution (see Fig. 10(b)). For $f = 0.5$, $n \approx 0.5$ that also coincides with both PD and MC simulations. Finally, for $f = 0.9$ the solution is unstable ($|y_1'| > 1$). However the fixed point corresponds to the value obtained with MC simulations (see Fig. 10), while PD simulations indicated the existence of oscillations. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5. Discussion and conclusions

An early microscopic model of decision making has been presented by Galam and Moscovici [17], using an Ising random field model, where they concluded that aggregation effects should be analyzed separately from the effects of the agents influence group. It is worth noting that the main ingredients of that early model, namely the external environment, the social influence, and the opinion divergency, are all represented in our model. In the present work, besides those basic elements, we focus on analyzing the combined effect of two important factors: the presence of contrarians and the possibility of repentance of the agents during the decision process.

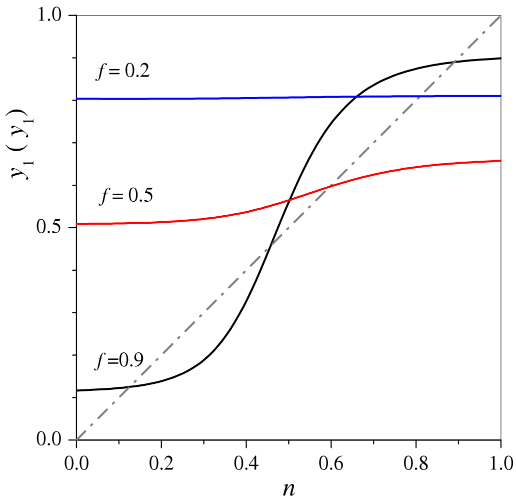

Fig. 12. Fixed points of $y_1(y_1(n))$ with $y_2(n)$ (gray dot-dashed line). We have plot just the case with $d = 0.4$. It is possible to observe that for $f = 0.2$ and $f = 0.5$ there is just one intersection, that corresponds to the stable solutions previously obtained. For $f = 0.9$ there are three intersections. The middle one corresponds to the fixed point of $y_1(n)$ while the other two represent the extremes of the oscillations. These extreme values are approximately 0.12 and 0.9 and correspond to the extreme value of the oscillations in the PD simulations, see Fig. 8(c). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In the previous sections we have discussed in detail the effect of repentance on the adoption of an innovation. Agents can adopt an innovation but they have the possibility of changing their minds and abandon the new technology or coming back to the old one, if any. We have also discussed in detail two distributions for the idiosyncratic resistance to adopt: a uniform distribution, that has been already utilized in Ref. [16], and a logistic one.

We have tried two different dynamics for the innovation model with repentance: Parallel Dynamics (synchronous update) and MC dynamics (asynchronous update). In the first case, PD, oscillations may appear which we have analyzed in detail in the present work. The oscillations are an effect of contrarians changing their minds when their payoffs become negative. One can consider these oscillations kind of artificial, because the PD implies a perfect synchronization of the population (all agents update their state simultaneously). While this is obviously unreal for a whole city or country, it could be possible for smaller groups like communities, neighborhoods, or internet groups. On the other side, the strict asynchronous update (MC), in which case no oscillations are observed, it is also artificial, because it means that each selected agent has an instantaneous complete knowledge of the system whence has to take its decision. So, both dynamics are equally artificial extremes of a reality which probably resides in some place in between. Indeed, we have checked some examples of an hybrid dynamics, in which a part of the population updates synchronously while the other part updates in MC fashion. There are oscillations for such an hybrid dynamics, even if, as expected, their amplitude decreases as the fraction of the population chosen for PD update decreases.

Another interesting point is that the temporal behavior of the adoption of innovations is very sensitive to the width of the distribution of the resistance to adopt u_i. In the case of a wide uniform distribution, as the one utilized in Ref. [16], damped oscillations appear as a transient state but, after a relatively short transient, the system converges to a fixed point. On the other side, with narrower uniform distributions of u_i, sustained oscillations appear, which are produced by the contrarians, whereas mimetic agents hardly change their decision.

For a bell shaped distribution, as it is the case of the logistic one presented in Section 4, we obtain similar results. That is, stable long term oscillations may appear for intermediate values of the advertising, d, or a large fraction of contrarians, f, as it is evident in Fig. 7. While a high number of contrarians may be unreal considering a novel technology, that could be the case regarding operating system choices, for example (*IOS* vs. *Android*). In any case, Fig. 7 shows that oscillations may appear with a relatively low fraction of contrarians, provided the advertising is strong: see for example that for a narrow distribution of idiosyncrasies ($\sigma = 0.05$) and for $d \approx 0.95$ the threshold is of the order of $f \approx 0.15$.

Finally, it is worth to remark that the coexistence of contrarians with the possibility of repentance makes the final total number of adopters lower than in the case with no regrets [16]. To check this point we have represented in Fig. 13 the present results for the uniform distribution of u_i together with those of Ref. [16]. It is possible to see that the shape is similar in both cases, but when the decisions are "reversible" the final adoption is lower than when not. To produce this comparison we considered the uniform distribution of idiosyncrasies because it was the one used in Ref. [16].

It is also important to note that the first oscillations are expected to be relevant ones, as it is not plausible that agents continue to change their minds *ad infinitum*. In any case, being done that the presence of contrarians plus repentance significantly reduces the final fraction of adopters, a direct consequence is that a stronger advertising campaign will be needed if it seeks to impose innovation.

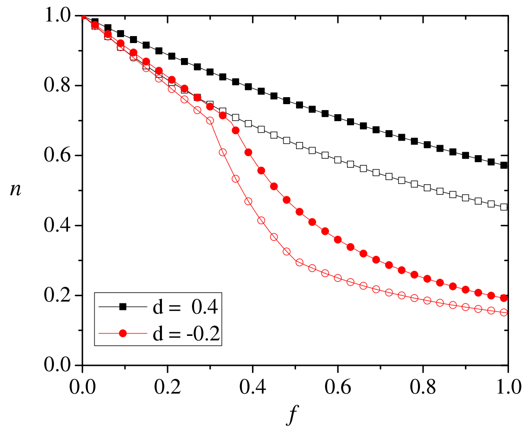

Fig. 13. Comparison between the results of Ref. [16] (without repentance) and the present ones with repentance: final number of adopters for two values of d ($d = 0.4$, squares, and $d = -0.2$, circles). Filled symbols correspond to no repentance and open ones with repentance (present contribution). Pairs of curves display similar behavior with always lower values of adoption for the case with repentance. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Summarizing, the effect of introducing repentance are oscillations, a decrease in the final number of adopters, but also an enhancement of the role of contrarians. Contrarians *per se* reduce the number of adopters. But if in addition they can change their opinions, they reduce the number of adopters even further, and such reduction may induce a change of opinion also in fraction of mimetic agents. Therefore, the presence of contrarians plus the possibility of repentance have an amplified effect of reducing the final number of adopters. Also, oscillations may appear when the distribution of resistances to adopt is smaller than 1, *i.e.* smaller than the social interaction, J, and such cycles are only possible if both features, contrarians and repentance, are present.

Work in progress include the presence of impulsive agents, i.e., people that can change their technology without assessing whether it is convenient. Preliminary results show that the introduction of such agents in the model accelerates the adoption process and increases the total number of adopters. In other words, a small number of impulsive agents in a society could be as efficient as a strong advertising. We are also considering a dynamic distribution of idiosyncrasies, the effect of distributing the agents on a network, and a non-linear term of social interaction that may describe the effects of fashion: people adopt a new fashion when there are a few followers but abandon when the number of adopters increases.

Concluding, we would like to point out that, despite its simplicity, the present microscopic model reproduce some known features of the innovation adoption process while given a venue for the study of more realistic cases.

Acknowledgments

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