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# Improvements on multivariable control strategies tested on the Petlyuk distillation column

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## HIGHLIGHTS

- ▸ New multivariable methodology for interacting chemical plants.
- ▸ Computer aided tools for supporting the calculations.
- ▸ Multivariable tuning method for testing the final control structure.
- ▸ Improvements for self optimizing control method.
- ▸ Comparisons with other strategies for Petlyuk column.

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## ABSTRACT

In this work, a multivariable control methodology, named minimum square deviation (MSD), is applied to the challenging Petlyuk distillation column. This process is able to separate a ternary mixture in the same shell reducing the capital investment cost respect to that needed if a classic layout of two integrated columns is used. By this way, the energy consumption can be reduced up to 30%. However, some problems have been reported, caused by the intrinsic coupling among the control loops implemented on this column. The MSD method selects a control structure systematically by minimizing the sum of square deviations (SSD) and net load evaluation (NLE) indexes. From SSD procedure a diagonal controller structure is achieved. The NLE stage is a valuable tool for deciding which is the best control configuration among full, diagonal or sparse. It depends on the weighting factors which quantify the relative importance between servo and regulator problems. On the other side, a well recognized multivariable control strategy, named self optimizing control (SOC), was previously implemented on the same Petlyuk distillation column. Different controlled variables were selected by applying singular value and null space methods which gave different decentralized control structures. Thus, the Petlyuk distillation column represents a proper example to highlight MSD achievements. Then, the first contribution of this work is to quantify the skills of both methods, MSD and SOC, through the use of the same complex case. The second contribution is to propose some improvements on SOC technique. The third contribution is to obtain new efficient control alternatives for the Petlyuk distillation column. Then, to give support to the final conclusions, a complete set of simulations and quantitative comparisons is given for several control structures and scenarios.


## 1. Introduction

Several researchers have developed multivariable control methodologies, in particular, those dedicated to systematic plant-wide control applied to chemical processes. These methodologies can be based on engineering experience (more heuristic) or based on mathematical support (Larsson and Skogestad, 2000). Furthermore, there is a third category connecting both mathematical and heuristic concepts (Skogestad, 2000, 2003; Alstad and Skogestad, 2007). These multivariable control strategies were implemented, compared and discussed in Suraj Vasudevan et al. (2009) for a vinyl acetate monomer plant. A brief review of the most important approaches included in these groups is given in the following paragraphs.

A pioneering work based on heuristic considerations is presented by Buckley (1964). The main issues on this area were introduced there. Similarly, another relevant work is the book of Luyben et al. (1998) which presented a nine-step approach based

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on engineering experience. More recently, Konda et al. (2005) proposed an integrated framework of simulation and heuristics to support the decisions. All of these previous methods are systematic in nature and represent a particular way to obtain acceptable control structures (CSs).

The second category relies on a rigorous mathematical framework based on dynamic systems theory, stability and sometimes optimization analysis too. These approaches are difficult to implement on large dimension processes because they can become very intensive computationally. Moreover, modeling errors affect the convergence and the solution. Nowadays, computing power favors the development in these areas and some medium-scale problems can be addressed properly. For example, Cao and Saha (2005) and Cao and Kariwala (2008) and more recently Kariwala and Cao (2010) proposed the use of combinatorial algorithms based on classical as well as bidirectional branch and bound routines for screening several control structures and CVs selection. An approach based on splitting the optimal controller is presented in Robinson et al. (2001). Assali and McAvoy (2010) proposed an approach based on optimal control to define dominant measurements and manipulated variables. Jørgensen and Jørgensen (2000) suggested a special mixed integer linear programming (MILP) tied to Parseval's theorem combined with relative gain array (RGA) and internal model control (IMC) concepts. Chen et al. (2011) proposed a method based on IMC theory and optimization for designing Smith predictor in a multivariable process.

The methodology of self-optimizing control (SOC) belongs to the third category mentioned above. This technique is thought to maintain the process working at its optimal state using linear combinations of measurements as CVs to reject disturbances effects.

It is important to note that most of the aforementioned works reported the use of the classic relative gain array (RGA) (Bristol, 1966) for input–output pairing purposes which drives to a decentralized control structure. In fact, several authors have analyzed the RGA properties, its implications on control performance, and its drawbacks when the process is ill-conditioned or close to the singularity (Grosdidier et al., 1985; Garcia and Morari, 1985; Skogetad and Morari, 1987; Skogestad and Postlethwaite, 2005). Other authors have proposed modifications to handle non-square processes (Chang and Yu, 1990; Khaki-Sedigh and Moaveni, 2009; Skogestad and Postlethwaite, 2005), disturbances (Chang and Yu, 1992, 1994; Lin et al., 2009) and dynamic information (McAvoy et al., 2003; He et al., 2009). A complete review and trends were given in Khaki-Sedigh and Moaveni (2009). In Yuan et al. (2011) was presented a technique for manipulated variables (MVs) selection, based on controllability for the multivariable chemical process. A criteria for measurements selection accounting only steady-state information and SOC was proposed in Hori et al. (2005) and Hori and Skogestad (2008). A systematic approach for MIMO controller design with different interaction levels was proposed in Shen et al. (2010) using the relative normalized gain array (RNGA) (He et al., 2009). All of these works demonstrate that multivariable control for chemical processes is still an open research field. The continuous emergence of new strategies is a clear demonstration of this issue.

The recently appeared minimum square deviation (MSD) methodology suggested by Molina et al. (2011) can be considered in the third category. The MSD approach can be divided in two sequential combinatorial problems addressing the CVs selection and controller design. The former is based on minimizing the sum of square deviations (SSD) of the uncontrolled variables (UVs) from their operating points assuming the perfect control for the CVs (Chang and Yu, 1990; Hori et al., 2005; Skogestad and Postlethwaite, 2005). From this step a diagonal control structure can be obtained. The MSD is extended with the net load evaluation (NLE) index to define a new multivariable control structure taking into account change of references and disturbances effects on CVs. This last stage can provide proper sparse control structures.

It is important to note that the SSD index is directly related to the non-square relative gain (NRG) array, suggested by Chang and Yu (1990) and perfect SS indirect control opportunely proposed by Hori et al. (2005). Hence, the SSD approach is used here as a tool for defining a proper CVs selection. Therefore, the main contribution of the MSD procedure is done by the use of the NLE index, which extends and generalizes the approach presented by Chang and Yu (1994). Hence, the new NLE scalar index allows to evaluate and sort different controller structures designed in the context of internal model control (IMC) theory. In addition, a stability test at steady-state (Garcia and Morari, 1985) to focus the search towards a feasible set of solutions is included. This work is an extended version of a previous one, Zumoffen et al. (2011), where MSD is applied in the Petlyuk distillation column. The Petlyuk column, also called the divided wall column (DWC), is a very good example of a process which requires on line optimization in order to obtain the potential energy savings in practice. However, there are only few implementations in the world because of their designs and operational difficulties. In fact, it represents an attractive case because its non-optimal operation may easily yield to worse performance, in terms of energy consumption, compared with the conventional arrangement of two integrated columns. Therefore, finding an efficient control structure (CS) for achieving a correct operation of this process represents a real challenge. Besides, since MSD and SOC belong to the third category, it is considered that implementing both methods on the same Petlyuk column with the same control objectives is the best way to rigorously evaluate the NLE potentiality. Then, the diagonal controller structures, obtained by SOC are taken into account as starting point to apply NLE analysis.

This paper is organized according to the following sequence: in Section 1 the MSD approach is introduced in the context of a deep review about other multivariable control methodologies. Section 2 gives information about several control structures proposed in the past specifically for different ternary mixtures separated by Petlyuk distillation columns. Section 3 gives the specific details about the Petlyuk distillation column used here together with the operation goals. In addition, in Section 4 the CSs presented in the literature concerning the SOC methodology are included to explain why particular combinations of CVs are chosen for doing the main comparisons with the MSD results. Section 5 gives a brief overview about the MSD strategy and the multivariable controller tuning used for the final dynamic tests. In Section 6 the application results obtained from each step of the MSD approach given here for the Petlyuk column are discussed. In Section 7, a complete set of simulations is given, based on some recommendations about how the SOC results could be improved. Interesting comparisons with the best control policies obtained by SOC implementation for this system are taken into account. The integral absolute error (IAE) indexes are shown for quantifying the achieved results. Finally, the conclusions are drawn for evaluating the real improvement achieved by the proposals recommended here together with future works are discussed in Section 8.

## 2. A brief review about control strategies applied on the Petlyuk column

The first work about control strategies for the DWC was presented by Wolf and Skogestad (1996) where a separation of ethanol, propanol and butanol was reported. They showed that the Petlyuk column has serious problems of operation and control, at least for high purity separations. Moreover, they proved that it is unfeasible to control four compositions. Servo and regulator performances were tested in that paper. Ling and Luyben (2009) worked with an industrially important ternary separation of benzene, toluene, and o-xylene (BTX) and compared the performance of DWC and two conventional integrated columns. Ling and Luyben (2009) recognized that the reported results presented a somewhat confusing picture, and they attempted to add more clarity for those problems. They proposed to control the composition of the heaviest product on the top of the pre-fractionator because it implicitly minimizes the energy consumption against composition disturbances. In addition, the impurities in the three product streams were controlled too. The manipulated variables were: liquid split, reflux flow rate, sidestream flow rate, and vapor boilup. This control structure was obtained from a deep knowledge of the process, taking into account their large experiences about successful control strategies for conventional distillation columns and a sensitivity analysis. They tested via dynamic simulations using rigorous distillation column models in Aspen Plus. In Ling and Luyben (2010) other alternative for controlling the same DWC was proposed. It consisted on exploring the use of temperatures to avoid expensive and high-maintenance composition analyzers. They found that four differential temperature control loops and the four manipulated variables such as reflux flow rate (R), sidestream flow rate (S), reboiler heat input (QR), and liquid split at the top of the wall provided good results for the most frequent disturbances. Kiss and Rewagad (2011) arrived to the same conclusion given by Ling and Luyben (2009) applied to a BTX-DWC system through rigorous simulations, carried out in Aspen Plus and Aspen Dynamics. They used the RGA number for all the frequencies for analyzing the dynamic behavior. They compared several conventional control structures based on PID control loops (DB/LSV, DV/LSB, LB/DSV, LV/DSB). More recently, Rewagad and Kiss (2012) explored dynamic optimization and advanced control structures based on MPC for a BTX-DWC. They concluded that MPC-based approach produced a significant increase in performance comparing to the classical decentralized PID strategies. Serra et al. (2001) have studied three systems and proposed several control structures (CSs) for the Petlyuk column with different relative volatilities. They analyzed several inventory control policies using linear analysis. In addition, they decided to apply dynamic matrix control (DMC) for controlling the three compositions. They obtained poor dynamic responses for disturbances rejection in comparison with a decentralized structure. Finally they concluded that PID gave better performance for perturbation treatment. In Wang and Wong (2007) the energy efficiency and controllability of a high-purity DWC system for separating ethanol, n-propanol, and n-butanol were addressed. They showed that high energy efficiency can only be obtained in a region with suitable combinations of liquid and vapor split ratios. Outside this region, the temperature profiles of the column sections were quite different. Thus, a temperature control was recommended to handle the disturbances.

The SOC approach (Skogestad, 2000) was analyzed and applied to a Petlyuk column for separating a ternary mixture of A, B and C components in the thesis of Halvorsen (2001). Here, the same Petlyuk column will be recognized as ABC-DWC. The SOC strategy is a general method for selecting CVs in order to obtain close top optimal operation based on a general profit criterion. This technique allows to keep near-optimal steady-state operation, with constant setpoints for the CVs, without needing to re-optimize when new disturbances affect the plant. In Alstad and Skogestad (2007) SOC was applied on the same ABC-DWC. They defined the CVs based on the null space method which yields locally optimal CVs as linear combinations of the available measurements. The requirement is to have at least the same number of measurements and the unconstrained degrees of freedom, including disturbances. It implies to obtain the optimal sensitivity matrix **F**, with respect to specific disturbances, taking into account a steady-state model of the plant. Additionally, the optimal matrix **H**, that satisfies $\mathbf{HF}=\mathbf{0}$; must be found in the left null space of **F**. Particularly, in Alstad and Skogestad (2007) SOC was used to obtain a good control structure for the Petlyuk distillation column mentioned above. They found an unconstrained nominal optimum by minimizing the energy consumption ($V$) to evaluate an optimal cost. Then, they arrived to four control alternative structures by applying null space method. To quantify the difference between these alternative control policies, they evaluated the loss of energy as it was defined in Halvorsen and Skogestad (2003). Hence, for a given disturbance $d$, the loss of energy was defined as the difference between the actual cost and the optimal cost.

In this work the MSD approach, introduced by Molina et al. (2011), is applied on the same challenging case of the Petlyuk column (ABC-DWC), given in Alstad and Skogestad (2007). They selected the SOC structures to guarantee the minimum vapor consumption which are briefly described in Section 4. Thus, to rigorously evaluate different control structures obtained by both techniques (SOC and MSD), it was considered necessary the use of the same code of the Petlyuk column model, implemented in MATLAB, as a benchmark. It allows performing a deep analysis about the potentiality of control structures such as decentralized, full and sparse.

## 3. Case study: Petlyuk distillation column

The Petlyuk distillation column (ABC-DWC) is shown in Fig. 1. It has six sections arranged in the same column shell with eight stages for each section. The ternary feed consists of components $\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$ with mole fraction $\mathbf{z}^T=[z_A,z_B,z_C]$, component $\mathbb{A}$ being the most volatile and component $\mathbb{C}$ the least volatile. The feed point is located between Sections 1 and 2 in the pre-fractionator. Ideally,

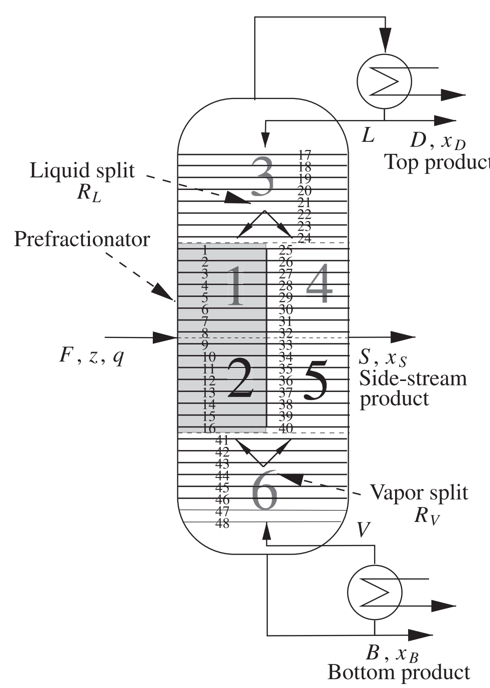


**Fig. 1.** Petlyuk distillation column.

**Table 1**
Study case data.

| | |
|---|---|
| **Column data** | |
| Relative volatilities | $\alpha^T=[9\ 3\ 1]$ |
| Liquid time constant | $\tau_L=0.063$ min |
| Holdup top and bottom | $M_B=M_D=20M_i$ |
| Holdup stages | $M_i=1$ kmol |
| Boiling point $\mathbb{A}$,$\mathbb{B}$, $\mathbb{C}$ | $T_B^T=[299.3\ 342.1\ 399.3]$ K |
| Antoine's parameters | $[2.86\ -1143\ -0.349]$ |
| **Feed** | |
| Flow | $F=1$ kmol/min |
| Composition | $z^T=[0.33\ 0.33\ 0.33]$ |
| Liquid fraction | $qF=0.477$ |
| Product specifications | $x_{A,D}=x_{B,S}=x_{C,B}=0.97\pm0.01$ |
| **Measurements delays** | |
| Compositions | 5 min |
| Flows | 1 min |
| Temperatures | 1 min |

components $\mathbb{A}$ and $\mathbb{B}$ (rich in $\mathbb{A}$) go over the top of the pre-fractionator, while a mixture of $\mathbb{B}$ and $\mathbb{C}$ (rich in $\mathbb{C}$) leaves the bottom. In the main Sections 3–6, three product streams are drawn off. The light component $\mathbb{A}$ with impurity $\mathbb{B}$ dominates the distillate stream $D$, component $\mathbb{B}$ dominates the side-stream $S$ while heavy component $\mathbb{C}$ with impurity $\mathbb{B}$ dominates the bottom stream $B$. The boilup ($V$) and reflux ($L$) streams are split over the dividing wall with split fractions $R_L$ and $R_V$, respectively, where $R_L=L_1/L_3$ and $R_V=V_2/V_6$, being $L_i$ and $V_i$ the molar liquid flow and vapor flow in section $i$, respectively.

For the ABC-DWC, three product specifications must be accomplished during the operation: (1) distillate purity ($x_{A,D}$), (2) bottom purity ($x_{C,B}$) and (3) side-stream ($x_{B,S}$), where $x_{i,j}$ denotes the mole fraction of component $i$ in stream $j$. In side-stream there are two impurities, so it is necessary to specify one of these, resulting in four purity specifications. However, based on the conclusions given in Wolf and Skogestad (1996) it is not possible to control properly the four compositions because it drives to some column operation problems.

The ABC-DWC model and all the related information was obtained from the Skogestad's web page (http://www.nt.ntnu.no/users/skoge). The equipment and the process requirements are detailed in Table 1.

## 4. Previous control structures considered for comparison purposes

In this section a deep analysis about the previous control structures proposed for the ABC-DWC case is done. This process has been studied by the Skogestad's group since more than ten years. Then, it is considered that important data exists to compare and obtain valuable conclusions about the achieved results. Mainly, the three most successful decentralized CSs, proposed in Alstad (2005) were selected here to be compared with the MSD results. As can be seen in Table 2, all these CSs suggest to control four variables, three of them correspond to the most important compositions paired with the same MVs. The only difference among the three structures is the fourth CV selection. Hence, in Table 2, structures $T_4-d$, $c_{DTS}-d$ and $c_{ODF}-d$ refer to the following selected fourth CV: (i) temperature $T_4$ (CS11), (ii) combination of variables named $c_{DTS}$ (CS3) and (iii) combination of variables given by null space method $c_{ODF}$ (CS2), respectively. The names between parenthesis correspond to those used for identifying the same control structures in Alstad (2005). Since all of CSs are diagonal, they will be recognized by ending the name with "$d$". Structure $T_4-d$ is proposed using the singular value method given in Skogestad and Postlethwaite (2005). It is based on ranking the candidate variables according to a large scaled steady-state gain (minimum singular value in the multiple input case). Since it was found that for some applications, there may not exist any SOC with single measurements, then, structure $c_{DTS}-d$ is proposed to consider measurements combination as CVs. In Halvorsen (2001) is recommended to use the average difference of temperature profiles on each side of the ABC-DWC (symmetric measurements), identified as $DTS$. The calculation of this variable can be done in the following way:

**Table 2**
PI-controller parameters.

| Control structure information | MV | CV | $K_C$ | $\tau_I$ |
|---|---|---|---|---|
| Loops common to all structures | $L$ | $x_{A,D}$ | 2.7 | 80 |
| | $V$ | $x_{C,B}$ | 2.1 | 110 |
| | $S$ | $x_{B,S}$ | −2.8 | 82 |
| Structure $T_4-d$ | $R_L$ | $T_4$ | −0.034 | 33 |
| Structure $c_{DTS}-d$ | $R_L$ | $c_{DTS}$ | −0.061 | 34 |
| Structure $c_{ODF}-d$ | $R_L$ | $c_{ODF}$ | −0.0175 | 21 |

$$c_{DTS}=(T_4-T_{28})+(T_{12}-T_{36}) \quad (1)$$

This combination is proposed after analyzing the steady-state behavior of the process. It is demonstrated that $c_{DTS}$ represents a good candidate to be a CV because it allows to keep the process close to the optimum when some disturbances are present. In addition, $DTS$ was observed to be constant along the direction of the minimum surface of $V(R_L, R_V)$. Hence, $DTS$ was evaluated as the best candidate based on the availability of temperature measurements and for having good self-optimizing properties.

The work of Alstad and Skogestad (2007), is also focused on the steady-state operation of the column. They proposed two possible sets of CVs which allow to keep the process near to the optimal point. One of these sets consists of three product compositions and a linear combination of temperature measurements. This selection was performed using the null space method. The resulting linear combination is controlled via the diagonal structure named by the authors $c_{ODF}-d$

$$c_{ODF}=0.448T_5-0.658T_{10}+0.421T_{12}-0.0471T_{27} -0.388T_{37}+0.192T_{43} \quad (2)$$

With respect to the available manipulated variables, in Halvorsen (2001) is recommended to keep $R_V$ with a fixed value, obtained by optimizing the process operation in the flat region. Generally, $R_V$ is fixed in the equipment during the design stage (Ling and Luyben, 2009) because it is very difficult to be manipulated in practice. Therefore, in all cases the fourth CV is controlled by manipulating $R_L$. In Alstad (2005) the best CSs are selected after analyzing operational objectives and active constraints, optimal operational point, energy losses using the non-linear model for the most significant disturbances, controllability and the impact of measurement errors. Finally, dynamic simulations with the non-linear model under control with the most promising CSs were done. Indeed, PI-based decentralized control structures were implemented and tuned using the recommendations presented by Skogestad (2003). Alstad (2005) checked dynamically the vapor consumptions and the main product compositions behavior. However, these dynamic simulations only show one disturbance scenario (for $z_A$ variations) because they found similar conclusions for the other disturbances. Based on both steady-state and dynamic performance, he concluded that structure $c_{ODF}-d$ (CS2) was the best self-optimizing control. Therefore, the selected diagonal CSs tested in that work and presented in Table 2, and implemented here for the sake of comparison.

## 5. Minimum square deviations approach: an overview

In the following subsections details about the MSD approach and its link with some existing methodologies are described. Recently the minimum square deviations (MSD) strategy for plant-wide control was introduced by Molina et al. (2011) in a particular application case, the Tennessee Eastman (TE) process. On the other hand, it was also presented succinctly in Zumoffen et al. (2011) for the Petlyuk distillation column. Hence, in the following a brief description of this methodology is given and then applied to the Petlyuk column in detail. Moreover a complete set of dynamic simulations and performance indexes are presented to support the proposed design.

### 5.1. Controlled variables selection

The outputs of the system are given by

$$\mathbf{y}(s) = \mathbf{G}(s)\mathbf{u}(s) + \mathbf{D}(s)\mathbf{d}_*(s) \tag{3}$$

where $\mathbf{G}(s)$ and $\mathbf{D}(s)$ are the process transfer functions (matrices) TFMs of $m \times n$ with $m > n$ and $m \times p$, respectively; $\mathbf{y}(s)$, $\mathbf{u}(s)$ and $\mathbf{d}_*(s)$ are the output, input and disturbance vectors of $m \times 1$, $n \times 1$ and $p \times 1$, respectively. The plant model can be divided in two subsystems: one square, which includes the $n$ output variables to be controlled (CVs); and other generally non-square, which includes the uncontrolled variables (UVs). Mathematically

$$\mathbf{y}_s(s) = \mathbf{G}_s(s)\mathbf{u}(s) + \mathbf{D}_s(s)\mathbf{d}_* \tag{4}$$

$$\mathbf{y}_r(s) = \mathbf{G}_r(s)\mathbf{u}(s) + \mathbf{D}_r(s)\mathbf{d}_* \tag{5}$$

where $\mathbf{G}_s(s)$, $\mathbf{D}_s(s)$, $\mathbf{G}_r(s)$ and $\mathbf{D}_r(s)$ are the process TFMs of $n \times n$, $n \times p$, $(m-n) \times n$ and $(m-n) \times p$, respectively. This generalized process is shown in Fig. 2 which is controlled via internal model control (IMC), where $\tilde{\mathbf{G}}_s(s)$ is a simplified model of $\mathbf{G}_s(s)$ and $\mathbf{G}_c(s)$ the controller transfer function.

The CVs are selected by accounting perfect control at steady-state. Considering that $n$ variables are perfectly controlled, the sum of square deviations of the remaining $m-n$ UVs is minimized respect to their operation points when disturbance and set point changes are considered. Hence, at steady-state ("$s$" is not written) the process outputs can be expressed as

$$\mathbf{y}_s = \mathbf{y}_s^{sp} \tag{6}$$

$$\mathbf{y}_r = \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{y}_s^{sp} + (\mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s)\mathbf{d}_* = \mathbf{S}_{sp}\mathbf{y}_s^{sp} + \mathbf{S}_d\mathbf{d}_* \tag{7}$$

with $\mathbf{G}_c = \tilde{\mathbf{G}}_s^{-1}$, $\mathbf{S}_{sp} = \mathbf{G}_r\mathbf{G}_s^{-1}$ and $\mathbf{S}_d = \mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s$.

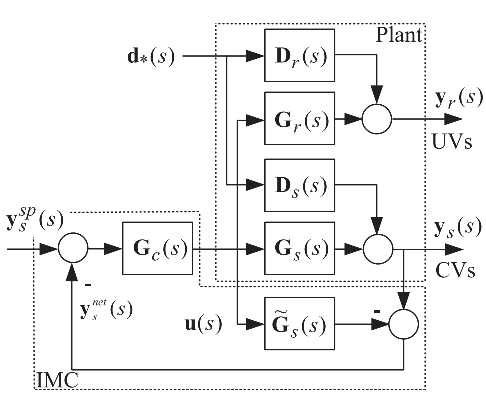


**Fig. 2.** IMC—control structure.

As it can be seen from Eqs. (6) and (7), the deviation of the outputs at steady-state when disturbances and set-point changes are present only depends on the specific selection of the CVs for a given plant. In other words, the main objective is finding the set of CVs that minimizes the UVs deviations

$$\text{SSD} = \sum_{i=1}^{n} \| \Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{sp}^n(i) \|_2^2 + \sum_{j=1}^{p} \| \Theta_2 \mathbf{S}_d \Theta_1 \mathbf{d}_*^p(j) \|_2^2 = \| \Lambda_2 \mathbf{S}_{sp} \Lambda_1 \|_F^2 + \| \Theta_2 \mathbf{S}_d \Theta_1 \|_F^2 \tag{8}$$

where $\Lambda_2 \mathbf{S}_{sp} \Lambda_1 \mathbf{y}_{sp}^n(i)$ and $\Theta_2 \mathbf{S}_d \Theta_1 \mathbf{d}_*^p(j)$ are the vectors of deviations corresponding to the $y_r$ outputs from their nominal operating point values when a unitary change happens in the $i$th set point and $j$th disturbance, respectively. The diagonal weighting matrices $\Lambda_1$ $(n \times n)$ and $\Theta_1$ $(p \times p)$ allow to include the process control objectives such as set point/disturbance magnitudes (this is important when the process model used is not normalized), similarly, $\Lambda_2$ $((m-n) \times (m-n))$ and $\Theta_2$ $((m-n) \times (m-n))$ take into account the relative degree of importance among the overall outputs. In addition, $\| \cdot \|_2$ represents the 2-norm for vectors and $\| \cdot \|_F$ the Frobenius norm for matrices. Thus, using a suitable parametrization of $\mathbf{G}_s$, based on the available measurements, the combinatorial problem can be stated as

$$\min_{\mathbf{G}_s}(\text{SSD}) \quad \text{subject to } \det(\mathbf{G}_s) \neq 0 \tag{9}$$

this minimization procedure has to evaluate $m!/(n!(m-n)!)$ combinations. The proposed methodology selects $n$ rows from $\mathbf{G}_{m \times n}$ to build $\mathbf{G}_s$ that minimizes the SSD index. The constraint, $\det(\mathbf{G}_s) \neq 0$, avoids the selection of unfeasible solutions from the IMC point of view. A similar methodology was successfully used by Molina et al. (2009) to determine the controlled variables for the TE process. In that work some valuable properties of $\mathbf{G}_s$ of Eq. (9) were discussed. Some preliminary results were also introduced in Zumoffen et al. (2010, 2011).

### 5.2. Controller structure design

Assuming that the CVs selection problem was solved successfully in the previous step (i.e. $\mathbf{G}_s(s)$ was selected), the next stage is to define the input-output pairing, i.e. the controller structure design. In fact, the possibilities are, diagonal (i.e. decentralized/without interaction), full (i.e. centralized/full interaction) or sparse (i.e. partial interaction). Here, the concept "interaction" refers to the controller topology itself. The closed-loop system always presents some global interaction degree due to the process. The MSD approach (Molina et al., 2009; Zumoffen et al., 2010, 2011) suggests, initially, two ways for obtaining a decentralized control structure: (1) based on RGA or (2) based on the emerging RNGA (He et al., 2009) which only can be implemented if dynamic information is available. However, it is important to note that the RNGA approach may give an erroneous pairing as it was pointed in Zumoffen et al. (2011).

The MSD approach includes the net load evaluation (NLE) strategy (Chang and Yu, 1992; Molina et al., 2009; Zumoffen et al., 2010) which allows handling servo and regulator problems according to the plant requirements. The NLE tries to minimize the disturbance effects over the CVs. The main concepts for the NLE are based on the GRDG approach given by Chang and Yu (1992). It was extended for including set point effects and to be calculated as a new scalar index, that is, in terms of the sum of square deviations. This new procedure was very helpful to deal with large-scale complex process in a generalized way. Hence, returning to Fig. 2 and the main mathematical statements given above, the controlled outputs can be expressed in Laplace domain as

$$\mathbf{y}_s(s)=\tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s)\mathbf{y}_s^{sp}(s)+(\mathbf{I}-\tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s))\mathbf{y}_s^{net}(s) \tag{10}$$

where

$$\mathbf{y}_s^{net}(s)=\mathbf{A}(s)\mathbf{y}_s^{sp}(s)+\mathbf{B}(s)\mathbf{d}_*(s) \tag{11}$$

$$\mathbf{A}(s)=[\mathbf{I}+(\mathbf{G}_s(s)-\tilde{\mathbf{G}}_s(s))\mathbf{G}_c(s)]^{-1}(\mathbf{G}_s(s)-\tilde{\mathbf{G}}_s(s))\mathbf{G}_c(s) \tag{12}$$

$$\mathbf{B}(s)=[\mathbf{I}+(\mathbf{G}_s(s)-\tilde{\mathbf{G}}(s))\mathbf{G}_c(s)]^{-1}\mathbf{D}_s(s) \tag{13}$$

Eq. (11) represents the so called net load effect. Note that $(\mathbf{I}-\tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s))$ produces an integral mode in the context of IMC design. Moreover, by manipulating Eq. (10), the main analysis relies on minimizing the effect of $(\mathbf{I}-\mathbf{F}(s))\mathbf{y}_s^{net}(s)$ since $\mathbf{G}_c=\tilde{\mathbf{G}}_s^{-1}(s)\mathbf{F}(s)$, with $\mathbf{F}(s)$ the low-pass TFM. This can be done by: (1) adjust a fast tuning in the filter matrix and/or (2) minimize the gain of $\mathbf{y}_s^{net}(s)$. The first option is limited by stability/robustness issues. Hence, the second alternative was selected here in a sum of square deviations sense. So, the real effect of $\mathbf{y}_s^{net}(s)$ is observed in the transient and it is directly proportional to its multivariable gain.

At steady-state Eqs. (12) and (13) are reduced to

$$\mathbf{A}=\mathbf{I}-\tilde{\mathbf{G}}_s\mathbf{G}_s^{-1},\quad \mathbf{B}=\tilde{\mathbf{G}}_s\mathbf{G}_s^{-1}\mathbf{D}_s \tag{14}$$

these equations show that perfect servo behavior can be achieved if a full model (eventually a full controller) is selected, $\mathbf{A}=\mathbf{0}_{n\times n}$. However, the disturbance effects $\mathbf{B}=\mathbf{D}_s$ have no attenuation. In this context, the search goes toward to find the best model $\tilde{\mathbf{G}}_s$ able to minimize both, the disturbance effects and set point changes in $\mathbf{y}_s^{net}$ (trade-off). So, parameterizing the selected model as

$$\tilde{\mathbf{G}}_{s\Gamma}=\mathbf{G}_s\otimes\Gamma \tag{15}$$

where $\otimes$ denotes the element-by-element product and $\Gamma$ is a $n\times n$ binary matrix (with elements 0 and 1) indicating which are the selected model components. So, the goal is to find the most suitable $\Gamma$ that produces the $\mathbf{y}_s^{net}$ minimization, in terms of the sum of square deviations, via the new scalar index called net load evaluation ($\text{NLE}_\Gamma$) shown in Eq. (16)

$$\text{NLE}_\Gamma=\sum_{i=1}^{n}\|\Delta_2\mathbf{A}_\Gamma\Delta_1\mathbf{y}_{set}^n(i)\|_2^2+\sum_{j=1}^{p}\|\Xi_2\mathbf{B}_\Gamma\Xi_1\mathbf{d}_*^p(j)\|_2^2=\|\Delta_2\mathbf{A}_\Gamma\Delta_1\|_F^2+\|\Xi_2\mathbf{B}_\Gamma\Xi_1\|_F^2 \tag{16}$$

where $\Delta_1$ $(n\times n)$, $\Delta_2$ $(n\times n)$, $\Xi_1$ $(p\times p)$ and $\Xi_2$ $(n\times n)$ are diagonal weighting matrices in which elements must be selected according to the relative degree of importance of the overall outputs. In addition, the weighting matrices allow to take into account the most probable expected events (set point or disturbance changes). The vectors $\mathbf{y}_{sp}^n(i)$ and $\mathbf{d}_*^p(j)$ have a unitary entry at the location $i$ and $j$, and zero elsewhere. $\mathbf{A}_\Gamma$ and $\mathbf{B}_\Gamma$ are the net load matrices shown in Eq. (14) parameterized with the model selection displayed in Eq. (15).

In this context, the problem is stated as a search of the best variable combination to obtain the optimal model parametrization. It is able to drive to the corresponding control structure, via IMC theory, as it is shown in Eqs. (17) and (18)

$$\min_\Gamma \text{NLE}_\Gamma=\min_\Gamma[\|\Delta_2\mathbf{A}_\Gamma\Delta_1\|_F^2+\|\Xi_2\mathbf{B}_\Gamma\Xi_1\|_F^2] \tag{17}$$

subject to

$$\text{Re}[\lambda_i(\mathbf{G}_s\tilde{\mathbf{G}}_{s\Gamma}^{-1})]>0\quad \text{with } i=1,\ldots,n \tag{18}$$

where $\text{Re}[\cdot]$ is the real part function, $\lambda_i(\cdot)$ is the $i$th eigenvalue, and $\tilde{\mathbf{G}}_{s\Gamma}$ the selected model parametrization. The inequality given in Eq. (18) corresponds to the stability/robustness criterion, developed by Garcia and Morari (1985), for multivariable control structures based on IMC theory. The optimization problem given by Eq. (17) has $2^{(n\times n)}$ potential solutions. According to the problem size, this minimization can be done by exhaustive search or by implementing some mixed-integer optimization routine (deterministic or stochastic). Good examples about this last option can be found in some previous works of the authors (Zumoffen and Basualdo, 2010; Molina et al., 2011) where genetic algorithms (GA) were used for defining sensor locations for large scale process.

### 5.3. Controller tuning

Considering the transfer function matrix of the square process model, factorized as $\tilde{\mathbf{G}}_{s\Gamma}(s)=\tilde{\mathbf{G}}_{s\Gamma}^{-}(s)\tilde{\mathbf{G}}_s^{+}(s)$ (Garcia and Morari, 1985), where $\tilde{\mathbf{G}}_{s\Gamma}^{-}(s)$ and $\tilde{\mathbf{G}}_s^{+}(s)$ correspond to the invertible and non-invertible parts, respectively. Then, according to the scheme given in Fig. 2 a practical IMC implementation suggests the following controller, $\mathbf{G}_c(s)=\tilde{\mathbf{G}}_{s\Gamma}^{-}(s)^{-1}\mathbf{F}(s)$, where $\mathbf{F}(s)=\text{diag}([f_1(s),\ldots,f_n(s)])$ with $f_i(s)=1/(\tau_{fi}s+1)$. It is the low-pass filter transfer function matrix, which must be designed accounting robustness for modeling errors. Initially, an IMC based on models without considering delay information is designed, i.e. $\tilde{\mathbf{G}}_s^{+}(s)=\mathbf{I}$. The filter time constant, $\tau_{fi}$, affects the $i$th input channel of the process and due to $\tilde{\mathbf{G}}_{s\Gamma}$ fulfills with Eq. (18). Hence, a preliminary and conservative multivariable tuning rule is proposed here such as, $\tau_{fi}\geq\max_j(\theta_{ji})$, with $j=1,\ldots,n$.

## 6. Proposed control structures for Petlyuk column based on the MSD methodology

In this section the MSD technique is applied, step by step, on the Petlyuk column for obtaining an efficient control structure. The MSD approach was conceived to be applied on stable processes. Indeed, the systematic selection of the stabilizing control loops is not addressed here. Hence, taking into account the description given in Section 3, the Petlyuk column has the condenser ($M_B$) and reboiler ($M_D$) levels without steady-state effect. Therefore, these variables are considered for stabilizing the process according to the inventory control rules. Serra et al. (2001) analyzed the controllability for different inventory CSs and proposed $M_B$ to be controlled with the bottom flow $B$ and $M_D$ with the distillate flow $D$.

Once the process is stabilized, the next step is obtaining a linear discrete state-space model of the plant by the subspace-based identification technique. The data base was obtained considering $R_V$ fixed (Halvorsen, 2001) and:

(A) 51 outputs: $T_1$ to $T_{48}$, $x_{A,D}$, $x_{C,B}$, $x_{B,S}$,
(B) 4 inputs: $L$, $V$, $S$, $R_L$,
(C) 4 disturbances: $F$, $z_A$, $z_B$, $qF$.

In this context, inputs and disturbances were excited with random signals which have uniform distribution around its working point ($\pm 0.5\%$) and a period of about 1000 min. Thus, the overall data were collected with a uniform sample time of 10 min and normalized to zero mean and unit variance. The result is a normalized discrete state-space linear model with 15 states.

### 6.1. SSD-based approach

The following objective is to find the set of CVs via the SSD approach application. According to the requisites of this column the compositions $x_{A,D}$, $x_{C,B}$ and $x_{B,S}$ must be controlled (original objective), so the search in Eq. (9) is reduced to find the best temperature measurement point, within 48 possibilities. Table 3 shows the first three best solutions when the adopted weighting

**Table 3**
SSD best solutions.

| No. | Controlled variables | | | | SSD |
|---|---|---|---|---|---|
| 1 | $x_{A,D}$ | $x_{C,B}$ | $x_{B,S}$ | $T_4$ | 42.80 |
| 2 | $x_{A,D}$ | $x_{C,B}$ | $x_{B,S}$ | $T_3$ | 42.84 |
| 3 | $x_{A,D}$ | $x_{C,B}$ | $x_{B,S}$ | $T_5$ | 42.99 |

**Table 4**
RGA for the best solution.

| Controlled variables | $L$ | $V$ | $S$ | $R_L$ |
|---|---|---|---|---|
| $x_{A,D}$ | 0.91 | 0.08 | 0.00 | 0.00 |
| $x_{C,B}$ | −1.72 | 2.04 | 0.68 | 0.00 |
| $x_{B,S}$ | 1.77 | −1.08 | 0.31 | 0.00 |
| $T_4$ | 0.03 | −0.03 | 0.00 | 0.99 |

matrices are, $\Lambda_1 = \mathbf{1}_4$, $\Lambda_2 = \mathbf{1}_{47}$, $\Theta_1 = \mathbf{1}_4$ and $\Theta_2 = \mathbf{1}_{47}$, where $\mathbf{1}_i$ represents the identity matrix of dimension $i \times i$. From this table it can be seen that the best temperature for sensing is $T_4$. Alternative solutions suggest to control the temperature $T_3$ or $T_5$ with similar SSD. Here it will be implemented on the structure based on solution no. 1.

It is important to note that, Alstad (2005) and Alstad and Skogestad (2007) using the singular value method, reported that $T_4$ (temperature in stage four) could be a solution as option number seven, among 52 possible candidates. After a careful evaluation based on the loss of energy analysis they concluded that $T_4$ is the best choice using a single temperature (from self-optimizing point of view) and is able to avoid that component C breaks through in the top of the pre-fractionator. From the non-linear dynamic simulations, the side-stream purity is least affected by a disturbance in $z_A$ while both, the bottom and top composition, show large deviations. Moreover, measurement errors in the purity side stream, $x_{B,S}$, give a higher boilup consumption than the other structures. Here, the selection of $T_4$ was made directly via SSD approach, avoiding the application of both singular value method and loss of energy evaluation. The SSD methodology gives $T_4$ as the best solution for minimal deviation on the uncontrolled variables based on the Frobenius norm calculations used here.

The RGA shown in Table 4 was computed from the information given by the linearized model. Therefore, the selected diagonal structure via SSD minimization agrees with structure $T_4-d$ from Alstad and Skogestad (2007). However, it must be noticed that the proposed pairing is a classical selection (considering previous works) but remains strong interactions for the loops of $x_{C,B}$ and $x_{B,S}$. This interaction may affect the dynamic behavior of the process when disturbances occur. Hence, in the following step, some interaction effects will be considered for designing a proper controller structure. It will be analyzed if some improvements can be obtained using sparse configuration policies.

### 6.2. NLE-based approach

In this section the NLE-based approach is applied to the Petlyuk column. The solution for this combinatorial problem is to find the best $\Gamma$ to minimize Eq. (17) subject to the feasibility (stability/robustness) constraint given in Eq. (18). It is possible to reduce the problem dimension to $2^{(n\times n-n)} = 2^{12} = 4096$, if remains of the diagonal controller structure are found in the previous step. Hence, the elements at the diagonal of $\Gamma$ are set as "one" during the search. It is possible to begin the next step by using as a starting point with other diagonal control structure obtained from any other methodology for multivariable controller design. Hence, the best model parametrization in terms of NLE magnitude is given by

$$\Gamma = \begin{bmatrix} 1 & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & 1 & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & 1 & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & 1 \end{bmatrix} \tag{19}$$

Thus, the NLE approach defines the best interaction level by selecting only the most suitable off-diagonal elements of $\Gamma$. To demonstrate the feasibility of the NLE-based method it will be applied either on the CVs suggested by the SSD approach ($T_4-d$) or the other selection ($c_{ODF}-d$ structure) detailed in Section 4. This proposal will be better understood if the controller structure design, based on the minimization of the NLE index, is made under the following contexts:

1. **START** $T_4-d$: In this case the diagonal control structure $T_4-d$, suggested by the SSD as well as that given by Alstad and Skogestad (2007) approaches, is considered as starting point. The resulting sparse controller structure will be named $T_4-s$.
2. **START** $c_{ODF}-d$: In this case the starting point for applying NLE-based approach is $c_{ODF}-d$. The resulting sparse controller structure will be named $c_{ODF}-s$.

**Table 5**
NLE-based approach for START $T_4-d$.

| Solutions from GA | $\Gamma$ | NLE |
|---|---|---|
| **1** | [1 0 0 0; 0 1 0 0; 0 0 1 1; 0 0 1 1] | 1.7958 |
| **2** | [1 0 0 0; 0 1 0 0; 0 0 1 1; 0 0 0 1] | 1.7962 |
| **3** | [1 0 1 0; 0 1 0 0; 0 0 1 1; 0 0 1 1] | 1.7966 |

**Table 6**
NLE-based approach for START $c_{ODF}-d$.

| Solutions from GA | $\Gamma$ | NLE |
|---|---|---|
| **1** | [1 0 0 1; 0 1 0 0; 0 0 1 1; 1 1 1 1] | 1.097157 |
| **2** | [1 0 0 0; 0 1 0 0; 0 0 1 1; 1 1 1 1] | 1.097188 |
| **3** | [1 0 0 0; 0 1 0 1; 0 0 1 1; 1 1 1 1] | 1.097443 |

**Table 7**
Controller structure and tuning for $T_4-s$.

| Manipulated variables | $x_{A,D}$ | $x_{C,B}$ | $x_{B,S}$ | $T_4$ |
|---|---|---|---|---|
| $L$ | $k_c$=6.259 $\tau_i = 169$ | – | – | – |
| $V$ | – | $k_c$=4.616 $\tau_i = 137$ | – | – |
| $S$ | – | – | $k_c = -5.762$ $\tau_i = 95.7$ | – |
| $R_L$ | – | – | $k_c$=0.003261 | $k_c = -0.01363$ $\tau_i = 34$ |

**Table 8**
Controller structure and tuning for $c_{ODF}-s$.

| Manipulated variables | $x_{A,D}$ | $x_{C,B}$ | $x_{B,S}$ | $c_{ODF}$ |
|---|---|---|---|---|
| $L$ | $k_c=6.255$ $\tau_i=169$ | $k_c=-0.00047$ | $k_c=-0.012$ $\tau_i=100$ | – |
| $V$ | – | $k_c=5.385$ $\tau_i=136$ | – | – |
| $S$ | $k_c=-0.217$ $\tau_i=181.6$ | $k_c=0.0146$ $\tau_i=25$ | $k_c=-7.421$ $\tau_i=100$ | $k_c=0.004503$ |
| $R_L$ | $k_c=1.682$ $\tau_i=94.9$ | $k_c=0.519$ | $k_c=4.621$ $\tau_i=69.7$ | $k_c=-0.0224$ $\tau_i=43$ |

In Tables 5 and 6 the best three optimal solutions obtained from the NLE approach for each START are shown. These results are achieved assuming equally weighting all the plant objectives (the same relative degree of importance among CVs) and weighting matrices focused on $z_A$, $z_B$, and $qF$ disturbances.

### 6.3. Petlyuk controller tuning

The tuning procedure for all controllers was done using the previously described methodology in Section 5.3. The final tuning parameters of the PI controllers, designed accordingly to the sparse structures recommended by NLE methodology are shown in Table 7 for $T_4-s$ and Table 8 for $c_{ODF}-s$.

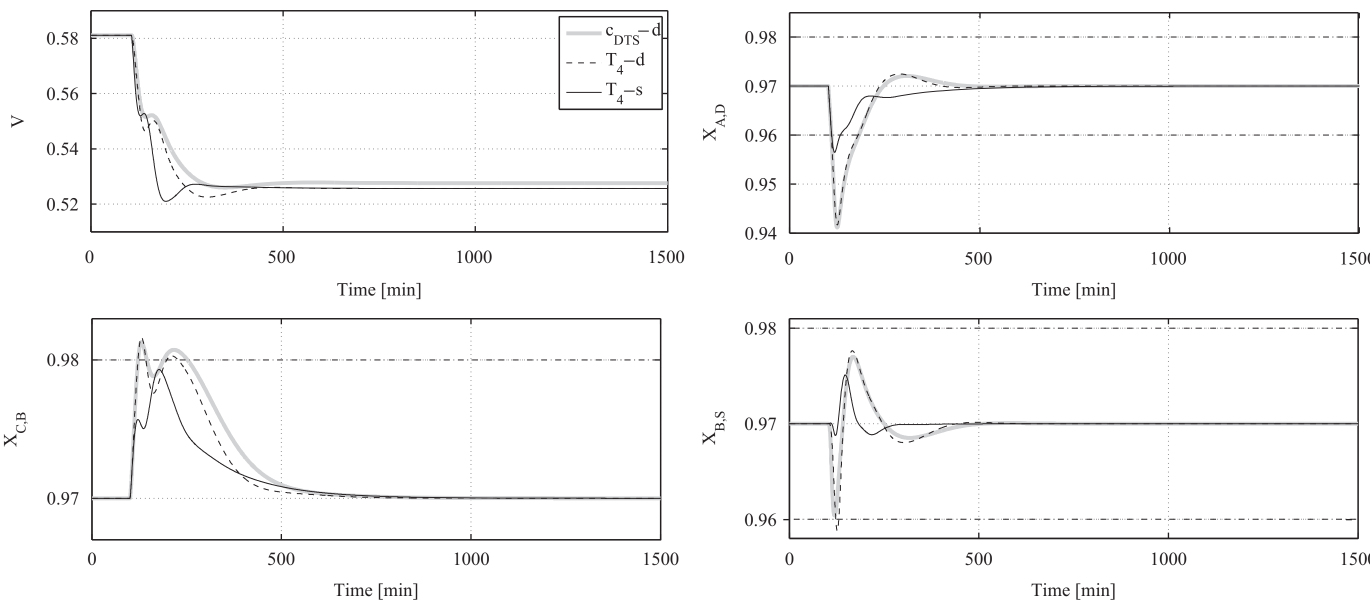


**Fig. 3.** Scenario no. 1—disturbance $\Delta qF=-0.1$.

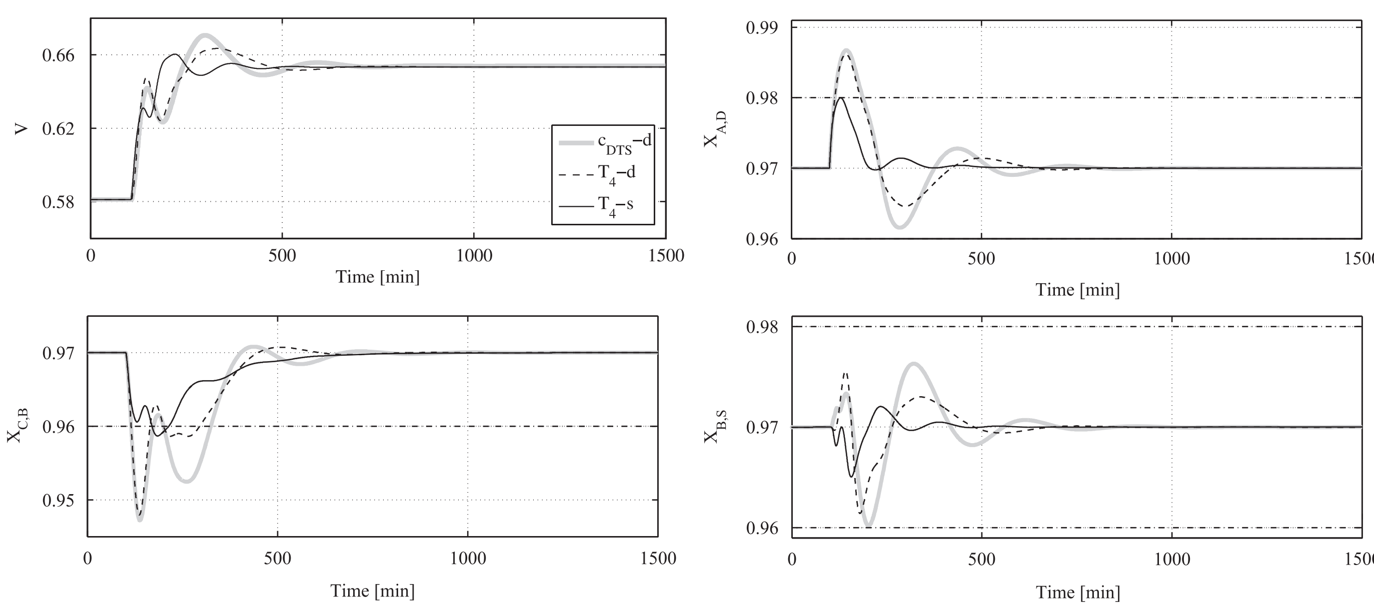


**Fig. 4.** Scenario no. 1—disturbance $\Delta qF=+0.1$.

## 7. Dynamic simulations

In this section three control policies are confronted for each scenario.

1. **Scenario no. 1**: In this case $T_4-d$, $T_4-s$ and $c_{DTS}-d$ CSs are dynamically compared.
2. **Scenario no. 2**: In this case $c_{ODF}-d$, $c_{ODF}-s$ and $T_4-s+c$ CSs are dynamically compared, where $T_4-s+c$ is the same case as $T_4-s$ in scenario no. 1, but here the set point for $T_4$ is given by a cascaded PI. This outer control loop uses the $c_{ODF}$ as CV. The tuning parameters are $k_c=3$ and $\tau_i=34$ min.

Figs. 3–8 summarize the dynamic behaviors of the controlled variables using the policies suggested for scenario no. 1. Similarly, from Figs. 9–14 the closed-loop responses are shown when the controller structures, detailed by scenario no. 2, are used. In all cases the regulator performance was tested when step disturbances occurred in $qF$, $z_A$ and $z_B$ with values of $\pm 0.1$ at 100 min. Moreover, the vapor boilup profile is shown for each case for evaluating the energy consumption during the process operation.

In Figs. 3 and 4 are shown the responses to step changes of $\pm 0.1$ in liquid fraction, $qF$, produced at $t=100$ min. From these figures it can be seen that the CSs ($T_4-d$ and $c_{DTS}-d$) present similar performances for the composition control loops. However, the sparse CS ($T_4-s$) proposed here, based on NLE minimization, improves the dynamic behavior since the peak values of the composition are reduced together with the time responses. The final magnitudes of the vapor boilup consumption are similar for the three structures.

The closed-loop process behavior when disturbances in feed flow composition $z_A$ happen are shown in Figs. 5 and 6. In these cases step changes of $\pm 0.1$ at $t=100$ min are proposed. The simulations show strong oscillations for the positive step

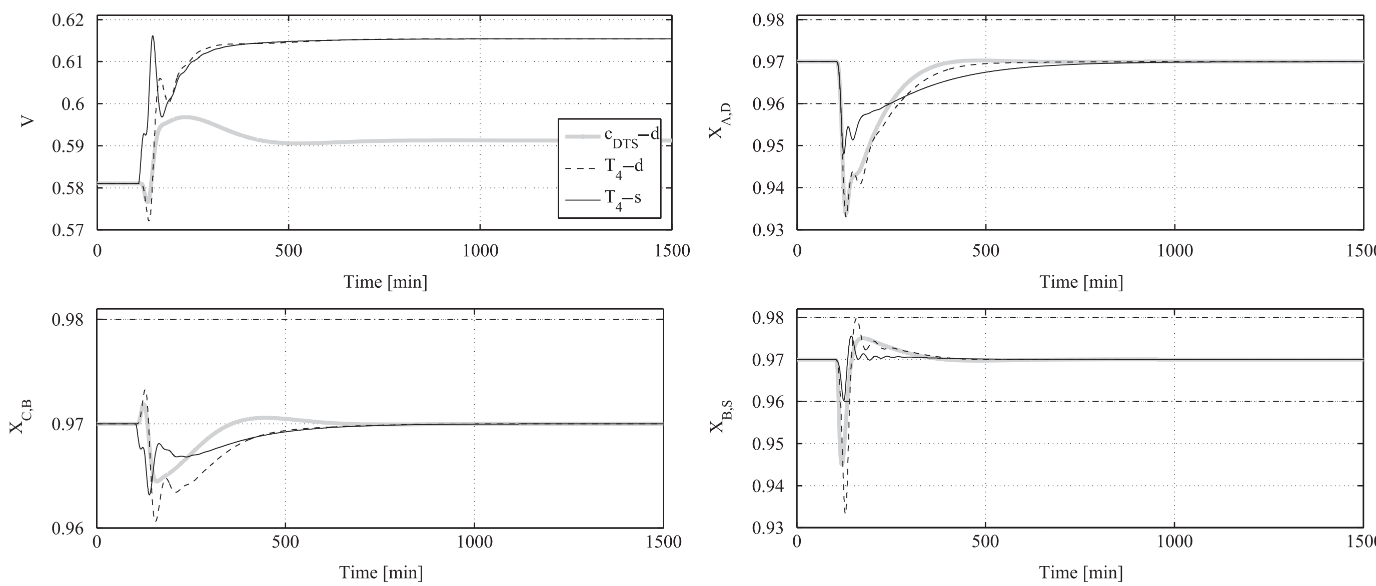


**Fig. 5.** Scenario no. 1—disturbance $\Delta z_A=-0.1$.

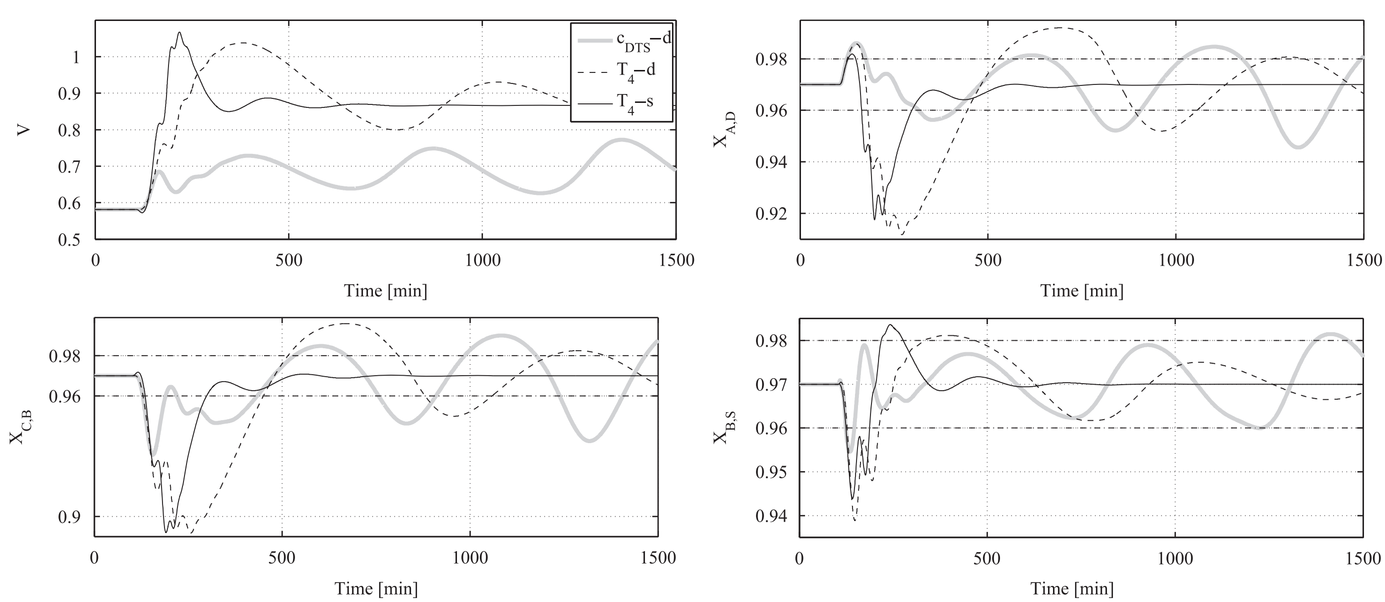


**Fig. 6.** Scenario no. 1—disturbance $\Delta z_A=+0.1$.

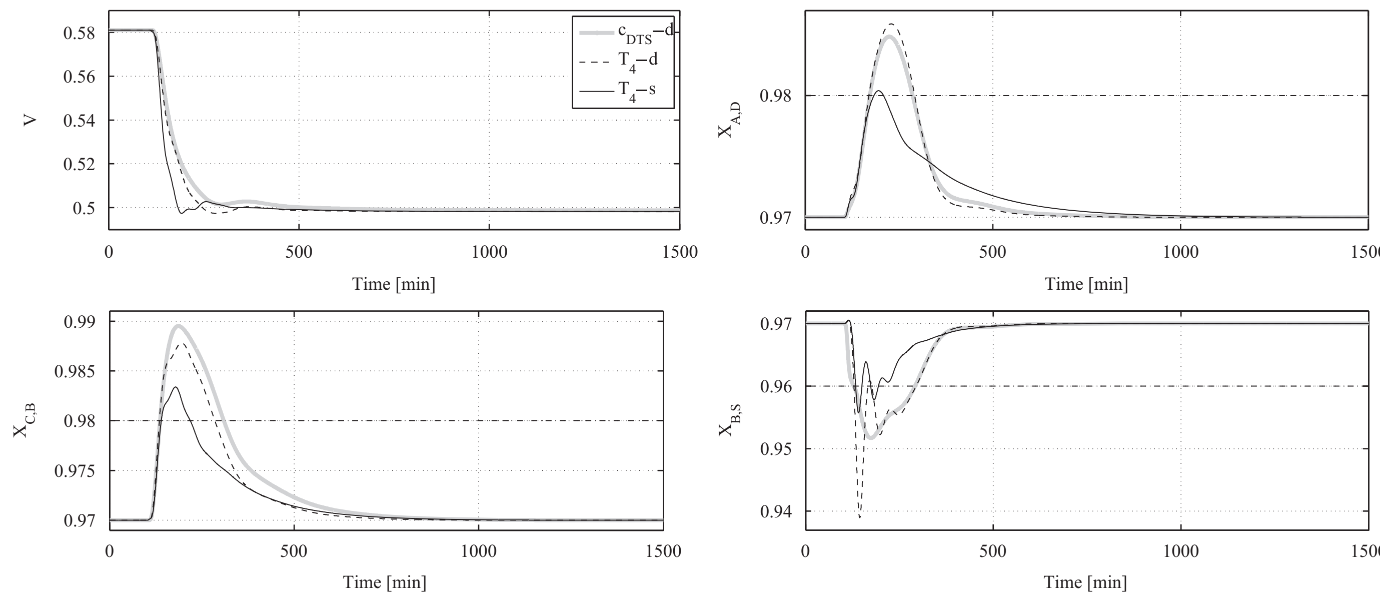


**Fig. 7.** Scenario no. 1—disturbance $\Delta z_B = -0.1$.

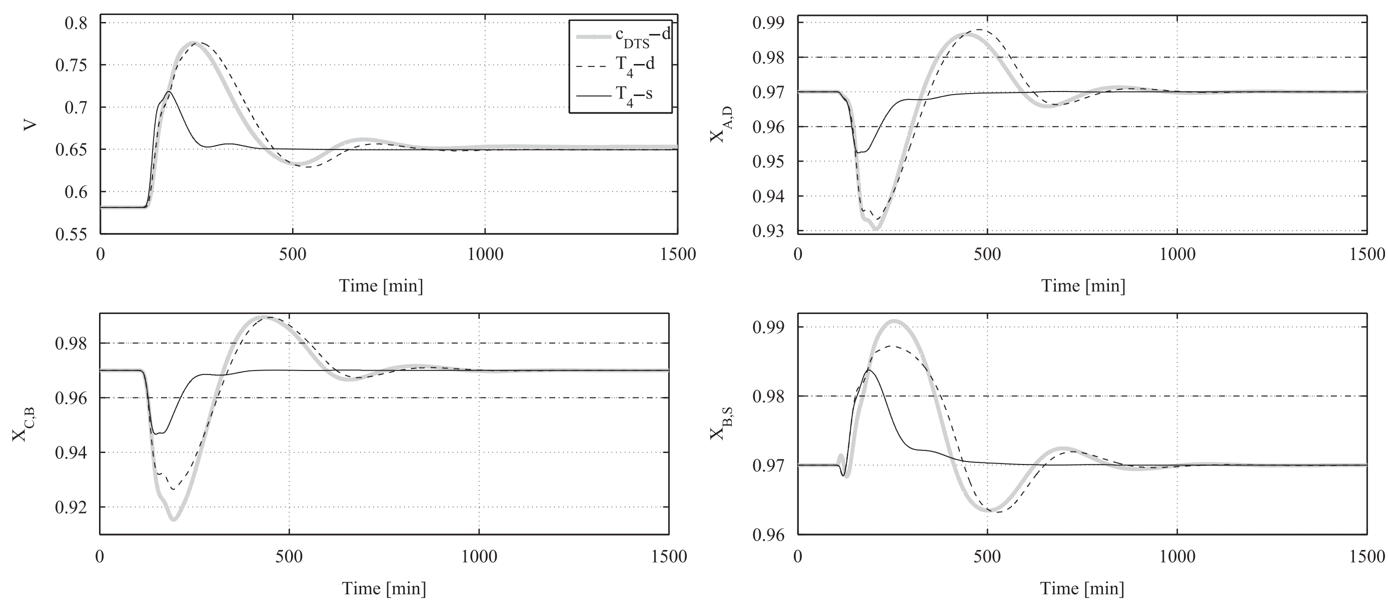


**Fig. 8.** Scenario no. 1—disturbance $\Delta z_B = +0.1$.

change in $T_4-d$ and $c_{DTS}-d$ cases. However, it is possible to note that $c_{DTS}-d$ structure tends to amplify the peak values, showing an unstable behavior, while $T_4-d$ tends to reduce them but with drastic oscillations. On the other hand, the sparse CS ($T_4-s$) proposed here significantly improves the dynamic behavior by reducing the oscillations and returning quickly the compositions into the required limits. Note that $T_4-s$ guarantees stable operation of the process when $\Delta z_A = +0.1$, but with a higher vapor boilup consumption when $\Delta z_A = -0.1$.

In Figs. 7 and 8 can be seen the simulation results of the controlled variables when disturbances occur in the feed flow composition, $z_B$. In these cases step changes of $\pm 0.1$ at $t=100$ min are proposed. Both CSs, $T_4-d$ and $c_{DTS}-d$, display a well regulated behavior. However, the sparse CS ($T_4-s$) proposed here is able to improve the dynamic performance by reducing peaks and time responses. The vapor boil up profiles are similar for all the structures.

Table 9 displays the resulting IAE values obtained as the sum of partial IAEs when regulatory performance is evaluated for three compositions ($x_{A,D}$, $x_{C,B}$, $x_{B,S}$). This table was developed by considering all CSs proposed for scenario no. 1 and four disturbances (with positive and negative variations). Note that the best performance is given by the NLE-based sparse control policy. In fact, the error improvement percentage index, $\text{EIP} = 100\,(\text{IAE}_i - \text{IAE}_j)/\text{IAE}_j$, is used here to quantify the improvements obtained by the $i$th CS respect to the $j$th one. Particularly, the mean (average) EIP along the potential disturbances was considered. The proposed sparse control structure called $T_4-s$ presents an average improvement of about 43.1% and 43.3% in respect to the $T_4-d$ and $c_{DTS}-d$ structures, respectively.

From Figs. 9–14 the closed-loop responses for scenario no. 2 are summarized. In all cases the regulator behavior was tested when step changes of disturbances $qF$, $z_A$ and $z_B$ of magnitudes of $\pm 0.1$ at 100 min occur. Moreover, the vapor boil up profile is also shown for each case.

Specifically, Figs. 9 and 10 show the dynamic behavior for a step change of $\pm 0.1$ at $t=100$ min in liquid fraction, $qF$. These

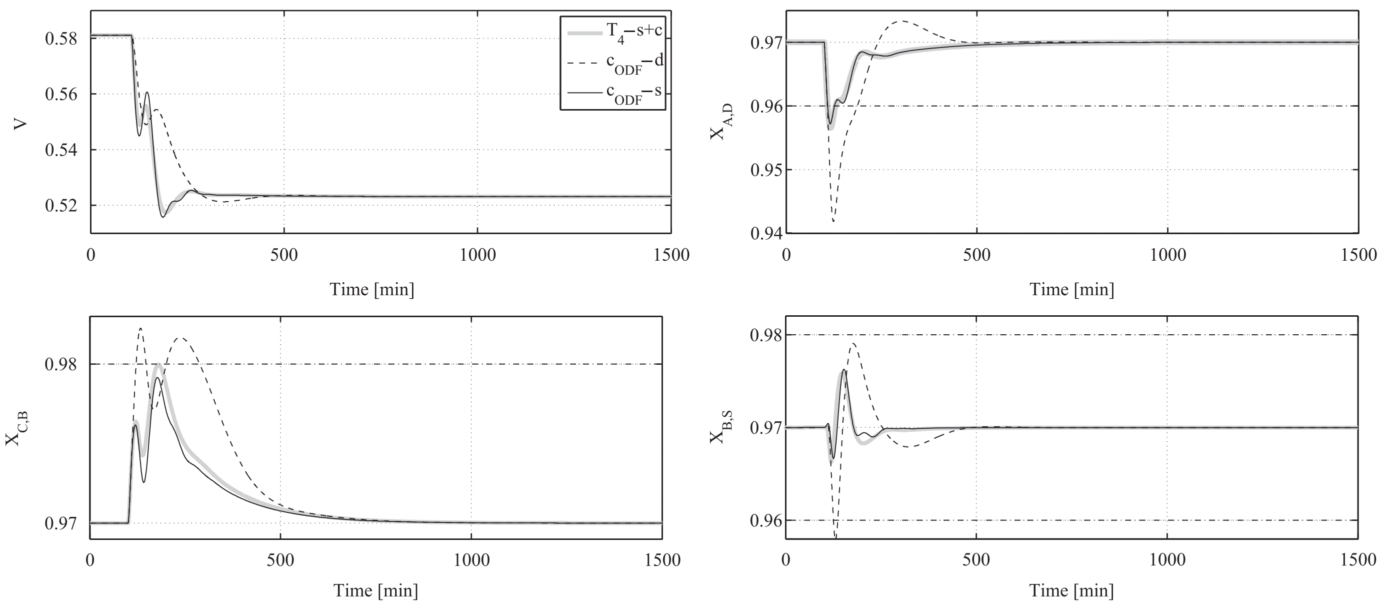


**Fig. 9.** Scenario no. 2—disturbance $\Delta qF = -0.1$.

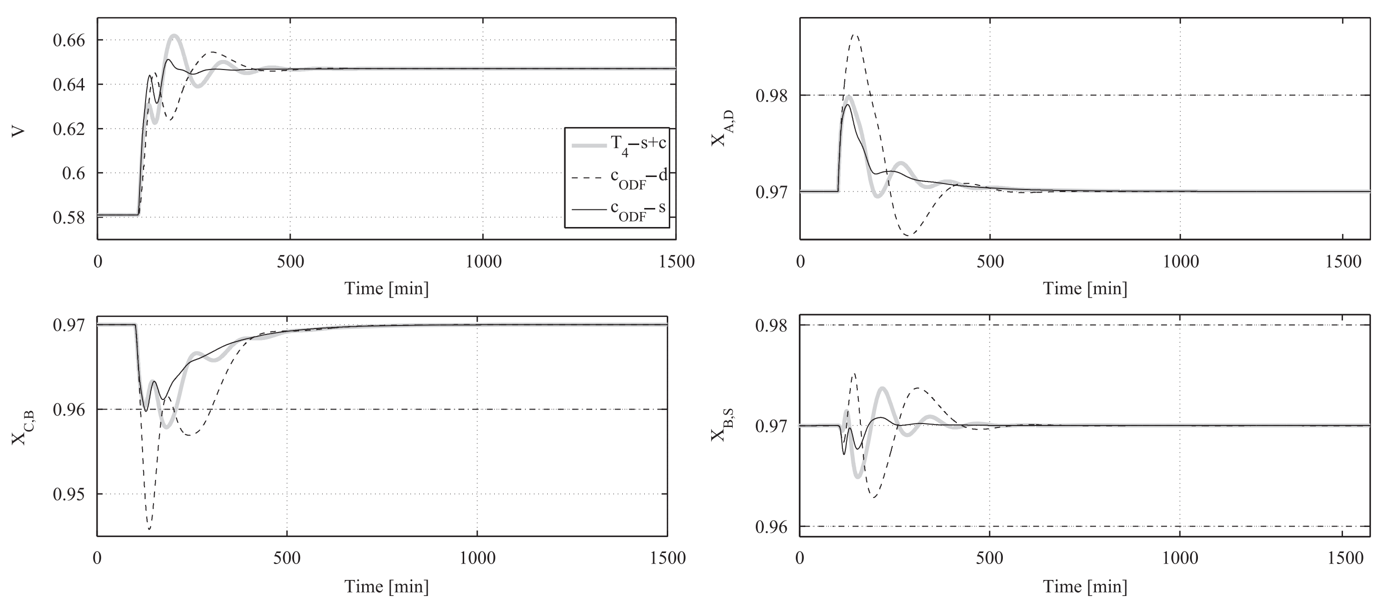


**Fig. 10.** Scenario no. 2—disturbance $\Delta qF = +0.1$.

figures show that the NLE-based sparse CS, $c_{ODF}-s$, has the best performance. The second place is for the NLE-based sparse CS for scenario no. 1 plus cascade loop, $T_4-s+c$. The final vapor boilup is the same for the three control configurations showing that losses for these disturbances are almost the same. The worst performance is given by the $c_{ODF}-d$ policy.

In Figs. 11 and 12 are displayed the closed-loop responses corresponding to a disturbance in the feed composition $z_A$. In this case step changes of $\pm 0.1$ at $t=100$ min are considered. From these figures can be seen that both sparse CSs, based on NLE minimization ($T_4-s+c$ and $c_{ODF}-s$) reduce the time responses for the composition control. Furthermore, in general, an important reduction on their peak values are detected. However, $c_{ODF}-s$ is slightly oscillating for the positive changes in $z_A$ disturbance. Anyway, this situation can be drastically reduced by re-tuning the controllers. Again, the worst performance is found for the $c_{ODF}-d$ policy.

Finally, in Figs. 13 and 14 the regulator profiles for a disturbance in the feed composition $z_B$ are shown. Here, step changes of $\pm 0.1$ magnitudes at $t=100$ min are simulated. It can be noticed that the improvements obtained by the NLE-based strategies ($T_4-s+c$ and $c_{ODF}-s$) are even more important than in the previous cases. Analogously, here the decentralized control policy presents the worst dynamic performance.

Similarly to Table 9, in Table 10 the resulting IAE for the three compositions ($x_{A,D}$, $x_{C,B}$, $x_{B,S}$) are shown. All the CSs proposed in scenario no. 2 and four disturbances (with positive and negative variations) are considered here for the sake of comparison. Note that the best performances are given by the NLE-based sparse control policies ($T_4-s+c$ and $c_{ODF}-s$). Moreover, the above CSs have very similar dynamic behaviors. In fact, using the mean error improvement percentage (EIP) index, $c_{ODF}-s$ has an improvement of about 45% in respect to the $c_{ODF}-d$ strategy and 3% compared with $T_4-s+c$ structure.

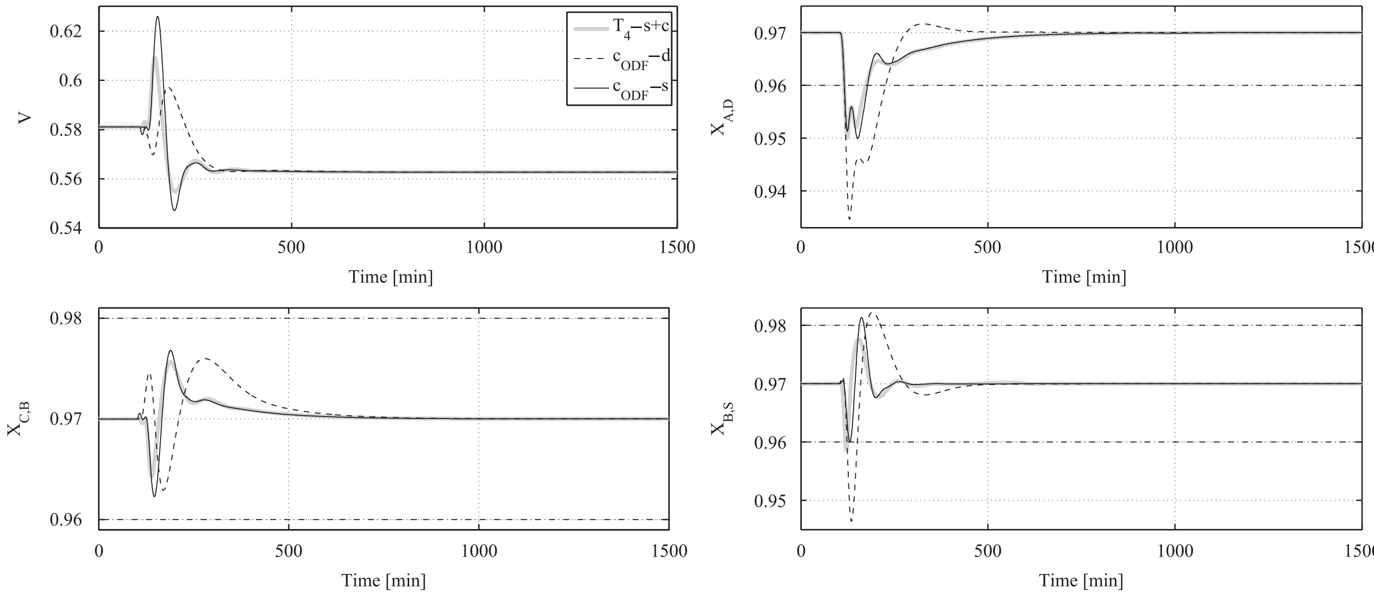


**Fig. 11.** Scenario no. 2—disturbance $\Delta z_A = -0.1$.

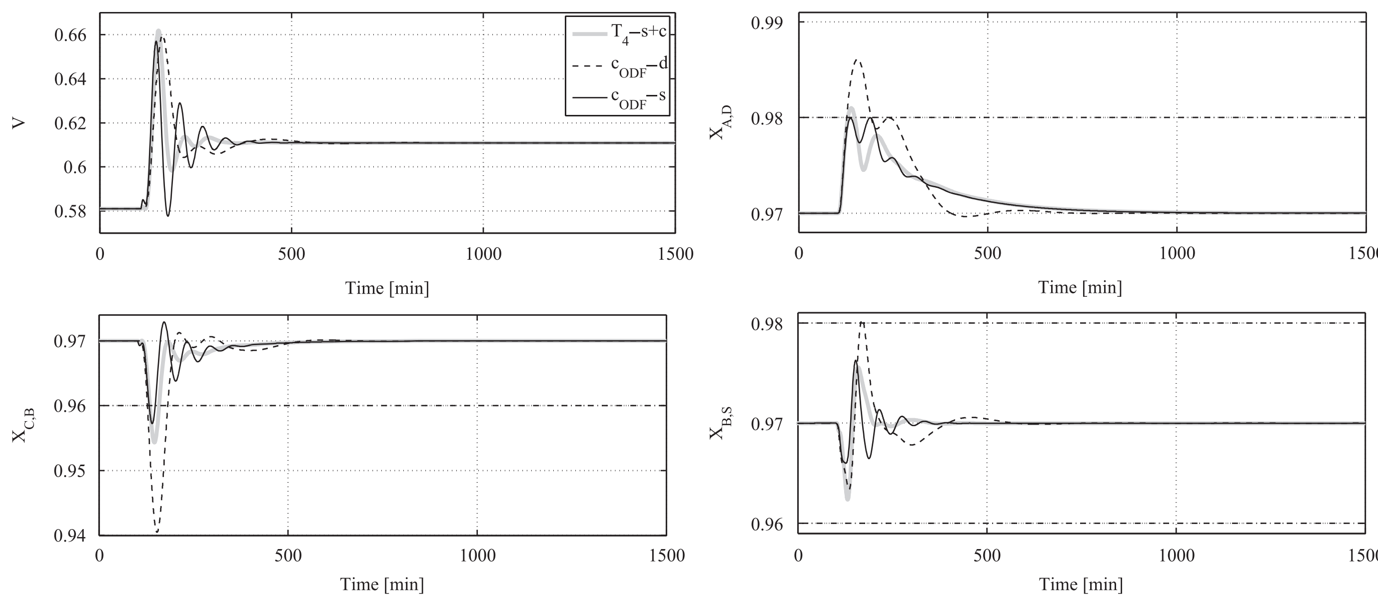


**Fig. 12.** Scenario no. 2—disturbance $\Delta z_A = +0.1$.

## 8. Conclusions and future work

The recently appeared MSD approach for multivariable control design was applied here to the complex Petlyuk distillation column (ABC-DWC). The best diagonal control structure, obtained after the SSD implementation, named $T_4{-}d$, was the first candidate to be chosen. It was the same result obtained by Alstad and Skogestad (2007) applying SOC and singular value techniques. However, they needed to perform several tests before arriving to the same conclusion obtained directly by SSD approach.

The second part of MSD, the NLE-based methodology, could drive to a good sparse option, named $T_4{-}s$, which demonstrated an important improvement of about 43.1% in respect to the diagonal version $T_4{-}d$ without additional vapor consumption. It must be noted that, even though SOC together with null space method drive to a proper combination of measurements able to deal with the disturbances impact, the diagonal control structure performance can be improved with sparse control structures ($c_{ODF}{-}s$). It was demonstrated that NLE procedure was able to achieve up to 43% EIP benefit starting from the $c_{ODF}{-}d$ structure obtained by SOC. This last statement represents a clear contribution of this work because it proposes a new way to improve the SOC methodology without reporting any extra vapor consumption than that reported by $c_{ODF}{-}d$. Hence, the sparse CSs obtained here presented the best dynamic performances as it was tested on two different scenarios. These preliminary results are encouraging since they were obtained from the application on a very complex case study. In addition, the NLE strategy seems to be able to improve the dynamic behavior when some interaction degree exists, independent of the selected starting decentralized structure. Thus, another good control alternative was presented here for the Petlyuk distillation column which contributes to a high product quality with lower operational costs. The promising results obtained here give support to extend the MSD methodology to other large scale benchmark plants to have more confidence on the statements given here. Other topics to study in

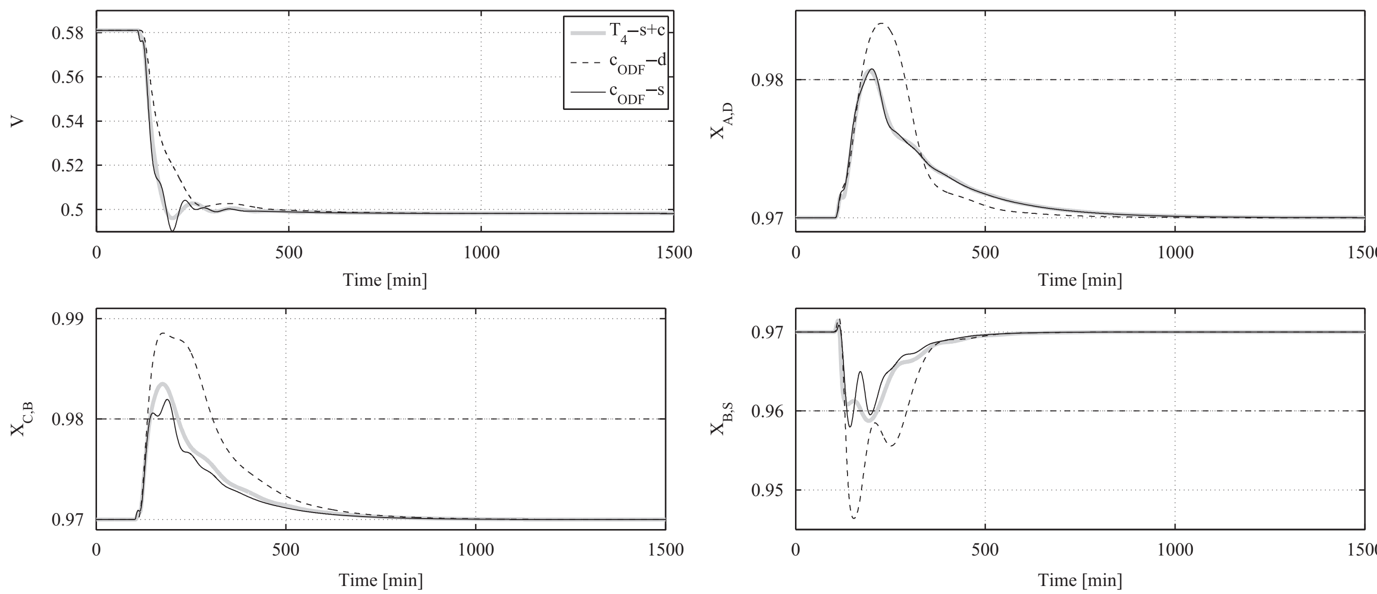


**Fig. 13.** Scenario no. 2—disturbance $\Delta z_B = -0.1$.

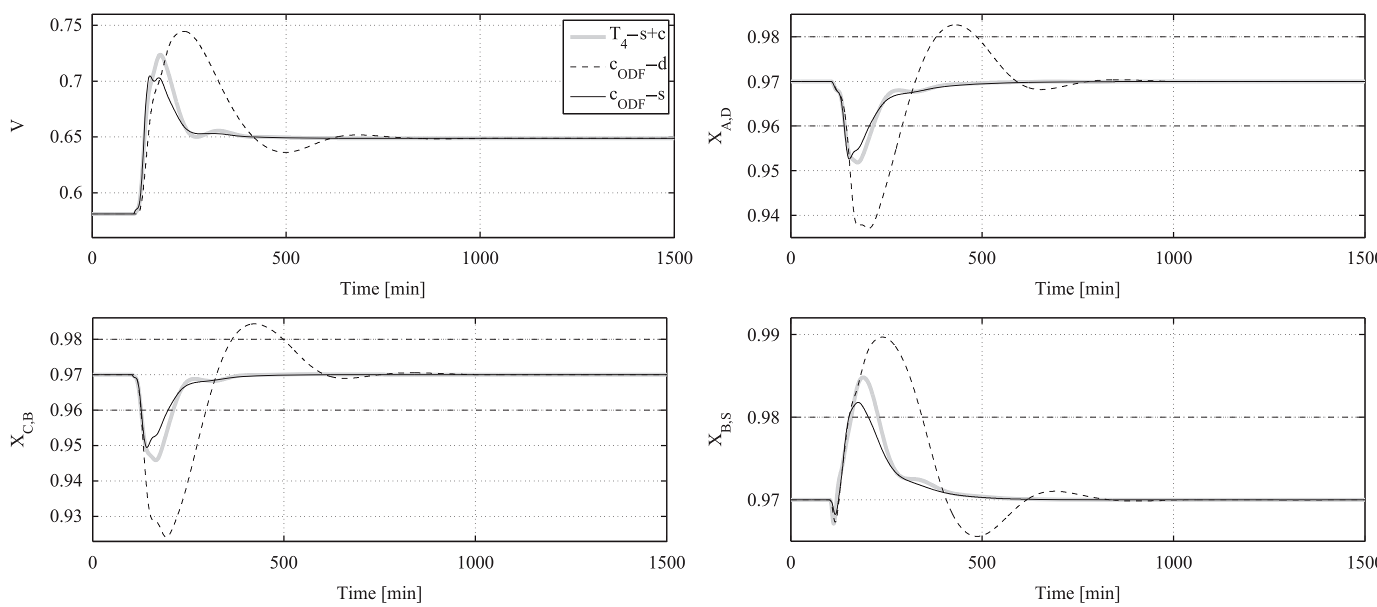


**Fig. 14.** Scenario no. 2—disturbance $\Delta z_B = +0.1$.

**Table 9**
Total IAE in scenario no. 1.

| Perturbations | $T_4-d$ | $c_{DTS}-d$ | $T_4-s$ |
|---|---|---|---|
| $\Delta F = +0.1$ | 4.91 | 5.40 | 3.23 |
| $\Delta F = -0.1$ | 6.39 | 6.81 | 4.36 |
| $\Delta z_A = +0.1$ | 59.51 | 41.85 | 16.91 |
| $\Delta z_A = -0.1$ | 6.95 | 5.36 | 5.14 |
| $\Delta z_B = +0.1$ | 23.51 | 24.15 | 5.85 |
| $\Delta z_B = -0.1$ | 8.63 | 9.70 | 6.49 |
| $\Delta qf = +0.1$ | 6.25 | 7.88 | 3.32 |
| $\Delta qf = -0.1$ | 4.91 | 5.40 | 3.23 |

**Table 10**
Total IAE in scenario no. 2.

| Perturbations | $T_4-s+c$ | $c_{ODF}-d$ | $c_{ODF}-s$ |
|---|---|---|---|
| $F_0+0.1$ | 3.55 | 6.55 | 4.07 |
| $F_0-0.1$ | 4.36 | 6.74 | 4.04 |
| $z_A+0.1$ | 3.20 | 4.65 | 3.21 |
| $z_A-0.1$ | 3.58 | 6.04 | 4.06 |
| $z_B+0.1$ | 5.76 | 19.01 | 5.07 |
| $z_B-0.1$ | 6.51 | 9.79 | 5.83 |
| $q_F+0.1$ | 3.37 | 6.13 | 2.86 |
| $q_F-0.1$ | 3.23 | 6.04 | 2.97 |

future are: (1) the extension of the MSD approach for analyzing the controller size, (2) the implementation of sparse control structures in the context of MPC technology and (3) fault tolerant control design. All of these topics are currently under development.

## Nomenclature

*Abbreviations*

BTX benzene, toluene and o-xylene
CS control structure

CSi — *i*th CS from Alstad (2005)
CSs — control structures
CV — controlled variable
CVs — controlled variables
DMC — dynamic matrix control
*DTS* — average difference of temperatures from Halvorsen and Skogestad (2003)
DWC — divided wall column
EIP — error improvement percent
IAE — integral absolute error
IMC — internal model control
MILP — mixer integer linear programing
MIMO — multi-input/multi-output
MSD — minimum square deviations
MV — manipulated variable
MVs — manipulated variables
NLE — net load evaluation
NRG — non-square relative gain
PID — proportional-integral-derivative
RGA — relative gain array
RNGA — relative normalized gain array
SOC — self-optimizing control
SSD — sum of square deviations
TFM — transfer function matrix
TFMs — transfer function matrices
TE — Tennessee Eastman
UVs — uncontrolled variables

*Variables*

$\mathbf{A}(s)$ — net load matrix for set points
$\mathbf{B}(s)$ — net load matrix for disturbances
$c_{DTS}$ — temperature combinations suggested by Halvorsen and Skogestad (2003)
$c_{ODF}$ — temperature combinations suggested by Alstad and Skogestad (2007)
$\mathbf{d}_*(s)$ — disturbance vector
$\mathbf{D}(s)$ — disturbance TFM
$\mathbf{D}_r(s)$ — disturbance TFM affecting $\mathbf{G}_r(s)$
$\mathbf{D}_s(s)$ — disturbance TFM affecting $\mathbf{G}_s(s)$
$\mathbf{F}(s)$ — low-pass filter TFM
$\mathbf{G}(s)$ — process TFM
$\mathbf{G}_c(s)$ — controller TFM
$\mathbf{G}_r(s)$ — process TFM for UVs
$\mathbf{G}_s(s)$ — process TFM for CVs
$\tilde{\mathbf{G}}_s(s)$ — process model TFM for CVs
$\tilde{\mathbf{G}}_{s\Gamma}(s)$ — process model parameterized with $\Gamma$
$\tilde{\mathbf{G}}^-_{s\Gamma}(s)$ — invertible model from $\tilde{\mathbf{G}}_{s\Gamma}(s)$
$\tilde{\mathbf{G}}^+_s(s)$ — non-invertible part from $\tilde{\mathbf{G}}_{s\Gamma}(s)$
$m$ — number of outputs
$n$ — number of inputs
$p$ — number of disturbances
$\mathbf{u}(s)$ — input vector
$\mathbf{y}(s)$ — output vector
$\mathbf{y}_r(s)$ — UVs
$\mathbf{y}_s(s)$ — CVs
$\mathbf{y}_s^{net}(s)$ — net load effect vector
$\mathbf{y}_s^{sp}(s)$ — Reference vector

*Greek symbols*

$\theta_{ji}$ — delay in component *ji*
$\lambda_i$ — *i*th eigenvalue
$\tau_{fi}$ — *i*th filter time constant
$\Gamma$ — parametrization matrix
$\Lambda_i$, $\Theta_i$ — weighting matrices for SSD approach
$\Delta_i$, $\Xi_i$ — weighting matrices for NLE approach

*Control structures*

$c_{DTS}-d$ — decentralized control suggested by Halvorsen and Skogestad (2003)
$c_{ODF}-d$ — decentralized control suggested by Alstad and Skogestad (2007)
$c_{ODF}-s$ — sparse control structure proposed here starting from $c_{ODF}-d$
$T_4-d$ — decentralized control suggested by Alstad (2005)
$T_4-s$ — sparse control structure proposed here starting from $T_4-d$
$T_4-s+c$ — sparse supervisory control structure proposed here by coupling $T_4-s$ and $c_{ODF}-d$

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