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Orbits of comets C/1848 P1 (Petersen) and C/1848 U1 (Petersen)

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ABSTRACT

Comets C/1848 P1 (Petersen) and C/1848 U1 (Petersen) are ones of a large number of comets with parabolic orbits. Given that there are sufficient observations, 67 in right ascencion and 61 in declination for the former and 144 in right ascension and 144 in declination for the latter, it proves to be possible to calculate better orbits. The C/1848 P1's orbit is hyperbolic, although statistically indistinguishable from a parabola. This object, therefore, cannot be considered as an NEO. C/1848 U1's orbit is also hyperbolic, but with no indication of a possible extrasolar origin.

1. Introduction

This paper continues a previous series on comets with catalogued parabolic orbits. Such orbits, however, although easier to calculate than an elliptical or hyperbolic orbits, particularly if one uses normal places, remain anathema to modern computing. Real cometary orbits are ellipses or hyperbolas. With modern computing power the conveniences of a parabolic orbit and normal places to reduce the size of the linear system solved by differential corrections are neither necessary nor desirable.

But why is it worthwhile to recalculate catalogued parabolic orbits, such as those in the Marsden and Williams catalog (2003)? A first explanation is professionalism. It is esthetically displeasing to leave an orbit as a parabola when better can be done. But perhaps even more importantly a parabolic orbit may, depending on factors such as perihelion distance, represent a near earth object (NEO). The probability is admittedly low, but the possibility cannot be discarded. Although certain celestially threatening events, such as the bolide that caused damage in Chelyabinsk, Russia, in February 2013 cannot be predicted, cometary threats *can* be predicted if an improved parabolic orbit becomes an ellipse. Should the improved orbit become a hyperbola, then one should investigate further to see if its origin might be extrasolar. This in fact occurred with comet C/1853 E1 (Secchi) whose initial parabolic orbit turned out to be a hyperbola with possible origin beyond the Oort cloud (Branham, 2012).

Comets observed between 1840 and 1860 are a particularly prolific source of parabolic orbits, see Marsden and Williams (2003), but with dozens to hundreds of observations. The precision of the observations, moreover, is sufficient to justify a more detailed treatment. Furthermore, all, or almost all, of the observations can be found on the internet, such as the ADS data base (http://adswww.harvard.edu/). The *Comptes rendus hebdomadaires des séances de l'Académie des sciences* (http://gallica.bnf.fr) represents another fruitful source of observations. Petersen discovered two comets in 1849, one in August, C/1848 P1, henceforth Petersen 1, and one in October, C/1848 U1, henceforth Petersen 2. Both of these comets have parabolic orbits, perihelion distances less than 1 AU, and a respectable number of observations: 67 in right ascension (α) and 61 in declination (δ) for Petersen 1 and 144 in α and 144 in δ for Petersen 2. Because both comets were discovered closely in time by the same observer, why not calculate both orbits? The orbit of Petersen 1, moreover, contains, as we shall see, a surprise.

2. The observations and ephemerides

A literature search of the journals published in the 19th century that include comet observations disclosed observations of both comets in *The Astronomical Journal, Astronomische Nachrichten, Monthly Notices RAS*, and *Astronomical Observations U.S. Naval Observatory*. Somewhat surprisingly, no observations were found for the Paris or Vienna observatories. Vienna, in particular, has always been a prolific source of cometary observations, but a literature search showed a discontinuity in observed comets

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between 1847 and 1850. Perhaps some modification was being made to the telescope or micrometer during the years 1848–1849. Tables 1 and 2 show the observatories and the number of observations made at each for each comet while Figs. 1 and 2 graph the observations.

Processing 19th century observations presents difficulties and becomes far from trivial. The observations are published in different languages, English, French, German, Italian, even Latin, do not conform to a standard format, and contain many errors. Rather than discuss these matters I refer the reader to an article of mine that goes into the details (Branham, 2011). The observations, reduced to the common format of: Julian Day (JD), Terrestrial Time (TT), α, and δ, are either given as differences $\Delta\alpha$ and $\Delta\delta$ with respect to a reference star, as α and δ with a reference star mentioned, or as α and δ with no mention of a reference star. With a reference star given a new position for the star was calculated from the Tycho-2 catalog (Høg et al., 2000) and $\Delta\alpha$ and $\Delta\delta$ applied to the star to calculate the comet's α and δ. If $\Delta\alpha$ and $\Delta\delta$ were not given, the difference between the Tycho-2 position and the position given in the catalog specified by the observer was applied to the α and δ published by the observer. If a reference star is not mentioned, the observation was taken as is.

Table 3 lists some missing information for the Petersen 1 comet and Table 4 the same plus some errors for the Petersen 2 comet.

The rectangular coordinates and velocities of the comet and the Earth were calculated by a program, used in numerous investigations previously, that treats the solar system as an n-body problem. The program is a 12-th order Lagrangian predictor–corrector that incorporates relativity by a Schwarzschild harmonic metric. To obtain coordinates and velocities for the Earth, the moon is carried as a separate body. This means a small step-size, 0.^{d}25. To correct the comet's orbit partial derivatives are

Table 1
Observations of comet C/1848 P1 (Petersen).

Observatory	Obsns. in α	Obsns. in δ	Reference[a]
Bishop's Observatory, England	4	4	AN, 1848, No. 27, pp. 365–366
Altona, Germany	52	46	AN, 1848, No. 27, pp. 363–364 MN, 1848, Vol. 8, p. 207
Berlin, Germany	2	2	AN, 1848, No. 27, pp. 365–366
Hamburg, Germany	7	7	AN, 1848, No. 27, pp. 365–366
Königsberg, Germany	2	2	AN, 1848, No. 27, pp. 371–372
Total	67	61	All

[a] AN: *Astronomische Nachrichten;* MN: *Monthly Notices RAS.*

Table 2
Observations of comet C/1848 U1 (Petersen).

Observatory	Obsns. in α	Obsns. in δ	Reference[a]
Cambridge, England	14	14	MN, 1848, Vol. 9, p. 25
Durham, England	6	6	MN, 1849, Vol. 9, p. 108
Liverpool, England	34	34	AN, 1849, No. 28, pp. 249–250
Altona, Germany	2	2	MN, 1848, Vol. 9, p. 11
Berlin, Germany	8	8	MN, 1848, Vol. 9, p. 11
Bonn, Germany	14	14	AN, 1848, No. 28, pp. 139–142
Hamburg, Germany	35	35	MN, 1848, Vol. 9, p. 11 MN, 1848, Vol. 9, p. 25 MN, 1849, Vol. 9, p. 47
Leipzig, Germany	3	3	MN, 1849, Vol. 9, p. 47
Markree, Ireland	14	14	AN, 1849, No. 28, pp. 303–304 MN, 1849, Vol. 9, p. 107
Dorpat, Russia	4	4	AN, 1849, No. 28, pp. 249–250
Cambridge, USA	8	8	MN, 1849, Vol. 9, p. 107
Washington DC, USA	2	2	WA, 1856, Vol. 4, p. 292
Total	144	144	All

[a] AN: *Astronomische Nachrichten;* MN: *Monthly Notices RAS;* WA: *Washington Observations.*

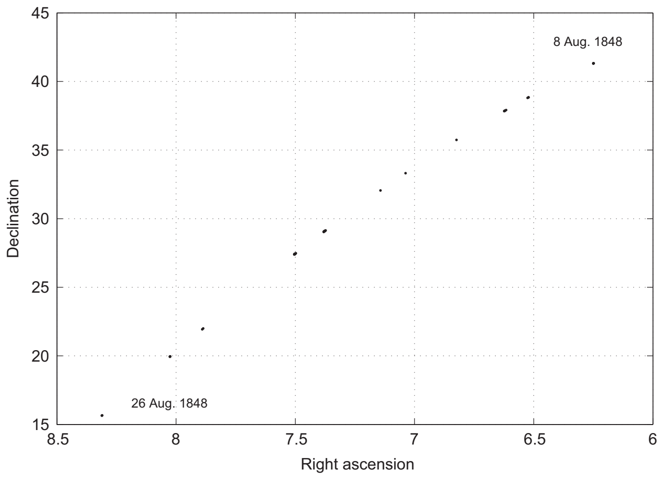

Fig. 1. The observations, comet C/1848 P1.

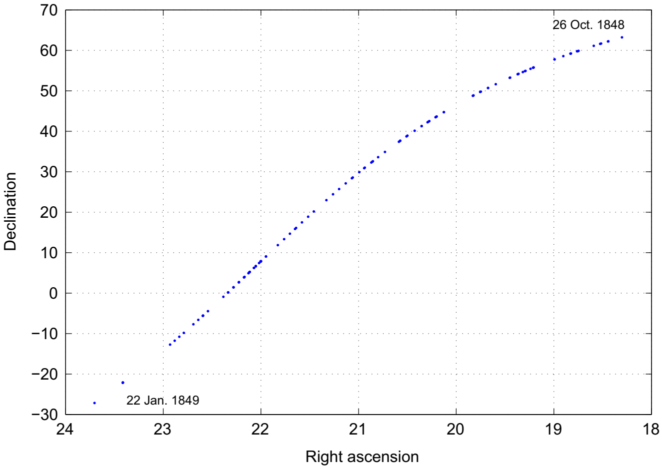

Fig. 2. The observations, comet C/1848 U1.

Table 3
Missing information for comet C/1848 P1.

Reference	Date	Error
AN, No. 27, pp. 371–372	August 15	Star a is Tycho 2442 01485 1
AN, No. 27, pp. 365–366	August 19	Star α is Tycho 1919 02071 1
AN, No. 27, pp. 365–366	August 19	Star β is Tycho 1919 01973 1

Table 4
Errors or missing information for comet C/1848 U1.

Reference	Date	Error or missing information
AN, No. 28, pp. 139–140	November 7	Star α is Tycho 3924 00228 1
AN, No. 28, pp. 139–140	November 7	Star β cannot be found
AN, No. 28, pp. 139–140	November 7	Star δ is Tycho 3924 00802 1
AN, No. 28, pp. 139–140	November 12	Star ζ is Tycho 3568 02286 1
AN, No. 28, pp. 249–250	December 17	Equinox of stars a, b is 1848
AN, No. 28, pp. 249–250	December 17	Star a is Tycho 1135 00027 1
AN, No. 28, pp. 249–250	December 17	Star b is Tycho 1137 00716 1
MN, Vol. 9, p. 108	November 3	Star is Tycho 3931 01125 1
MN, Vol. 9, p. 108	November 7	Star is Tycho 3924 00228 1
MN, Vol. 9, p. 108	November 29	Star is Tycho 2692 00938 1
MN, Vol. 9, p. 108	December 23	Star is Tycho 0560 00894 1

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calculated by Moulton's method (Herget, 1968), which integrates the partial derivatives to correct for the osculating rectangular coordinates and velocities at the epoch, JD2396240.5 for Petersen 1 comet and JD2396400.5 for Petersen 2, along with the coordinates and velocities. The rectangular coordinates, after interpolation to the moment of observation for the Earth and to the moment of observation antedated by the light time correction to allow for planetary aberration, are then converted to a unit vector that is transformed to a mean or apparent place in α and δ by application of precession, nutation, annual aberration, relativity, and so forth. Because we are dealing with 19-th century observations it is necessary to correct for the E-terms of the aberration during the calculation of a mean place. See Scott (1964) for a discussion of the E-terms. The final step calculates an observed minus a computed place, $(O-C)$, in α and δ.

To assign weights to the observations I use the Welsch weighting function, which assumes nothing as to the quality of a given type of observation, but rather assigns weight based on the magnitude of the post-fit residual (Branham, 1990, Sec. 5.5). This recognizes two features from statistical analysis of data: smaller residuals are more probable than larger residuals and hence assigns them higher weight; extremely large residuals are errors, not genuine but improbable residuals, and assigns them low weight, so low as to be virtually 0 for such large residuals. One scales the post-fit residual r_i by the median of the absolute values of the residuals and assigns a weight wt as

$$wt = \exp\left[-(r_i/2.985)^2\right] \qquad (1)$$

Rather than start from a least squares solution, I used the robust L_1 criterion (Branham, 1990, Ch. 6) for the first approximation, calculate the residuals from this solution, compute the weights, and then calculate a least squares solution. Because the first approximation is good, it generally becomes unnecessary to iterate the solutions more than once.

Fig. 3 shows the weights from Eq. (1) for comet Petersen 1. Although Eq. (1) in theory assigns weight 0 to no residual, any residual less than the machine $\epsilon, 2.2 \times 10^{-16}$ for the Intel processor used on my computer, is in effect 0. For Petersen 1 four of the weights are less than ϵ. Fig. 4 gives the weights for Petersen 2 of which five are less than the machine ϵ.

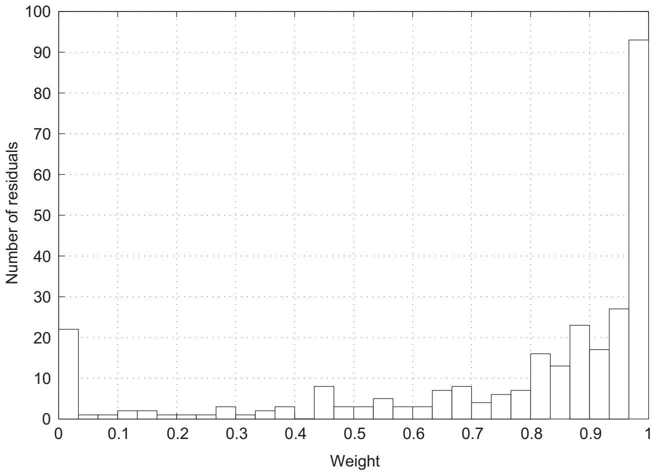

Fig. 4. Weights, comet C/1848 U1.

Table 5
Solution for rectangular coordinates and velocities comet C/1848 P1; epoch JD 2396240.5.

Unknown	Value	Mean error
x_0 (*AU*)	7.03524×10^{-1}	1.95961×10^{-5}
y_0 (*AU*)	2.75247×10^{-1}	1.02993×10^{-5}
z_0 (*AU*)	6.99647×10^{-1}	1.87639×10^{-5}
$\dot{x}_0$ (*AU day*$^{-1}$)	-7.20123×10^{-3}	1.83179×10^{-6}
$\dot{y}_0$ (*AU day*$^{-1}$)	1.87890×10^{-3}	3.764809×10^{-6}
$\dot{z}_0$ (*AU day*$^{-1}$)	-2.27907×10^{-2}	1.15501×10^{-6}
$\sigma(1)$	4.″15	

Table 6
Solution for rectangular coordinates and velocities for comet C/1848 U1; JD 2396400.5.

Unknown	Value	Mean error
x_0 (*AU*)	7.58417×10^{-1}	2.25040×10^{-5}
y_0 (*AU*)	6.20427×10^{-1}	8.68662×10^{-6}
z_0 (*AU*)	7.06778×10^{-2}	5.52511×10^{-2}
$\dot{x}_0$ (*AU day*$^{-1}$)	-7.67556×10^{-3}	3.44878×10^{-7}
$\dot{y}_0$ (*AU day*$^{-1}$)	6.12239×10^{-3}	3.71537×10^{-7}
$\dot{z}_0$ (*AU day*$^{-1}$)	-2.25115×10^{-2}	1.89605×10^{-7}
$\sigma(1)$	5.″15	

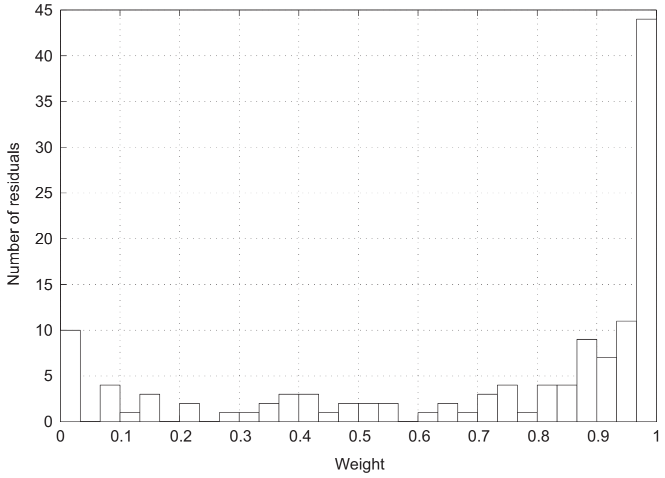

Fig. 3. Weights, comet C/1848 P1.

3. The solutions

Table 5 shows the final solution for the rectangular coordinates, x_0, y_0, z_0, and velocities, $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$, along with their mean errors and also the mean error of unit weight $\sigma(1)$ for the Petersen 1 comet; Table 6 exhibits the same entities for Petersen 2. Regarding correlations for comet Petersen 1 three of the correlations are greater than 50%: -76.2% between x_0 and $\dot{x}_0$; 58.1% between x_0 and $\dot{y}_0$; and -88.9% between z_0 and $\dot{z}_0$. But the condition number of 775 for the data matrix shows that the solution is stable. For comet Petersen 2 there are four correlations exceeding 50%: -75.6% between x_0 and y_0; 94.2% between x_0 and $\dot{x}_0$; 60.0% between x_0 and $\dot{y}_0$; and 80.7% between z_0 and $\dot{z}_0$. Despite the highest correlation being higher than that for Petersen 1, the condition number of the matrix decreased to 476. One may consider, once again, that the solution is stable.

Tables 7 and 8 give the orbital elements corresponding to the rectangular coordinates of Tables 5 and 6: the time of perihelion passage, T_0; the eccentricity, e; the semi-major axis, a; perihelion distance, q; the inclination, i; the node, Ω; and the argument of perihelion, ω. The calculation of the mean errors of the orbital elements proceeds via Rice's procedure (1902). Let $\mathbf{C}$ be the covariance matrix for the least squares solution for the rectangular

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Table 7
Hyperbolic orbital elements and mean errors for comet C/1848 P1.

Unknown	Value	Mean error
T_0	*JD* 2396253.45212 13.95219 August 1848	$0.^d00013$
a (*AU*)	−9110.401	21481.44
e	1.000035	0.000083
q (*AU*)	0.3200049	0.1769668
Ω	214.°6084	0.°5812702
i	76.°10868	0.°6000177
ω	23.°34118	0.°6877538

Table 8
Hyperbolic orbital elements and mean errors for comet C/1848 U1.

Unknown	Value	Mean error
T_0	*JD* 2396401.88317 09.38317 January 1849	$0.^d00566$
a (*AU*)	−399.1451	8.055121
e	1.002406	0.000049
q (*AU*)	0.9603926	0.0127546
Ω	221.°0584	0.°0644552
i	66.°77932	0.°0445548
ω	158.°1583022	0.°0762036

coordinates and velocities. Identify the errors in a quantity such as the node Ω with the differential of the quantity, $d\Omega$. The error can be found from

$$(d\Omega)^2 = \sigma^2(1)\left(\partial\Omega/\partial x_0 \;\; \partial\Omega/\partial y_0 \;\; \cdots \;\; \partial\Omega/\partial \dot{z}_0\right)\cdot \mathbf{C}\cdot \begin{pmatrix} \partial\Omega/\partial x_0 \\ \partial\Omega/\partial y_0 \\ \vdots \\ \partial\Omega/\partial \dot{z}_0 \end{pmatrix}. \qquad (2)$$

The partial derivatives in Eq. (2) are calculated from the well known expressions linking elliptical or hyperbolic orbital elements with their rectangular counterparts. For the Petersen 1 comet the orbit represents a hyperbola, but considering the mean error, and this is the surprise mentioned previously, statistically indistinguishable from a parabola. For Petersen's October comet the orbit becomes a hyperbola, but this time statistically distinguishable from a parabola.

4. Discussion

Although a nonparabolic orbit has been calculated for Petersen 2 but a parabola for Petersen 1, how good are they? Various statistical tests can be applied to test the goodness of an orbit. The runs test measures how often a variable, distributed about the mean, changes sign from positive to negative or negative to positive, called the runs. Runs with n data points have a mean of $n/2$ and a variance of $n(n-2)/4(n-1)$. The runs test, being nonparametric, makes no assumption about the normality of the data, an advantage over competing tests for nonrandomness, although to actually calculate probabilities for the observed runs one does assume approximate normality. For a discussion of the runs test see Wonnacott and Wonnacott (1972, pp. 409–411). The runs test was applied to the residuals, weighted by Eq. (1), calculated from the solutions of Tables 5 and 6. Fig. 5 shows the residuals for Petersen 1 and Fig. 6 those for Petersen 2.

The Petersen 1 comet evinces 59 runs out of an expected 64 with standard deviation 5.635. The residuals, therefore, may be considered random: there is a 37.5% chance with a 2-sided probability of their being random. For Petersen 2 there are 130 runs out of an expected 144 with standard deviation 8.470, giving a 9.8% chance of randomness. Knuth (1981, pp. 64–67) has developed a more sophisticated version of the runs test that calculates a covariance matrix and a chi-squared statistic for the probability of the null hypothesis: the observed residuals are drawn randomly from a normal distribution. For Petersen 1 the probability is 15.2% and for Petersen 2 11.4%. Although none of these probabilities can be considered overwhelmingly in favor of randomness, they are sufficient to rule out the possibility of serious systematic errors in the results.

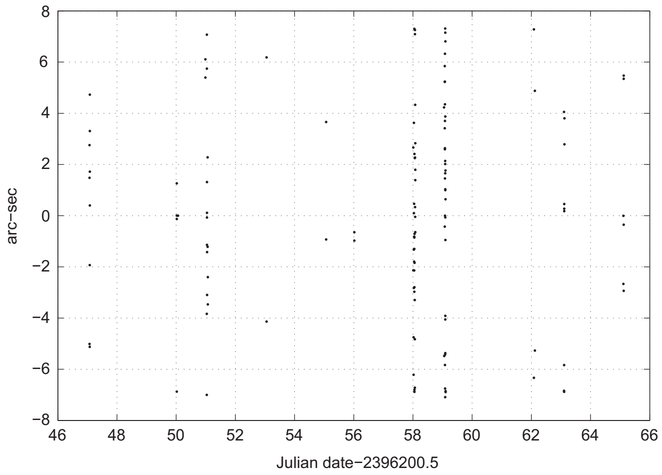

Fig. 5. Weighted residuals, comet C/1848 P1.

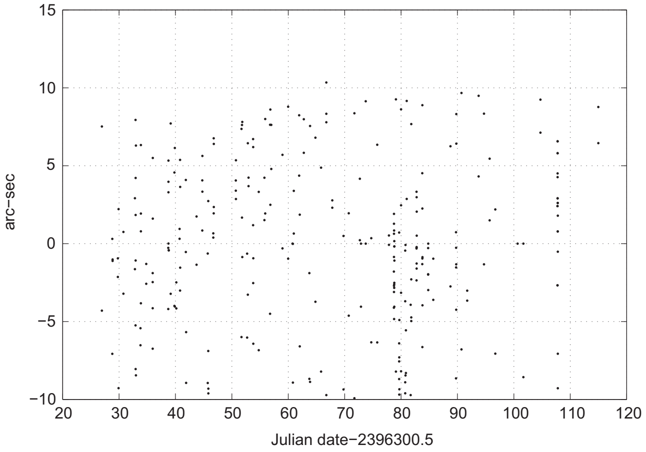

Fig. 6. Weighted residuals, comet C/1848 U1.

Because one orbit is parabolic and the other hyperbolic, neither comet represents an NEO threat. The mean error of the eccentricity, it is true, does not preclude comet Petersen 1 from being a high eccentricity ellipse. To be precise given the size of the eccentricity and its mean error and if we assume a normal distribution for the residuals, there is a 28.2% chance that the orbit may be elliptical. I integrated the orbit backwards to JD −9999999.5 ($\approx$ 32000 B.C.) after converting the coordinates of Table 5 to barycentric coordinates. At this time, with the comet at 4316 AU from the solar system center of mass, the orbit becomes a high eccentricity ellipse, $e=0.99987$, with $a=2549$ AU and $P=1.24\times10^5$ year. This comet, therefore, could not return for a long time, if it ever does return. It is extremely unlikely to have an extrasolar origin. Although there is a 71.8% chance of the orbit

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being hyperbolic, the shortness of the observational time span precludes inferring much about the origin of the orbit.

But could comet Petersen 2 come from beyond the Oort cloud? Integrating the orbit backwards to JD -999999.5, 7451 B.C., the comet finds itself at 3677 AU from the Sun with a still hyperbolic orbit, $a = -429.7$ AU, and a positive velocity of 1.596 km s^{-1}. The velocity, however, is decreasing at a rate of -3.8×10^{-4} km s^{-1} per day, which implies becoming velocity 0 at about a further 1900 AU. This behavior contrasts with that of Comet Secchi (Branham, 2012), which a had a velocity of 2.91 km s^{-1} at over 10^5 AU from the Sun and no sign of decreasing.

Could these conclusions change if one were to refine the reduction procedure by adding Galactic tidal effects and more perturbing objects? To be specific I added perturbations by the minor planet Ceres and the plutoids Eris, Makemake, Haumea, and Sedna. For the details of how to include the Galactic tidal modeling and the additional objects see Branham (2012). For comet Petersen 1 at JD -9999999.5 the distance from the barycenter becomes 4318 AU, a=2486 AU, e=0.99913, and the period still 1.24×10^5. These value differ little from the previous ones. For comet Petersen 2 at JD -999999.5 the comet finds itself at 3678 AU from the Sun in a hyperbolic orbit with $a = -437.0$ AU and a positive velocity of 1.585 km s^{-1} decreasing at the slower rate of -2.0×10^{-7} km s^{-1} per day. After 24,780 years the velocity becomes 0 at a further 5700 AU, well into the Oort cloud. But, as before, the orbit still becomes elliptical. Thus, the comet's origin seems to be the Oort cloud at a distance of 5000–10,000 AU.

5. Conclusions

Both comet C/1848 P1 (Petersen) and comet C/1848 U1 (Petersen) have non-elliptical orbits, the former a parabola and the latter a hyperbola. Thus, neither comet can be considered as an NEO. Nor is there evidence for either comet having an extrasolar origin.

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