Journal of Process Control 57 (2017) 26–37

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Economic performance assessment and monitoring in LP-DMC type controller applications

J.L. Godoy[a,b,*], A. Ferramosca[c], A.H. González[a]

[a] INTEC (CONICET and Universidad Nacional del Litoral), Santa Fe, Argentina
[b] Universidad Tecnológica Nacional (UTN), Facultad Regional Paraná. Argentina
[c] CONICET, Universidad Tecnológica Nacional (UTN), Facultad Regional Reconquista. Reconquista, Argentina

ARTICLE INFO

Article history:
Received 25 August 2016
Received in revised form 16 May 2017
Accepted 7 June 2017

Keywords:
Linear programming-dynamic matrix control
Performance assessment
Underlying optimization problem
Key performance indicators

ABSTRACT

One of the main reported problems in petrochemical applications of Linear Programming-Dynamic Matrix Control (LP-DMC) type controllers is their global performance assessment. Since the stationary optimization and dynamic control blocks have not a transparent link between them, it is not easy to find appropriate references to evaluate the overall performance, and more important, to clearly determine the causes of an eventual loss of performance. In this work, the whole LP-DMC problem is analyzed and an off-line underlying optimization problem is defined, whose solution is used as a benchmark for the global closed-loop behaviors study. Based on this idea three global indicators, which evaluate and diagnose poor controller performance, are proposed. The indicators were successfully tested in two case studies: an integrating system and the well-known Shell heavy oil fractionator.

1. Introduction

1.1. Theretical background of MPC performance assessment techniques

Model Predictive Control (MPC) is probably the most used advanced control strategy in industrial applications, that has also reached a consistent theoretical development in the academic field. Aspects as stability, feasibility, inclusion of additional (economic) objectives, robustness, etc., have been successfully tackled in recent years [1,2]. However, the MPC-type controllers currently used in industry do not necessarily take advantages of these benefits. In fact, most of the MPC-type controllers currently used in large scale industries are still DMC-type controller (Dynamic Matrix Control [3,4]), and thus, they have a structure that significantly differs – in many aspect – from the theoretic formulations found in the academic literature. The DMC-type controller, for instance, separates the constrained on-line MPC optimization problem into two linked stages: (i) a stationary constrained optimization – based on a Linear or Quadratic Programing (LP or QP), which calculates a set of stationary targets to be sent to the next stage, and (ii) an unconstrained dynamic optimization (DMC), which computes the control actions necessary to reach the stationary targets, and injects these controls to the plant [5–7]. Furthermore, in the particular case of LP-DMC – and opposite to what is usually assumed in the MPC literature [8–10] – the two optimization problems do not represent the typical separation between the stationary-economic optimization (real time optimizer, RTO) and the dynamic one (advanced control), or at least, they are not posed in a transparent form, as it is desired in industrial environments. Instead, the stationary optimization problem (i.e., the LP problem), incorporates movement constraints and it is devoted to iteratively steers the system to a final setpoint that is "implicitly" defined by the linear cost function and by the system constraints. This way, there is not a transparent interpretation of what the controller is really doing regarding the stationary optimality: the cost function only provides a direction in a usually large input space, and given that the optima will be at the boundary of the feasible set, small modifications of the constraints values may produce a large variation of the optimal operating point.

DMC controllers generally provide outstanding performances just after the controllers' commissioning. However, the closed-loop performance may deteriorate rapidly, due to set-points changes or disturbances, leading to counterintuitive scenarios where the operators tend to incorrectly modify the main DMC parameters, or even, to turn off the controller. As a matter of fact, without monitoring and ongoing support, the performance and benefits of DMC applications are generally reduced by 40% after the first or second year of operation [11–15]. This clearly indicates the importance

* Corresponding author at: INTEC (CONICET and Universidad Nacional del Litoral), Güemes 3450, Santa Fe (3000), Argentina.
E-mail address: jlgodoy@santafe-conicet.gov.ar (J.L. Godoy).

http://dx.doi.org/10.1016/j.jprocont.2017.06.007
0959-1524/

of assessing and monitoring the performance of DMC controllers to ensure an effective control and, consequently, a safe and profitable operation of the plant. However, due to the limitations of the DMC structure presented above, a proper performance assessment of the economic/stationary behavior of LP-DMC controller is both, counterintuitive and difficult to implement.

Despite the wide diversity of techniques for MPC performance assessment, most of them can be grouped according to some common features, related to the provided information and the required input information. A group of methods is based on the comparison of current data with an historical benchmark ([16–18]); for instance, some methods rely on Minimal Variance Controller as a benchmark ([19,20]). Others are focused on investigating the quality of the process model by using the prediction error as an index to assess the controller performance ([21,22]). In fact, the aim of these latter techniques is to assess (mainly) the quality of the prediction models ([16,23–25]), according to the (not always true) belief that the limitation for a better closed-loop performance comes only from the inaccuracy of the models (several studies have shown that the control-relevant characteristics of the model are also crucial).

Although several approaches have been proposed in the literature to formulate proper performance indicators and benchmarks [12,26], most of them appear to be incomplete, inefficient or impractical. As a consequence, only few performance indicators have been actually implemented in industries [12]: one exception is given by Lee et al. [27], which presents an industrial implementation of the assessment approach proposed in [28]. Another exception is the indicators presented in [7], which are the most commonly used indicators in LP-DMC industrial applications.

1.2. The key performance indicators used in industry

LP-DMC controllers are commonly used in large-scale processes (such as oil and gas refiniries), where processes have usually more controlled variables (CVs) than manipulated variables (MVs). These CVs can be separated into two groups: CVs with frequently active (binding or more relevant) constraints and CVs with frequently inactive (non-binding or less relevant) constraints.

As any practitioner know, the real process must be pushed toward the bending CV constraints to reach better plant economic operations, avoiding, if possible, any MV constraint activation, since MV saturation means a loss of degrees of freedom or controllability. On the other hand, in LP problems the number of active constraints at the optimum is given by the degree of freedom of the process (i.e., the number of MV variables). So, the most used key performance indicators (KPIs) in industry are: *(i)* the CV constraints percentage of activation (*pCVac*), defined as the ratio between the number of CVs' active constraints and the number of MVs; and *(ii)* the MV constraints percentage of activation (*pMVac*), defined as the ratio between the number of MVs' active constraints and the number of MVs [7]. Clearly, the *pCVac* indicator measures if the desired implicit objective of reaching the bending CV constraints is achieved, while *pMVac* measures the loss of degrees of freedom, being $pCVac + pMVac = 100\%$ (the ideal scenario is then given by $pMVac = 0$).

However, given its simplicity, these indicators are rather poor to realistically assess the whole operation, deriving many times in completely wrong operating decisions. The *pCVac* indicator, for instance, does not take into account neither the proximity of inactive limits nor the model deterioration and disturbances. This indicator considers an objective achieved only when the corresponding CV continuously meets the limit, while unmeasured disturbances (or plant-model mismatches) could produces small variations around the limits, that should not be considered a problem. This way, this kind of indicators usually results in unpleasing low assessment values, typically 25%–50%, even for newly commissioned controllers [7]. Some improvement to the classical *pCVac and pMVac* indicators, based on heuristic fundamentals, were proposed in [11], but the results were not the expected ones.

1.3. The proposed KPIs

In this work, based on a subjacent stationary optimization problem that correctly describes the underlying economic stationary optimum, some novel KPIs are proposed to assess the whole controller performance from a more realistic point of view. The proposed indicators accounts for both, stable and integrating outputs, explotes the distance of the operating point to the constraints and explicitly considers disturbances and model-plant mismatches. Furthermore, a pure economic performance indicator is proposed, which enables more reliable management decisions.

This paper is organized as follows: section 2 briefly describes the structure of a typical LP-DMC type commercial controller. Section 3 presents an underlying optimization problem, which describes the theoretical details necessary to formulate the indicators presented in Section 4. In Section 5, several simulations results are shown to test the proposed indicators. Finally, in Section 6, some conclusions to this work are provided.

2. LP-DMC controller description

The LP-DMC controller separates its main tasks into [6]: *(i)* a predictor block, which performs all the prediction necessary for the subsequent stages; *(ii)* a steady state optimization block, that computes at every time step the incremental stationary targets that should converge (if properly tuned) to the economically optimal target, and are sent to the dynamic block; and *(iii)* a dynamic controller block, that calculates the control actions to be applied to the plant (or even, to low-level controllers), and try to reach the targets received from the incremental stationary optimizer. Next, a brief description of each stage is presented.

2.1. Predictor

In DMC type commercial controllers, the model to perform the future output predictions is given in the step response form. This way, the future outputs, predicted at time k, using information up to (and including) time k-1, $\hat{\mathbf{Y}}(\mathrm{k|k-1}) = \left[\hat{\boldsymbol{y}}(\mathrm{k|k-1})' \cdots \hat{\boldsymbol{y}}(\mathrm{k+N-2|k-1})' \ \mathbf{x}(\mathrm{k|k-1})'\right]'$, are given by

$$\hat{\mathbf{Y}}(k|k-1) = \mathbf{M}\,\hat{\mathbf{Y}}(k-1|k-1) + \mathbf{A}\Delta\mathbf{u}(k-1), \tag{1}$$

where $\hat{\mathbf{Y}}(\mathrm{k-1|k-1}) = \left[\hat{\boldsymbol{y}}(\mathrm{k-1|k-1})' \cdots \hat{\boldsymbol{y}}(\mathrm{k+N-3|k-1})' \ \mathbf{x}(\mathrm{k-1|k-1})'\right]'$ is the N-horizon predicted output vector at time k-1, assuming constant future inputs (with $\mathbf{x}$ being the state vector for residual plant dynamics neglected when the step response is truncated after N steps, and only used for integrating outputs); $\Delta\mathbf{u}(k-1) = \mathbf{u}(k-1) - \mathbf{u}(k-2)$ is a vector of changes in the manipulated input, $\mathbf{u}(k-1)$; matrices $\mathbf{M}$ and $\mathbf{A}$ are given by

$$\mathbf{M} = \begin{bmatrix} 0 & \mathbf{I}_{ny} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I}_{ny} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \mathbf{I}_{ny} & 0 \\ 0 & 0 & 0 & \cdots & 0 & \mathbf{I}_{ny} & \mathbf{I}_{ny} \\ 0 & 0 & 0 & \cdots & 0 & 0 & \mathbf{A}_R \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_{N-1} \\ \mathbf{A}_N - \mathbf{A}_{N-1} \end{bmatrix}, \tag{2}$$

where $\mathbf{A}_i$, with $i = 1,\ldots,N$, is a matrix containing the step response coefficients at time i corresponding to each output and input; and

$\mathbf{A}_R$ is a matrix devoted to separate the effect of stable modes from that of integrating ones. Furthermore, the correction of $\hat{\mathbf{Y}}(k|k-1)$ based on output measurements at time k, $\mathbf{y}(k)$, is given by

$$\hat{\mathbf{Y}}(k|k) = \hat{\mathbf{Y}}(k|k-1) + \mathbf{K}\left(\mathbf{y}(k) - \hat{\mathbf{y}}(k|k-1)\right), \tag{3}$$

where $\mathbf{K}$ is the filter gain. For further details on this formulation, the interested reader is referred to [29]. Each element $\hat{\mathbf{y}}(k+l|k)$ of $\hat{\mathbf{Y}}(k|k)$ is the estimate of the future output vector $\mathbf{y}(k+l)$ based on measurements up to time k. The two predicted outputs to be sent to the steady state optimizer block are

$$\begin{aligned}\hat{\mathbf{y}}(k+N-2|k) &= \mathbf{M}_{N-2}\,\hat{\mathbf{Y}}(k|k),\\ \hat{\mathbf{y}}(k+N-3|k) &= \mathbf{M}_{N-3}\,\hat{\mathbf{Y}}(k|k),\end{aligned} \tag{4}$$

where $\mathbf{M}_{N-2} = \begin{bmatrix}0 & \cdots & 0 & \mathbf{I}_{ny} & 0\end{bmatrix}$, $\mathbf{M}_{N-3} = \begin{bmatrix}0 & \cdots & \mathbf{I}_{ny} & 0 & 0\end{bmatrix}$. Notice that $\hat{\mathbf{y}}(k+N-2|k)$ is the stationary prediction of future outputs after an input change $\Delta\mathbf{u}(k-1)$. On the other hand, the predicted output vector to be sent to the dynamic controller block is

$$\hat{\mathbf{Y}}_p = \begin{bmatrix}\hat{\mathbf{y}}(\mathrm{k}+1|\mathrm{k})' & \cdots & \hat{\mathbf{y}}(\mathrm{k}+\mathrm{N}-2|\mathrm{k})'\end{bmatrix}' = \mathbf{M}_p\,\hat{\mathbf{Y}}(\mathrm{k}|\mathrm{k}), \tag{5}$$

where $\mathbf{M}_p$ is the first $(N-2)\times ny$ rows of $\mathbf{M}$ (Eq. (2)). Given that N cover the system estimated settling time, the prediction horizon N is usually selected as $N = Hc + Hss$, where Hc is the control horizon and Hss is the steady state horizon. The control horizon, on the other hand, is usually selected as $Hc = 0.5Hss$, which leads to prediction horizon of $N = 1.5Hss$.

2.2. Incremental LP optimizer

For the sake of clarity, we will denote with the superscript r the components of matrices or vectors that correspond exclusively to integrating (ramps) outputs. Based on the stationary prediction of future outputs, the Incremental LP formulation for a given time k is defined as follows [5]:

$$\begin{aligned}\Delta\mathbf{u}_{ss}(k) = \underset{\Delta\mathbf{u}}{arg\,min}\ & \mathbf{f}'\Delta\mathbf{u}\\ s.t.\\ & -Hc\,\Delta\mathbf{u}_{\max} \le \Delta\mathbf{u} \le Hc\,\Delta\mathbf{u}_{\max}\\ & \mathbf{u}_{\min} \le \mathbf{u}(k-1) + \Delta\mathbf{u} \le \mathbf{u}_{\max}\\ & \mathbf{y}_{\min} \le \hat{\mathbf{y}}(k+N-2|k) + \mathbf{A}_{N-1}\Delta\mathbf{u} \le \mathbf{y}_{\max}\\ & \mathbf{S}\Delta\mathbf{u} + \Delta\hat{\mathbf{y}}^r(k+N-2|k) = 0\end{aligned} \tag{6}$$

where $\mathbf{f}$ (usually called LP cost) is a constant vector defining the cost function gradient (which means that $-\mathbf{f}$ is the direction of maximum decrease), $\Delta\mathbf{u}_{\max}$, $\mathbf{u}_{\max}$, $\mathbf{u}_{\min}$, $\mathbf{y}_{\max}$, $\mathbf{y}_{\min}$ represent the constraints on the input move, the input and the output, respectively, $\mathbf{S} = (\mathbf{A}^r_{N-1} - \mathbf{A}^r_{N-2})$ represent the system slopes of integrating outputs and $\Delta\hat{\mathbf{y}}^r(k+N-2|k) = \hat{\mathbf{y}}^r(k+N-2|k) - \hat{\mathbf{y}}^r(k+N-3|k)$ are the latest integrating output changes. The equality constraint basically forces the integrating outputs to remain constant at the end of the prediction horizon. In this LP problem, $\Delta\mathbf{u}$ is the optimization variable, and it is associated to the future input moves necessary to achieve an optimal stationary value. Based on the LP problem solution, $\Delta\mathbf{u}_{ss}(k)$, the inputs and outputs to be sent to the dynamic stage as targets, are then given by:

$$\begin{aligned}\mathbf{u}_{ss}(k) &= \mathbf{u}(k-1) + \Delta\mathbf{u}_{ss}(k),\\ \mathbf{y}_{ss}(k) &= \hat{\mathbf{y}}(k+N-2|k) + \mathbf{A}_{N-1}\Delta\mathbf{u}_{ss}(k)\end{aligned} \tag{7}$$

where it should be noted that $\mathbf{u}_{ss}(k)$ represents the target for the last inputs at the end of the control horizon, k–1 + Hc, and $\mathbf{y}_{ss}(k)$ represents the stationary output target at the end of the prediction horizon, k + N-2.

2.3. Dynamic matrix controller (DMC)

The dynamic block (or DMC) computes the best current and future control moves based on the transitory prediction of future outputs [5,29]. The DMC block solves the following quadratic unconstrained optimization:

$$\begin{aligned}\Delta\mathbf{U}(k) = arg\min_{\Delta\mathbf{U}}\ & \|\mathbf{Q}\left(\mathbf{Y}_{ss} - \hat{\mathbf{Y}}_p - \mathbf{A}^{Hc}_{N-2}\Delta\mathbf{U}\right)\|^2 + \|\lambda\mathbf{P}\mathbf{R}_R\Delta\mathbf{U}\|^2\\ & + \|\lambda\mathbf{R}_{\mathbf{u}}\left(\Delta\mathbf{u}_{ss}(k) - [\mathbf{I}_{nu}\ \ldots\ \mathbf{I}_{nu}]\,\Delta\mathbf{U}\right)\|^2\end{aligned} \tag{8}$$

where $\Delta\mathbf{U}(\mathrm{k}) = \left[\Delta\mathbf{u}(\mathrm{k})'\ \cdots\ \Delta\mathbf{u}(\mathrm{k}+\mathrm{Hc}-1)'\right]'$ is the sequence of Hc future input movements, $\mathbf{Y}_{ss} = \left[\mathbf{y}_{ss}(\mathrm{k})'\ \ldots\ \mathbf{y}_{ss}(\mathrm{k})'\right]'$ is the sequence of N-2 future output targets (Eq. (7)) coming from the Incremental LP optimizer, $\hat{\mathbf{Y}}_p$ comes from the predictor (Eq. (5)), and $\mathbf{A}^{Hc}_{N-2}$ is the so-called dynamic matrix [29]. $\mathbf{R}_{\mathbf{u}} = diag(\mathbf{u}_{\max} - \mathbf{u}_{\min})^{-1}$ and $\mathbf{R}_R = diag\,(\mathbf{R}_{\mathbf{u}}\ldots\mathbf{R}_{\mathbf{u}})$ are normalization matrices. $\mathbf{Q}$and $\mathbf{P}$ are weighting diagonal matrices of appropriated dimension, and λ is a weighting coefficient. In particular, $\mathbf{Q} = diag\left(1/ECE_1 \ldots\ 1/ECE_{ny}\ldots\right)$, where ECE_j is the Equal Concern Error (ECE) of the jth CV, which is defined as the acceptable error level of the jth CV with regard to its target [7]. Notice that this optimization (Eq. (8)) attempt to force the final dynamic value $\mathbf{u}(k+Hc-1)$ to be equal to input target $\mathbf{u}_{ss}(k)$; i.e. this is the way the complete LP solution (i.e., $\mathbf{u}_{ss}(k)$ and $\mathbf{y}_{ss}(k)$) is passed to the dynamic controller block. According to the receding horizon policy, only the current control move $\Delta\mathbf{u}(k)$ is applied to plant. Note that in most of the cases (most of commercial DMC-type controllers) the problem in Eq. (8) is unconstrained, and only the current control move is limited to be $-\Delta\mathbf{u}_{\max} \le \Delta\mathbf{u}(k) \le \Delta\mathbf{u}_{\max}$, by means of a saturation function at the output of the dynamic controller block.

3. Underlying optimization problem

The incremental stationary optimization problem (Eq. (6)) not only includes input move constraints, but also has a cost function written in terms of these input moves. So, this cost will be null every time the system reaches a steady state. This means that the cost of problem (Eq. (6)) does not represent itself an economic or meaningful cost, since its value at steady state will be always null, independently of the stationary point the system has reached. However, there exists an underlying (hidden) optimization problem which the Incremental LP problem (Eq. (6)) will iteratively tend to. The new underlying LP problem will be called *Overall LP problem*. This problem does not depend on the time k, and in fact it can be solved off-line, since it attempts to find a true stationary point that minimizes a given cost, independently of where the system currently is. Given that the vector $\mathbf{f}$ of the cost is given by economically-selected weights, it is reasonable to use this vector to define the Overall LP problem. Furthermore, the optimization variables of this problem should be the inputs, and the constraints of this problem should define the entire region where the system can be moved (i.e. no input move constraints). This way, the proposed Overall LP problem (which does not depend on k, but only on the system stationary model) is given by:

$$\begin{aligned}\mathbf{u}^* = \underset{\mathbf{u}}{arg\,min}\ & \mathbf{f}'\mathbf{u}\\ s.t.\ \ & \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}\\ & \mathbf{y}_{\min} \le \mathbf{A}_{N-1}\mathbf{u} + \mathbf{d}_{ss} \le \mathbf{y}_{\max}\\ & \mathbf{S}\mathbf{u} - \mathbf{r}_{ss} = 0\end{aligned} \tag{9}$$

where $\mathbf{d}_{ss}$ and $\mathbf{r}_{ss}$ are model-plant mismatches at steady state, which are estimated using the stationary pair $(\mathbf{y}_{ss}, \mathbf{u}_{ss})$, as follows: $\mathbf{d}_{ss} = \mathbf{y}_{ss} - \mathbf{A}_{N-1}\mathbf{u}_{ss}$, $\mathbf{r}_{ss} = \mathbf{S}\mathbf{u}_{ss}$. The measurements at steady state $(\mathbf{y}_{ss}, \mathbf{u}_{ss})$ are obtained on a time interval next to controller com-

missioning where the controller is working well. It is important to note that $\mathbf{d}_{ss}$ and $\mathbf{r}_{ss}$ are due to gains and slopes mismatches, respectively; and these parameters are a benchmark of the model-plant mismatches (MPMs).

3.1. The economically optimal target

In many industrial application of interest, the best steady state from an economic point of view is not defined by a fixed point in the input/output space, but (simply) by maximizing or minimizing some meaningful CVs. This means that the economically optimal target is defined as a set of *nu* desired active constraints [30–32]. Let us denote with the superscript t those *nu* binding CV constraints involved in the economically optimal target. Then, the optimal input target, $\mathbf{u}^t$, is defined in terms of these binding CV constraints, as the one fulfilling

$$\mathbf{H}^t\mathbf{u}^t = \mathbf{b}^t, \tag{10}$$

where $\mathbf{H}^t \in \Re^{nu\times nu}$ and $\mathbf{b}^t \in \Re^{nu}$ are a matrix and a vector selecting the desired active inequality and equlity constraints of $\mathbf{H}$ and $\mathbf{b}$, which are given by:

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}_{N-1} \\ -\mathbf{A}_{N-1} \\ \mathbf{S} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{y}_{\max} - \mathbf{d}_{ss} \\ -\mathbf{y}_{\min} + \mathbf{d}_{ss} \\ \mathbf{r}_{ss} \end{bmatrix}.$$

Equivalently, the optimal input target, $\mathbf{u}^t$, is such that it satisfies the following equation:

$$\mathbf{y}_{\lim} = \begin{bmatrix} \mathbf{A}^s_{N-1} \\ \mathbf{S} \end{bmatrix}\mathbf{u}^t + \begin{bmatrix} \mathbf{d}^s_{ss} \\ -\mathbf{r}_{ss} \end{bmatrix} = \mathbf{A}^t\mathbf{u}^t + \mathbf{d}^t, \tag{11}$$

where $\mathbf{y}_{\lim}$ is a limits vector composed by the desired upper limits, desired lower limits and zeros (for integrating CVs); and $\mathbf{A}^s_{N-1}$ and $\mathbf{d}^s_{ss}$ are the gains and the respective mismatches corresponding to the involved stable CVs.

Assuming that $\mathbf{H}^t$ is full rank, the optimal input target, $\mathbf{u}^t$, can be computed by solving Eq. (10) or Eq. (11). In this context, the main objective for the Overall LP problem (Eq. (9)) is to reach $\mathbf{u}^t$ as its optimal solution, which can be written as $\mathbf{u}^* = \mathbf{u}^t$. However, this situation may be obstructed by undesired constraint activation (that prevent the feasibility of the desired vertex) or by new MPMs that shift the mapped CV constraint limits, preventing the controller from reaching the economically optimal target.

Remark. Note that in principle, the Incremental LP problem is the one devoted to find the economically optimal target input, $\mathbf{u}^t$, by means of the penalties in $\mathbf{f}$, which include some economic information. However, given that this is a very simplified problem,[1] the point iteratively reached (i.e., its optimum) is usually far from the desired one. This is the reason way most of the industrial application of LP-DMC Type controllers shows frequently low economic performance.

3.2. Convergence of the incrememntal to the overall LP problem

First we present a remark that relates both, the Incremental and Overall LP problems.

[1] Note that although the Incremental LP problem is usually commissioned such that the Overall LP problem has $\mathbf{u}^t$as its minimum, this setup only works for the first months. Then, because of the simplicity and the lack of robustness of the whole formulation, this problem becomes to have a minimum far from the desired one.

Remark. Provided that $\Delta\mathbf{u} = \mathbf{u} - \mathbf{u}(k-1)$ in Eq. (6), the Incremental LP problem can be rewritten in terms of input variables (instead of input movements) as follows

$$\begin{aligned} \Delta\mathbf{u}_{ss}(k) = arg \min_{(\mathbf{u}-\mathbf{u}(k-1))} & \mathbf{f}'(\mathbf{u}-\mathbf{u}(k-1)) \\ s.t. & \\ -Hc\,\Delta\mathbf{u}_{\max} \le \mathbf{u} - \mathbf{u}(k-1) \le Hc\,\Delta\mathbf{u}_{\max} & \\ \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} & \\ \mathbf{y}_{\min} \le \hat{\mathbf{y}}(k+N-2|k) + \mathbf{A}_{N-1}\mathbf{u} - \mathbf{A}_{N-1}\mathbf{u}(k-1) \le \mathbf{y}_{\max} & \\ \mathbf{S}u - \mathbf{S}u(k-1) + \Delta\hat{\mathbf{y}}^r(k+N-2|k) = 0 & \end{aligned}$$

This problem, given that $\mathbf{u}(k-1)$ is a fixed value, and that minimizing $\mathbf{f}'(\mathbf{u}-\mathbf{u}(k-1))$ is the same as minimizing $\mathbf{f}'\mathbf{u}$ can be expressed as

$$\begin{aligned} \mathbf{u}_{ss}(k) = arg\min_{\mathbf{u}} & \mathbf{f}'\mathbf{u} \\ s.t. & \\ -Hc\,\Delta\mathbf{u}_{\max} + \mathbf{u}(k-1) \le \mathbf{u} \le Hc\,\Delta\mathbf{u}_{\max} + \mathbf{u}(k-1) & \\ \mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max} & \\ \mathbf{y}_{\min} \le \mathbf{A}_{N-1}\mathbf{u} + \mathbf{d}_{ss}(k) \le \mathbf{y}_{\max} & \\ \mathbf{S}\mathbf{u} - \mathbf{r}_{ss}(k) = 0 & \end{aligned} \tag{12}$$

where we recall that $\mathbf{d}_{ss}(k) = \hat{\mathbf{y}}(k+N-2|k) - \mathbf{A}_{N-1}\mathbf{u}(k-1)$ and $\mathbf{r}_{ss}(k) = \mathbf{S}u(k-1) - \Delta\hat{\mathbf{y}}^r(k+N-2|k)$. Notice also that when the controlled process reaches a stationary point with the following measurements: $(\mathbf{y}(k),\ \mathbf{u}(k-1)) = (\mathbf{y}_{ss},\ \mathbf{u}_{ss})$, then it holds that $\hat{\mathbf{y}}(k+N-2|k) = \mathbf{y}_{ss}$ and $\hat{\mathbf{y}}^r(k+N-2|k) = \hat{\mathbf{y}}^r(k+N-3|k) = \mathbf{y}^r_{ss}$, which implies that $\mathbf{d}_{ss}(k)$ and $\mathbf{r}_{ss}(k)$ of Eq. (12) become equal to $\mathbf{d}_{ss}$ and $\mathbf{r}_{ss}$ of (Eq. (9)), respectively. As a consequence, the last solution of the Incremental LP problem (Eq. (12)) meets the condition: $\mathbf{H}^t\mathbf{u}_{ss}(k) = \mathbf{b}^t$ (see Eq. (10)).

The latter optimization problem looks like the Overall LP problem (it has the same cost and almost the same feasible region/set), except for the input movement constraints. In this context, we can denote as $U^I_{feas}(k)$ the **time-dependent** feasible set of the (rewritten) Incremental LP problem (Eq. (12)) at the time k and as U^O_{feas} the **time-independent** feasible set of the Overall LP problem (Eq. (9)). Let us assume that $\mathbf{d}_{ss}(k)$ and $\mathbf{r}_{ss}(k)$ are approximately constant over the time, then the only **parameter** of the problem in Eq. (12) that depends on time k, is $\mathbf{u}(k-1)$, which modifies the input movement constraint. If we define now the box-type set of maximum input movements as $U^\Delta_{feas} = \left\{\mathbf{u}_\Delta \mid -Hc\,\Delta\mathbf{u}_{\max} \le \mathbf{u}_\Delta \le Hc\,\Delta\mathbf{u}_{\max}\right\}$, then $U^I_{feas}(k) = \left(\mathbf{u}(k-1) \oplus U^\Delta_{feas}\right) \cap U^O_{feas}$, which implies that $U^I_{feas}(k) \subseteq U^O_{feas}$ for all $k \ge 0$ (provided that $\mathbf{u}(k-1) \in U^O_{feas}$). Notice that U^Δ_{feas} is a **time-independent** set. In the Overall LP problem (Eq. (9)), the solution $\mathbf{u}^*$ is not – in general – the input target obtained using the solution of the Incremental LP problem (Eq. (12)), $\mathbf{u}_{ss}(k)$. What the next Theorem states, is precisely that $\mathbf{u}_{ss}(k) = \mathbf{u}^*$ and $\mathbf{y}_{ss}(k) = \mathbf{A}_{N-1}\mathbf{u}^* + \mathbf{d}_{ss}$, in a finite number of steps. Fig. 1 shows a schematic plot of the result presented in Theorem 1. This figure shows the feasible set U^O_{feas} (dashed line) of the Overall LP problem and the feasible sets sequence $U^I_{feas}(k)$ of the Incremental LP problems, from initial set to the final set.

Theorem 1. Let us consider the sequence of Incremental LP solutions $\left\{\mathbf{u}_{ss}(k),\ \ k \ge 0\right\}$, where $\mathbf{u}_{ss}(0) = \mathbf{u}(0)$ is the initial condition and whose successive terms $\mathbf{u}_{ss}(k+1)$ and $\mathbf{u}_{ss}(k)$ are related by the following iteration function: $\mathbf{u}_{ss}(k+1) =$

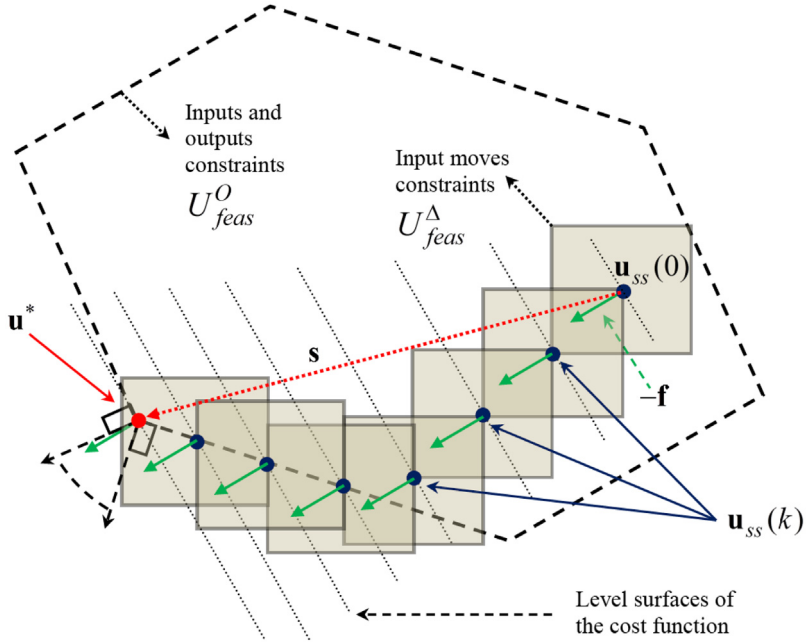

Fig. 1. Convergence of Incremental LP to the Overall LP solution.

$$\underset{\mathbf{u}_I}{\operatorname{argmin}} \left\{ \mathbf{f}'\mathbf{u}_I \;\middle|\; \mathbf{u}_I \in \overbrace{\left(\mathbf{u}_{ss}(k) \oplus U^{\Delta}_{feas}\right) \cap U^{O}_{feas}}^{U^{I}_{feas}(k+1)} \right\}, \quad k \geq 0.$$

Then, the sequence converges to $\mathbf{u}^* = \underset{\mathbf{u}}{\operatorname{argmin}}\{\mathbf{f}'\mathbf{u} \mid \mathbf{u} \in U^O_{feas}\}$ (i.e., to the Overall LP solution) in a finite number of steps.

3.3. Sketch of the proof

Let us consider the vectors $\mathbf{w}(k) = \mathbf{u}_{ss}(k+1) - \mathbf{u}_{ss}(k)$, which go from $\mathbf{u}_{ss}(k)$ to $\mathbf{u}_{ss}(k+1)$, for every $k \geq 0$, where $\mathbf{u}_{ss}(k+1)$ corresponds to the vertex of $U^I_{feas}(k+1)$ that maximizes $-\mathbf{f}'\mathbf{w}(k)$. The set U^Δ_{feas} is a box-type set, given by the limits $\pm Hc\,\Delta\mathbf{u}_{\max}$, which means that the faces of this set (polytope) are in the direction of the coordinate axes (two faces per axis). Let us define the vector $\mathbf{s} = \mathbf{u}^* - \mathbf{u}_{ss}(0)$, which goes form the initial condition $\mathbf{u}_{ss}(0) = \mathbf{u}(0)$ to $\mathbf{u}^*$ (see Fig. 1). The angles between this latter vector and the coordinate versors (or their opposite ones), $\mathbf{v}_i$, are denoted by θ_i, where $i = 1, \ldots, 2nu$ denote the index of each coordinate versor and its opposite one (i.e., $\mathbf{v}_n = -\mathbf{v}_{n+nu}$, for all $1 \leq n \leq nu$). Let $\mathbf{v}_i$ be the coordinate versor corresponding to the smallest angle θ_i, for some (not necessarily unique) $1 \leq i \leq 2nu$. Then, define the projection of $\mathbf{w}(k)$ on $\mathbf{s}$ and $\mathbf{v}_i$, along the normal direction to coordinate versor $\mathbf{v}_i$, denoted as $\mathbf{w}_\mathbf{s}(k)$ and $\mathbf{w}_{\mathbf{v}i}(k)$, respectively. These latter projections fulfill the condition $\|\mathbf{w}_\mathbf{s}(k)\| \geq \|\mathbf{w}_{\mathbf{v}i}(k)\|$, for all $k \geq 0$ (see Fig. 2 for a schematic plot). If $\mathbf{u}_{ss}(k)$ is not at its last step (i.e. if it is not at a distance from $\mathbf{u}^*$ smaller than $\|Hc\,\Delta\mathbf{u}_{\max}\|$, in which case $\mathbf{u}^*$ can be reached in one step), then, the ith input movement constraint will be active. This means that $\|\mathbf{w}_{\mathbf{v}i}(k)\| = Hc\,\Delta u_{\max,i}$. Then, given that vectors $\mathbf{w}_\mathbf{s}(k)$ are taken along vector $\mathbf{s}$, which goes form $\mathbf{u}_{ss}(0)$ to $\mathbf{u}^*$, and they are consecutive, the number of steps for $\mathbf{u}_{ss}(k)$ to reach $\mathbf{u}^*$ cannot be larger than $ceil\left(\|\mathbf{s}\|/Hc\,\Delta u_{\max,i}\right) + 1$, which is a finite integer.

The results in Theorem 1 are only an approximation to the complete problem of the convergence of the Incremental LP-DMC solutions to the Overall LP Problem solution, since the Dynamic Controller block (that is the one that actually applies the control action) and the transient regime of the system are ignored. However, it is useful for two main reasons: once we establish conditions to guarantee that the Dynamic Controller block applies the control actions according to the successive stationary values given by the Incremtal Optimizer block, then there is a guarantee that the closed loop will converge to the desired point. On the other hand, the transient regime of the system should not affect the stationary results; it only delays the convergence. An important point to be considered next is the disturbance scenarios (disturbance and model mismatch), since disturbances could in fact prevent the convergence.

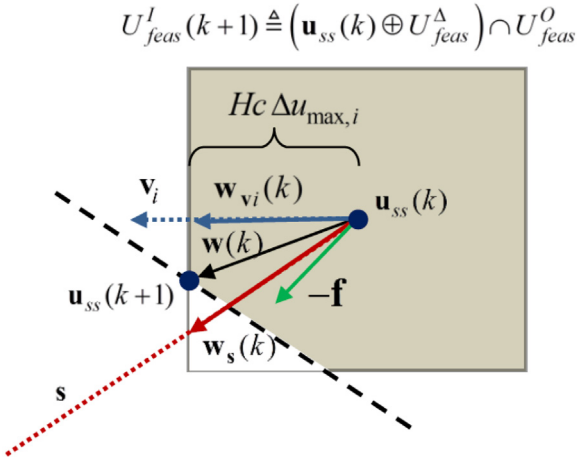

Fig. 2. Schematic plot of the projection vectors used in the Proof of Theorem 1. This scenario approximately corresponds to the forth step ($k = 3$) in Fig. 2.

4. Performance indicators based on the underlying overall optimization problem

Based on the results of the last section, we propose next three indicators devoted to online assess the LP-DMC type controller performance. The main idea is to use the Overall LP problem and the

economically optimal target to understand the whole closed-loop behaviour.

1) *Distance to economic optimum target normalized by the allowable deviations*

The normalized distance to the economic optimum target (or desired target) is calculated as follows:

$$D_t(k) = \|\mathbf{Q}^t[\mathbf{y}_{\lim} - \mathbf{y}^t(k)]\|, \quad \mathbf{Q}^t = diag(\mathbf{s}_t)^{-1}, \quad \mathbf{s}_t = [sgn(l)\,ECE_l]_{l\in t}, \tag{13}$$

where $\mathbf{y}^t(k)$ are the measurements of the *nu* CVs involved in the desired target, $\mathbf{y}_{\lim}$ are the corresponding limits to be achieved in the target (Eq. (11)), and $\mathbf{s}_t$ are the corresponding scales with $sgn(l) = 1$ for upper limit and $sgn(l) = -1$ for lower limit. This distance conveniently weighs a deviation from each individual target. The measurement unit is the average number of allowable deviations (or of ECEs). This indicator describes the online proximity of the controller to the desired point (economic optimum).

2) *SS prediction degradation.*

As for the overall optimization problem, when the operating point approaches the optimum, then the MVs tend to its optimal target, i.e. $\mathbf{u}(k) \to \mathbf{u}^t$, and the stationary predictions of the *nu* binding CVs tend to its respective constraint limits (CV targets), i.e. $\hat{\mathbf{y}}^t(k) = \mathbf{A}^t\mathbf{u}(k) + \mathbf{d}^t \to \mathbf{y}_{\lim}$ (see Eq. (11)). Hence, we can define a normalized distance to the desired target as follows:

$$Degra(k) = \|\mathbf{Q}^t\left[\mathbf{y}_{\lim} - \hat{\mathbf{y}}^t(k)\right]\| = \|\mathbf{Q}^t\left[\mathbf{b}^t - \mathbf{H}^t\mathbf{u}(k)\right]\|, \tag{14}$$

where $\mathbf{A}^t$ and $\mathbf{d}^t$ determine the benchmark respect to which the stationary model degradation will be assessed. This distance was conveniently weighted to scale the relative importance of each target constraint. For example, if a current input $\mathbf{u}(k)$ meets all active constraints in Eq. (14) minus the *j*-th maximum constraint (with an distancing equal to one ECE_j), then $Degra(k) = |\mathbf{b}^t(j) - \mathbf{H}^t(j,:)\mathbf{u}(k)|/ECE_j = 1$. Furthermore, the following relationship between $Degra(k)$ and $D_t(k)$ is fulfilled:

$$\begin{aligned} \|\mathbf{y}_{\lim} - \hat{\mathbf{y}}^t(k)\|_{\mathbf{Q}^t} &\le \|\mathbf{y}_{\lim} - \mathbf{y}^t(k)\|_{\mathbf{Q}^t} + \|\mathbf{y}^t(k) - \hat{\mathbf{y}}^t(k)\|_{\mathbf{Q}^t}, \\ Degra(k) &\le D_t(k) + \|\mathbf{y}^t(k) - \hat{\mathbf{y}}^t(k)\|_{\mathbf{Q}^t}, \end{aligned} \tag{15}$$

where $\|\mathbf{y}^t(k) - \hat{\mathbf{y}}^t(k)\|_{\mathbf{Q}^t}$ is a normalized measure of the model-plant mismatch at steady state. Hence, when there is no model-plant mismatch (i.e. $\hat{\mathbf{y}}^t(k) = \mathbf{y}^t(k)$), then $Degra(k) = D_t(k)$. Furthermore, when $D_t(k) = 0$, then $Degra(k)$ quantifies only the deterioration of the stationary model. Therefore, the proposed indicator considers two aspects: distance to the target and stationary model deterioration.

The indicator D_t describes properly the online proximity of the controller to the desired target. However, the target may include operating objectives; therefore this distance does not report properly the economic performance of the controller. Notice also that the objective function of the Incremental LP problem does not represent a proper performance indicator since it only goes to zero when the system reaches any stationary point. On the other hand, the cost function of the Overall LP problem would be a measurement of the economic profit.

3) *Economic performance.*

To evaluate online the economic performance of a controller, it is necessary to compare its current performance with some reference point or benchmark. Based on the underlying (overall) optimization problem (Eq. (9)) and the desired target (Eq. (11)), we can define an overall and reliable benchmark given by the overall cost in the desired target (i.e. the optimal cost): $Cost_{Op} = \boldsymbol{\beta}'(\mathbf{A}^t\mathbf{u}^t + \mathbf{d}^t) + \boldsymbol{\alpha}'\mathbf{u}^t = \mathbf{f}'\mathbf{u}^t + cte > 0$, where $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ have the profits or costs of the products (or CVs) and inputs (or MVs). Then, we can define an economic performance indicator as follows:

$$EP(k) = \frac{Cost_{Op}}{Cost(k)} \le 1, \tag{16}$$

where $Cost(k) = \beta'\hat{\mathbf{y}}^t(k) + \alpha'\mathbf{u}(k) = \mathbf{f}'\mathbf{u}(k) + cte > 0$ is the current overall cost of the desired target stationary prediction $\hat{\mathbf{y}}^t(k) = \mathbf{A}^t\mathbf{u}(k) + \mathbf{d}^t$. The indicator $EP(k)$ will be reliable only if $\mathbf{f}$ is in the feasible cone of the desired vertex; otherwise, it could be confusing. Moreover, when the system is far from $\mathbf{u}^t$, $Cost_{Op}$ and $Cost(k)$ are yet not comparable, i.e. *EP* is meaningful when the system is close to a $\mathbf{u}^t$. Then, the indicator D_t (measuring the distance to the desired target) determines the confidence of the indicator *EP* (measuring the economic performance). The economic loss, $1 - EP(k)$, averaged during a useful period, can facilitate the decision of when to apply maintenance.

Remark. Note that opposite to QP-DMC (or other similar strategies), the overall economic target point is not explicit in the LP-DMC structure. So, it is crucial to determine *a priori* this target point, which is the point where the Incremental LP solution will iteratively converge to. This is the main objective of proposing an underlying optimization problem, which in turns permits to have proper references for the new KPIs: i.e., the underlying problem provides the overall target $\mathbf{y}_{\lim}$ of D_t, the overall stationary model of *Degra* and the overall cost of *EP*.

5. Simulation results

The objective of thi section is to analyze the proposed KPIs by means of the simulation of an integrating system and of the well-known Shell heavy oil fractionator. Emphasis is put on the convergence of the Incremental LP to the Overall LP solution, and on the KPIs evolution.

5.1. Case study 1: integrating process

Let us consider a simple plant with 3 inputs and 3 outputs, so that the variables' subspaces can be graphically represented. The transfer function matrix is given by:

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{-0.2}{s} & \dfrac{0.45}{s} & 0 \\ \dfrac{0.3}{15s^2+3s+1} & \dfrac{0.7}{10s^2+7.8s+1} & \dfrac{-0.14}{9.14s+1} \\ \dfrac{-0.2}{75s+s} & \dfrac{0.17}{6s^2+4s+1} & \dfrac{-0.5}{15s^2+10s+1} \end{bmatrix}$$

The constraints on outputs and inputs are given respectively by

$$\begin{bmatrix} -6.5 \\ -3 \\ -3 \end{bmatrix} \le \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \le \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}, \quad \begin{bmatrix} -10 \\ -6 \\ -11.8 \end{bmatrix} \le \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \le \begin{bmatrix} 12 \\ 10 \\ 10 \end{bmatrix}.$$

The steady state horizon is $Hss = 50$ for a sampling time $Ts = 1$, then $Hc = 25$ and $N = 75$. The weightings for the dynamic control block are: $y_{1,SP} = 1.5$ (setpoint of y_1), $\lambda = 40$, $\mathbf{P} = diag([60,\ 40,\ 30])$, and $ECE_1 = ECE_2 = ECE_3 = 0.5$. The desired target that will define the point $\mathbf{u}^t$ is given by maintaining the integrating output y_1 at its setpoint, maximizing y_2 and minimizing y_3; hence $\boldsymbol{\beta}' = [0\ -1\ 1]$. The vector $\mathbf{f}$ (that passes through the bisecting line of the target feasible cone) is given by: $\mathbf{f}' = [-12.6457\ \ -5.6203\ \ -10.5691]$. Then, the vector $\boldsymbol{\alpha}$ can be obtained from the following relationship: $\boldsymbol{\alpha}' = \boldsymbol{\beta}'\mathbf{A}_{N-1} + \mathbf{f}'$. Note that the output y_1 is an integrative CV with the following slopes: $[-0.2\ \ 0.45\ \ 0]$. Then, solving Eq. (10), the obtained desired target is: $\mathbf{u}^t = [7.924\ \ 3.522\ \ 3.136]'$. Fig. 3 shows the feasible region of the Overall LP problem (Eq. (9)). The polytope represents the set of inequality constraints on the inputs and on the outputs projected into the input space. The plane intersecting such polytope repre-

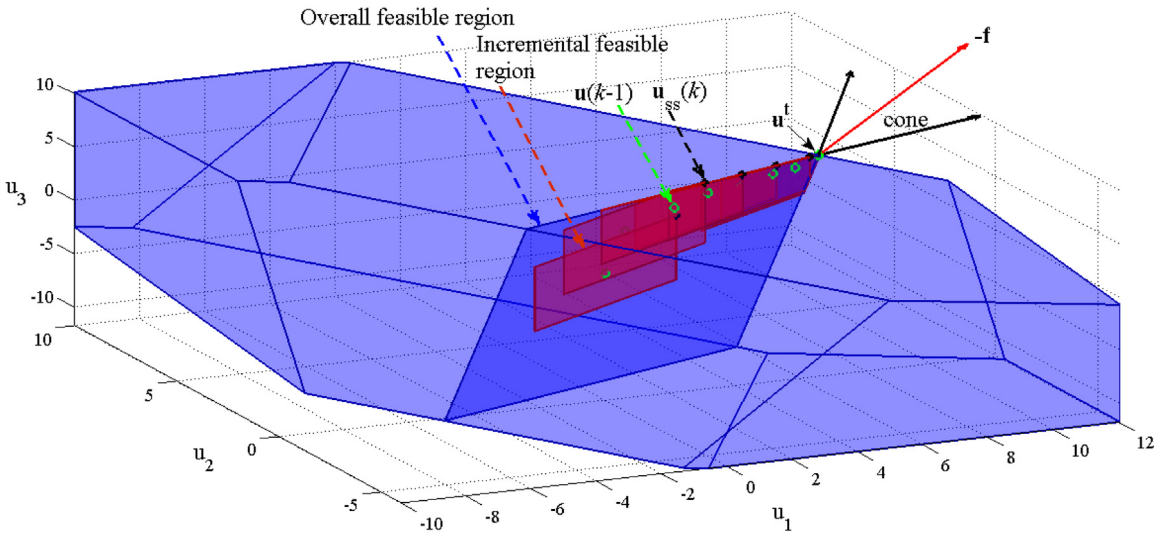

Fig. 3. Feasible region of the Overall LP problem and convergence of the Incremental LP problem, when model-plant mismatches are not considered.

sents the equality constraint. The intersection of these two sets is the feasible region of the Overall LP problem. The red dot represents the desired target, $\mathbf{u}^t$, which is a vertex of the feasible region. Fig. 3 shows also the evolution of the solutions to the Incremental LP problem, which tend to the solution to the Overall LP problem for k increasing. Note that the constraints on the input moves in the Incremental LP problem are taken as $Hc\,\Delta\mathbf{u}_{\max} = [3\ \ 3\ \ 3]'$.

To study the effect of model-plant mismatches (MPMs) and unmeasured disturbances (UDs), errors of +10% in the gains and slopes are considered and UDs in the CVs are simulated. Moreover, random output noises with deviations equal to 0.0447 are also considered in the simulations. Given that $\hat{\mathbf{G}}(s) = 1.1\,\mathbf{G}(s)$, then $\mathbf{A}_{N-1} = 1.1\,\mathbf{G}(0)$ and $\mathbf{S} = 1.1[-0.2\ \ 0.45\ \ 0]$. Consequently, the biases $\mathbf{d}_{ss}$ and $\mathbf{r}_{ss}$ are nonzero and can be calculated using the following stationary point: $\mathbf{u}_{ss} = [-3.25\ \ -1.4444\ \ -3]'$, $\mathbf{y}_{ss} = [0\ \ -1.5661\ \ 1.9045]'$. Fig. 4a shows the actual (underlying) feasible region (the red region in Fig. 4a) of the Overall LP problem and the estimated feasible region (the blue region in Fig. 4a), which is modified/shifted due to the MPM. The UDs also influence the shift of the underlying feasible region from the estimated one (however, this new actual region is not shown in Fig. 4a for the sake of clarity). An equivalent way to interpret this fact is to see that the actual desired target is shifted from the estimated one (see Fig. 4b). The LP-DMC controller regulates the actual system based on the actual (underlying) stationary model by guiding the operating point to the underlying desired target. The MPMs and sustained UDs produce a modification of the actual stationary model with respect to the estimated one, which is equivalent to produced separation of the actual (underlying) desired target from the estimated one. This means that the target where the Incremental LP will steer the system is no longer the estimated target.

Fig. 4a and b show how the shift in the underlying feasible region (or equivalently the displacement of the underlying desired target) – due to MPMs and UDs – pushes the operating point $\mathbf{u}(k)$ out of the feasible region estimated at the beginning. This is a consequence of the following facts: *i*) the MPMs modify the stationary gains, and so the underlying region has a different shape; *ii*) the UDs modify the actual biases $\mathbf{d}_{ss}$ and $\mathbf{r}_{ss}$, and hence the underlying feasible region is shifted from the estimated region. This way, the system reaches a point that lies outside the initially estimated region.

The general objective of the LP-DMC is to steer the system to the desired target (or to keep the system sufficiently close to it) fulfilling the constraints. Fig. 5 shows the time evolution of the variables. Note that, while the CVs return to the target after an UD enters the system, the MVs go to different targets (because of the changes in the gain). Therefore, the feasible region also changes, and the location of the point $\mathbf{u}(k)$ is different from the initial one. Fig. 6 shows the time evolution of the proposed indicators. The first graphic in Fig. 6 confirms that the Incremental LP cost does not represent a proper performance indicator because it only goes to zero when the system reaches any stationary point. The following plots in Fig. 6 show the evolution of the proposed indicators. The results should be considered from time $k = 110$ forward, because the controlled system reaches a stationary regime. The UDs enters the system at time $k = 400$ (see Fig. 5a), moving the underling point $\mathbf{u}^t$ (see Fig. 4) to a more favorable economically location (see Fig. 6d). Then, they return to zero at time $k = 1400$. Next, the UDs enters the system again at time $k = 1600$, but it this case the point $\mathbf{u}^t$ is moved to a less convenient location (see Fig. 6d). Therefore, the indicator *EP* properly denounces this operation change with respect to the economic optimality.

The desired target is given by maintaining the integrating output y_1 at its setpoint, maximizing y_2, and minimizing y_3. Therefore, $\mathbf{y}_{\lim} = \begin{bmatrix} 0 & y_{2,\max} & y_{3,\min} \end{bmatrix}'$ and $\mathbf{Q}^t = diag\left(1/ECE_1\ \ 1/ECE_2\ \ 1/ECE_3\right)$. Then, the indicators D_t and *Degra* take the followings forms:

$$D_t(k) = \left[\left(\frac{y_1(k) - y_{1,SP}}{ECE_1} \right)^2 + \left(\frac{y_{2,\max} - y_2(k)}{ECE_2} \right)^2 + \left(\frac{y_3(k) - y_{3,\min}}{ECE_3} \right)^2 \right]^{1/2}$$

$$Degra(k) = \left[\left(\frac{\Delta\hat{y}_1^t(k)}{ECE_1} \right)^2 + \left(\frac{y_{2,\max} - \hat{y}_2^t(k)}{ECE_2} \right)^2 + \left(\frac{\hat{y}_3^t(k) - y_{3,\min}}{ECE_3} \right)^2 \right]^{1/2}$$

where $\Delta\hat{y}_1^t(k) = [\mathbf{A}_{N-1}(1,:) - \mathbf{A}_{N-2}(1,:)]\mathbf{u}(k) - \mathbf{r}_{ss}$, $\hat{y}_2^t(k) = \mathbf{A}_{N-1}(2,:)\mathbf{u}(k) + \mathbf{d}_{ss}(2)$, and $\hat{y}_3^t(k) = \mathbf{A}_{N-1}(3,:)\mathbf{u}(k) + \mathbf{d}_{ss}(3)$. Notice that in $D_t(k)$, the value of $\mathbf{y}^t(k)$, for the involved integrating CV, is the movement relative to its setpoint, i.e. $\Delta y_1(k) = y_1(k) - y_{1,SP}$. Then, by comparing both indicators, for instance at time $k = 800$ (see Figs. 5 and 6), it results that: $Degra(800) > D_t(800) \approx 0$. This means that the controller meets the desired target (with $D_t(800) \approx 0$) although the model suffers a degradation which is denounced by $Degra(800) > 0$. This implies that the MVs need to make a greater effort to achieve the desired target (see Fig. 5). The initial MPM prevents to obtain $Degra = 0$ (see before time $k = 400$ in Fig. 6). Then, when the UDs enter the system, *Degra* measures the model deterioration in allowable deviation units (or ECEs). However, for this simulated case, a better estimation of the MPM,

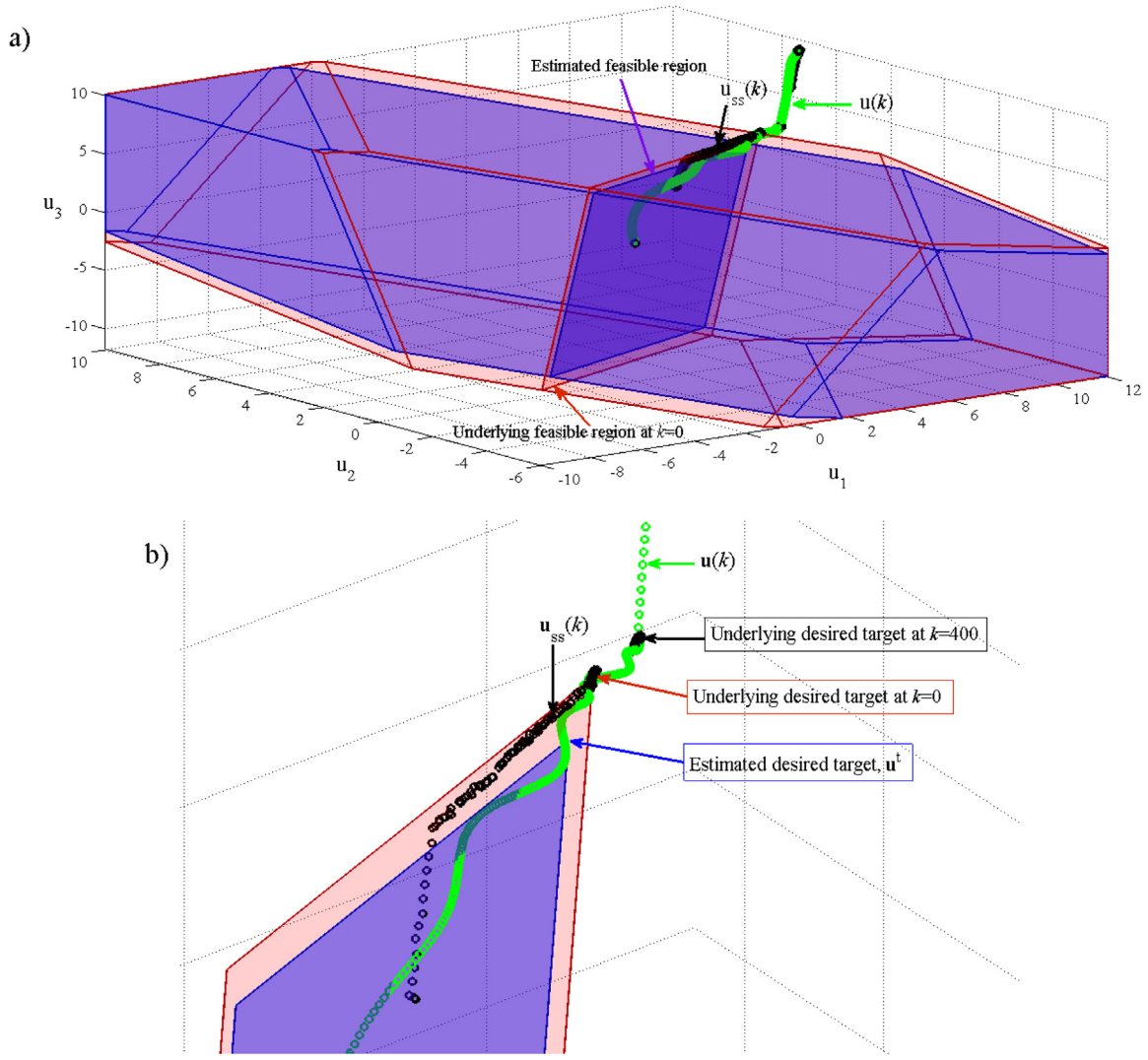

Fig. 4. a) Shift of the underlying feasible region due to disturbances or new plant-model mismatches. b) Zoom of the region close to the desired target.

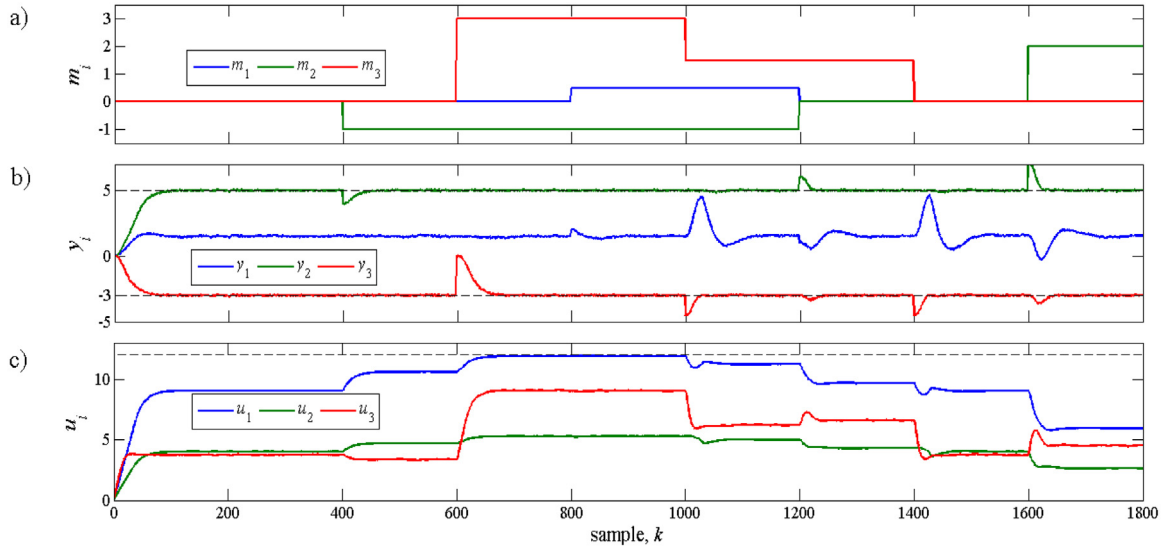

Fig. 5. Time evolution of: a) unmeasured disturbances (UDs), b) controlled variables (CVs), and c) manipulated variables (MDs).

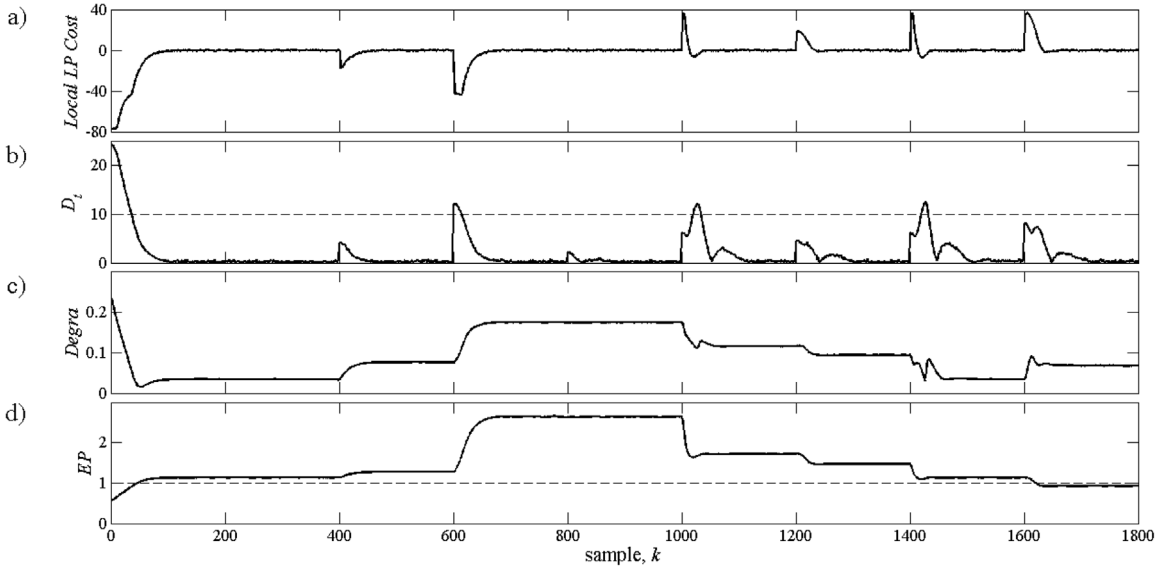

Fig. 6. Time evolution of the Incremental LP cost and of the proposed indicators.

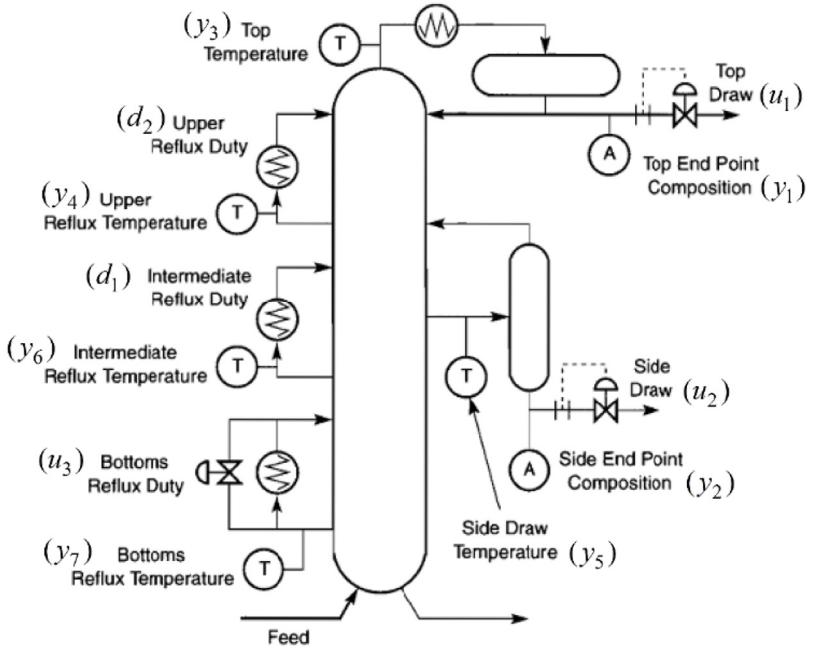

Fig. 7. Schematic representation of Shell Heavy Oil Fractionator [34].

or even a re-identification, is not needed because the UDs shift the CV limits (projected on the input subspace) in a direction where the profit increases (Fig. 6d).

Many factors may prevent the constraint activation at the operating point. However, this point is frequently close to the desired constraints. The indicator D_t confidently measures this distance (Fig. 6) without counting the constraint activation episodes. This way, it is clear that D_t is a better indicator than *pCVac* and *pMVac* (which are based on counting constraint activations [7] and are commonly used as KPI in industries).

Also, a frequent KPI used in industries to estimate the profit of the process is the one given by $profit = \gamma\, pCVac/pCVac_{optimal}$, where$pCVac_{optimal}$ is estimated just after the system is commissioned, by averaging *pCVac*, and γ is a fixed value [7]. In contrast, the proposed indicator *EP* is a true ratio of economic global costs, which enables more reliable management decisions. In addition, the optimal cost (benchmark) used by *EP* does not need to be estimated statistically as $pCVac_{optimal}$does.

5.2. Case study 2: Shell heavy Oil fractionator

The Shell Benchmark Process was proposed by Prett & Morari [33] and is composed by a heavy oil fractionator, as represented in Fig. 7. The main feature of this process is the high interaction among the variables as well as large time delays. The fractionator is characterized by three product draws and three side circulating loops. The heat requirement of the column enters with the feed, which is a gaseous stream. Product specifications for the top and side draw streams are determined by economics and operating requirements. There is no product specification for the bottom draw, but there is an operating constraint on the temperature in the lower part of

Table 1
The Shell heavy fractionator case: LP-DMC tuning parameters and constraints.

Sampling time	2 min
Steady State Horizon (*Hss*)	120 samples (considered settling time: 240 min)
Control Horizon (*Hc*)	60 samples
Prediction Horizon (*N*)	180 samples
Global Move Suppression (λ)	20
Move Suppression factors ($\mathbf{P}$)	*diag* (10, 10, 10)
MVs upper limits ($\mathbf{u}_{max}$)	[0.5, 0.5, 0.7]
MVs lower limits ($\mathbf{u}_{min}$)	[−1.2, −0.5, −0.5]
Maximum Movements ($Hc\ \Delta\mathbf{u}_{max}$)	[0.3, 0.3, 0.3]
CVs upper limits ($\mathbf{y}_{max}$)	[0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]
CVs lower limits ($\mathbf{y}_{min}$)	[−0.5, −0.5, −0.5, −1.5, −0.5, −0.5, −0.5]
CV Weight (ECEs)	[0.05, 0.05, 0.2, 0.2, 0.2, 0.2, 0.05]

the column. The three circulating loops remove heat to achieve the desired product separation.

The controller is composed by 7 controlled variables (CVs), 3 manipulated variables (MVs) and 2 measured disturbances (MDs), which are presented in Fig. 7. The linear model for this process is given by:

$$\mathbf{y} = \mathbf{G}\,\mathbf{u} + \mathbf{G}_d\,\mathbf{d}, \tag{17}$$

$$\mathbf{G} = \begin{bmatrix} 4.05e^{-27s}\frac{1}{50s+1} & 1.77e^{-28s}\frac{1}{60s+1} & 5.88e^{-27s}\frac{1}{50s+1} \\ 5.39e^{-18s}\frac{1}{50s+1} & 5.72e^{-14s}\frac{1}{60s+1} & 6.90e^{-15s}\frac{1}{40s+1} \\ 3.66e^{-2s}\frac{1}{9s+1} & 1.65e^{-20s}\frac{1}{30s+1} & 5.53e^{-2s}\frac{1}{40s+1} \\ 5.92e^{-11s}\frac{1}{12s+1} & 2.54e^{-12s}\frac{1}{27s+1} & 8.10e^{-2s}\frac{1}{20s+1} \\ 4.13e^{-5s}\frac{1}{8s+1} & 2.38e^{-7s}\frac{1}{19s+1} & 6.23e^{-2s}\frac{1}{10s+1} \\ 4.06e^{-8s}\frac{1}{13s+1} & 4.18e^{-4s}\frac{1}{33s+1} & 6.53e^{-1s}\frac{1}{9s+1} \\ 4.38e^{-20s}\frac{1}{33s+1} & 4.42e^{-22s}\frac{1}{44s+1} & 7.20\frac{1}{19s+1} \end{bmatrix}, \quad \mathbf{G}_d = \begin{bmatrix} 1.20e^{-27s}\frac{1}{45s+1} & 1.44e^{-27s}\frac{1}{40s+1} \\ 1.52e^{-15s}\frac{1}{25s+1} & 1.83e^{-15s}\frac{1}{20s+1} \\ 1.16\frac{1}{11s+1} & 1.27\frac{1}{6s+1} \\ 1.73\frac{1}{5s+1} & 1.79\frac{1}{19s+1} \\ 1.31\frac{1}{2s+1} & 1.26\frac{1}{22s+1} \\ 1.19\frac{1}{19s+1} & 1.17\frac{1}{24s+1} \\ 1.14\frac{1}{27s+1} & 1.26\frac{1}{32s+1} \end{bmatrix}.$$

where each row is associated with a CV (y_1 to y_7), each column of $\mathbf{G}$ is associated with a MV (u_1 to u_3) and each column of $\mathbf{G}_d$ with a MD (d_1 to d_2). The main objective is to control the top composition (y_1), the side composition (y_2) and the bottom reflux temperature (y_7) by the manipulation of the top draw (u_1), side draw (u_2) and bottom reflux duty (u_3). Given that this process has measured disturbances $\mathbf{d}(k) = [d_1(k)\ \ d_2(k)]'$, the prediction is performed by modifying Eq. (1) as follows:

$$\hat{\mathbf{Y}}(k|k-1) = \mathbf{M}\,\hat{\mathbf{Y}}(k-1|k-1) + \mathbf{A}\Delta\mathbf{u}(k-1) + \mathbf{A}_d\Delta\mathbf{d}(k-1), \tag{18}$$

where

$$\mathbf{A}_d = \begin{bmatrix} \mathbf{A}_1^d \\ \vdots \\ \mathbf{A}_{N-1}^d \\ \mathbf{A}_N^d - \mathbf{A}_{N-1}^d \end{bmatrix}.$$

The matrix $\mathbf{A}_i^d$ (with $i = 1,\ldots,N$) contains the step response coefficients at time i corresponding to each CV and MD. The controller tuning parameters and the constraints are presented in Table 1. The economic objective is given by minimizing y_1, minimizing y_2, and maximizing y_7. To achieve this desired target is required minimizing u_1, maximizing u_2, and maximizing u_3. Fig. 8 shows the feasible region of the Overall LP problem and the black dot/circle represents the desired target (vertex). The same constraint limits as in [34] were used except for three limits ($u_{1,\,min} = -1.2$, $u_{3,\,max} = 0.7$, and $y_{4,\,min} = -1.5$) that were relaxed for the desired target to be feasible (see Table 1). To achieve the economic objective the following overall cost is proposed:

$$Cost(k) = \beta'(\mathbf{A}_{N-1}\mathbf{u}(k) + \mathbf{A}_{N-1}^d\mathbf{d}(k) + \mathbf{d}_{ss}) + \alpha'\mathrm{u}(k) \tag{19}$$

where $\beta' = [1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ -1]$, $\alpha' = [1\ \ -1\ \ -1]$, $\mathbf{d}_{ss} = 0$ and $\mathrm{f}' = \beta'\mathbf{A}_{N-1} + \alpha' = [0.606,\ \ 0.316,\ \ 0.707]$.

In order to test the proposed indicators in this case study, the simulation was performed considering the addition of measured disturbances (MDs), model-plant mismatches (MPMs) or unmeasured disturbances (UDs). A white noise with zero mean and standard deviation of 0.02 was added to the measured outputs. Fig. 9 shows the simulated data in the nominal case (i.e., in absence of MPM or UD and MD) from the beginning to the time 1300, then the addition of MDs and finally the addition of an UD equal to 0.2 on y_1 from 4300 to the end. Fig. 10 shows the time evolution of the proposed indicators. During the addition of MDs, the indicator *EP* does not change significantly, but when the UD is injected into the system, this indicator denounces its presence (see Fig. 10d).

The MDs at the times 1650 and 3750 (see Fig. 9) produce $D_t(1650) = 3 < Degra(1650) = 3.3$ and $D_t(3750) = 2 < Degra(3750) = 2.15$, respectively (see Fig. 10). This result denounces that the three limits that define the desired target cannot be satisfied with an average deviation of 3 and 2 ECEs, respectively. When the MD $d_2 > 0$, the MV u_1 get saturated (which prevents y_7 from reaching its upper limit) and a new vertex appears, which is located at: y_1 lower limit, y_2 lower limit and u_1 lower limit (see Fig. 9). However, at these times the MDs shift the operating point to a location with greater profit, with values of $EP(1650) = 1.03$ and $EP(3750) = 1.015$, respectively. On the other hand, at time 4980 the MPM produces the followings values of indicators: $D_t(4980) = 0$, $Degra(4980) = 4$, and $EP(4980) = 0.94$. That is, a stationary model degradation of 4 ECEs caused an economic loss of 6%, while the controlled system satisfied the desired target. It is important to remark that the model degradation is assessed by means of the percentage of economic losses that such degradation causes. The indicators proposed here allow one to quantify the impact that MDs and MPMs have on the economic cost. The UDs (or MPMs) are cumulative over the time and the proposed indicators will report when the controller is no longer tuned to maintain an acceptable economic performance. As a consequence, the control system will need maintenance, such as a re-identification.

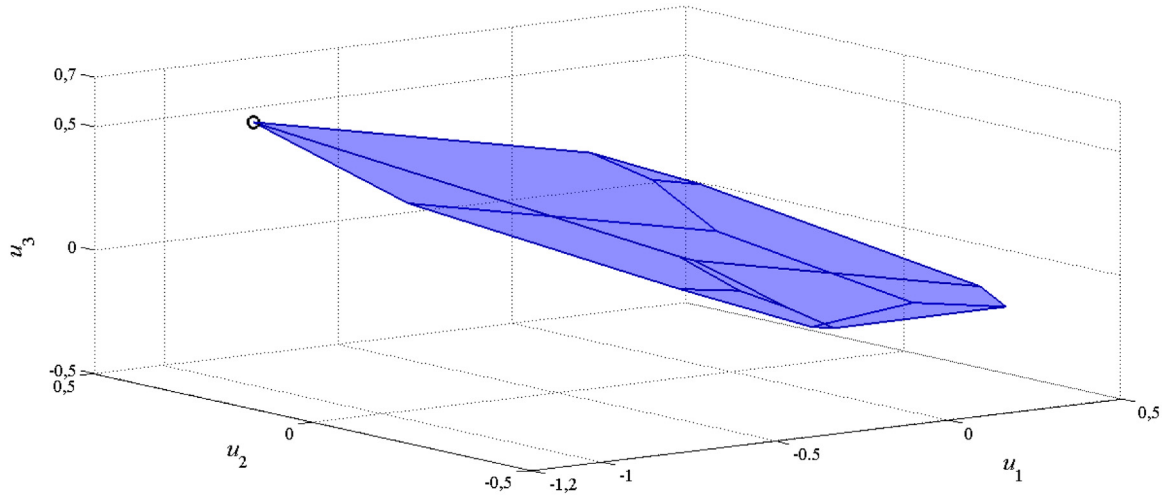

Fig. 8. Feasible region of the overall optimization problem and desired target (in circle).

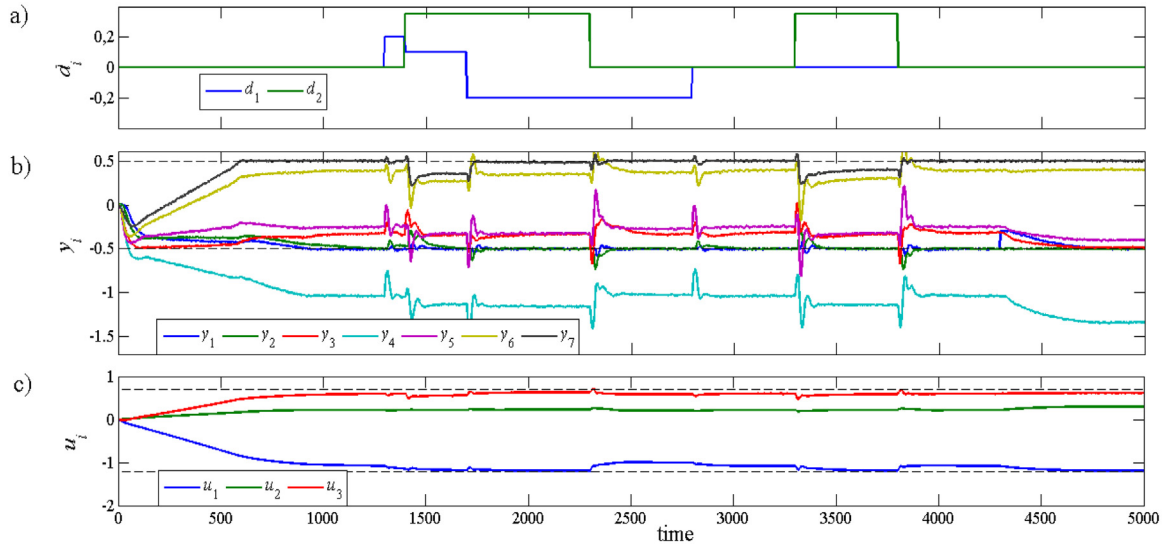

Fig. 9. Time evolution of: a) measured disturbances, b) controlled variables, and c) manipulated variables.

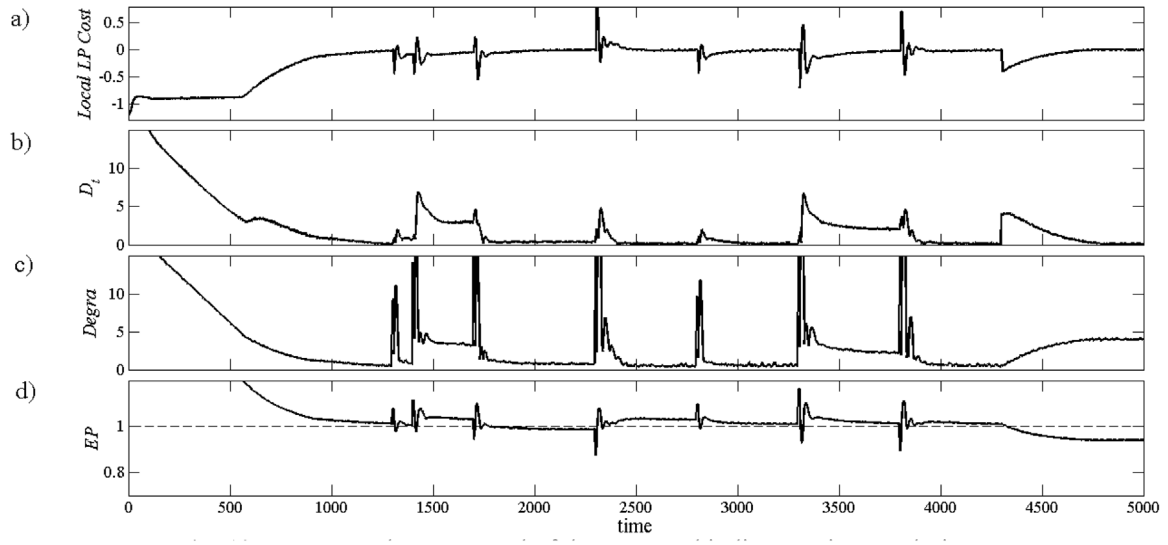

Fig. 10. Incremental LP cost and of the proposed indicators time evolution.

To conclude, we propose to use the indicators D_t, *Degra* and *EP* for performance assessment and monitoring in LP-DMC applications. These indicators are generic, easy to implement, responsive, reliable and serve to the purposes of the KPIs by providing an historical reference which can highlight unusual improvements or losses of performance.

6. Conclusion

In this work an analysis of DMC/MPC-type controllers was done, in order to propose some tools for assessment and monitoring the closed-loop performance. In addition to propose an iterative scheme to reproduce appropriately the dynamic controller predictions, the stationary optimization was analyzed in detail. In this regards, an underlying global LP problem was proposed that permits to develop several online performance indicators that accurately discriminate the source of potential problems.

In a future work, a LP-DMC tuning technique will be developed by using the underlying optimization problem presented in this work.

Acknowledgement

The authors are grateful for the financial support received from CONICET.

References

[1] E.F. Camacho, C. Bordons, Model Predictive Control, 2nd ed., Springer-Verlag, 2004.
[2] J.B. Rawlings, D.Q. Mayne, Model Predictive Control: Theory and Design, 1st ed., Nob-Hill Publishing, 2009, 2017.
[3] C.R. Cutler, B.L. Ramker, Dynamic matrix control—a computer control algorithm, in: Proc. Joint Automatic Control Conference, WP5-B, 1980.
[4] C.E. García, A.M. Morshedi, Quadratic programming solution of dynamic matrix control (QDMC), Chem. Eng. Commun. 46 (1986) 73–87.
[5] A.M. Morshedi, C.R., Cutler, T.J., Fitzpatrick, T.A. Skrovanek Dynamic process control. US Patent 4,616,308A, 1986.
[6] S. Goodhart, J. Nishizawa, K. Yano, H. Yada, Advanced control in cogeneration utility management, Computing and Control Engineering Journal 11 (2000) 273–282.
[7] M. Campos, M., Gomes, J. Perez, Controle Avançado e Optimização na Indústria do Petróleo. Interciência, Rio de Janeiro, 2013.
[8] S.J. Qin, T.A. Badgwell, A survey of industrial model predictive control technology, Control Eng. Pract. 11 (2003) 733–764.
[9] J.B. Rawlings, D. Angeli, C.N. Bates, Fundamentals of economic model predictive control, in Proceedings of the 51 st IEEE Conference on Decision and Control (2012) 3851–3861.
[10] J.V. Kadam, W. Marquardt, Integration of economical optimization and control for intentionally transient process operation, in Assessment and Future Directions of Nonlinear Model Predictive Control (R. Findeisen, F. Allgöwer and L. T. Biegler, Eds.), 419–434, 2007.
[11] A.G. Kern, Online Monitoring of Multivariable Control Utilization and Benefits. Hydrocarbon Processing, Publishing Company, Gulf, 2005, pp. 43–47.
[12] M. Jelali, An overview of control performance assessment technology and industrial applications, Control Eng. Pract. 14 (2006) 441–466.
[13] A. Cinar, A. Palazoglu, F. Kayihan, Chemical Process Performance Evaluation, CRC Press, 2007.
[14] B. Huang, R. Kadali, Dynamic Modeling, Predictive Control and Performance Monitoring: A Data-driven Subspace Approach, Springer-Verlag, 2008.
[15] M. Jelali, Control Performance Management in Industrial Automation: Assessment, Diagnosis and Improvement of Control Loop Performance, Springer-Verlag, 2013.
[16] J. Schäfer, A. Cinar, Multivariable MPC system performance assessment, monitoring, and diagnosis, J. Process Control 14 (2004) 113–129.
[17] A. AlGhazzawi, B. Lennox, Model predictive control monitoring using multivariate statistics, J. Process Control 19 (2009) 314–327.
[18] N. Agarwal, B. Huang, E.C. Tamayo, Assessing model prediction control (MPC) performance: probabilistic approach for constraint analysis, Ind. Chem. Res. 46 (2007) 8101–8111.
[19] B. Huang, S.X. Ding, N. Thornhill, Alternative solutions to multi-variate control performance assessment problems, J. Process Control 16 (2006) 457–471.
[20] Y. Zhao, J. Chu, H. Su, B. Huang, Multi-step prediction error approach for controller performance monitoring, Control Eng. Pract. 18 (2010) 1–12.
[21] H. Jiang, S.L. Shah, B. Huang, B. Wilson, R. Patwardhan, F. Szeto, Model analysis and performance analysis of two industrial MPCs, Control Eng. Pract. 20 (2012) 219–235.
[22] G. Pannocchia, A. De Luca, Performance degradation diagnosis and remedies in offset-free MPC, In 2012 American Control Conference (ACC) (2012) 429–434.
[23] A.S. Badwe, R.D. Gudi, R.S. Patwardhan, S.L. Shah, S.C. Patwardhan, Detection of model-plant mismatch in MPC applications, J. Process Control 19 (2009) 1305–1313.
[24] Z. Sun, S.J. Qin, A. Singhal, L. Megan, Performance monitoring of model-predictive controllers via model residual assessment, J. Process Control 23 (2013) 473–482.
[25] M. Kano, Y. Shigi, S. Hasebe, S. Ooyama, Detection of significant model-plant mismatch from routine operation data of model predictive control system, IFAC Proc. 43 (2010) 685–690.
[26] M. Bauer, I.K. Craig, Economic assessment of advanced process control: a survey and framework, J. Process Control 18 (2008) 2–18.
[27] K.H. Lee, E.C. Tamayo, B. Huang, Industrial implementation of controller performance analysis technology, Control Eng. Pract. 18 (2010) 147–158.
[28] F. Xu, B. Huang, S. Akande, Performance assessment of model predictive control for variability and constraint tuning, Ind. Eng. Chem. Res. 46 (2007) 1208–1219.
[29] J.H. Lee, M. Morari, C.E. Garcia, State-space interpretation of model predictive control, Automatica 30 (1994) 707–717.
[30] B. Chachuat, A. Marchetti, D. Bonvin, Process optimization via constraints adaptation, J. Process Control 18 (2008) 244–257.
[31] A. Maarleveld, J.E. Rijnsdorp, Constraint control on distillation columns, Automatica 6 (1970) 51–58.
[32] C.E. Garcia, M. Morari, Optimal operation of integrated processing systems. Part II: Closed-loop on-line optimizing control, AIChE J. 30 (1984) 226–234.
[33] D.M. Prett, M. Morari, The Shell Process Control Workshop, Butterworths, USA, 1987.
[34] J.M. Maciejowski, Predictive Control with Constraints, Prentice Hall, 2002.