A Petri Net Model of Argumentation Dynamics

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Abstract. Petri nets are a mathematical modelling tool suitable for describing dynamic computational systems. In this work we present a formalization of abstract argumentation frameworks using Petri nets, where arguments and attacks are represented as places and transitions. This provides a formalism to study the semantic consequences of a procedural evaluation of argument attacks. The relation between markings of the net and argument extensions is analysed.

1 Introduction

Roughly speaking, *argumentation* is the study of arguments and their relationships. It is a form of reasoning suitable to deal with incomplete and contradictory information in dynamic domains. Although several proposals of argumentation systems are available, it is possible to study pure semantic notions in a general framework with a high level of abstraction. In abstract argumentation formalisms some components remain unspecified, being the structure of an argument the main abstraction. In this kind of systems, the emphasis is put on the semantic notion of finding the set of accepted arguments. Most of these *abstract argumentation frameworks* are based on the single concept of information conflict called *attack*, represented as an abstract relation, and extensions are defined as sets of possibly accepted arguments. The study of *dynamics* of argumentation has been an important topic in this area. An initial proposal of dynamic argumentation is presented in [3] using situation calculus. In recent formalisms [5,4,2,11] the semantic study of arguments and attacks is addressed under a temporal perspective, where arguments and attacks are progressively considered as the framework evolves through time. This is important since argumentation is intrinsically tied to dialectic activities, like dialogues, debates and even introspection.

Then it is possible to study how the process of argumentation advances while arguments and attacks are selected or provided in a sequential manner, which is relevant in systems with a large amounts of arguments. For instance, a DeLP program [7] of few defasible rules may produce hundreds of defeasible arguments. Even more, in some contexts it is not necessary to consider all of these arguments as a whole while reasoning. Gradual consideration of arguments and attacks is interesting. However, the fact that some controversies between arguments may be addressed in a sequential, distributed and concurrent operation with different semantic consequences was not previously studied under a suitable mathematical model in the argumentation community.

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Argumentation processes can be naturally complex. For instance, when a set of topics is discussed by several participants, some of the debates may occur at the same time. This is how many legislative corps works, such as in the United Nations or several state Congresses around the world. A formal model for the study of a procedural, timed interpretation of the act of attacking an argument is novel and interesting in the context of timed argumentation, where it is uncertain what arguments are going to be addressed next. In this work we are not interested in the logical aspects of argumentation as in [3], but in the abstract characterization of potentially distributed, asynchronous argumentation.

Petri nets are a mathematical modelling tool suitable for describing concurrent, asynchronous, non-deterministic computational systems [1,10,8]. The basic formalism is simple and it provides a sound framework for the study of properties of discrete events systems, yet in the last years several extensions were proposed in order to provide models for different characterizations of dynamic systems. Petri nets provide a description of independence and causal relations between system actions, which allows to reason about partially ordered sets of actions without having to consider their interleavings [8]. Properties of Petri nets were studied for half a century around the globe and it is widely considered a mature discipline.

In this work we propose a Petri net representation of an abstract argumentation framework. This provides a formalism to study the semantic consequences of a procedural evaluation of individual argument attacks. Being a classic engineering-oriented formalism, Petri nets are appropriate for *argumentation process analysis* which is an important direction of this line of research. In this paper we introduce the formalism and we show there is a correspondence between the evolution of the net and the underlying argumentation semantics.

This paper is organized as follows. In Section 2 we recall the basic notions of classical abstract argumentation frameworks. In Section 3 a brief, general description of Petri nets is included. In Section 4 we present a Petri net model for abstract argumentation and related semantic notions are introduced in Section 5. In Section 6 the dialectical interpretation of the argumentation net is discussed. Finally, conclusions and future work is discussed.

2 Classic Abstract Argumentation

Dung defines several argument extensions that are used as a reference for many authors. The formal definition of the classic argumentation framework follows.

Definition 1 [6] *An argumentation framework is a pair* $AF = \langle AR, attacks \rangle$ *where* AR *is a set of arguments, and* $attacks$ *is a binary relation on* AR*, i.e.* $attacks \subseteq AR \times AR$*.*

Arguments are denoted by labels starting with an upper-case letter, leaving the underlying logic unspecified. A set of accepted arguments is characterized in [6] using the concept of *acceptability*, which is a central notion in argumentation, formalized by Dung in the following definition.

Definition 2 [6] *An argument $\mathcal{A} \in AR$ is acceptable with respect to a set of arguments S if and only if every argument $\mathcal{B}$ attacking $\mathcal{A}$ is attacked by an argument in S.*

If an argument $\mathcal{A}$ is acceptable with respect to a set of arguments S then it is also said that S *defends* $\mathcal{A}$. Also, the attackers of the attackers of $\mathcal{A}$ are called *defenders* of $\mathcal{A}$. We will use these terms throughout this paper.

Acceptability is the main property of Dung's semantic notions, some of them summarized in the following definition.

Definition 3 *A set of arguments S is said to be*
– conflict-free if there are no arguments $\mathcal{A}, \mathcal{B}$ in S such that $\mathcal{A}$ attacks $\mathcal{B}$.
– admissible if it is conflict-free and defends all its elements.
– a preferred extension if S is a maximal (for set inclusion) admissible set.

In [6], theorems stating conditions of existence and equivalence between these and other extensions are also introduced.

Example 1 *Consider the argumentation framework $AF_1 = \langle AR, attacks \rangle$, where $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}\}$ and $attacks = \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B}), (\mathcal{D}, \mathcal{A}), (\mathcal{E}, \mathcal{D}), (\mathcal{G}, \mathcal{H}), (\mathcal{H}, \mathcal{G})\}$. Then*
– $\{\mathcal{A}, \mathcal{C}, \mathcal{E}\}$ is an admissible set of arguments.
– $\{\mathcal{A}, \mathcal{C}, \mathcal{E}, \mathcal{F}, \mathcal{G}\}$ is a preferred extension.

In the following section we recall the basic definitions of Petri nets, as needed later. For a more detailed introduction to Petri nets, the reader may refer to [10].

3 Petri Nets

A Petri net is a directed, weighted, bipartite graph consisting of two kind of nodes called *places* and *transitions*. Usually places are represented as circles and transitions are represented as boxes or bars. Arcs connect transitions and places and have a weight (positive integers). A *marking M* of the net assigns a nonnegative integer to each place in the net. If a marking M assigns to place p an integer k, it is said that p has (or is marked with) k *tokens*. This tokens are graphically represented as dots inside a place, or just simply a number.

Definition 4 *A Petri net is a 5-tuple $PN = (P, T, F, W, M_0)$ where*

- *$P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places and $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions, with $P \cap T = \emptyset$ and $P \cup T = \emptyset$,*
- *$F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs,*
- *$W : F \to \{1, 2, 3, \ldots\}$ is a weight function,*
- *$M_0 : P \to \{1, 2, 3, \ldots\}$ is the initial marking.*

A transition t is said to be *enabled* at marking M if each input place p of t is such that $M(p) \geq W(p,t)$. Enabled transitions may be *fired*. A firing of a transition t removes $W(p,T)$ tokens from each input place p and adds $W(t,p)$ tokens to each output place. A sequence of firings leads to a sequence of markings. A marking M_i is said to be *reachable* from marking M_j if there exists a sequence of firings that transforms M_j to M_i. The set of all reachable markings of a net P is called the *reachability space* of P. A *vanishing state* of the net is a marking that can be changed since it enable some transitions. A *tangible state* is a marking in which no transition is enabled. The number of tokens in a place p at marking M_k is denoted as $M_k(p)$. A Petri net is said to be *k-bounded* if the number of tokens in each place does not exceed a finite number *k* for any marking reachable from M_0. A Petri net is said to be *safe* if it is 1-bounded. A *self loop* is a transition with an output and an input from the same place. Two transitions that output to the same place are said to be in *backward conflict*. Two transitions are in *forward conflict* if, being both of them enabled by a common place, only one can be fired.

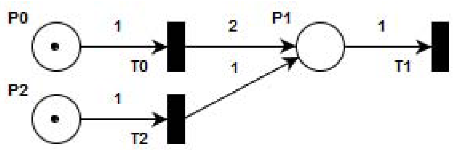

Fig. 1. A simple Petri net

In Figure 1 a simple Petri net is depicted, where places are circles and transitions are black bars. The marking is represented by black dots inside places, showing $M(P0) = 1$, $M(P1) = 0$ and $M(P2) = 1$. Transitions $T0$ and $T2$ are enabled and can be fired. If transition $T0$ is fired, it consumes the only token of place $P0$ and produces two tokens in place $P1$. After that, transition $T1$ is enabled and can be fired twice, each one consuming one token of $P1$. Some firing sequences on this net are $\{T0, T1, T1, T2, T1\}$ and $\{T2, T1, T0, T1, T1\}$. Both of them lead to the only *tangible state* of the net, with all the places free of tokens. Between the initial state and this tangible state, it is possible to generate eight vanishing states. This is because the firing order of some transitions is interchangeable, as $T0$ and $T2$. Transition $T1$ consumes all the tokens produced by these two transitions and can only be fired after them.

An extension of Petri nets distinguishes a special kind of arcs called *inhibitor arcs*. This set of arcs appears as a new component in the formal definition of the Petri net, being then a 6-tuple. An inhibitor arc connects a place to a transition and it is graphically represented as a line with a white circle in its end. This kind of arc disables the transition when the input place has a token, and enables the transition when the input place has no token and any other normal input has the required tokens. This extension allows the test for absence of tokens and this simple fact makes Petri nets as expressive as Turing Machines.

In Petri nets there are several definitions of fairness. Two transitions t_1 and t_2 are said to be in a bounded-fair relation if the maximum number of times that either one can fire while the other is not firing is bounded. Hence, one transition cannot block the other by firing infinitely. A Petri net is said to be a bounded-fair net if every pair of transitions in the net are in a bounded-fair relation. A firing sequence T is said to be unconditionally fair if it is finite or every transition in the net appears infinitely often in T. A Petri net is said to be an unconditionally fair net if every firing sequence from the initial state is unconditionally fair.

In the following section we present a formalization of argumentation frameworks using Petri nets with inhibitor arcs. Semantic notions are discussed in subsequent sections.

4 Argumentation Nets

An abstract argumentation framework as in Definition 1 induces a Petri net where places are arguments and transitions represent the conflict between arguments. This is formalized in the following definition.

Definition 5 *Let $\Phi = \langle AR, attacks \rangle$ be an argumentation framework. The argumentation net of Φ, or simply argnet, is a Petri net $\mathcal{V}_\Phi = (AR, T, F, H, W, M)$ where*

- *AR is the set of places*
- *$T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions.*
- *$F \subseteq (AR \times T) \cup (T \times AR)$ is a set of arcs.*
- *$H \subseteq (AR \times T) \cup (T \times AR)$ is a set of inhibitors arcs.*
- *$W : F \to \{1, 2, 3 \ldots\}$ is the weight function.*
- *$M : AR \to \{0, 1, 2, 3 \ldots\}$ is the initial marking.*

such that

- *for any attack $(\mathcal{A}, \mathcal{B}) \in attacks$, there exist a transition t_{AB} and the arcs $(\mathcal{A}, t_{AB})$,$(t_{AB}, \mathcal{A})$,$(\mathcal{B}, t_{AB}) \subseteq F$. This transition is called an attack transition.*
- *for any argument $\mathcal{A}$ with attackers $\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n$ there exists the transition t_A with the inhibitor arcs $(\mathcal{A}, t_A)$ and $(\mathcal{X}_i, t_A)$ for $1 \leq i \leq n$, and the arc $(t_A, \mathcal{A})$. This transition is called a restoring transition.*
- *$M(\mathcal{X}) = 1$ for any argument $\mathcal{X} \in AR$.*

Tokens represent potential strength of arguments for attacking each other. There is an *attack* transition whenever an argument $\mathcal{A}$ attacks another argument $\mathcal{B}$. Sometimes in this text arguments and places are treated as equivalents. In this paper the weight of every transition is 1, and then transitions remove or add only one token at a time. When referring to attack transitions, the corresponding place for $\mathcal{A}$ will be called the *attacking* or input place and the corresponding place for $\mathcal{B}$ will be called the *attacked* or output place. Such a transition can be fired when a token is available in both the *attacking* and *attacked* place. The attack transition consumes the tokens of both places, and restores the consumed token in the attacking place. Restoring transitions links attacked arguments with all of its attackers. Such a transition places a token in an empty attacked place $\mathcal{X}$ whenever all the attacker places are empty. This models the fact that, since every attacker of $\mathcal{X}$ has no strength, then the strength of $\mathcal{X}$ can be reinstated.

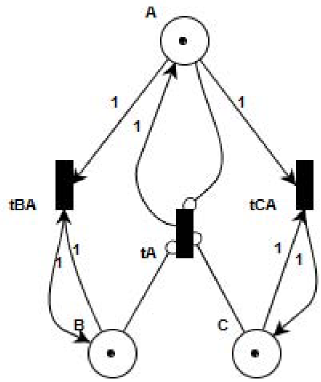

Fig. 2. Arguments $\mathcal{B}$ and $\mathcal{C}$ attack argument $\mathcal{A}$

Example 2 *Let $\Phi = (\{\mathcal{A}, \mathcal{B}, \mathcal{C}\}, \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{A})\})$ be an argumentation framework. The corresponding argnet $\mathcal{V}_\Phi$ is depicted in Figure 2. Arguments are represented as places and the attacks are represented by transitions t_{BA} and t_{CA}. The restoring transition t_A adds a token to the place $\mathcal{A}$ only when $\mathcal{A}$ and its attackers have no tokens.*

Petri nets are mainly a model of computation. In this argumentation net our *units of computation* are the transitions, either attacking or restoring ones. Therefore, we are interested in the evolution of the strength of arguments as transitions are fired, $i.e.$ when tokens are consumed and restored in argument places. In the initial marking M_0 every place has a token, since no attack is considered yet and then every argument has the potential strength of affecting other arguments. Note that transitions can put tokens in a place whenever (a) a token is removed from the same place as in attacking transitions or (b) when there is no token in the place as in restoring transitions. Hence no place can hold more than one token, as stated in the following Proposition.

Proposition 1 *For any argumentation framework Φ, the argnet $\mathcal{V}_\Phi$ is safe (1-bounded).*

If the Petri net is k-bounded, then the reachability space is finite. In this work we are interested in the connection between the firing of transitions, the evolution of markings and the underlying argumentation semantics. Consider the net of Figure 2. The enabled transitions are tBA and tCA. Restoring transition tA is not enabled since places A, B and C are not free of tokens. There are two possible firing sequences in this net: $\{tBA\}$ and $\{tCA\}$. Since the firing of an attacking transition removes the token in the attacker argument, then the firing of tCA inhibits the firing of tBA and viceversa. This happens with attacking transitions for the same arguments, as stated in the following Proposition.

Proposition 2 *Let $\mathcal{A}$ be an argument attacked by arguments $\mathcal{D}_1, \mathcal{D}_2 \ldots \mathcal{D}_n$,. The transitions $tD_1A, tD_2A \ldots tD_nA$ are all in forward conflict.*

However, whether tBA or tCA are fired in the net of Figure 2 the final marking M of $\mathcal{V}_\Phi$ is the same: $M(A) = 0$, $M(B) = 1$ and $M(C) = 1$. In other words, any of these attacks can be applied and the final outcome is the same, but only one of them is firable since these transitions are not independent of each other. Note that the maximal admissible set of Φ is $\{\mathcal{B}, \mathcal{C}\}$, the only places with a token in the tangible state.

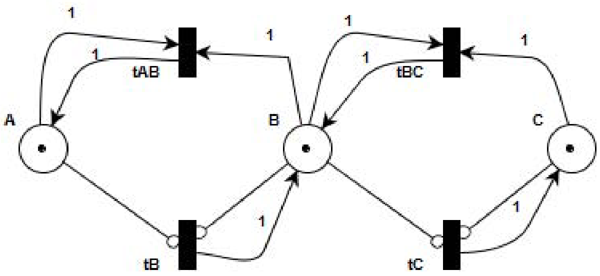

Fig. 3. Argument $\mathcal{A}$ attacks $\mathcal{B}$. Argument $\mathcal{B}$ attacks $\mathcal{C}$.

Consider the net of Figure 3 where a situation of argument defense is presented. The initial enabled transitions are tAB and tBC. This means that both attacks are fireable at the beginning. If transition tAB is fired first, then no transition is later enabled, leading to a tangible state. If transition tBC is fired first, transition tAB is still enabled. After firing transition tAB, the restoring transition tC becomes enabled, since neither B nor C have tokens. After firing transition tC, place (argument) C gains a token and then no transition is enabled after that. Thus, a tangible state can be reached by firing tAB or by firing the sequence $\{tBC, tAB, tC\}$. Moreover, there is only one tangible state with marking $M(A) = 1$, $M(B) = 0$ and $M(C) = 1$. Note that the maximal admissible set of the corresponding abstract framework is $\{\mathcal{A}, \mathcal{C}\}$.

In the following section we present semantic notions for argumentation Petri nets, based on markings and sequence of transitions.

5 Argumentation Semantics

As stated before, Petri nets are a model of computation for concurrent and distributed systems, where the emphasis is put in the firing of transitions and how the marking of the net evolves as a consequence. Argnets provide an interesting model for procedural argumentation semantics. In this section we consider nets without isolated parts. As shown in the example of Figure 3 a sequence of firing of transitions leads to a sequence of markings, which can be interpreted as an evolution of the strength of arguments as attacks take place. As long as there are enabled transitions, an attack or a restoration can occur and then there are still arguments able to loose or gain strength. There is, however, a set of arguments that never loose its tokens. A *trap* of a Petri net is a set of places S such that any transition with an input in S has also an output in S and if S is marked under some marking M, it is still marked under any succesor marking of M.

Proposition 3 *Let $\Phi = \langle AR, attacks \rangle$ be an argumentation framework with corresponding argnet $\mathcal{V}_\Phi$ and let $D_f \subseteq AR$ be the set of all defeater-free arguments in AF. Then D_f is a trap of $\mathcal{V}_\Phi$.*

Proof: Defeater-free arguments can only attack other arguments and then every outgoing transition of the corresponding place is an attack transition, conforming a self-loop. These are the only transitions that will be enabled and then the token is never lost. In some formalisms, this kind of loop if represented as a single transition called read-transition. □

Hence, defeater-free arguments always have a token in the corresponding place. Because of this, any attack transition from a defeater-free argument is enabled at M_0 and it will be enabled as long as the attacked argument still possesses its token. That means that the attack is enabled while it has an actual impact on the attacked argument (otherwise the attack is not necessary). In the net of Figure 2, places $\mathcal{B}$ and $\mathcal{C}$ never loose their tokens, yet only one attack is enough to suppress argument $\mathcal{A}$. Under the interpretation of attacks as actions, in this example only one attack is sufficient to reach a tangible state.

An admissible extension is basically a set of arguments defending each other. In the Petri net this is interpreted as a distribution of tokens over the net, with a particular condition. This marking can be reached by firing transitions until no transition is enabled, as stated in the following proposition.

Proposition 4 *Let $\Phi = \langle AR, attacks \rangle$ be an argumentation framework with argnet $\mathcal{V}_\Phi = (P, T, F, H, W, M)$. Let T be a sequence of firing transitions $\{t_1, t_2, \ldots, t_n\}$ that transforms M to M_n such that every transition in $\mathcal{V}_\Phi$ after M_n is not enabled. Then the set of arguments $S = \{\mathcal{A} \in AR | M_n(\mathcal{A}) = 1\}$ is an admissible set of Φ.*

Proof: Suppose no transitions are enabled and S is not admissible. Then either (a) it is not free of conflict or (b) at least one argument $\mathcal{A} \in S$ is not defended by S. If (a) is the case, then at least two arguments $\mathcal{X}$ and $\mathcal{Y}$ are such that $M(\mathcal{X}) = M(\mathcal{Y}) = 1$ are in conflict. But then the attack transition between them is enabled, which is a contradiction. If (b) is the case, then at least an argument $\mathcal{X} \in S$ is attacked by an argument $\mathcal{Y}$, but not defended by an argument in S. But $\mathcal{Y}$ must be free of tokens, otherwise (a) is the case. Since $M(\mathcal{Y}) = 0$, then $\mathcal{Y}$ is not free of attackers and then there is at least one argument attacking $\mathcal{Y}$. But, since $\mathcal{Y}$ has no token and no transitions are enabled, then it is not possible for all the attackers of $\mathcal{Y}$ to have no tokens. Then at least one argument $\mathcal{Z}$ attacks $\mathcal{Y}$ such that $M(\mathcal{Z}) = 1$. But then $\mathcal{X}$ is actually defended by S, which is a contradiction. Since (a) and (b) cannot be the case, then S is admissible whenever the transitions are not enabled. □

Hence, a tangible state in the net corresponds to a distribution of tokens signalling arguments in an admissible extension. If no tangible state can be reached through a sequence of firings of transitions, then there is always an enabled attack or restoring transition, $i.e.$ a token can be placed or removed somewhere in the net.

Proposition 5 *If at least one transition in $\mathcal{V}_\Phi$ is potentially fireable in any marking M_k then AF is not well-formed.*

Proof: If at least one transition is potentially fireable, then every state of the net is a vanishing state. Since there is a finite set of possible states and the net is safe, then at least one argument is gaining and loosing a token repeatedly (although not necessarily in consecutive markings). □

Consider the net of Figure 4 where an odd cycle of attacks is present. Starting from the initial marking, no tangible state can be produced. In fact, there is an infinite sequence of attacking and restoration transitions. Whenever a token is restored in a place $\mathcal{X}$, it enables an attack transition tXY from that place. There are six vanishing states in this net and the only admissible set in the corresponding argumentation framework is the empty set.

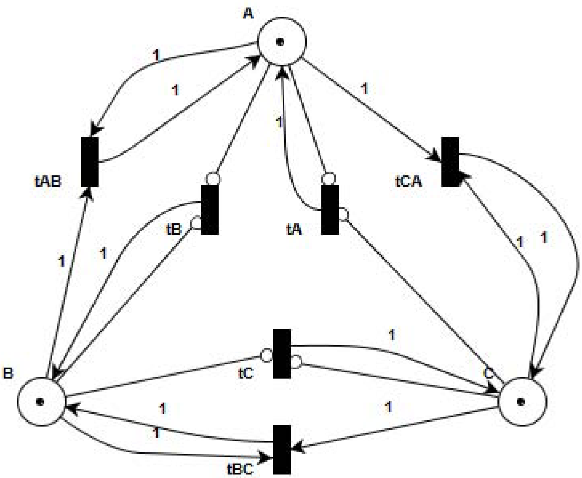

Fig. 4. Argument cycle between $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$

The converse of Proposition 5 is not true, as shown in the net of Figure 5, corresponding to the argumentation framework $\Phi = \langle\{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}, \{(\mathcal{A}, \mathcal{B}), (\mathcal{B}, \mathcal{C}), (\mathcal{C}, \mathcal{D}), (\mathcal{D}, \mathcal{A})\}\rangle$ with a cycle of attacks between the four arguments. Starting from the initial marking, it is possible to reach thirteen different states, with only two of them being tangible. As stated in Proposition 4 these tangible states correspond to the admissible sets $S_1 = \{\mathcal{A}, \mathcal{C}\}$ and $S_2 = \{\mathcal{B}, \mathcal{D}\}$. The set S_1 can be reached, for instance, by the sequence of firings $T_1 = \{tAB, tCD\}$ but also by $T_1' = \{tCD, tBC, tAB, tC\}$. The set S_2 can be reached, for instance, by $T_2 = \{tBC, tDA\}$ or $T_2' = \{tBC, tAB, tDA, tB\}$. Note that in T_1' and T_2' a restoration transition is needed.

Consider the following sequences of transitions in the net of Figure 5:

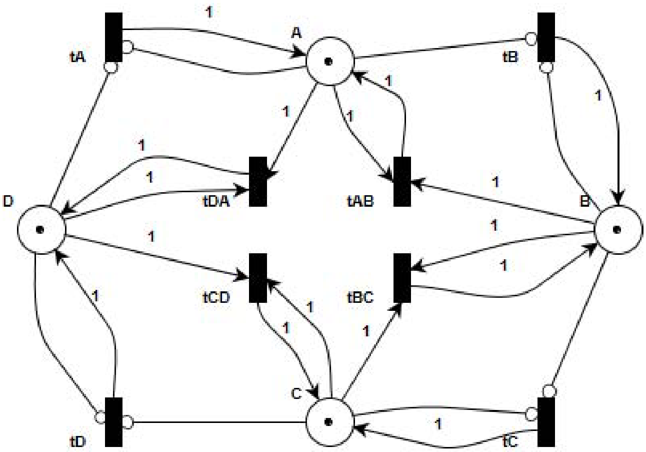

Fig. 5. Four arguments $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ in a cycle

- $T = \{tDA, tCD, tA, tBC, tD, tDA\}$
- $T' = \{tDA, tCD, tBC, tA, tAB, tC\}$

Although both sequences start firing transitions $\{tDA, tCD\}$, they lead to different tangible states. Sequence T leaves tokens in $\mathcal{D}$ and $\mathcal{B}$ while the sequence T' leaves tokens in $\mathcal{A}$ and $\mathcal{C}$. The former decides to reinstate $\mathcal{A}$ (by firing tA) before triggering an attack to $\mathcal{C}$ from $\mathcal{B}$ (by firing tBC). The sequence T', however, decides to attack $\mathcal{C}$ before reinstate $\mathcal{A}$. This shows, of course, that the choice of transitions to fire may completely change the outcome of the process. But the most interesting aspect is that the reinstatement of tA and the attack tBC are interchangeable, since both T and T' lead to a state of the net in which only tAB and tD are the enabled transitions after the fourth transition. In other words, $\{tDA, tCD, tA, tBC\}$ and $\{tDA, tCD, tBC, tA\}$ lead to the same vanishing state. After that, both sequences take different paths: T reinstantes $\mathcal{D}$ to attack $\mathcal{A}$ later, while T' attacks $\mathcal{B}$ to later reinstate $\mathcal{C}$. As expected in argumentation, a restoring transition can be fired only after other transitions are fired. For a given sequence of transitions, we will denote $t_1 \ll t_2$ if transition t_1 occurs before transition t_2. In order to reinstate an argument $\mathcal{X}$, all of its attackers must loose their tokens. This is formalized in the following proposition.

Proposition 6 *Let Φ be an argumentation framework with argnet $\mathcal{V}_\Phi$. For any sequence of firings of transitions T, then*

1. *$tXY \ll tY$ in T and*
2. *for every argument W_i attacking Y, $tZ_iW_i \ll tY$ in T.*

Proof: Trivial. Place Y must loose its token in order to fire its reinstate transition. This can only be achieved by an attack transition with input Y. The same is true for all of its attackers. □

Moreover, if $tZ_iW_i \ll tY$ in a sequence T then there is a sub-sequence $T' = [tZ_iW_i \dots tY]$ such that there are no extra occurrences of tZ_iW_i in T' and $tW_i \not\ll tY$. In other words, although W_i loose its token after the attacking transition, it is not reinstated before Y is. Note that in Proposition 6 there is no specific restriction about the order of tXY and every tZ_iW_i. This is consistent with the notion of attack and defense in argumentation frameworks.

Another important aspect of Petri nets is the *reachability problem*. This is a *decision problem* about deciding, for a given marking M, whether it is reachable in a particular net. In our formalism, the reachability graph is finite since the net is safe. What is interesting is to prove that certain relevant markings are in the reachability graph induced by a net.

Proposition 7 *Let $\Phi = \langle AR, attacks \rangle$ be an argumentation framework with argnet $\mathcal{V}_\Phi = (P, T, F, H, W, M)$. If $S \subseteq AR$ is a preferred extension of Φ, $S \neq \emptyset$, then there exists a sequence of firings $\{t_1, t_2, \dots, t_n\}$ that transforms M to M_n such that $M_n(\mathcal{A}) = 1$ if $\mathcal{A} \in S$ and $M_n(\mathcal{A}) = 0$ if $\mathcal{A} \notin S$.*

Proof: Let M_S be the marking such that only arguments in the preferred extension S have tokens. Suppose M_S is not in the reachability space of the net $\mathcal{V}_\Phi$. This means that there exists at least one argument $\mathcal{A}$, such that $\mathcal{A}$ cannot (a) acquire a token if $M_S(\mathcal{A}) = 1$ or (b) loose a token if $M_S(\mathcal{A}) = 0$ in any sequence of firing transitions. Suppose (a) is the case. Since the initial marking assigns tokens for every place, then $\mathcal{A}$ looses its token and it is not able to recover it in any sequence of transitions. However, since $\mathcal{A}$ is in the preferred extension, then it is defended by arguments in S. But if every defender D_i is such that $M(D_i) = 1$, then after firing the outgoing attack transitions, every attacker of $\mathcal{A}$ looses its token, which after the restoring transition makes $M(\mathcal{A}) = 1$. Then clearly at least one defender $\mathcal{D}_k$ of $\mathcal{A}$ has no token, otherwise $\mathcal{A}$ could recover its token. Now $\mathcal{D}_k$ and $\mathcal{A}$ are two arguments that M_S assigns tokens to, but they cannot acquire them. The same analysis can be made for $\mathcal{D}_k$. □

It could be the case that some controversies are present in the framework, as shown in Example 4, where an infinite sequence of transitions can be fired. In this particular case of a three-argument cycle, every transition leads to a vanishing state. The same is true for longer odd cycles, when an argument attacks its own (indirect) defender. This is sometimes called a *contradictory argumentation line*, since every argument indirectly attacks itself. An argument $\mathcal{A}$ is said to be *controversial* with respect to another argument $\mathcal{B}$ if $\mathcal{B}$ indirectly attacks and indirectly defendes $\mathcal{A}$ [6].

Proposition 8 *Let Φ be an argumentation framework with argnet $\mathcal{V}_\Phi$. If there exists an infinite sequence of firings of transitions in $\mathcal{V}_\Phi$, then Φ is controversial.*

Proof: If there exists an infinite sequence of transitions, then some arguments are repeatedly loosing and gaining their tokens. Thus, there is a cycle of attacking and restoring transitions. It means that at least two transitions tXY and tY are involved a bounded-fair relation. Hence the attack of $\mathcal{X}$ on $\mathcal{Y}$ indirectly causes the restoration of $\mathcal{Y}$. It means that $\mathcal{X}$ attacks and indirectly defends $\mathcal{Y}$, and then Φ is controversial. □

A Petri net may have an isolated sub-graph with a cycle causing an infinite sequences of firings of transitions. Since there is always a transition that is potentially fireable, then the net cannot reach a tangible state. There may be, however, a subnet such that no transition is enabled and there are no other, external transitions that can change that fact. A subnet generated by a set of transitions T is another net formed by T and all of its input and output places with its corresponding arcs. In argumentation nets, every transition of a net $\mathcal{V}_\Phi$ that is not potentially fireable at a given marking M, forms a subnet $\mathcal{V}'_\Phi$ such that the restriction of M to $\mathcal{V}'_\Phi$ is a potential admissible subset of arguments. Although there are no tangible states in the whole net, some transitions will be never fired and thus some places are not receiving or loosing tokens any more.

In the following section we discuss a dialectical interpretation of sequences of firings of transitions in an argumentation net.

6 Transitions as Dialogue Acts

An argumentation system may produce thousands of arguments from a knowledge base. In Defeasible Logic Programming [7], the addition of a simple defeasible rule may cause new derivation trees, thus incrementing the set of arguments. The size and complexity of argumentation makes procedural evaluation of arguments and its relationship an important topic. It is interesting to evaluate the role of transitions under procedural models of argumentation. A sequence of firings can be considered as a sequence of moves in an argumentation game, where two participants (*agents*) decide what attack is considered next. This is basically a dialogue that last until some particular condition is reached. Several forms of dialogue games may be defined, for instance:

– *Single-topic*: an agent P proposes an argument $\mathcal{A}$ to agent O. The goal of P is to defend $\mathcal{A}$, $i.e.$ to keep the corresponding place tokenized. The goal of O is to de-tokenize $\mathcal{A}$.
– *Set-of-beliefs*: both agents propose a set of arguments S as a set of beliefs in contention. The goal of the dialogue is to collaboratively analyse the acceptance of arguments in S by highlighting attacks and restorations until some condition is reached.

The first dialogue is competitive, while the other is collaborative. Both dialogues may run until no transitions are enabled, or for a finite period of time, or until a maximal number of transitions were fired. In any case, a restriction of valid sequences of firings can be considered. For instance, an agent that proposes tXY cannot propose tY later. This means that an agent that causes the disqualification of an argument in the dialogue (by deleting its token) cannot provoke the restoration of the same argument. This restriction is probably more reasonable in single-topic dialogues. Also some transitions may be completely forbidden. This may be the case when some arguments and some relations are previously agreed to be off-topic.

It is also possible to consider restoration transitions as an automatic consequence of the last transition that enables such a restoration, an then restoring transitions are not a move by itself in the dialogue. Thus, a restoration transition is not a legal single move. It must be always preceded by an attack transition. According to this form of dialogue, there are only two possible moves for an agent: (tXY) and $(tXY; tZ)$ for any arguments $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{Z}$ such that $\mathcal{Y}$ is an attacker of $\mathcal{Z}$

Another interpretation may be to provide restoring transitions with special, additional conditions for firing. Here restoration is not automatic, but reserved for particular moments in the dialogue. Then there is a priority between transitions, being attack ones preferred over restoring ones. When no attack is possible, a restoration of a place may be considered by any agent, even when that agent previously removed the token of the same place. This is a sort of belief revision made by the agent, by consenting the validity of a previously challenged argument.

In the Petri nets model of argumentation this restrictions to the dialogue can be achieved by the notion of *supervisors* [9], as shown in the following Definition.

Definition 6 [9] *Let $\mathcal{V}_\Phi = (AR, T, F, W)$ be a Petri net, $\mathcal{M}$ the set of all markings of $\mathcal{V}_\Phi$ and $U \subseteq \mathcal{M}$. A* ***supervisor*** *Ξ is a function $\Xi : U \to 2^T$ that maps to every marking a set of transitions that the Petri net is allowed to fire.*

The notion of supervisor is usually associated with the task of preventing deadlock in Petri nets. However, the same formalism may be used to direct the dialogue to specific purposes.

Consider the net of Figure 4, where there is no tangible state. A dialogue of transitions engaged in this net requires additional controls to avoid circular argumentation. A possible supervisor for this situation may be $\Xi(M) = \emptyset$ for any marking $M \subseteq \{(1,0,0),(0,1,0),(0,0,1)\}$ $i.e.$ markings with only one token. This means that no transition is enabled after removing two tokens. In other words, the last argument to survive leads to a special kind of tangible state since no transition is legally fireable. The notion of tangible state is now contextual to the overall state of enabled transitions and the supervisor restrictions. Even more, it is possible to use more than one supervisor, with priorities. One supervisor define legal attack transitions and the other define legal restorations. The change of supervisor takes place when the net reaches a tangible state under supervisors restriction.

7 Conclusions and Future Work

Petri nets are a model of computation for concurrent and distributed systems, where the emphasis is put in the firing of transitions and how the marking of the net evolves as a consequence. In this work we proposed a Petri net representation of abstract argumentation frameworks as an approach to the study of procedural interpretation of attacks, $i.e.$ the consideration of argument attacks as actions in an argumentation system. Given this new Petri net model, we have proved that there is a relation between tangible states of the net and admissible sets of the corresponding framework. We also discussed that the procedural profile of Petri nets makes this formalism suitable to provide a framework for the study of argumentation dialogues, under the formal regulation of a supervisor.

Future work has several directions, as intended in this seminal proposal. A Petri net model of argumentation frameworks that allow places to have more than one token is being studied. Tokens here are intended to represent the strength of an argument, and a single attack weakens such an argument by removing one token. An argument is considered rejected if it has no tokens. In other direction, it is important to study the relation between partial repetitive nets and the existence of admissible extensions in the corresponding argumentation framework. Dialectical strategies to avoid cyclic transitions in an argumentation dialogue are interesting. Finally, the addition of *timed* transitions is important to model the dynamics of timed argumentation formalisms [5].

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