STRATEGIC ANALYSIS OF FOREST INVESTMENTS USING REAL OPTION: THE FUZZY PAY-OFF MODEL (FPOM)

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ABSTRACT

We introduce here a forestry investment decision-making tool based on a fuzzy sets approach. Three scenarios are considered: a base case, an optimistic and a pessimistic one. Two roles are conceived for decision-makers, either as owners or as investors. For either one of these two situations, the possibility degrees of the scenarios may be represented by means of a fuzzy numbers, representing ambiguous net present values (NPV). Real option values (ROV) are computed based on them. An application to a potential forestry project in Argentina shows that while in the case of an owner of forestry project the expected benefits are both positive under NPV and ROV, an investor would discard the project if she assumes equal weights for the scenarios in a traditional analysis but would accept it under the fuzzy approach.

Keywords: Fuzzy number, NPV (net present value), ROV (real option value).

INTRODUCTION

The forestry sector exhibits a remarkably distinct investment profile that has attracted recently the interest of institutional investors. The attractive investment features of this industry are based on their tangibility as assets, seen in the land that sustains the forests, the standing timber as well as in the associated milling facilities. In countries like Argentina, forestry is becoming increasingly important, not only in the economic arena but also for its social and environmental impact. That is, the productive side of the industry generates wealth but at certain social and environmental costs. For this reason decision-making in forestry require solid management tools, to overcome the weaknesses of the established methods. In fact, they are usually very simple, to the point to lead to mistaken decisions. This is particularly true of financial tools as Net Present Value (NPV) assessments (Milanesi et al. 2012) and its variants, like Faustmann's formula (Bettinger, et al. 2009). Even their proponents point out the problems associated with their application in forestry contexts under time-varying conditions.

As it is typical of highly uncertain and complex activities, the valuation of capital investment in the forestry industry, unlike the classical approaches, should emphasize on a flexible management of resources (Carmona and Aranda 2003, Milanesi et al. 2012). This goal can be attained by resorting to the method of Real Options Valuation (ROV), which overcomes the weaknesses of conventional models (Milanesi et al. 2012) introducing strategic flexibility in the assessment of investment projects (Smit and Trigeorgis 2004). The method, based on the Black-Merton-Scholes model (Black et al. 1973, Merton 1973) tha characterizes the values of options as solutions of stochastic differential equations, can be modified and adjusted in different ways, like allowing the selection of particular stochastic processes and increasing the complexity and structure of options.

In its non-fuzzy version, ROV has been applied to forestry problems by Thomson (1992), Yin and Newman (1997) and Milanesi et al. (2012). The latter, in particular, used it to determine an optimal period of harvest. On the other hand, Petrasek and Perez (2010) assessed harvest contracts with American options. While other alternatives are possible, binomial models are a preferred choice for most of these applications, like in other areas (Trigeorgis 1995, Mun 2002). They may be either represented as grids or trees (Brandao et al., 2005, Smith 2005), or captured in the *certainty equivalents* of implicit probability distributions (Rubinstein 1994).

ROV, like all the known assessments of risk can be modified introducing fuzzy concepts (Kahraman et al., 2002, Fuller and Majlender 2003). The formal approach involves the representation of possibilities instead of probabilities (Zadeh 1965, Dubois and Prade 1980, Carlsson and Fuller 2001). There are three main ways in which this "fuzzyfication" of ROV can be carried out:

a. *Fuzzy continuous time model* (FCM): it is an adaptation of the classic Black-Merton-Scholes in which trapezoidal[1] numbers represent the fuzzy values of both the underlying asset (real or financial) and the option price (Carlsson and Fuller 2003, Carlsson et al. 2007).
b. *Fuzzy discrete-time model* (FDM): it adapts the traditional binomial model to fuzzy logic. This allows operating and defining the underlying ambiguity using triangular or trapezoidal fuzzy numbers to estimate the upward and downward movements of values (Muzzioli and Torricelli 2004, Yoshida et al. 2006, Zdnek 2010, Liao and Ho S 2010).
c. *Fuzzy Pay-Off Method* (FPOM): it has been developed by Collan et al. (2009) as a way of analyzing scenarios using fuzzy triangular distributions. The results obtained with this method are consistent with

[1] A variant of the triangular fuzzy numbers discussed below.

a more classical method developed by (Datar and Mathews 2004, Datar et al. 2007).

The first step in the implementation of the fuzzy approach is the determination of uncertainty degrees (Landro 2010). The second step involves the characterization of a semantic scale of ambiguity-vagueness levels in terms of those degrees. This scale must be suitable for situations where the lack of information transforms an uncertain setting into an ambiguous one.

By means of the fuzzy scale the probabilistic valuation approach becomes complemented by one based on possibility degrees, which is perhaps more appropriate according to the semantics of business decision-making processes (Kinnunen 2010). Besides, the fuzzy option pricing models enhance the advantages of strategic flexibility (Carlsson and Fuller 2003).

Even if these models show the aforementioned advantages, the literature shows only a few applications to forestry. So, for instance, Liliadis (2005) uses a fuzzy model for the analysis of forest fire risk, while Mitra et al. (1998) uses fuzzy logic to predict soil erosion. Closer to our study, Mendoza (1989) introduces forest planning in a fuzzy environment.

In this paper we improve upon the NPV method using a FPOM approach to forestry planning. First we simulate scenarios of the growth of forest biomass, derive triangular fuzzy numbers for the representation of possible outcomes and apply the Fuzzy Pay-Off Method (FPOM). Then as a test case we focus on a forestry investment in the Province of Misiones, Argentina. Finally we present the results and key findings.

FUZZY CONCEPTS

Fuzzy sets and fuzzy numbers are the basic notions on which we will build the version of ROV to be applied to a forestry problem. They capture, in mathematical terms, ambiguity and vagueness.

More precisely, let X be a discrete or continuous set, the universe in which fuzzy entities live. A fuzzy subset A of X is:

$$A=\{\mathrm{x}\in \mathrm{X}: \mu_A(x)> 0\} \tag{1}$$

where μ_A: $\mathrm{X}\rightarrow [0,1]$ is a membership function, such that $\mu_A(x)$ represents the degree of membership of an element x to A. In the interval $[0,1]$ the extremes represent clearly defined situations: 0 represents non-membership while 1 indicates full membership. Intermediate values, instead, represent degrees of partial inclusion. If the range of μ_A is $\{0, 1\}$, we say that A is a *crisp* set.

A fuzzy set can be written as $A=\cup_{\gamma \in [0,1]} A_\gamma$ where

$$A_\gamma = \{\mathrm{x}\in \mathrm{X}: \mu_A(\mathrm{x})>\gamma\} \tag{2}$$

A_γ is known as the γ-*cut* of A. It consists of all the elements in X that belong to A in a degree at least γ. Thus, any fuzzy subset is expressed as the union of its γ-cuts. A fuzzy subset A of $\Re$ is convex if and only if, for all $\gamma \in [0,1]$ each γ-cut is a closed interval of $\Re$. It is said *normal* if at least one element $\mathrm{x} \in A$ is such that $\mu_A(\mathrm{x})=1$.

A fuzzy *number* A is a fuzzy subset of $\Re$ which is convex for every $\gamma \in [0,1]$ as well as normal. This implies, in particular, that for each $\gamma > 0$, A_γ can be expressed as the interval $[a_1(\gamma), a_2(\gamma)]$, where $a_1(\gamma) = min(x \in A_\gamma)$ and $a_2(\gamma) = max(x \in A_\gamma)$.

A refinement of the concept of fuzzy number is obtained when a fuzzy set A is a *triangular fuzzy number* with center a, and left and right amplitudes $\alpha > 0$, $\beta > 0$. In particular $\mu_A(a)=1$, which makes A normal. Defining $A(x)$ (which instantiates $\mu_A(x)$ for triangular fuzzy numbers) as its membership function, we have:

$$A(x) = \left\{ \begin{array}{ll} 1 - \frac{a-x}{\alpha} & if\ a - \alpha \le x \le a \\ 1 - \frac{x-a}{\beta} & if\ a \le x \le a + \beta \\ 0\ \ in\ another\ case & \end{array} \right\} \quad (3)$$

This implies that A is such that for any $\gamma > 0$, the corresponding γ-cut is $A_\gamma = [a - (1-\gamma)\,\alpha, a + (1-\gamma)\,\beta]$.

Finally, the *fuzzy mean value* of a fuzzy number A, with γ-cuts $A_\gamma = [a_1(\gamma), a_2(\gamma)]$ is (Carlsson and Fuller 2003):

$$\frac{\int_0^1 \frac{a_1(\gamma)+a_2(\gamma)}{2}\gamma d\gamma}{\int_0^1 \gamma d\gamma} = \int_0^1 \left[a_1(\gamma)+a_2(\gamma)\right] \gamma d\gamma \qquad (4)$$

That is, the mean value weighs the average value of the γ-cuts A_γ by γ. The denominator is introduced in order to normalize the result.

FUZZY MODEL OF THE CASH FLOWS

All these concepts can be applied in the field of financial theory, in particular, for the estimation of the cash flows generated by an investment. The replacement of *crisp* numbers (i.e. real numbers) by fuzzy numbers allows the incorporation the ambiguity and vagueness into value assessments.

One way of incorporating fuzzy numbers into valuation models of real options (ROV) is the *Fuzzy Pay-Off Method* (FPOM) (Collan et al. 2009). It assumes the existence of different scenarios, describing the evolution and outcomes of an investment. The projected cash flows of the different scenarios are used to calculate a real option value for each of them. The possibilities of each scenario are represented by a triangular fuzzy number.

Let us consider here three scenarios and their corresponding triangular fuzzy numbers. On one hand we have a *base case*, in which its triangular fuzzy number is centered on the initially projected value (under normal circumstances), denoted a. The *best case* scenario yields the highest projected income at the minimum cost while the *worst case* scenario yields, instead, the lowest projected income at the highest cost.

The value of the real option is calculated by considering the positive net present values as triangular fuzzy numbers. More precisely NPV is identified with *A* such that, if $x < 0$, $A(x)=0$, representing the possibility of dropping the project. According to Collan et al. (2009) the ROV obtains as follows:

$$ROV = \frac{\int_0^{\infty} A(x)dx}{\int_{-\infty}^{\infty} A(x)dx} E(A_+) \tag{5}$$

Where ROV is the Fuzzy Expected Expanded Value; $E(A_+)$ is the fuzzy mean value of the positive values of A; $\int_0^{\infty} A(x)dx$ computes the area below the positive part of A and $\int_{-\infty}^{\infty} A(x)dx$ computes the area below the whole fuzzy number A.

Notice that the ROV requires the computation of the total area of the triangular number (the denominator of (5)), the area of A over the positive real numbers and the fuzzy mean value of the positive part of A.

Table 1 reports $E(A_+)$ in four possible contexts, depending on where a, α and β stand with respect to 0. The first case obtains when a, α and β are all positive:

$$E(A_+) = a + \frac{\beta - \alpha}{6} \tag{6}$$

In the second case a is positive but $(a - \alpha)$ is negative. Thus we have:

$$E(A_{+}) = \int_{0}^{\frac{\alpha-a}{\alpha}} \left[0 + a + (1-\gamma)\beta\right] \gamma d\gamma + \int_{\frac{\alpha-a}{\alpha}}^{0} \left[a - (1-\gamma)\alpha + a + (1-\gamma)\beta\right] \gamma d\gamma$$

$$E(A_{+}) = a\frac{\beta-\alpha}{6} + \frac{(\alpha-a)^{3}}{6\alpha^{2}} \qquad (7)$$

In the third case $a < 0$ but $a + \beta > 0$:

$$E(A_{+}) = \int_{0}^{\frac{a+\beta}{\beta}} \left[0 + a + (1+\gamma)\beta\right] \gamma d\gamma = \frac{(a+\beta)^{3}}{6\beta^{2}} \qquad (8)$$

Finally, the fourth case arises when the entire distribution is below zero. In this case there are no positive values to exploit, and so the project should be discarded:

$$E(A_{+}) = 0 \qquad (9)$$

This is summarized in Table 1.

APPLYING THE MODEL TO A FORESTRY SETTING

We can apply these results to analyze a forestry investment project. Let us consider three scenarios for a forest of *Pinus taeda* L. covering an area of 38 hectares in Misiones, Argentina: favorable, unfavorable and the base case. The growth and production of timber for the three cases were simulated with the SisPinus (EMBRAPA-FORESTS) software for a period of 20 years (Oliveira 2011). The scenario simulation parameters are reported in Table 2 and the respective cash flows are presented in Table 3.

Two situations can be considered: a) the agent owns the site or b) at period 0 the agent invests on the site. To assess the net present value corresponding to each scenario we need the *risk-free rate* (*rf*), the *risk premium* (*RP*), the *cost of capital* (*k*) determined according the Capital Asset Pricing Model (CAPM) (Milanesi et al 2012), the *acquisition cost of land* per hectare (*$AR / ha*), the *exchange rate* peso-dollar (*AR / UD*), the *surface* in hectares (*ha*) and *value of the initial investment* in case b). The data is presented in Table 4.

Table 5 summarizes the cash flow at each scenario for case a) and case b).

Traditionally, the expected values are weighed the probabilities of the scenarios (Carmona and Aranda 2003). If we assume equal weights for each scenario, it follows trivially that the average expected values are $AR 792,700 for the owner and -$AR 252,300 for the investor. Thus, investing on the forest is discouraged. Note that this result is obtained disregarding the possibility of profiting from strategic flexibility (i.e. to abandon the project when negative values arise). This limitation can be overcome by taking into account a triangular fuzzy number representing the spectrum of possible outcomes. The center (a) represents the base case, while the ends of the triangular fuzzy number correspond, in the optimistic case, to $a + \beta$ and in the pessimistic case, $a - \alpha$. Table 6 shows and Figure 1 illustrates this procedure. Notice that we consider only cases 2 and 3 and discard case 1 (the domain of the triangular number is positive) and case 4 (negative domain).

With the information available we evaluate the project. Table 1 provides the expressions for $E(A_+)$ while the value of the real option is obtained according to equation (5). As said, we consider only the cases where positive and negative values are mixed. Thus, we need the ratio of area between the area of the positive fragment of the triangle and the entire area under the triangle (λ). This area is determined in terms of the point of intersection of the triangle sides with the $x=0$ axis. In case (2) this intercept is at $A(0) = 1 - a/\alpha$ and thus we have:

$$\begin{cases} \lambda(+) = \dfrac{3a^2 + 2a\alpha + \alpha\beta}{2\alpha} \\ \\ \lambda(-) = \dfrac{(\alpha - a)^2}{2\alpha} \end{cases} \tag{10}$$

In case (3) the intercept obtains at $A(0) = 1 + a/\beta$ and so we have:

$$\begin{cases} \lambda(+) = \dfrac{(a+\beta)^2}{2\beta} \\ \\ \lambda(-) = \dfrac{(\beta+\alpha)}{2} - \lambda(+) \end{cases} \tag{11}$$

Table 7 presents the ROV values for the owner and the investor using the actual parameters of our example on the expressions of Table 1 and in (10) and (11).

As it can be seen in the case of the owner (a), the project takes on negative values if the scenario is unfavorable. In the case of the investor (b), instead, the negative values obtain in the base and unfavorable scenarios. Nevertheless, unlike the results presented in Table 5, if strategic flexibility (i.e. the possibility of abandon the project) is considered, ROV is positive for both the owner ($AR 724,002) and the investor ($AR 79,533).

CONCLUSIONS

We presented a model for decision making with a fuzzy representation of the net present value (NPV) in which real options (ROV) grant strategic flexibility to the project. The importance of this tool can be seen in the application to a forestry context. While traditional approaches rule out the alternative of investing in the project, ROV indicates that there is still an expected benefit to be obtained from the investment.

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Table 1: Possible Valuations in the FPOM model

Cases	Parameters	Fuzzy Mean Value of A_+
1	$0 < (\alpha - a)$	$a + (\beta - \alpha) / 6$
2	$(\alpha - a) < 0 < a$	$a + (\beta - \alpha) / 6 + (\alpha - a)^3 / 6\alpha^2$
3	$a < 0 < a + \beta$	$(a + \beta)^3 / 6\beta^2$
4	$a + \beta < 0$	0

Table 2. Initial parameters of the simulation.

	Unfavorable	Base	Favorable
Site Index [m]	18	22	24
Initial Density [tree/ha]	1666	1666	1111
Survival [%]	70	85	95
N° of thinnings	1	2	3

Table 3. Information based on the three scenarios studied ($AR)

Period	Base			Favorable			Unfavorable		
	Revenues	Cost	Fund Flow	Revenues	Cost	Fund Flow	Revenues	Cost	Fund Flow
0	0	144,378	-144,378	0	115,819	-115,819	0	172,938	-172,938
1	0	176,373	-176,373	0	140,497	-140,497	0	212,250	-212,250
2	0	107,515	-107,515	0	87,376	-87,376	0	127,654	-127,654
3	0	86,799	-86,799	0	71,397	-71,397	0	102,202	-102,202
4	18,135	56,636	-38,501	63,066	49,097	13,969	0	19,400	-19,400
5	140,658	62,683	77,975	274,728	19,400	255,328	20,154	19,400	754
6	330,495	19,400	311,095	512,647	106,693	405,953	74,655	19,400	55,255
7	538,338	19,400	518,938	580,135	19,400	560,735	149,481	90,948	58,533
8	804,196	131,319	672,877	889,820	147,480	742,340	227,881	19,400	208,481
9	722,844	19,400	703,444	1,016,385	19,400	996,985	335,003	19,400	315,603
10	984,069	19,400	964,669	1,416,958	19,400	1,397,558	477,947	19,400	458,547
11	1,232,032	19,400	1,212,632	1,823,842	19,400	1,804,442	606,359	19,400	586,959
12	1,512,265	19,400	1,492,865	2,232,820	19,400	2,213,420	741,787	19,400	722,387
13	1,780,687	19,400	1,761,287	2,628,867	19,400	2,609,467	880,086	19,400	860,686
14	2,086,536	19,400	2,067,136	3,016,971	19,400	2,997,571	1,048,927	19,400	1,029,527
15	2,362,745	19,400	2,343,345	3,389,436	19,400	3,370,036	1,207,881	19,400	1,188,481
16	2,700,899	19,400	2,681,499	3,734,726	19,400	3,715,326	1,344,264	19,400	1,324,864
17	2,999,003	19,400	2,979,603	4,076,146	19,400	4,056,746	1,488,341	19,400	1,468,941
18	3,272,846	19,400	3,253,446	4,436,494	19,400	4,417,094	1,627,640	19,400	1,608,240
19	3,539,899	19,400	3,520,499	4,753,709	19,400	4,734,309	1,781,890	19,400	1,762,490
20	3,799,886	575,559	3,224,327	5,062,442	548,554	4,513,887	1,945,280	460,672	1,484,608

Table 4. Variables to estimate the present value of projected cash flows

Scenarios	*rf*	*RP*	*k*	*$AR / ha*	*AR$/ U$D*	*ha*	*Initial Investment [AR$]*
Favorable	11.00%	12.00%	23.00%	5,500	4.80	38	-1,003,200
Base	11.00%	16.00%	27.00%	5,000	5.10	38	-969,000
Unfavorable	11.00%	20.00%	31.00%	4,500	6.80	38	-1,162,800

Table 5. Net present value by scenario: Owner (a) and Investor (b)

Scenarios	Without Investment NPV [AR$]	With Investment NPV [AR$]
Favorable	1,881,583	878,383
Base	664,921	-304,079
Unfavorable	-168,405	-1,331,205
NPV average	792,700	-252,300

Table 6. Triangular fuzzy number of possible cases (a) Owner and (b) Investor

Parameters of the triangular fuzzy number	Owner(a) Case 2 [$AR]	Investor(b) Case 3 [$AR]
β	1,216,661	1,182,461
α	833,325	1,027,125
A	664,920	-304,079
$\beta + a$	1,881,583	878,382
$a - \alpha$	-168,405	-1,331,205

Table 7. ROV for Owner (a) and Investor (b)

ROV	Owner (a)	Investor (b)
$\lambda(+)$(positive area)	2,069,074	326,249
$\lambda(-)$(negative area)	17,016	112,941
$\lambda = \lambda(+)/(\lambda(+)+\lambda(-))$	0.991	0.742
E(A+)	729,956	107,066
ROV = E(A+) $*$ λ	724,002	79,533

$\lambda(+)$: positive area; $\lambda(-)$: negative area

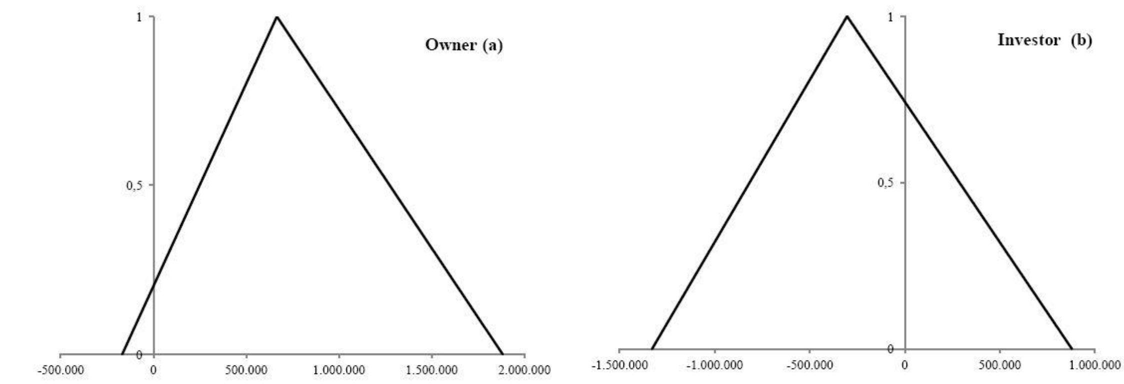

Figure 1. Triangular fuzzy number for the possible cases: owners and investor in ($AR)