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An approach to improve the performance of adaptive predictive control systems: theory, simulations and experiments

M. A. JORDÁN*†, M. S. BASUALDO‡ and D. ZUMOFFEN§

†Universidad Nacional del Sur (UNS), Dto. Ing. Eléctrica y Computadoras,
Av. Alem 1253, 8000 Bahía Blanca, Argentina
‡Inst. de Física Rosario (IFIR-CONICET), Grupo de Sistemas Inteligentes y Control, Rosario,
§Universidad Tecnológica Nacional (UTN), Fac. Regional Rosario.
Dto de Ing. Eléctrica, Rosario

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In this paper an approach to improve the overall performance of indirect adaptive control systems tailored for non-linear stable plants is presented. The approach involves a commutation of a linear time-varying robustness filter in the feedback path of the control loop in synchronization with an adaptive controller. The algorithm is framed in the celebrated IMC structure for predictive control systems. It can automatically suit to structural changes in the system as order and dead-time, and can deal with plants with zero dynamics. The convergence and stability of the system is analysed in details. It is shown through numeric simulations and experimentation on a heat exchanger with cooling system, that undesired transients due to abrupt and significative changes in the dynamics can be efficiently damped down by the developed control algorithm, achieving a high-quality performance in steady state.

1. Introduction

With the growing interests on high-performance and fault-tolerant non-linear control systems, more accurate and robust control algorithms in the presence of external disturbances and model uncertainties are required. It is well known that performance and robustness in control systems have some antagonistic character and a common way to counterbalance this reciprocal goals is through trade-offs between achievable performance and robustness against external disturbances and model uncertainties (Zhou and Doyle 1998).

In a more evolved sense, the concept of "robust performance" has been introduced in the literature of robust control by determining the worst performance over the specified uncertainty range of the model (Morari and Lee 1999). Other techniques that overcome the conflict between performance and robustness has been oriented to the design of two-loop structures as for instance the enhanced internal model control IMC (Zhu *et al.* 1995) and the well-known Youla controller parameterization (Zhou and Ren 2001).

A quite different point of view for controller design that ensures good trade-offs rests on the use of robust adaptive laws for direct or indirect controllers is aimed to obtain a progressively better performance up to a bound given by the metric of a residual set of the estimates (Ioannou and Sun 1995, Du and Nair 2002, Moon *et al.* 2003, Xu and Ioannou 2004). Several techniques help to make the control laws more robust with respect to structural uncertainties maintaining their global convergence. Other uncertainties such as dead time, structural variation of dominant dynamics, appearance of non-minimal phase or zero dynamics, cannot be handled easily in the unifying theory.

In all these approaches the emphasis of the designs are placed on asymptotic performance and stability. As in every robust control design, the nominal performance has to be traded off with robust stability and the

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*Corresponding author. Email: mjordan@criba.edu.ar

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improvement in performance achieved by the proposed schemes are limited by the size of the unmodelled dynamics.

It is a well-known fact that persistently-exciting conditions (PE) decisively improve the convergence rate of adaptive control systems, although their effect on the behaviour of the closed-loop adaptive system during the transient phase is not so obvious (Sun 1993). Traditional analysis based on boundness of all closed-loop signals does not exclude large oscillations of them and the possibility of bursting at steady state in the presence of small modelling errors. There exist general non-linear theories to quantify transient response such as those based on small gain arguments or passivity (Zang and Bitmead 1994). Even when they can provide guidelines to practical control design and analysis of both the steady-state and transient response, this appears possible at the moment only to have a local practical validity.

To compensate for the transient error caused by misidentification, the design should take not only the model into consideration, but also the information about the quality of this model. Some compensation mechanism has to be furnished with the certainty equivalence principle design to address the transient performance issue. Bearing this idea in mind, this paper presents a mechanism to attenuate the effect of great uncertainties in the dynamics in the transient stage achieving the ideal behaviour asymptotically .

Other related works in the past propose the use of multiple models, each one of which corresponds to a different environment in which the plant may have to operate. So the objective is to determine the most appropriate model at any instant, using a suitable performance criterion based on the identification error (Narendra and Balakrishnan 1997). Other techniques are more specific and employ, for instance, the design of robust non-linear damping terms to attenuate the effects of parameter uncertainties (Yang and Tateishi 2001): The direct use of the estimation error as control signal in a modified MRAC scheme to compensate the error resulting from the certainty equivalence design (Sun 1993); a parameter-dependent measure of the PE condition to give a guaranteed transient performance of tracking a smooth desired trajectory (Arteaga and Tang 2002); and finally, the mean square tracking error criterion and the L_∞ tracking error bound criterion in order to modify a standard MRAC that can have an arbitrarily improved nominal performance in the ideal case and in the presence of bounded input disturbances (Datta and Ioannou 1994).

In this paper we present a new approach to robust adaptive control that is not based on a modification of some underlying adaptive law. Rather, it is based on stability properties that motivate the construction of an adaptive filter in the feedback to redesign the controller self. The approach can be introduced better under the known framework of predictive controllers like the celebrated IMC-approach (García and Morari 1982). The scheme is constructed in a modular form composed of two stages. In the first stage the effect of large uncertainties is strongly attenuated, and in the second stage the achievement of ideal asymptotic performance is intended. The stages are automatically synchronized with commutation.

2. The internal model control approach

The philosophy of the predictive control was introduced at the end of the seventies by Richalet *et al.* (1978) and Cutler and Ramaker (1980). Early applications and theoretical basis were established later by García and Morari (1982) and Clarke *et al.* (1985a, b). Here the internal model control (IMC) approach is taken as basis for the next development.

Let us consider a system with single input, single output and linearizable dynamics about every operation point in the work region. The IMC controller design is based on the minimization of the following energy criterion applied at every step k

$$J(k)=\sum_{i=N_1}^{N_2}\alpha_i^2 e^2(k+i)+\sum_{i=0}^{N_u-1}\beta_i^2\bar{u}^2(k+i), \qquad (1)$$

where e is a tracking error between a desired trajectory y_r and the predicted system output $\hat{y}$ evaluated on a so-called prediction horizon via model, u and y are the past values of the system output and control action, respectively, and $\bar{u}$ is the future control action calculated over the so-called control horizon, see figure 1.

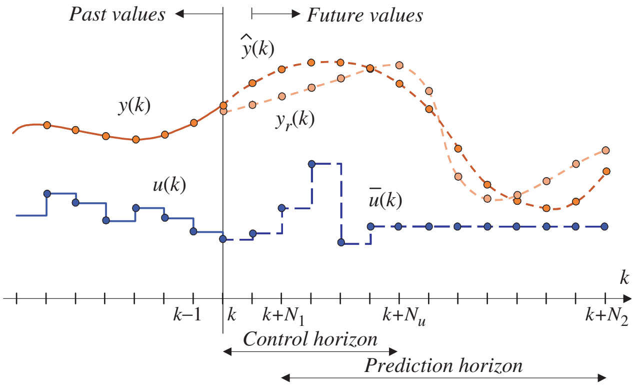

Figure 1. Predictive control strategy.

Equation (1) can be accompanied with restrictions on y and u.

Here N_u defines the length of the control horizon in steps while N_1 and N_2 are referred to as the lower and the upper limits of the prediction horizon in steps, respectively. Finally α_{ij} and β_{ij} are penalty coefficients of the energies of e and $\bar{u}$, respectively.

The future output trajectory is originally calculated by means of a FIR model (discrete impulse response: $\hat{g}(i)$, $i = 1, \ldots, N$) of the system. It is assumed that the original plant in Laplace transfer function $G(s)$ is approximated around a given operation point and contains a zero-order holder e^{-sT_0}/s for the sampling time T_0, with which the discrete transfer function $G(z^{-1})$ is built up. The sequence $\bar{u}$ of control actions is generally let to vary over the control horizon only, and thereafter maintained constant till the end of the prediction horizon.

The optimal control sequence $\bar{u}$ can be easily deduced for the unconstrained case by searching for the global minimum of (1) with respect to $\bar{u}$ over N_u. As the functional (1) is quadratic, the minimum can be analytically calculated as a linear optimization process without restrictions. So the output prediction at time k is defined as

$$\hat{y}(k+i/k) = \sum_{j=1+d}^{N} \hat{g}(j)u(k+i-j) + \hat{n}(k+i/k) \quad (2)$$

for i some positive integer and $\hat{n}$ being a perturbation signal predicted at k. Using (2) on the prediction horizon, the tracking error can be expressed in vector form as (Jordán 1992)

$$\mathbf{e}(k) = \mathbf{y}_r(k) - \hat{\mathbf{n}}(k) - \mathbf{T}_1\mathbf{G}\mathbf{T}_2\mathbf{u}(k) - \mathbf{T}_3\mathbf{S}\mathbf{T}_4\boldsymbol{\psi}(k), \quad (3)$$

with the signal vectors

$$\mathbf{y}_r(k) = [y_r(k+N_1), \ldots, y_r(k+N_2)]^T \quad (4)$$

$$\hat{\mathbf{n}}(k) = [y(k) - \hat{y}(k), \ldots, y(k) - \hat{y}(k)]^T \quad (5)$$

$$\mathbf{u}(k) = [\bar{u}(k), \ldots, \bar{u}(k+N_u-1]^T \quad (6)$$

$$\boldsymbol{\psi}(k) = [u(k-1), u(k-2), \ldots, u(k-N_1-N)]^T, \quad (7)$$

and the matrices

$$\mathbf{G} = \begin{bmatrix} \hat{g}(1) & 0 & \cdots & 0 & 0 \\ \hat{g}(2) & \hat{g}(1) & \ddots & \vdots & \vdots \\ \hat{g}(3) & \hat{g}(2) & \ddots & 0 & \vdots \\ \vdots & \hat{g}(3) & \ddots & \hat{g}(1) & 0 \\ \hat{g}(N) & \vdots & \ddots & \hat{g}(2) & \hat{g}(1) \\ 0 & \hat{g}(N) & \ddots & \hat{g}(3) & \hat{g}(1)+\hat{g}(2) \\ 0 & 0 & \ddots & \vdots & \hat{g}(1)+\hat{g}(2)+\hat{g}(3) \\ \vdots & \vdots & \ddots & \hat{g}(N) & \vdots \\ 0 & 0 & \cdots & 0 & \sum_{i=1}^{N}\hat{g}(i) \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} \hat{g}(2) & \hat{g}(3) & \hat{g}(4) & \cdots & \hat{g}(N) \\ \hat{g}(3) & \hat{g}(4) & \vdots & \iddots & 0 \\ \hat{g}(4) & \vdots & \hat{g}(N) & \iddots & 0 \\ \vdots & \hat{g}(N) & \iddots & \iddots & \vdots \\ \hat{g}(N) & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{A} = \mathrm{diag}\left[\alpha_{N_1}, \ldots, \alpha_{N_2}\right]$$

$$\mathbf{B} = \mathrm{diag}\left[\beta_1, \ldots, \beta_{N_u}\right]$$

$$\mathbf{T}_1 = [\,0 \mid \underset{\xleftrightarrow{\;N-N_1\cdot N_u\;}}{I \mid \overset{\overset{N-N_2\cdot N_u-1}{\longleftrightarrow}}{0}}\,], \qquad \mathbf{T}_2 = \overset{\overset{N_u}{\longleftrightarrow}}{\begin{bmatrix} 0 \\ \hline I \end{bmatrix}}, \qquad \mathbf{T}_3 = [\,\overset{\overset{N_2-N_1\cdot 1}{\longleftrightarrow}}{I} \mid 0\,], \qquad \mathbf{T}_4 = \overset{\overset{N-N_1}{\longleftrightarrow}}{\begin{bmatrix} 0 \\ \hline I \end{bmatrix}}, \quad (8)$$

where **G** and **S** are dynamic matrices depending on the plant weighting function, $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{T}_3$ and $\mathbf{T}_4$ are transform matrices. The dimensions of them are $(2N-1) \times N$ for **G**, $(N-1)^2$ for **S**, $(N_2 - N_1 + 1) \times (2N-1)$ for $\mathbf{T}_1$, $N \times N_u$ for $\mathbf{T}_2$, $(N_2 - N_1 + 1) \times (N-1)$ for $\mathbf{T}_3$, and $(N-1) \times (N-N_1)$ for $\mathbf{T}_4$. It is noticing in (4) that the perturbation signal is considered constant over the prediction horizon because it is the best estimation that one can achieve without using noise filtering.

So (1) can be put into vector form as

$$J(k) = \mathbf{e}^T(k)\mathbf{A}^2\mathbf{e}(k) + \bar{\mathbf{u}}^T(k)\mathbf{B}^2\bar{\mathbf{u}}(k), \quad (9)$$

where **A** and **B** are tuning matrices of the IMC algorithm with dimensions $(N_2 - N_1 + 1)^2$ for **A** and $(N_u)^2$ for **B**.

Thus the control action sequence can be inferred by calculating $\partial J/\partial \bar{u} = 0$ on (9) with (2). In this way the following equation arises in the discrete variable z

$$D(z^{-1})\,u(z) = z^{N_1}\,R(z)\,y_r(z) \;-\; R(1)\,[y(z) - \hat{y}(z)], \quad (10)$$

with the complex polynomials

$$D(z^{-1}) = 1 + d_1 z^{-1} + \cdots + d_{N-N_1} z^{-(N-N_1)} \quad (11)$$

$$R(z) = r_1 + r_2 z + \cdots + r_{N_2-N_1+1} z^{N_2-N_1} \quad (12)$$

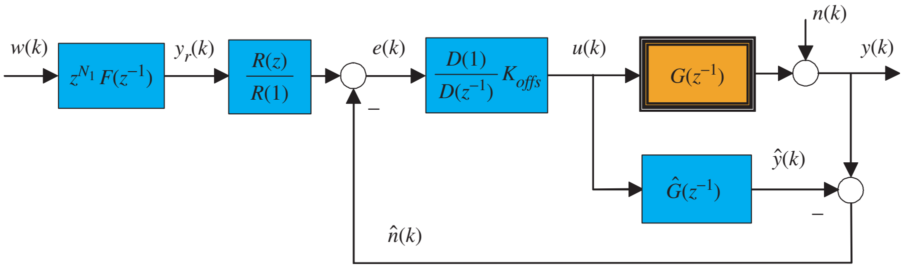

Figure 2. IMC control system.

and the coefficients d_is and r_is arranged in vectors

$$\begin{bmatrix} d_1 \\ \vdots \\ d_{N-N_1} \end{bmatrix}^T = \left[r_1, \ldots, r_{N_2-N_1+1}\right]\mathbf{T}_3\mathbf{S}\mathbf{T}_4 \qquad (13)$$

$$\begin{bmatrix} r_1 \\ \vdots \\ r_{N_2-N_1+1} \end{bmatrix}^T = [1, 0, \ldots, 0]\left[\mathbf{T}_2^T\mathbf{G}^T\mathbf{T}_1^T\mathbf{A}^2\mathbf{T}_1\mathbf{G}\mathbf{T}_2 + \mathbf{B}^2\right]^{-1} \times \mathbf{T}_2^T\mathbf{G}^T\mathbf{T}_1^T\mathbf{A}^2. \qquad (14)$$

The dynamic and transform matrices above are suitable for the design of optimal adaptive controllers with time-varying horizons which change the structure of $\mathbf{G}$ and $\mathbf{S}$ through the transform matrices $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{T}_3$ and $\mathbf{T}_4$ according to some optimality criterion such as for instance $\min_{N_1, N_2, N_u} J(k)$. Also the tuning matrices $\mathbf{A}$ and $\mathbf{B}$ can be parallel modified in their coefficients as well as in their structures using a similar criterion.

Equation (10) is quite appropriate for its implementation in the so-called parallel compensation technique suggested by Brosilow (1979) that leads to the IMC strategy of García and Morari (1982). The structure of the controller is shown in figure 2. Herein $\hat{G}$ is referred to as the FIR model of the plant that supports the design of the IMC controller expressed in a truncated impulse response transformation

$$\hat{G}\left(z^{-1}\right) = \hat{g}(0) + \hat{g}(1)\,z^{-1} + \cdots + \hat{g}(N)\,z^{-N}. \qquad (15)$$

The variable $\hat{y}(k)$ is a prediction of $y(k)$ based on $\hat{G}$, while $w(k)$ and $n(k)$ are the set point and an external perturbation, respectively. It can be deduced from (10) and (13)–(14) that the control law $\bar{u}(k)$ requires static compensation for achieving $e(\infty) = 0$ only when the β_is in the functional J are greater than zero (Jordán 1991). The gain compensation factor referred to as K_{offs} is given by

$$K_{offs} = \frac{1}{\sum_{i=1}^{N} \hat{g}(i)}, \qquad i = 1, \ldots, N. \qquad (16)$$

The block $z^{N_1}F(z^{-1})$ in figure 2 is a prefilter for smoothing $w(k)$ so as to generate a flat reference trajectory for $y(k)$ over the prediction horizon. A simple algorithm can be used for this purpose to generate $y_r(k)$ according to

$$y_r(k) = \alpha_r y_r(k-1) + (1-\alpha_r)\,w(k+N_1), \qquad (17)$$

where α_r is a smoothing coefficient between 0 and 1, which is selected close to the limit 0 for conferring the closed loop dynamics a rapid response of and vice versa for the limit 1.

In contrast to the block $D(1)/D(z^{-1})$, which describes a low-pass filter, the block $R(z)/R(1)$ has a non-causal nature and acts on y_r according to the predictive strategy carried out on known disturbances. In applying this prefiltering on $w(k)$, the performance of the control loop can be suitably modified on-line without affecting the stability.

One important particularity in the implementation of the controller in the figure 2 is that usually only the first element in the calculated sequence $\bar{u}(k+i)$ is applied at k, and afterward the calculation is started anew in the next step $k+1$.

An important feature of the IMC structure is that by coincidence of the process $G(z^{-1})$ with the model $\hat{G}(z^{-1})$, the closed loop stability is determined only by the zeros of $D(z^{-1})$. The locations of these depend on the tuning parameters $\{N_1, N_2, N_u, \alpha_i, \beta_i\}$. Suitable selections of them for robustness and stabilization rest on several theorems (García and Morari 1982, Clarke and Mohtadi 1985a, b, Jordán 1992).

In general, small values of N_u and N_2, with $N_u < (N_2 - N_1 + 1)$ give high performance closed loops at low computational costs, but the controller may be sensitive to model errors. By increasing the distance between N_u and $N_2 - N_1 + 1$ with $(N_u < N_2 - N_1 + 1)$, or increasing the penalty coefficients β_i on the future control action, the closed-loop response becomes sluggish but the robustness increases. The choice $N_1 = d + 1$ is a typical selection that provides a way to advance the set point so many steps ahead as the pure delay of the plant. To attain this goal, the dead time d of G has to be identified from the series $\hat{g}(i)$.

A quite similar structure, known as the dynamic matrix controller (DMC) control (Cutler and Ramaker 1980), arises when the differences of the control variables are taken in J rather than their absolute values. By this means, the controller introduces an integral behaviour and no static compensation is necessary. In this case (10)–(14), with $\mathbf{G}$, $\mathbf{S}$, $\mathbf{A}$, $\mathbf{T}_1$, $\mathbf{T}_2$, $\mathbf{T}_3$, $\mathbf{T}_4$ and $\boldsymbol{\psi}$ remain the same, only the matrix $\mathbf{B}$ changes simply its structure to (Jordán 1992)

$$\mathbf{B} = \begin{bmatrix} \beta_1 & 0 & \cdots & & 0 \\ -\beta_2 & \beta_2 & 0 & & \vdots \\ 0 & -\beta_3 & \beta_3 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \beta_{N_u} & \beta_{N_u} \end{bmatrix}.$$

In this way an adaptive controller can easily change the control strategy within the same framework.

3. The adaptive control approach

A quite effective way to provide autotuning of the controller or to adapt for possible changes of the system dynamics in the system operation is provided by indirect adaptive control methods, in which a model of the system is explicitly identified and permanently updated. The efficiency of any adaptive method is mainly judged by the ability of the controller to maintain the performance optimal and additionally to provide the best model structure with a set of parameters that fit the system dynamics optimally by sudden changes of the operation points of the plant or of coupled dynamics.

In this paper optimal, numerically robust estimation methods based on the factorization algorithms of the error covariance matrix are proposed for providing time-variant non-parametric models of the weighting coefficients $\hat{g}(i)$. Thus the parameter vector to be estimated is arranged as

$$\hat{\boldsymbol{\theta}} = [\hat{g}(1), \hat{g}(2), \ldots, \hat{g}(N)]^T, \tag{18}$$

where $\hat{g}(0)$ is usually zero for processes. For updating $\boldsymbol{\theta}(k)$, following recursive algorithm known as the UD factorization (Biermann 1977) is employed

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{U}(k)\mathbf{D}(k)\mathbf{U}^T(k)\,\boldsymbol{\psi}(k)\,\tilde{e}(k) \tag{19}$$

$$\tilde{e}(k) = y(k) - \boldsymbol{\psi}^T(k)\,\hat{\boldsymbol{\theta}}(k-1) \tag{20}$$

$$\mathbf{U}(k) = \mathbf{f}_{\mathrm{U}}(\mathbf{U}(k-1), \boldsymbol{\psi}(k), \lambda(k)) \tag{21}$$

$$\mathbf{D}(k) = \mathbf{f}_{\mathrm{D}}(\mathbf{D}(k-1), \boldsymbol{\psi}(k), \lambda(k)), \tag{22}$$

where λ is a forgetting factor, $\mathbf{U}$ and $\mathbf{D}$ are matrix factors of the error covariance matrix $\mathbf{P} = \mathbf{U}\mathbf{D}\mathbf{U}^T$, $\boldsymbol{\psi}$ is the regressor, $\tilde{e}$ the *a priori* estimation error and $\mathbf{f}_{\mathrm{U}}$ and $\mathbf{f}_{\mathrm{D}}$ are recursive formulae that will recursively provide $\mathbf{U}$ and $\mathbf{D}$, respectively (see for instance Isermann *et al.* 1992).

The regressor is conformed as

$$\boldsymbol{\psi}(k) = [u(k-1), \ldots, u(k-N)]^T, \tag{23}$$

where $u(k-i)$ are the N past values of the control action.

The initial conditions are set properly as (Jordán 1991, 1992)

$$\mathbf{D}(0) = \mathrm{diag}\left[\frac{N+1}{N}, \frac{N}{N-1}, \ldots, \frac{3}{2}, 2\right] \tag{24}$$

$$\mathbf{U}(0) = \begin{bmatrix} 1 & -\dfrac{N-1}{N} & 0 & \cdots & 0 \\ 0 & 1 & -\dfrac{N-2}{N-1} & \ddots & \vdots \\ 0 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -\dfrac{1}{2} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

$$\hat{\boldsymbol{\theta}}(0) = \frac{1}{\Delta u}[(h(1) - h(0)), \ldots, (h(N) - h(N-1))]^T \tag{26}$$

$$\boldsymbol{\psi}(0) = [u(0), u(0), \ldots, u(0)]^T, \tag{27}$$

where $h(i)$ is the measured discrete-time unit step response of y in open loop to a stepwise change Δu of the input from one initial operation point in steady-state to another end point in open-loop operation. Thereby it is supposed that h has approximately settled after N steps. If there are no possibility to provide $h(i)$ by measuring, one selects $\hat{\boldsymbol{\theta}}(0) = \mathbf{0}$, $\mathbf{D}(0) = \delta\mathbf{I}$ and $\mathbf{U}(0) = \mathbf{I}$, where $\mathbf{I}$ is the unit matrix, $\delta = 10^2 \div 10^4/\sigma_u^2$ and $\sigma_u^2 = \boldsymbol{\psi}^T(0)\boldsymbol{\psi}(0)$ is an estimate of the variance of the input $u(k)$.

Once $\hat{\boldsymbol{\theta}}(k)$ is estimated and the $\hat{g}(i)$s are available at every k, the actual control action is calculated

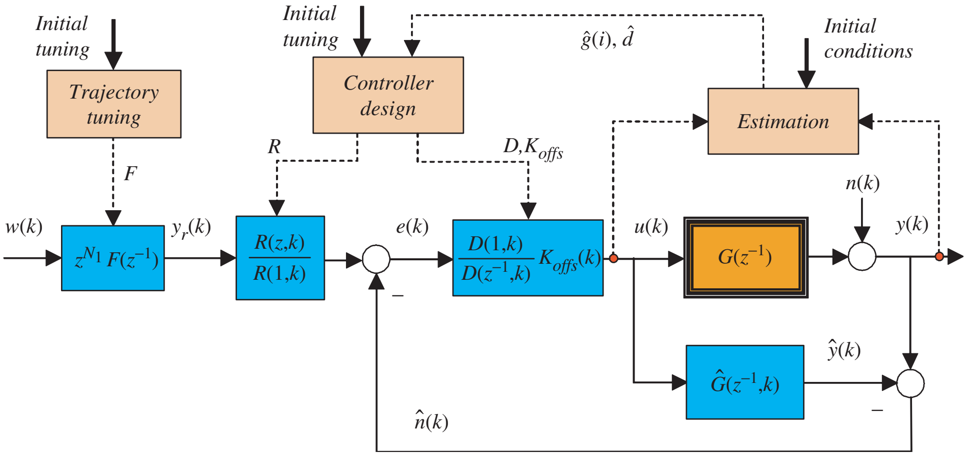

Figure 3. Adaptive predictive control in the IMC structure.

according to (10) as

$$u(k) = \sum_{i=1}^{N-N_1} d_i(k)\, u(k-i) + \sum_{j=1}^{N_2} r_{j-N_1+1}(k) \times [y_r(k+j) - [y(k) - \hat{y}(k)]], \tag{28}$$

where the $d_i(k)$s and $r_j(k)$s are the coefficients in (13) and (14), respectively. So the on-line predicted output is

$$\hat{y}(k) = \boldsymbol{\psi}^T(k)\, \hat{\boldsymbol{\theta}}(k). \tag{29}$$

The adaptive mechanism based on the Certainty Equivalence Principle is depicted in figure 3. It is composed of blocks representing the estimator and the controller design including the reference trajectory generation.

3.1 *Guidelines for tuning parameters*

It is well known that adaptive control systems suffer from long-term instability when the manipulated variable $u(k)$ is not rich enough in order to ensure a good persistent excited regressor $\boldsymbol{\psi}(k)$ in the space $\Re^N$.

In the absence of external perturbations $w(k)$ and/or $n(k)$, the excitation begins to decay and the covariance matrix will not tend to zero asymptotically at all. One way to supervise this misbehaviour is to use indicators that can generate symptoms in advance to stop on time the estimation. For instance, the evolution of the eigenvalues of $\mathbf{U}(k)\mathbf{D}(k)\mathbf{U}^T(k)$ or $\mathbf{D}(k)$ is found suitable to detect a future degradation of the estimates $\hat{\boldsymbol{\theta}}(k)$.

Another suitable indicator is the real variable (Kofahl 1992)

$$z_N(k) = \frac{\lambda(k)}{\lambda(k) + \boldsymbol{\psi}^T(k)\,\mathbf{U}(k-1)\mathbf{D}(k-1)\mathbf{U}^T(k-1)\,\boldsymbol{\psi}(k)}, \tag{30}$$

which fulfills $0 \le z_N \le 1$ and is small close to 0 by a good excited system, and is large next to 1 by a poorly excited system. In the case where clearly $z_N \gtrsim 0$ the estimation laws (19)–(22) can be applied successfully. When z_N ascends over a certain confidence bound, then the estimation is switched off.

For optimal update of the controller, a continuous search for suitable tuning of the prediction horizon N_2 is suggested. This can be increased stepwise starting from $N_u + N_1$ upwards, and subject to the constraint $N_u < N_2 - N_1 + 1$.

It is worth noticing that the non-parametric model (15) can embrace a great family of stable plants ranging from well-behaved systems up to systems with non-minimal phase, under-damping, dead time and high order. The only condition is that the settling time is not too large in terms of N, say, for instance, not larger than 50, so that the convergence of the estimates is possible after an acceptable time. When the system dynamics change, the non-parametric model can adapt, automatically, for parameters and structure as well, e.g., order m and dead time d. As the horizon extreme N_1 is usually set in $N_1 = 1 + d$, the dead time d has to be indirectly estimated from the $g(i)$s. For instance, by means of

$$\hat{d} = \min i_0 \in \Im^+ / |g(i_0 + 1)| > \xi, \tag{31}$$

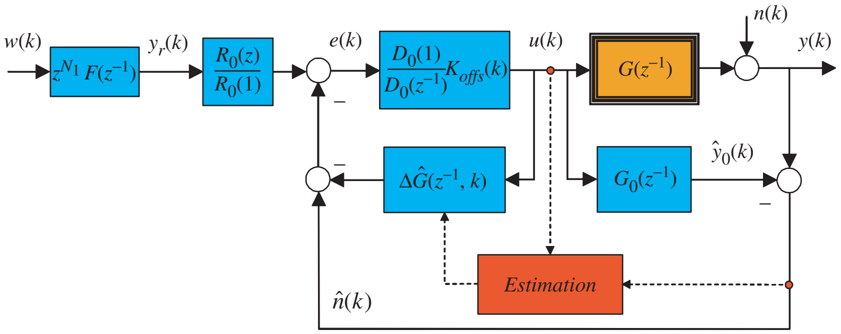

Figure 4. Adaptive robustness filter in the IMC-structure.

where ξ is a small bound over which it can be considered that the system output does react to a stimulus.

4. Adaptive robustness-filter

4.1 *The approach*

In the case of a process-model mismatch $\Delta\hat{G} = G - \hat{G} \neq 0$, the parallel compensation structure provides a direct way to achieve robustness of the closed loop by inserting a filter in the feedback path. For instance, a simple exponential filter is used in the model algorithmic controller MAC by Richalet *et al.* (1978). Later, García and Morari (1985), have similarly suggested a low-pass filtering in the feedback to improve the overall performance. By adjusting a sufficiently large time constant in the filter, robustness can mostly be enhanced.

A new method consisting in identifying the model mismatch and inserting it as a negative feedback in the IMC structure is presented in this paper and depicted in figure 4 (cf. figure 3). The basic idea is to develop an adaptive modification in the fixed control feedback that leads to a controller, about which it can be ensured in advance will be asymptotically stabilizing without restrictions. Ideally, the objective would be to ensure the regional stability in an invariant that contains this asymptotic convergent point.

The heuristics is described in the following (originally in Jordán (1991)). Consider the IMC control system in figure 2. It is assumed that there exists a stabilizing controller set $(D_0(z^{-1}), R_0(z), K_{offs})$ for the underlaying plant with an exact model G_0. So $D_0(z^{-1}) = 0$ has stable zeros. If now the system changes its dynamics with a new model $G(z^{-1})$, the stability is determined by

$$D_0(z^{-1}) + q_0\big[G(z^{-1}) - G_0(z^{-1})\big] = 0, \qquad (32)$$

with $q_0 = D_0(1)/G_0(1)$. Implementing now an adaptive law as indicated in figure 4 for providing the estimation of the model mismatch $\Delta\hat{G}$ with the property

$$\lim_{k\to\infty} \Delta\hat{G}(z^{-1}, k) = G\big(z^{-1}\big) - G_0\big(z^{-1}\big), \qquad (33)$$

the stability of the new loop is asymptotically defined by the zeros of $D_0(z^{-1})$ which are originally stable. Thus the control loop remains stable at the convergence point no matter how large is the difference $(G - G_0)$ as long as G describes a new stable system.

Another feature of the robustness filter approach is that, under convergence, the new control action fulfils

$$\lim_{k\to\infty} u(k) = c\, u_0(k), \qquad (34)$$

where $c = 1/G(1)$ and $u_0(k)$ is the sequence generated by the previous controller designed for G_0 subject to the same excitation $w(k)$.

The adaptive filter in the feedback is

$$\Delta\hat{G}(z^{-1}, k) = \Delta\hat{g}(1, k)\, z^{-1} + \Delta\hat{g}(2, k)\, z^{-2} + \cdots + \Delta\hat{g}(N, k)\, z^{-N}, \qquad (35)$$

in which the coefficients $\Delta\hat{g}(i)$s are provided by estimation in

$$\Delta\hat{\boldsymbol{\theta}}(k) = [\Delta\hat{g}(1, k), \Delta\hat{g}(2, k), \ldots, \Delta\hat{g}(N, k)]^T. \qquad (36)$$

In this occasion also the UD-factorization described previously in (19), (21) and (22) can be used with the *a priori* error defined as (cf. figure 4)

$$\tilde{e}(k) = (y(k) - \hat{y}_0(k)) - \boldsymbol{\psi}^T(k)\, \Delta\hat{\boldsymbol{\theta}}(k-1). \qquad (37)$$

The initial conditions are set in $\hat{\boldsymbol{\theta}}(0) = \mathbf{0}$, $\mathbf{D}(0) = \delta\mathbf{I}$, $\delta = 10^2/(\boldsymbol{\psi}^T(0)\boldsymbol{\psi}(0)) \div 10^4/(\boldsymbol{\psi}^T(0)\boldsymbol{\psi}(0))$ and $\mathbf{U}(0) = \mathbf{I}$. Finally, the regressor is the same as in (23). The proof of convergence is in the next section.

In order to eliminate the emerging offset in the output, also the gain of the controller has to be corrected adaptively as

$$\hat{K}_{offs}(k) = \frac{1}{G_0(1) + \Delta\hat{G}(1,k)}. \qquad (38)$$

The asymptotic performance of the control loop is different in steady state since D_0 was designed for G_0 but not for G. However, the performance drop can be attenuated by the robustness conferred by suitable control settings of a commissioning phase. Another possibility is to carry out a new optimization of the horizons via N_2 and N_u for the latest dynamics.

It is pointed out that one of the main advantages of the robustness filter is its ability to deal with strong and abrupt dynamics transitions and to damp down undesired transient behaviours. In addition, no design parameters are needed as in the case of adaptive control.

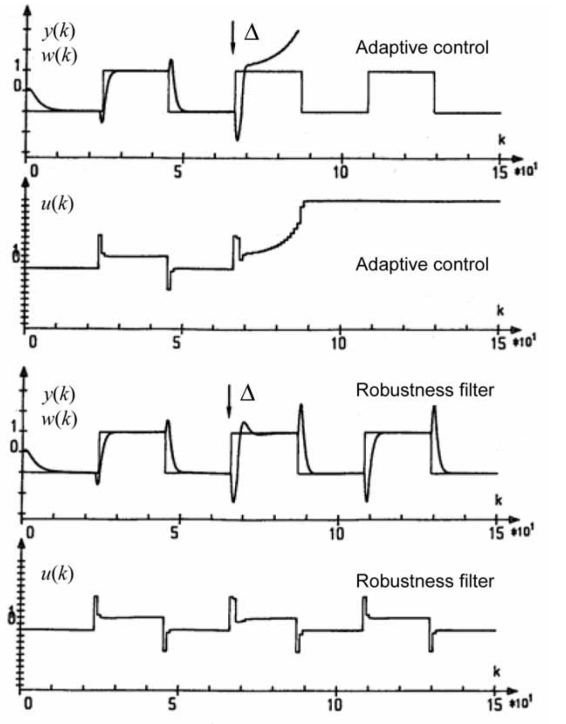

Figure 5. Comparison of the transient performance between adaptive control and robustness.

4.2 *Feature comparison by simulations*

The proposed adaptive algorithm is simulated for different dynamics with changing both parameters and model structure. In order to highlight the improvements and benefits of the proposed approach with respect to adaptive control schemes, a comparative study of the behaviour under the same perturbations is carried out for adaptive control and adaptive filter separately. For the numerical simulations, a rich excitation of sufficient order is applied through $w(k)$ such that no supervision for the adaptive control is needed. In both cases an initial FIR model is captured by directly measuring the system step response in open loop from $k = 0$ up to $k = 20$, and immediately after that, a controller is designed and the loop is closed.

In figure 5 a non-minimal-phase system with an original Laplace transfer function $G(s) = (1 - 4s)/(1 + 4s)(1 + 10s)$, sampling time $T_0 = 4$ s, $N = 20$, and a forgetting factor equal to $\lambda = 0.9$ is simulated. The unstable zero of the system $s_0 = 1/4$ is displaced at $k = 65$ to $s_0 = 1/7.5$, accentuating in this way the inverse time response of the system in open loop. The previous performance of the adaptive controller is highly satisfactory in the starting phase in comparison to the system response in open loop. Shortly after the change, the adaptive control is not able to stabilize the new dynamics any longer with the particular control settings and then the signals evolve divergently. It is also seen clearly that the control action increases progressively up to the saturation value.

On the contrary, the control loop with the robustness filter can maintain the stability satisfactory. Also the control performance could be preserved in the transient and in the steady state. It is remarkable that the evolution of the control action is asymptotically the same as that generated previous to the change since the process gain has remained constant (see equation (34)). In contrast to the case of adaptive control, where it is no *a priori* certainty whether the final controller will be stabilizing or not, in this case of robustness filter, the asymptotic stability is guaranteed in advance by design.

Next a second simulation is drawn up. It concerns a third-order system with an original Laplace transfer function $G(s) = (1 + 2s)/(1 + 10s)(1 + 7s)(1 + 3s)$, sampling time $T_0 = 4$ s, $N = 20$, and $\lambda = 0.9$ is simulated and depicted in figure 6. At $k = 65$ a dead time equal to 16 s, it is $d = 4$, is suddenly incorporated in the dynamics. The previous evolution of the adaptive controller is satisfactory with an all-round high performance. After introducing the delay d in the dynamics, the adaptive control reacts with large amplitudes reaching the saturation zones and causing a large oscillation for 20 seconds. On the other hand the intervention of the approach with robustness filter is effective to avoid

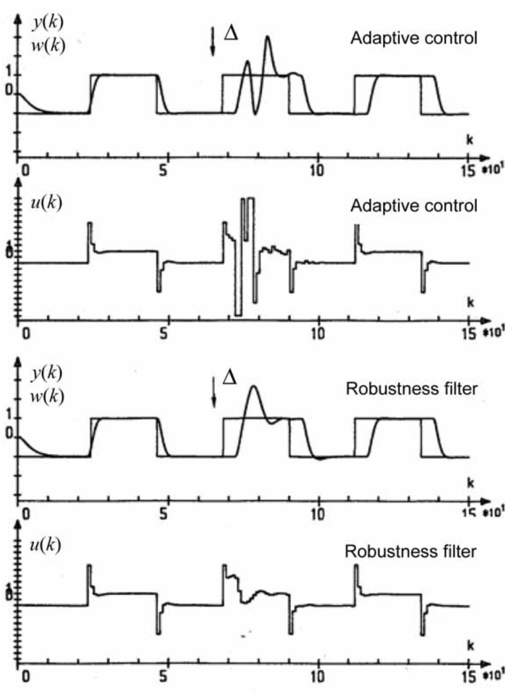

Figure 6. Comparison of the transient performance between adaptive control and robustness filter for a dynamic change in the dead time.

large oscillations and achieve a control performance that is identical to that achieved with the adaptive control in steady state. The evolution of the control action is much better than the adaptive control and remains smooth and bounded during the dynamic transition.

5. Convergence analysis

At this point, the analysis of trajectories of the estimates of the robustness-filter approach in the parameter space and in the state space of the regressor is presented. Consider the control system of the figure 4 with $w(k) \equiv 0$ and the original controller $\big(D_0, R_0, K_{offs_0}\big)$ adapted for G_0. Assume that at time $k = 0$ an arbitrary abrupt change of the dynamics with model G takes place and hereafter the robustness filter (35) is switched on. The estimation algorithm is subject to (19), (21), (22) and (37) with the required initial conditions given earlier.

Let the system output is expressed by

$$y(k) - \hat{y}_0(k) = \boldsymbol{\psi}^T(k)\,\Delta\hat{\boldsymbol{\theta}}(k-1) + \tilde{e}(k), \tag{39}$$

with

$$\tilde{e}(k) = \boldsymbol{\psi}^T(k)\Big(\big(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0\big) - \Delta\hat{\boldsymbol{\theta}}(k-1)\Big) + n(k) + e_N(k) \tag{40}$$

$$e_N(k) = \sum_{i=1+N}^{\infty} g(i)\,u(k-i), \tag{41}$$

where $\tilde{e}$ is the *a priori* error and e_N the truncation error of the weighting coefficients. The vector $\boldsymbol{\theta}$ contains the true weighting coefficients of G from 1 up to N, and $\hat{\boldsymbol{\theta}}_0$ the weighting coefficients of the model $\hat{G}_0$. The control action in figure 4 can be put into the form

$$u(k) = -\sum_{i=1}^{N}\big(d_{0_i} + q_0(k)\,\Delta\hat{g}(i,k)\big)\,u(k-i) \\ + q_0(k)(y(k) - \hat{y}_0(k)), \tag{42}$$

with $q_0(k) = D_0(1)\hat{K}_{offs}(k) = D_0(1)/(G_0(1) + \Delta\hat{G}(1,k))$, $\Delta\hat{g}(i,k)$ the estimate $\Delta\hat{G}(z^{-1},k)$ at k and d_{0_i} the coefficient of $D_0(z^{-1})$. Here it was supposed the most general case when $N \geq N_2 - N_1 + 1$.

5.1 *Convergence of the estimator*

Let the candidate Lyapunov function be

$$V(k) = \frac{1}{2}\breve{\boldsymbol{\theta}}^T(k)\big[\mathbf{U}(k)\mathbf{D}(k)\mathbf{U}^T(k)\big]^{-1}\breve{\boldsymbol{\theta}}(k), \tag{43}$$

with

$$\breve{\boldsymbol{\theta}} = \big(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0\big) - \Delta\hat{\boldsymbol{\theta}}(k) \tag{44}$$

$$\big[\mathbf{U}(k)\mathbf{D}(k)\mathbf{U}^T(k)\big]^{-1} = \sum_{i=0}^{k}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i). \tag{45}$$

It can be deduced that (Isermann *et al.* 1992)

$$V(k) = \lambda(k)V(k-1) - \mu(k)\,(\tilde{e}(k) - n(k) - e_N(k))^2 \tag{46}$$

$$\mu(k) = \frac{1}{\lambda(k) + \boldsymbol{\psi}^T(k)\Big[\sum_{i=0}^{k}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i)\Big]^{-1}\boldsymbol{\psi}(k)} \tag{47}$$

$$\sum_{i=0}^{k-1}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i) = \mu(k)\sum_{i=0}^{k}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i), \tag{48}$$

where $0 < \lambda(k) \leq 1$. For $\boldsymbol{\psi}(k)$ bounded and for

$$\left[\sum_{k=0}^{N}\boldsymbol{\psi}(k)\boldsymbol{\psi}^T(k)\right]^{-1} \geq 0, \quad \text{for all } k, \tag{49}$$

i.e., the matrix be uniformly definite positive for all k, it is valid

$$V(k-1) - V(k) \geq \mu(k)\,(\tilde{e}(k) - n(k) - e_N(k))^2. \tag{50}$$

As the right member of (50) is positive and bounded for all k, so the trajectory $\boldsymbol{\theta}(k)$ decreases permanently. Moreover

$$\begin{aligned}\tilde{\boldsymbol{\theta}}(k) - \tilde{\boldsymbol{\theta}}(k-1) = {}& \left[\mathbf{U}(k)\mathbf{D}(k)\mathbf{U}^T(k)\right]^{-1}\boldsymbol{\psi}(k) \\ & \times (\tilde{e}(k) - n(k) - e_N(k))\end{aligned} \tag{51}$$

tends to zero for $k \to \infty$ with (49) being accomplished uniformly in time.

Now necessary and sufficient conditions for the regressor $\boldsymbol{\psi}(k)$ in order to fulfil (49) are found. Equation (49) is equivalent to demand that $\boldsymbol{\psi}(k)$ spans uniformly a space of dimension N. In order to prove this statement, let the first element in $\boldsymbol{\psi}(k)$ be rewritten in terms of the other elements. So

$$\begin{aligned}
u(k-1) = {}& \left[-d_{0_1} - q_0(k-1)\,\Delta\hat{g}(1,k-1)\right]u(k-2) && (52)\\
& + \left[-d_{0_2} - q_0(k-1)\,\Delta\hat{g}(2,k-1)\right]u(k-3) + \cdots + && (53)\\
& + \left[-d_{0_{N_2-N_1+1}} - q_0(k{-}1)\,\Delta\hat{g}(N_2 - N_1 + 1, k-1)\right]u(k - N_2 + N_1 - 2) && (54)\\
& + [-q_0(k-1)\,\Delta\hat{g}(N_2 - N_1 + 2, k-1)]\,u(k - N_2 + N_1 - 3) + \cdots + && (55)\\
& + [-q_0(k-1)\,\Delta\hat{g}(N-1, k-1)]\,u(k-N) && (56)\\
& ---------------------------------- && \\
& + [-q_0(k-1)\,\Delta\hat{g}(N, k-1)]\,u(k-N-1) && (57)\\
& - q_0(k-1)\sum_{j=N+1}^{\infty} g(j)\,u(k-j-1) && (58)\\
& + q_0(k-1)\big(\boldsymbol{\psi}^T(k)\boldsymbol{\theta} + n(k-1) - \hat{y}_0(k-1)\big), && (59)
\end{aligned}$$

with the $\Delta\hat{g}(i,k-1)$s and $q_0(k-1)$ depending on past information $u(k-2), u(k-3), \ldots$. It can be seen that $u(k-1)$ stays in relation with all elements in $\boldsymbol{\psi}(k)$, but additionally on elements that belong to the past information, for instance, with $u(k-i)$, $i > N$ and $n(k-1)$ as seen in the terms (57)–(59). Also the factors $\Delta\hat{g}(i,k-1)$ and $q_0(k-1)$ depend on the past history of the input at N steps behind $(k-1)$.

Therefore $u(k-1)$ is not a linear combination of the other terms in $\boldsymbol{\psi}(k)$. The same is deduced for the other elements $u(k-i)$ in $\boldsymbol{\psi}$. Hence $\boldsymbol{\psi}(k)$ is said persistently excited in the period $[0,k]$ and can uniformly span a space of dimension N only if the input is rich at least of order N.

Finally this guarantees the existence of two real-valued positive constants such that it is valid

$$\kappa_0\mathbf{I} \geq \left[\sum_{i=k}^{k+T}\boldsymbol{\psi}(i)\psi^T(i)\right]^{-1} \geq \kappa_1\mathbf{I} \tag{60}$$

uniformly in k for every $T > 1$. Particularly, the constant κ_1 is known as the level of persistency of excitation (Ioannou and Sun 1995). Thus, the larger κ_1 is, the faster the convergence of the estimates will take place. Essentially, the family of inputs $u(k)$ that are not rich enough of order N is restrained to periodic signal with $n < N$ harmonics, including constant signals given on operation points.

It can be seen from (47) and (48) that for $\lambda(k) < 1$ the matrix $[\sum_{k=0}^{N}\boldsymbol{\psi}(k)\boldsymbol{\psi}^T(k)]^{-1}$ does not go to zero asymptotically and consequently the parameter error (51) is bounded but non-zero at the limit.

So for the matrix $[\sum_{k=0}^{N}\boldsymbol{\psi}(k)\boldsymbol{\psi}^T(k)]^{-1}$ being uniformly definite positive for all k, it follows

$$\lim_{k\to\infty}\mu(k) = \begin{cases}\mu_0 < 1, & \text{for } \lim\limits_{k\to\infty}\lambda(k) = \lambda_0 \leq 1\\ 1, & \text{for } \lambda(k) \equiv 1 \text{ for all } k\end{cases} \tag{61}$$

$$\lim_{k\to\infty}\left[\sum_{i=0}^{k}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i)\right]^{-1} = 0. \tag{62}$$

Thus, for a some time at infinity referred to as k_∞, it is valid for the steady state

$$\lim_{k\to\infty}\tilde{e}(k) = \boldsymbol{\psi}^T(k_\infty)\Big(\big(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0\big) - \Delta\hat{\boldsymbol{\theta}}(\infty)\Big) + n(k_\infty) + e_N(k_\infty), \tag{63}$$

which states that the parameter error $((\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0)-\Delta\hat{\boldsymbol{\theta}}(\infty))$ is bounded with an order of magnitude equal to those of the perturbation and the truncation error. Clearly, from (61) and (62) one achieves for the Cauchy series (51) that

$$\lim_{k\to\infty}\left(\breve{\boldsymbol{\theta}}(k)-\breve{\boldsymbol{\theta}}(k-1)\right)=\mathbf{0}, \tag{64}$$

i.e., the parameter trajectory vector $\Delta\hat{\boldsymbol{\theta}}(k)$ tends to a fixed convergence point $\Delta\hat{\boldsymbol{\theta}}(\infty)$ which stays in a residual set k_∞, whose measure in steady state depends on the relation

$$\left\|\boldsymbol{\psi}^T(k_\infty)\left(\left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right)-\Delta\hat{\boldsymbol{\theta}}(\infty)\right)\right\| \le |n(k_\infty)|+|e_N(k_\infty)| \tag{65}$$

for $n(k)$ and $u(k)$ being stationary signals. The truncation error e_N can be made as small as wished by increasing N or T_0 as large as needed, so that, in general, the measure of the residual set is only proportional to the noise-input rate.

The conditions for exact convergence, i.e.,

$$\lim_{k\to\infty}\left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right)-\Delta\hat{\boldsymbol{\theta}}(k)=\mathbf{0} \tag{66}$$

are imposed by the statistic relation $E\{\boldsymbol{\psi}(k)\breve{e}(k)\}=0$ (see for instance, Isermann *et al.* (1992)), where $E\{\cdot\}$ is the expectative operator. Equivalently

$$E\{u(k-i)\tilde{e}(k)\}=0, \quad \text{for } i=1,\ldots,N. \tag{67}$$

From (52)–(59), (40) and the related analysis afterwards, it is deduced that for $n(k)$ being white noise and $e_N(k)$ zero, the convergence in the sense of (66) is achieved. Besides, from (40) and (69) and $\lambda=1$, it is valid in steady state at time k_∞

$$\tilde{e}(k_\infty)=n(k_\infty). \tag{68}$$

It can be seen that the fulfillment of the whiteness condition is often in many discrete-time systems. Otherwise, a bias proportional to $|n(k_\infty)+e_N(k_\infty)|$ will be present in the estimates. Moreover, an additional advantage of FIR models against rational-transfer-function models is pointed out, in which the first class is able to provide a biasfree convergence in the presence of white noise, while the later ones will require a quit specific noise filter to accomplish this feature.

5.2 *Convergence of the closed loop*

In this part, the stability analysis of the control system with time-varying feedback with the robustness filter is carried out. Towards this goal, let the basic structure of the robustness filter system in figure 4 be equivalently redesigned to conform to a new structure as shown in figure 7. This is constituted by the known asymptotically stable forward linear block $D_0(1)/D_0(z^{-1})$ and a time-varying non-linear feedback block. Besides, an external disturbance is considered, composed of the noise signal n and the FIR truncation error e_N, both modulated by the time-varying gain $\hat{K}_{offs}(k)$ in (38). No external set point signal other than $n(k)$ is taken into account to provide excitation of the loop. Finally, the analysis is performed with forgetting factor $\lambda(k)\equiv 1$.

The signal in the forward path in figure 7 involves the so-called *a posteriori* error defined as

$$\breve{e}(k)=\boldsymbol{\psi}^T(k)\left(\left(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}}_0\right)-\Delta\hat{\boldsymbol{\theta}}(k)\right)+n(k)+e_N(k), \tag{69}$$

which satisfies the well-known relation with the *a priori* error (Isermann *et al.* 1992)

$$\breve{e}(k)=\lambda(k)\,\mu(k)\,\tilde{e}(k). \tag{70}$$

The following convergence proof is based on the Hyperstability Theory. The condition for a control system to be hyperstable is the so-called Popov condition (Narendra *et al.* 1985)

$$\sum_{k=k_1}^{k_2} u(k)\,\varepsilon(k)\ge -\eta_0^2 \tag{71}$$

for every time points k_1, k_2 with $k_2>k_1\ge 0$ and η_0 a real constant. According to the figure 7, it is globally asymptotically stable.

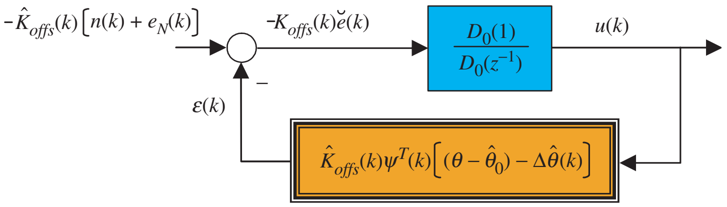

Figure 7. Robustness filter system redesigned as a non-linear time-varying feedback system.

Let us divide the analysis into three cases. First assume the case in which $n(k)$ and $e_N(k)$ are null and negligible, respectively, then, the cases with the presence of external perturbations are considered.

5.2.1 Case: $n(k) \equiv 0$ and $e_N(k)$ negligible.

Consider initial conditions $\Delta\hat{\boldsymbol{\theta}}(0), \boldsymbol{\psi}(0) = [u_0, \dots, u_0]^T$ with u_0 constant, and $P(0)$ and $U(0)$ as indicated in the previous section. The adaptive laws (19), (21), (22) and (37) will generate the update of $\Delta\hat{\boldsymbol{\theta}}$ for the robustness filter. As stated in the previous section, the generation of series $\{u(\tau)\}$ in the loop for implementing the regressor $\boldsymbol{\psi}(k)$ will generally produce conditions for achieving persistency of excitation, i.e., $\sum_{i=0}^{k} \boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i) \geq 0$, except for a family series of measure zero comprehending signals $u(k)$ of richness of order smaller than N, i.e., a periodic behaviour with a number of spectral lines less than N.

Thus, using (42), (69) and (70) in (71),

$$\sum_{k=k_1}^{k_2} u(k)\,\varepsilon(k) = \frac{D_0(1)}{D_0(z^{-1})} \sum_{k=k_1}^{k_2} \frac{1}{\hat{G}_0(1) + \Delta\hat{G}(k)} \times \left[(y(k) - \hat{y}_0(k)) - \Delta\hat{\boldsymbol{\theta}}^T(k)\,\boldsymbol{\psi}(k) \right]^2, \quad (72)$$

with $\hat{G}_0(1) + \Delta\hat{G}(k) = \sum_{i=1}^{N} \hat{g}_0(i) + \Delta\hat{g}(i,k) = 1/\hat{K}_{offs}(k)$.

Clearly, for every k_1 and k_2, in case of persistency of excitation in the period $[k_1, k_2]$, both $\Delta\hat{\boldsymbol{\theta}}(k)$ and $\hat{K}_{offs}(k)$ result bounded. Besides, for bounded $u(k)$ the filter in the feedback generates a bounded $\varepsilon(k)$. If the level of magnitude of $u(k)$ ascends to the level of persistency of excitation and $\mu(k)$ in (47) descends causing the parameter error vector $((\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0) - \Delta\hat{\boldsymbol{\theta}}(\infty))$ to decrease and so $u(k)$ to drop, there exists a time point k_0 where from hereafter the polynomial

$$D_0(z^{-1}) + D_0(1)K_{offs} \times \left[\left(G(z^{-1}) - \hat{G}_0(z^{-1}) \right) - \Delta\hat{G}(z^{-1}, k) \right] = 0 \quad (73)$$

has only stable zeros for every $k \geq k_0$. Up to this point the dynamics of the adaptive loop in figure 4 can be analysed in the state space of dimension N of the regressor $\boldsymbol{\psi}$. To this end, let the control action in (42) be put compactly into the form

$$u(k) = -\boldsymbol{\psi}^T(k) \left[\boldsymbol{\Gamma}_0 + q_0(k)\,\Delta\hat{\boldsymbol{\theta}}(k) \right] + q_0(k)(y(k) - \hat{y}_0(k)), \quad (74)$$

with $\boldsymbol{\Gamma}_0 = [d_{0_1}, \dots, d_{0_{N_1-N_2+1}}, 0, \dots, 0]^T$. So one can obtain the recursive equation

$$\boldsymbol{\psi}(k+1) = \boldsymbol{\Phi}(k)\,\boldsymbol{\psi}(k) + \mathbf{h}(k)\,(y(k) - \hat{y}_0(k)), \quad (75)$$

where

$$\boldsymbol{\Phi}(k) = \left[\begin{array}{ccccc} \multicolumn{5}{c}{-\boldsymbol{\psi}^T(k)\left[\boldsymbol{\Gamma}_0 - q_0(k-1)\,\Delta\hat{\boldsymbol{\theta}}(k-1)\right]^T} \\ \hdashline 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 \end{array} \right],$$

$$\mathbf{h}(k) = \left[\begin{array}{c} q_0(k) \\ \hdashline 0 \\ \vdots \\ 0 \end{array} \right] \quad (76)$$

are the transition matrix and input vector, respectively. Since the process is stable, $\boldsymbol{\psi}$ is bounded and the time-varying loop fulfils (73) with stable zeros, then $\boldsymbol{\Phi}(k)$ has only stable eigenvalues. Thus for M steps ahead it is valid

$$\boldsymbol{\Phi}(k+M) = \prod_{i=0}^{M-1} \boldsymbol{\Phi}(k+i)\,\boldsymbol{\psi}(k) + \sum_{i=0}^{M-1} (y(k+i) - \hat{y}_0(k+i))\,\mathbf{h}(k+i) \prod_{j=1}^{M-1-i} \boldsymbol{\Phi}(k+j). \quad (77)$$

For a sufficiently large M one achieves

$$\left\| \prod_{i=0}^{M-1} \boldsymbol{\Phi}(k+i) \right\| < 1, \quad (78)$$

and then

$$\left\| \boldsymbol{\psi}(k+M) \right\| < \left\| \boldsymbol{\psi}(k) \right\| \quad \text{for } k \geq k_0. \quad (79)$$

Thus for arbitrary finite time points k_1 and k_2 it is clear that $[(\hat{y}_0 - y) - \Delta\hat{\boldsymbol{\theta}}^T \boldsymbol{\psi}]^2$ in (72) remains bounded. Now for the case with the singular situation that $\hat{G}_0(1) = -\Delta\hat{G}(k)$ and taking into account that $D_0(1)/D_0(z^{-1})$ is an asymptotically stable system, the sum in (72) will fulfil condition (71) in $[k_1, k_2]$. The cross-energy (72) in this period has a lower bound which is uniform in k for $[k_1, k_2] \subseteq (0, \infty]$.

However, the level of persistency of excitation κ_1 will, in general decrease during the time when $\boldsymbol{\psi}$ arrives at the equilibrium point, so that for $k_1, k_2 \to \infty$, with $k_1 > k_2$ the parameter trajectory of the robustness filter satisfies asymptotically the manifold

$$\varepsilon(\infty) - \hat{K}_{offs}(\infty)\Big[\boldsymbol{\psi}^T(\infty)\Big(\Big(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0\Big) - \Delta\hat{\boldsymbol{\theta}}(\infty)\Big)\Big] = 0 \quad (80)$$

$$u(\infty) - D(1)\varepsilon(\infty) = 0, \quad (81)$$

where $\varepsilon(\infty)$, $u(\infty)$ and $\boldsymbol{\psi}(\infty)$ are limiting, not necessary zero constants.

All these constants define the possible equilibrium points $\Delta\hat{\boldsymbol{\theta}}(\infty)$. The measure of the asymptotic parameter error will depend on the maximal level of persistency reached in the estimation. It can be seen that the convergent points of the robustness filter lie in the residual set defined as

$$\mathcal{M}_{\Delta\hat{\boldsymbol{\theta}}(\infty)} = \Big\{\Delta\hat{\boldsymbol{\theta}}(\infty) \in \Re^N/\text{equations (80) and (81) fulfilled}\Big\}. \quad (82)$$

As $\boldsymbol{\psi}(k_\infty) = \text{constant}$ is no longer persistent exciting, the vector $[\Delta\hat{\boldsymbol{\theta}}^T(\infty), \boldsymbol{\psi}^T(\infty)]^T$ constitutes an unstable equilibrium point. So round-off errors present in by the practical implementation in computation of the adaptive algorithm will produce the violation of the positiveness condition (49) before converging to zero (bursting phenomenon). So begins the state $[\Delta\hat{\boldsymbol{\theta}}^T(k), \boldsymbol{\psi}^T(k)]^T$ to increase erratically untill the condition (49) is generally achieved again. This rather chaotic behaviour will be treated in the next section in the context of other practical situations such as in the presence of disturbances and truncation errors.

5.2.2 Case: $\boldsymbol{n(k) \neq 0}$ and $\boldsymbol{e_N(k)}$ small. On one side, this case is more practical because a FIR model constitutes a truncation of the transfer function represented by an impulse response series and so there always exists e_N. Nevertheless, it is expected that the FIR series embraces the settling time of the system over the whole range of variation of its dynamics, such that $e_N(k)$ be considered small or negligible in terms of the magnitude of discrete-time input pulse. On the other hand, measures are frequently corrupted with noise, so that the disturbance $n(k)$ is commonly present at the output system. One supposes, herein, that $n(k)$ and $e_N(k)$ are bounded in the sense

$$\sum_{k=k_1}^{k_2}(n(k) + e_N(k))^2 \leq \zeta_0^2, \quad (83)$$

for every k_1 and k_2 with $0 \leq k_1 < k_2 \leq \infty$ and ζ_0 a finite real constant. Additionally, it is common to assume that $n(k)$ is a stochastic signal (not necessary white noise) of a stationary process. Due to the feedback, $n(k)$ is statistically independent of the system output $G(z^{-1})\,u(k)$ only in the case that n is white noise. On the contrary, for $n(k)$ being a correlated signal, there will exist the internal coherence between $n(k)$ and $\boldsymbol{\psi}(k)$. But if the innovation present in $n(k)$ is dominant against computer round-off errors, this can guarantee the positive definiteness of $\sum_{i=k_1}^{k_2}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i)$ uniformly in time. Also in many practical situations the whiteness condition is satisfied for sampled systems. In this case a strong innovation is ensured.

So according to the last terms (58) and (59) in the composition of $u(k-1)$ (it is, those containing e_N and $n(k)$, respectively), and similarly in the other terms $u(k-2), \ldots, u(k-N)$ in the regressor $\psi(k)$, this vector spans the whole space of parameters. Therefore, the persistency of excitation is guaranteed with a uniform level of persistency dependent on the innovation of $n(k) + e_N(k)$.

One states

$$\sum_{k=k_1}^{k_2} u(k)\,\varepsilon(k) = \sum_{k=k_1}^{k_2} u(k)\,\hat{K}_{offs}(k)\,(n(k) + e_N(k) - \breve{e}(k)), \quad (84)$$

with $\breve{e}(k)$ the *a posteriori* error. Hence, bearing (83) in mind and with $\sum_{i=k_1}^{k_2}\boldsymbol{\psi}(i)\boldsymbol{\psi}^T(i) \geq 0$, and with parameters bounded with bounded $u(k)$, condition (71) is fulfilled for every pair of values k_1 and k_2, also for limiting k_1 and k_2.

In this way the coefficients of the robustness filter converge to a constant vector $\Delta\hat{\boldsymbol{\theta}}(\infty)$ lying in the residual set

$$\mathcal{M}_{\Delta\hat{\boldsymbol{\theta}}(\infty)} = \Big\{\Delta\hat{\boldsymbol{\theta}}(\infty) \in \Re^N/\boldsymbol{\psi}^T(k_\infty)\Big(\Big(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_0\Big) - \Delta\hat{\boldsymbol{\theta}}(\infty)\Big) + n(k_\infty) + e_N(k_\infty) = 0\Big\} \quad (85)$$

In contrast to the previous case without perturbation, all the equilibria $\Delta\hat{\boldsymbol{\theta}}(\infty)$ in $\mathcal{M}_{\Delta\hat{\theta}(\infty)}$ are stable, since the persistency-of-excitation condition (49) is also valid uniformly in steady state.

In the case where $n(k)$ is white noise and $e_N(k)$ is zero, an exact convergence in the sense of (66) occurs and

$$\lim_{\substack{k_1, k_2 \to \infty \\ k_2 > k_1}} \sum_{k=k_1}^{k_2} u(k)\,\varepsilon(k) = 0, \quad (86)$$

since $n(k_\infty) = \tilde{e}(k_\infty) = \breve{e}(k_\infty)$ in (72). This result is independent on the mean energy ζ_0^2 of the noise in (83). Consequently it is also inferred that the feedback is asymptotically turned off (see figure 7), i.e., it is virtually switched off and that the regressor accomplishes

$$\psi(k_\infty) = \frac{D_0(1)}{D_0(z^{-1})}[n(k_\infty - 1), \ldots, n(k_\infty - N)]^T.$$

5.2.3 Case: external excitation $w(k) \neq 0$. Persistently exciting signals are often present in the control system and eventual changes of the operation points can also be used as excitation. Also they are generally more energetic than noise signals. Hence, the positiveness condition can be guaranteed at least during the periods these signals are actuating.

Now it is supposed that the action of a persistently exciting signal $w(k)$, previously filtered by $z^{N_1}F(z^{-1})R_0(1)/R(z^{-1})$ (see figure 4), is added to the closed-loop input $-\hat{K}_{offs}(k)(n(k) + e_N(k))$ (see figure 7). Clearly, the convergence in the sense of (71) takes place to a stable equilibrium point in a residual set in case the sufficient richness of $u(k)$ continues to infinity. The more the statistical internal coherence between $n(k)$ and $\psi(k)$ diminishes, the larger is the energy of $w(k)$, so that the measure of the residual set for the asymptotic parameter vector decreases accordingly to this relation.

6. Adaptive control with robustness filter

As seen in the numeric simulations, the asymptotic performance of the adaptive control system, in general, gives better results than obtained by a robustness filter system, mainly if the particular settings of the tuning coefficients allow the adaptive control to reach asymptotic stability in steady state. However, sudden dynamic changes affecting the behaviour of the closed loop significantly are much more efficiently damped down by a robustness-filter system with the supplementary feature of always achieving an asymptotic stable behaviour without extra tuning parameters.

Hence, in order to achieve quality in both transient and asymptotic performance, it seems suitable to synchronize both approaches in order to share the manifested advantages of both sides. Also an approach is described as a simple combination of algorithms for adaptive control and robustness filter as depicted in figure 8.

The problem now is to find an appropriate indicator function referred to as $\upsilon(k)$ that enables commuting between the algorithms automatically. The selection carried out in the paper is based on the certainly equivalence principle that sustains designs of indirect adaptive control systems. So let us suppose the control stays at the beginning in an adaptive modus as indicated in figure 8 for $\upsilon = 0$. One of the conditions demanded by indirect adaptive controls in the IMC structure for ensuring convergence is that the roots of the time-varying characteristic polynomial

$$D(z^{-1}, k) + D(1)K_{offs}(k)\Big(G(z^{-1}) - \hat{G}(z^{-1}, k)\Big) = 0 \quad (87)$$

lie in the unit circle of the z^{-1} domain. As $D(z^{-1}, k)$ is designed under the assumption that $\hat{G}(z^{-1}, k)$ is identical to the plant $G(z^{-1})$ (certainly equivalence principle) a plausible way to test (87) could consist in checking

$$D(z^{-1}, k) = 0 \quad (88)$$

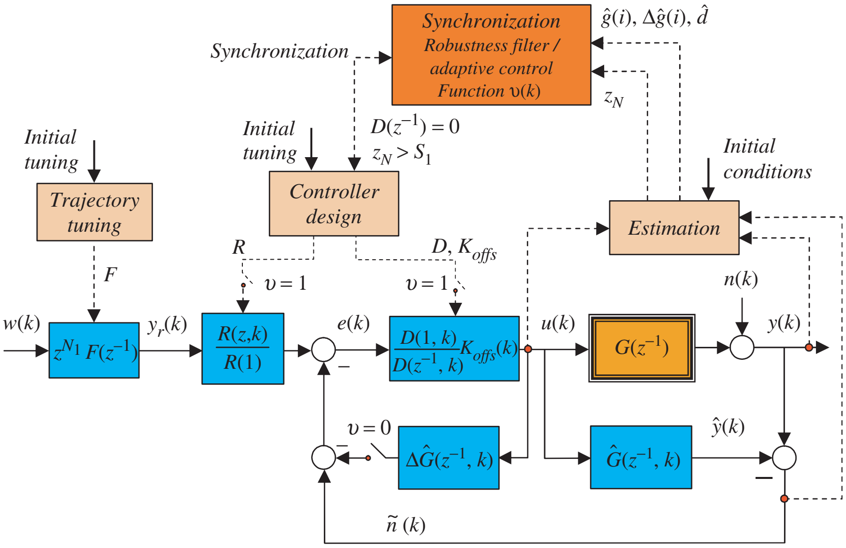

Figure 8. Syncronization of robustness filtering and adaptive control in the IMC structure.

and inquiring for unstable zeros. So, when (88) have only stable zeros one can argue that the adaptive controller parameters stay in a domain of attraction and the update of $\hat{\boldsymbol{\theta}}(k)$ can prosecute. In this case, the indicator is set $\upsilon = 1$. On the contrary, when this condition is violated, one sets $\upsilon = 0$ and the adaptive-control modus is commuted to the robustness-filter modus, till υ turns to 1 again, i.e., until $D(z^{-1}, k)$ be stable once again for new controller designs.

It is noted that in both systems, measures for supervision of the positiveness condition (49) have to be taken in parallel to this commutation. These may be implemented by means of (30) in both approaches and the parameter estimation could focus on one model only, for instance, $\Delta\hat{\boldsymbol{\theta}}(k)$ for the robustness filter, and then using this, construct $\hat{\boldsymbol{\theta}}(k) = \Delta\hat{\boldsymbol{\theta}}(k) + \boldsymbol{\theta}_0$ for the adaptive control, with $\boldsymbol{\theta}_0$ being the plant model at the beginning.

In order to compare the effectiveness of the combination, in contrast to the pure adaptive control system, simulations are next presented in two numeric experiments. A test process with Laplace transfer function $G(s) = (1 + 2s)/(1 + 10s)(1 + 7s)(1 + 3s)$, sampling time $T_0 = 4\,\text{s}$ and $N = 20$ is simulated in a control loop with an IMC controller with parameters $N_1 = N_u = 1$, $N_2 = 3$, $\beta_1 = 0$, $\alpha_i = 1$ and a forgetting factor $\lambda = 0.9$. In the first experiment no perturbation is included, and in the second one a correlated noise is introduced with a relation equal to $\sqrt{\sigma_n/\sigma_w} = 0.2$, with σ_w^2 and (σ_n^2, the variances of the set point and of the noise, respectively.

The commissioning phase in both cases consists of measuring the step response in open loop and immediately afterwards switching on the adaptive loop. Three dynamics changes of $G(s)$ are introduced at regular intervals, namely, at $k = 45$ an increment of the dead time from $d = 0$ to $d = 4$ (i.e., 16 s) occurs, at $k = 105$ an increment of the process gain from 1 to 1.5 takes place, and finally at $k = 165$ a change of the minimal phase into a non-minimal one happens by moving the zero of $G(s)$ from the place 2 to the place -5.

In figure 9 one can see a parallel to the evolution of the pure adaptive control (above) and of the adaptive control with robustness filtering (below). It is clear that, in the first case, the performance is seriously affected by the appearance of large oscillations after the changes in the dead-time and in the gain, but specially it appears a tendency to instability when the non-minimal phase takes place (see figure 9, top). In the case of the combination (see figure 9, bottom), the commutation between algorithms takes place automatically subject to the permanent test of (88) immediately after the changes are introduced. Even when the last dynamics change involves the transition to a non-minimal phase system, the transient performance can achieve a high control quality.

Similar features are reflected in figure 10 in the evolution of the respective control actions, where in the case of the pure adaptive control, $u(k)$ goes repeatedly to saturation, and shows a tendency to instability after the third dynamics change. On the other hand, the combination demonstrates at times, an irregular but always bounded and much better behaviour after the changes. Also, after the third change, a quite smooth response is noticed.

The second numeric experiment with noise is shown in figures 11 and 12. Similar characteristics are noted for comparison. However, the intensity of the transient behaviour and performance drop is more accentuated now in the adaptive-control case, while in the combination case the behaviour remains of high quality with minor disturbances and short transients. These characteristics are also valid for the evolution of the control actions.

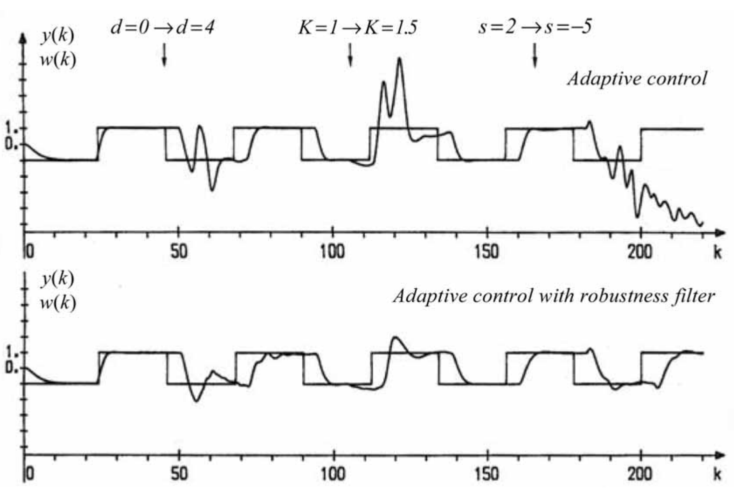

Figure 9. Comparison of the control performance for three different dynamics changes with perturbed measure.

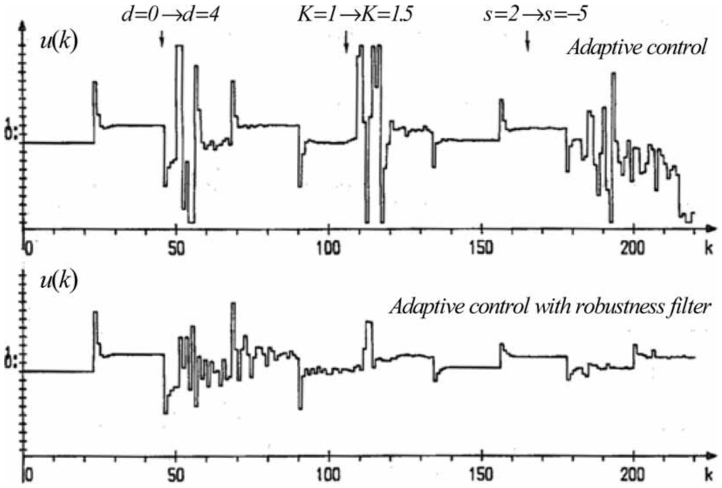

Figure 10. Comparison of the control action for three different dynamics changes with unperturbed measure.

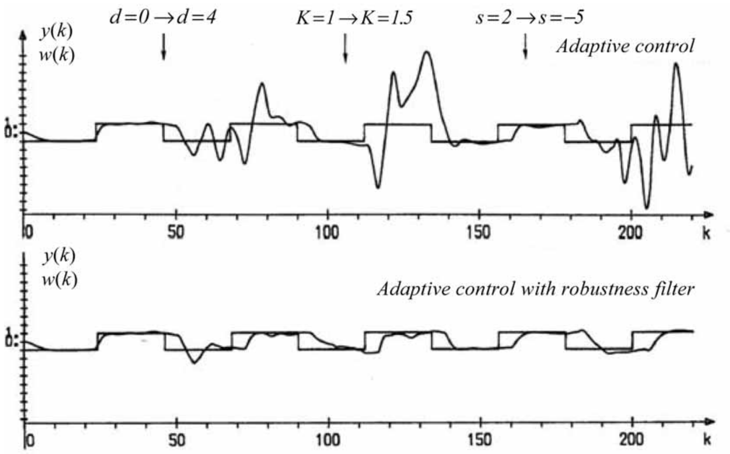

Figure 11. Comparison of the control performance for three different dynamics changes with perturbed measure.

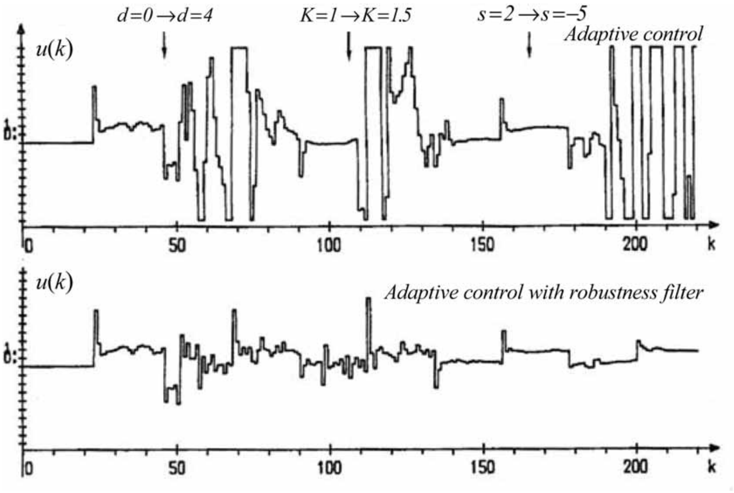

Figure 12. Comparison of the control action comparison for three different dynamics changes with perturbed measure.

In summary, a combination of adaptive control with robustness filter is clearly superior in quality, and requires little more calculation than in the adaptive-control case. Besides, no more design parameters than are required by the adaptive control are needed in the combination.

7. Experiments

We applied our approach to a non-linear process in order to illustrate its features on a real dynamics. A pilot plant composed of a heat exchanger with cool system (see figure 13) was chosen for doing the experiments in order to compare with recent studies of adaptive control applied to similar heat exchangers, see for instance Fink *et al.* (2001) and Škrjanc *et al.* (2003), among others.

The pilot plant used in this application consists of three circuits: the primary steam circuit between the steam boiler and the heat exchanger the secondary water circuit between the heat exchanger; and the cooler, and the air circuit between the cooler and the atmosphere. The saturated steam stays at 7.5 bar and 175°C. The input and output water temperature in the secondary circuit, ϑ_{FE} and ϑ_{FA}, respectively, are regulated independently. For this goal the angular velocity of the cooling ventilator and the valve in the water circuit will be manipulated.

The dynamics of the system is strong non-linear in a large region of the variable domains, namely, the saturated steam flow $\dot{M}_D$: 0–78 kg/h, the water flow $\dot{M}_F$: 0–10000 l/h and the air flow $\dot{M}_L$: 0–8000 m^3/h. A characteristic in the dynamics is that by a stepwise change in the steam flow $\dot{M}_D$, the water temperatures go in opposite directions (ϑ_{FA} increasing and ϑ_{FE} decreasing) with a dead time depending inversely on $\dot{M}_F$, for instance 30 s for $\dot{M}_F = 3000$ l/h. The coupling between the variables ϑ_{FA} and ϑ_{FE} is pronounced.

Figure 14 shows the evolution of ϑ_{FA} for a stepwise change of the steam flow $\dot{M}_D$ in both directions by different rates of $\dot{M}_F$. The asymmetry between responses in opposite directions, specially for relatively small and medium values of $\dot{M}_F$ is seen. Similar significant differences are reflected in the weighting functions $g(i)$ of the plant for a sampling time $T_0 = 5$ s as inferred from figure 15.

The control system is designed for ϑ_{FA} as the system output, u_D as the control action in the water circuit (see TC31 in figure 13) and $\dot{M}_F$ as a perturbation, while ϑ_{FE} is regulated independently with a PID controller over the r.p.m. of the cooling ventilator (see TC41 in figure 13). Intervention on any one control loop influences the other one due to the strong couplings between ϑ_{FA} and ϑ_{FE}.

The features of the proposed approach are comparatively analysed in experiments showing the behaviours of the controlled heat exchanger for 3 cases, namely for

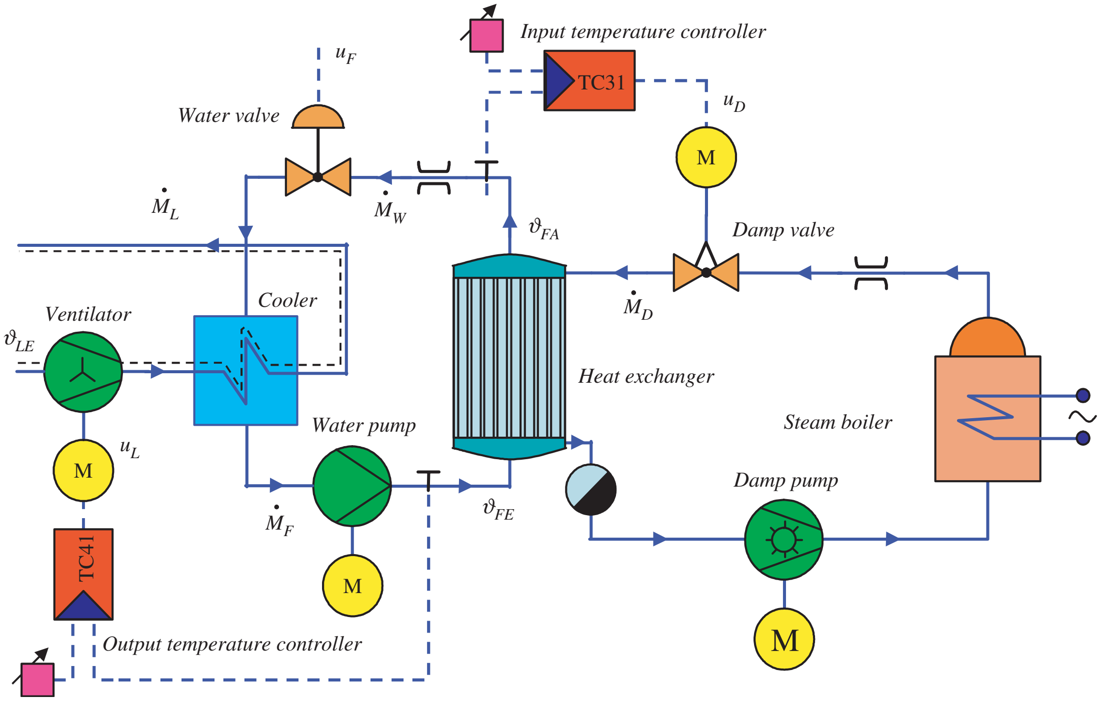

Figure 13. Heat exchanger pilot plant with cooling system at the Technische Hochschule Darmstadt, Germany.

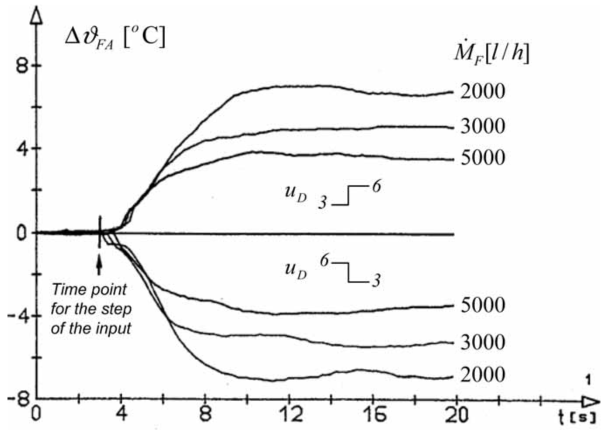

Figure 14. Step responses of the output temperature for different water flow values.

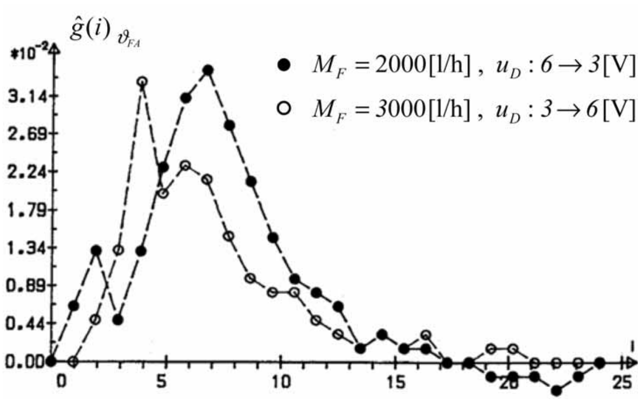

Figure 15. Two weighting functions for different operation points.

a fixed controller, an adaptive controller and an adaptive controller with robustness filter. These experiments are summarized in figures 16 and 17 for the regulated outputs and the control actions, respectively.

Before starting the controller design, the water flow was set in 5000 l/h at $k = 0$. In a commissioning phase from $k = 5$ to $k = 30$, the step response is sampled. From this curve a first model $\hat{g}(i)$ is built up and upon this a controller is designed. At $k = 100$ the water flow $\dot{M}_F$ was suddenly decreased to the value of 2700 l/h. Besides, the set point is varied stepwise at different levels of the output temperature ϑ_{FA}, while the set point for the input temperature ϑ_{FE} is set constant to 39.0°C. The described operation sequence was repeated three times for the different controllers. For all them, an IMC control law with a common set of tuning parameters composed of $N_1 = 1 + \hat{d}$, $N_2 = N_1 + 7$, $N_u = 1$, $\alpha_i = 1$ and $\beta_1 = 0.03$ was used.

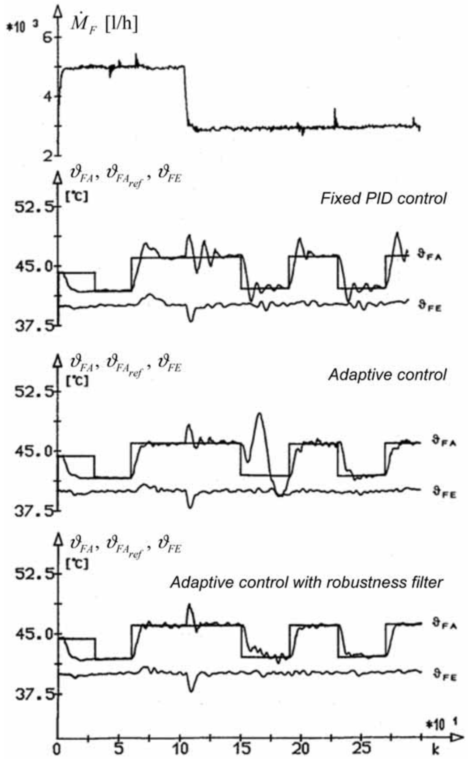

Figure 16. Time evolution of the water flow $\dot{M}_F$ and of the controlled temperatures ϑ_{FA} and ϑ_{FE} in three control systems.

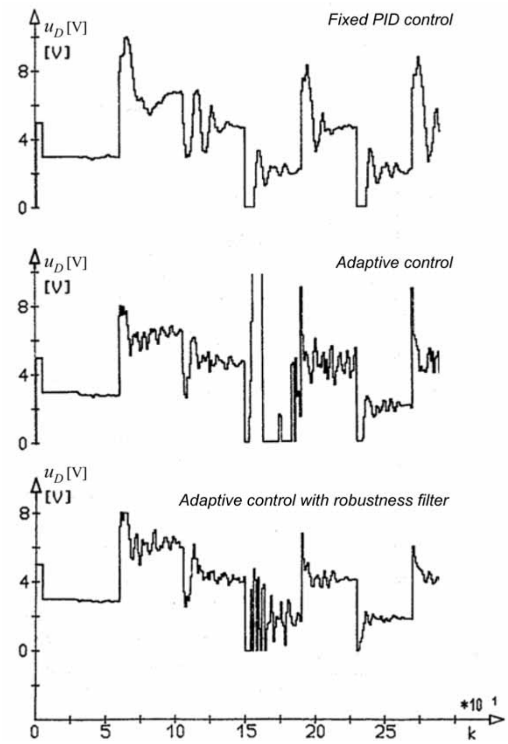

Figure 17. Time evolution of the control actions in three control systems.

The performance of the fixed predictive control is appreciated at the top place in figures 16 and 17. The behaviour is stable, yet after the first step change of the set point, the control performance degrades, showing a great overshoot of about 40% of its end value. Later, after the change of $\dot{M}_F$, the performance remains deficient, with a lower damping and a great overshoot in both directions. Also the evolution of ϑ_{FE} in the air circuit is significantly perturbed by the interventions in the water circuit.

The performance of the adaptive predictive control without robustness filter is shown in the middle in figures 16 and 17. The behaviour is stable with a good-quality performance after the first step change of the set point. After the stepwise variation of $\dot{M}_F$ has occurred, the regulation on ϑ_{FA} is affected, but the transient is shorter and with lower energy than in the previous case. However, when the next change of the set point takes place, the performance degrades significantly, showing an unstable behaviour because of the appearance of unstable zeros in $D(z^{-1}, k)$ during a relatively short period. It takes a certain time (about one minute) for the dynamics to be completely controlled again. From this point the performance looks satisfactory up to the end of the experiment time. The behaviour of the control action is markedly irregular and fluctuating. On the other side, the evolution of ϑ_{FE}, despite the unstable period, remains much more regular than in the previous case with fixed controller.

In the third case the performance of the adaptive predictive control with robustness filter is depicted at the bottom of figures 16 and 17. The robustness filter is automatically made active immediately after the polynomial $D(z^{-1}, k)$ presents unstable zeros. This occurs at $k = 151$, where the robustness filter, generated by identification of the model discrepancies, is implemented. It is seen clearly that the overall behaviour is much better than in the two previous cases, reaching a rapid and accurate tracking of the set point. The variable ϑ_{FE} evolves as in the previous case.

In the last two cases a common exponential, forgetting factor $\lambda = 0.95$ and an initial covariance matrix $\mathbf{P}(0) = \mathbf{I}$, were applied. The identification process was switched on/off according to the commutation function $\upsilon(k)$. On one side, this depends on the zeros of $D(z^{-1}, k)$ as seen in (88), and on the other side, on the excitation indicator $z_N(k)$ (see (30)) and a bound $S_1(k)$ as shown in figure 18, in the second curve set. The dependence of $\upsilon(k)$ is logical of type "AND", i.e., $\upsilon(k) = 1$ if $D(z^{-1}, k) = 0$ has only stable zeros and $z_N(k) \leq S_1(k)$, and $\upsilon(k) = 0$ otherwise. The bound $S_1(k)$ is constructed as the output of a low-pass filter with input $z_N^2(k)$.

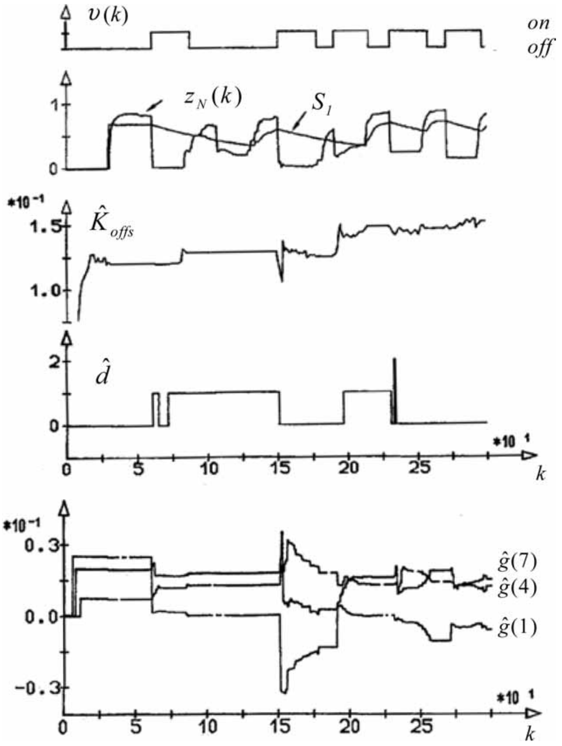

Figure 18. Supervision of the identification and controller design.

The case of adaptive control with robustness filter the evolutions of the estimated gain $\hat{K}_{offs}$ and dead time $\hat{d}$ are presented separately (below in the same figure). It shows a variation of the gain of about 20% and a dead time $\hat{d}$ varying between 0 and 2. Next, the evolution of three estimated coefficients $\hat{g}(i)$ selected arbitrarily are depicted on the bottom. The time-variation of the coefficients and of the model structure depend on the changing operation points, which actually reflect the existence of strong dynamic non-linearities and state couplings of the system behaviour.

In a comparison with the cited literature (Fink *et al.* 2001, Škrjanc *et al.* 2003), one can find qualitatively-similar high-performance responses of the control-loop using non-linear controllers, also for large excursions of the setpoint. However, the price paid for this performance is the need to count on an accurate description of the plant by means of a non-linear model or having a proper knowledge to generate fuzzy rules or neuronal training. The approach here is basically linear with minimal required knowledge of the plant. One is able to compensate large excursions of the setpoint with the combination of laws for pure adaptive control with robustness filter in a quite simple manner.

8. Conclusions and future work

8.1 *Conclusions*

In this paper a new approach to improve the overall performance of indirect adaptive control systems tailored for non-linear stable plants has been presented. The approach involves a commutation of a time-varying linear robustness filter in the feedback path of the control loop in synchronization with an adaptive controller.

The proposed modification is presented under the celebrated IMC structure for predictive control systems. Essentially, the robustness filter is a mechanism that tries to null asymptotically the fed signal of the IMC control system. When this is achieved, the resulting control loop is guaranteed to be asymptotically stable independent of the design parameter settings of the previous adaptive control.

The robustness filter is implemented by using on-line estimation of the difference between the dynamics of the plant and of a previous nominal model. The last model could be obtained in a commissioning phase simply by measuring the step response of the plant. The commutation into the robustness filter occurs basically when some pole of the adaptive IMC controller moves outside the unit circle. The converse switching takes place otherwise. So the approach aims to share the advantages of both strategies, on one side the high-quality steady-state performance of the adaptive control at steady state, and on the other side, the ability of the robustness filtering to damped transients and ensuring stability asymptotically. Also the commutation requires a sufficient level of persistence of excitation.

The two main assumptions for the application of the approach are first that the plant must be stable with a settling time that is embraced by the FIR sequence, and secondly that dynamics is linearizable at every operation point. It is plausible that both assumptions can be satisfied generally by processes. It is pointed out that the approach does not demand much more calculation than for the adaptive-control case and absolutely no more design parameters than those required by the adaptive controller.

It was indicated that the implemented algorithm based on a FIR model can automatically adopt to any change in the system order and dead-time for dynamics around operation points. It also has no restriction for plants with zero dynamics and, in this respect, it can deal with unstable dynamics of the adaptive control loop caused either by improper settings of the design parameters or due to instability of the adaptive loop.

The convergence of the algorithm was an important part of the paper. It was shown that, as in the majority of the indirect adaptive controllers, the condition of persistency of excitation is also necessary for the proposed approach. Nevertheless, there is one advantage, namely that by stationary white-noise perturbation on the output, the positiveness condition of the covariance matrix is ensured regularly. This is a feature of the FIR model structure. Moreover, it can be expected that the presence of non-linearities in the dynamics can reinforce the level of persistent excitation of the regressor. However, the estimation has to be supervised as in conventional adaptive control systems to avoid numeric long-term instability.

It is shown through many numeric simulations and experimentation on a heat exchanger with cooling system, that undesired transients of the controlled output due to abrupt and significative changes to the operation points and coupled dynamics can be efficiently damped down by the developed approach, achieving a high-quality performance in steady state.

8.2 *Future work*

Finally, this work could extend the main concepts to other possible automatization areas like supervision of technical processes and fault tolerant control (FTC) systems. Our present aim is oriented to include parameter transient behaviour analysis for fault diagnosis identification (FDI) and symptom generation of system faults concerning, typically, sensor and actuator failures, as well as changes affecting the system dynamics, like structural variations of order and dead time. On account of links between FDI and FTC techniques are still lacking and few recent results can be found, for instance in Zhang *et al.* (2004), we believe that the benefits of our approach in the context of FTC could be double. On one side, a great part of dynamical changes is detected and quantified by the indirect adaptive control law, which also adapt the response, automatically achieving a good performance. The complementary set of possible failures that are not detected by the adaptation (for instance, sensor offset or extra delay in actuators), could be detected and identified by ad-hoc procedures of symptoms generation for diagnosis. On the other side, it is seen in our testing, the approach can ensure bounded process signals where changes are occurring. One can take advantage of this particular feature by using the period after the commutation of the adaptive control into the robustness filter just to active mechanisms for detection and isolation that finally can lead to a proper therapy or automatic plant reconfiguration.

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