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An efficient MILP continuous-time formulation for short-term scheduling of multiproduct continuous facilities

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Abstract

This paper presents a new MILP mathematical formulation for the scheduling of resource-constrained multiproduct plants involving continuous processes. In such facilities, a sequence of continuous processing steps is usually carried out to produce a significant number of final products and required intermediates. In order to reduce equipment idle time due to unbalanced stage capacities, storage tanks are available for temporary inventory of intermediates. The problem goal is to maximize the plant economic output while satisfying specified minimum product requirements. The proposed approach relies on a continuous time domain representation that accounts for sequence-dependent changeover times and storage limitations without considering additional tasks. The MILP formulation was applied to a real-world manufacturing facility producing seven intermediates and fifteen final products. Compared with previous scheduling methodologies, the proposed approach yields a much simpler problem representation with a significant saving in 0–1 variables and sequencing constraints. Moreover, it provides a more realistic and profitable production schedule at lower computational cost. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Multiproduct facilities; MILP optimization model; Optimal operation; Continuous processes

Nomenclature

(*a*) *Sets*

I production runs
I_j campaigns that can be processed in unit j ($I_j \subseteq I$)
I_s campaigns producing or consuming state s ($I_s \subseteq I$)
I_s^+ campaigns producing state s ($I_s^+ \subseteq I_s$)
I_s^- campaigns consuming state s ($I_s^- \subseteq I_s$)
J continuous processing units
J_s available units to run tasks producing state s ($J_i = J_s$, for any campaign $i \in I_s^+$)
$J_{ss'}$ available units to run tasks producing state s or state s' ($J_{ii'} = J_{ss'}$, for campaigns $i \in I_s^+$ and $i' \in I_{s'}^+$)
S states (intermediates or final states)
S^I intermediate states ($S^I \subseteq S$)
S^P final states ($S^P \subseteq S$)
T storage tanks for intermediates
T_s available tanks to store the intermediate $s \in S^I$ ($T_I = T_s$ for any campaign $i \in I_s^+$)
$T_{ss'}$ available tanks to store both states s and s' ($T_{ii'} = T_{ss'}$ for any pair of campaigns $i \in I_s^+$ and $i' \in I_{s'}^+$)

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(*b*) *Parameters*

d_s minimum requirement of final product $s \in S^P$
H time horizon length
$l_{sj}^{\min}$ minimum allowed length of a campaign $i \in I_s^+$ producing state s and running at unit j
M a very large number
ρ_{is} amount of state s required per unit size of campaign $i \in I_s^-$
p_s price of final state $s \in S^P$
$r_{sj}^{\min}$ minimum production rate of state s at any campaign $i \in I_s^+$ running at unit j
$r_{sj}^{\max}$ maximum production rate of state s at any campaign $i \in I_s^+$ running at unit j
ru_j ready time of unit j
ro_s release time for any campaign producing state s
$\tau_{ss'j}$ changeover time between runs $i \in I_s^+$ and $i' \in I_{s'}^+$ at unit j ($\tau_{ii'j} = \tau_{ss'j}$ for a pair of campaigns $i \in I_s^+$ and $i' \in I_{s'}^+$)
$\sigma_{ii't}$ changeover time between campaigns $i \in I_s^+$ and $i' \in I_{s'}^+$, both assigned to tank $t \in T_{ss'}$
v_t maximum capacity of tank $t \in T$

(*c*) *Variables*

C_i completion time for campaign i
CT_i completion time for the storage task of intermediate s provided by campaign $i \in I_s^+$
$F_{ii'}$ amount of intermediate s supplied by run $i \in I_s^+$ to the consuming campaign $i' \in I_s^-$
IT_i starting time for the storage task of intermediate s supplied by campaign $i \in I_s^+$
L_{ij} length of campaign $i \in I_s^+$ in unit $j \in J_s$
MK makespan
Q_i production size of campaign i
$U_{ii'}$ binary variable denoting that campaign $i \in I_s^+$ supplies intermediate s to campaign $i' \in I_s^-$
$V_{ii'}$ accumulated amount of state s consumed by campaign $i' \in I_s^-$ at the completion time of run $i \in I_s^+$
W_{it} binary variable denoting that the intermediate $s \in S^I$ produced by run $i \in I_s^+$ has been assigned to tank $t \in T_s$
$X_{ii'}$ binary variable denoting that campaign i is run/stored before ($X_{ii'} = 1$) or after ($X_{ii'} = 0$) campaign i' in some available unit/tank
Y_{ij} binary variable denoting that campaign $i \in I_s^+$ is run in unit $j \in J_s$
$Z_{ii'}$ binary variable denoting that campaign $i' \in I_s^-$ starts after campaign $i \in I_s^+$ has ended

1. Introduction

Much of the work in the area of production scheduling has been focused on batch processing facilities. Recently, however, several publications have paid attention to the scheduling of multiproduct facilities involving continuous processes. Continuous time formulations for the short-term scheduling of continuous processes, all of them based on the RTN/STN representation, have been developed by Schilling and Pantelides (1996), Ierapetritou and Floudas (1998), Zhang and Sargent (1998). Though proposed as general-purpose frameworks that can be applied to mixed production facilities involving batch and continuous processes, they present two major drawbacks when real industrial problems are tackled. First, they provide large-size problem formulations with a high number of 0–1 variables and constraints that considerably increases with the number of event points or event-times being used. Second, they have a weak control on the number of campaigns being run for each production task. As a result, non-realistic production schedules involving an excessive, non-economic number of campaigns for every required task are usually generated. This happens even if no intermediate due dates are to be satisfied. Many of such runs are consecutively accomplished in the same unit, but at different processing rates. This is not a usual industrial practice since it implies a much higher demand of manpower and costly equipment idle time. Instead, a quite few campaigns per task are weekly run in industry. In turn, Karimi and McDonald (1997) proposed a couple of slot-based MILP formulations to deal with the scheduling of single-stage multiproduct facilities involving no storage requirements and multiple product demands at specified due dates. The present work generalizes the MILP algorithmic methodology for the short-term scheduling of batch processing facilities, introduced by Méndez and Cerdá (2000), to now account for continuous processing units. Rather than relying on the notion of time-slots or event points, the model basic block is the set of (direct/non-direct) predecessors for any campaign at each processing/storage unit. Therefore, no effort should be applied to properly select the number of slots or event times and a better control on the number of campaigns is achieved.

2. Problem definition

The short-term scheduling problem considered in this work can be stated as follows. Given are: (a) a processing facility that involves a set of continuous operations producing intermediate or final products; (b) a set of available equipment items performing the required processing tasks; (c) a set of tanks of limited capacity for temporary storage of intermediates; (d) a predefined set of runs for each intermediate and final product; (e) minimum demands of final products to be satisfied; (f) a given scheduling horizon. The problem objective is to find: (i) the optimal sequence of runs to be executed in every continuous unit; (ii) the production task being performed and the amount of intermediate/final product yielded by each one; and (iii) the campaign starting and completion times, in order to maximize the economic return from production sales while satisfying allocation constraints, storage limitations and end-product minimum requirements.

3. Model assumptions

(a) Several intermediates can be required to run a continuous processing task but each task just produces a single intermediate/final product.
(b) A sth-producing run $i \in I_s^+$ can supply intermediate s to one or several production tasks $i' \in I_s^-$ consuming s.
(c) An intermediate s required by a production task $i' \in I_s^-$ can be directly provided by run $i \in I_s^+$ bypassing storage and/or taken from the tank assigned to run i.
(d) Minimum run lengths can vary with the product being manufactured and the unit assigned.
(e) Minimum and maximum equipment processing rates can vary with the product and the unit assigned.
(f) Changeover times between campaigns are sequence-dependent in processing units and storage tanks.
(g) The time interval during which a suitable tank $t \in T_s$ is assigned to a particular campaign $i \in I_s^+$ (i.e. the time interval for the storage task) begins at the starting time of run i and ends as soon as the amount of intermediate s produced by run i has been totally consumed.
(h) No initial inventory of any intermediate $s \in S^I$ is on hand.
(i) Unlimited storage capacity is available for final products whereas tanks for intermediates have limited capacities.
(j) The plant is operated on a closed-shop mode.

4. The mathematical model

4.1. Problem constraints

4.1.1. Scheduling horizon constraints

Every production run should be completed within the specified scheduling horizon of length H.

$$C_i \le H \quad \forall i \in I \tag{1}$$

4.1.2. Campaign/unit allocation constraints

A single unit can at most be assigned to campaign i.

$$\sum_{j \in J_s} Y_{ij} \le 1 \quad \forall i \in I_s^+,\ s \in S. \tag{2}$$

4.1.3. Minimum run length constraint

Every campaign producing an intermediate or a final product s in unit $j \in J_s$ should have a length L_{ij} never lower than the specified minimum duration $l_{sj}^{\min}$.

$$l_{sj}^{\min} Y_{ij} \le L_{ij} \le H Y_{ij} \quad \forall i \in I_s^+,\ j \in J_s,\ s \in S. \tag{3}$$

4.1.4. Considering finite product release times and unit ready times to compute the starting times of initial campaigns

$$C_i - \sum_{j \in J_s} L_{ij} \ge \sum_{j \in J_s} \max[ru_j, ro_s] Y_{ij} \quad \forall i \in I_s^+,\ s \in S. \tag{4}$$

4.1.5. Relationship between the length L_{ij} and the amount of material s produced by campaign $i \in I_s^+$

The amount of material s produced by a campaign $i \in I_s^+$ running at unit $j \in J_s$ not only depends on its duration L_{ij} but also on the selected unit processing rate lying within the feasible range $[r_{sj}^{\min}, r_{sj}^{\max}]$.

$$\sum_{j \in J_s} r_{sj}^{\min} L_{ij} \le Q_i \le \sum_{j \in J_s} r_{sj}^{\max} L_{ij} \quad \forall i \in I_s^+,\ s \in S. \tag{5}$$

4.1.6. Final product demand constraints

One or several production campaigns can be run to meet the specified minimum demand for each final product.

$$d_s \le \sum_{i \in I_s^+} Q_i \quad \forall s \in S^P. \tag{6}$$

4.1.7. Production run sequencing constraints

The completion time of any precedence campaign i running before in the same unit $j \in J$ is a lower bound on the starting time of a production run i'. The sequencing variable $X_{ii'}$ will be equal to one whenever both campaigns i and i' are assigned to the same unit $j \in J_{ii'} = J_i \cap J_{i'}$ and, in addition, campaign i is run before. In such a case, constraint (7) is enforced while Eq. (8) becomes redundant. If the assignment variables Y_{ij} and $Y_{i'j}$ are still equal to 1 but campaign i' is first executed, then $X_{ii'} = 0$ and, consequently, constraint (8) will be enforced. In any other case, the value of $X_{ii'}$ is meaningless. By considering the pair of Eqs. (7) and (8) rather than only constraint (7), a single variable $X_{ii'}$ is just needed to describe the relative locations of runs (i, i') in the same processing sequence. In this way, the number of sequencing variables $X_{ii'}$ is reduced by half. In addition, sequencing constraints (7)–(8) explicitly account for sequence-dependent transition times.

$$C_{i'} - L_{i'j} \geq C_i + \tau_{ii'j} - H(1 - X_{ii'}) - H(2 - Y_{ij} - Y_{i'j}) \quad \forall i, i' \in I,\ i < i',\ j \in J_{ii'}, \tag{7}$$

$$C_i - L_{ij} \geq C_{i'} + \tau_{i'ij} - HX_{ii'} - H(2 - Y_{ij} - Y_{i'j}) \quad \forall i, i' \in I,\ i < i',\ j \in J_{ii'}. \tag{8}$$

4.1.8. Material balances

Material balances are included to guarantee that enough material $s \in S^I$ is produced to run every campaign $i \in I_s^-$ requiring s.

4.1.8.1. Sinks for the intermediate s produced by campaign $i \in I_s^+$.

The amount of intermediate s produced by campaign $i \in I_s^+$ can be supplied to one or several sth-consuming runs $i' \in I_s^-$ featuring $F_{ii'} > 0$. The continuous variable $F_{ii'}$ denotes the amount of material s provided by run i to campaign i'.

$$Q_i = \sum_{i' \in I_s^-} F_{ii'} \quad \forall i \in I_s^+,\ s \in S^I. \tag{9}$$

4.1.8.2. Sources of intermediate s for campaign $i \in I_s^-$.

The amount of intermediate s required by production run $i \in I_s^-$ is provided by those campaigns $i' \in I_s^+$ featuring $F_{i'i} > 0$.

$$\rho_{is} Q_i = \sum_{i' \in I_s^+} F_{i'i} \quad \forall i \in I_s^-,\ s \in S, \tag{10}$$

ρ_{si} is the amount of s consumed by unit size of run i.

4.1.9. Source/sink campaign matching conditions

If run $i \in I_s^+$ supplies intermediate s to campaign $i' \in I_s^-$, then the matching 0–1 variable $U_{ii'}$ is made equal to one and $F_{ii'} > 0$.

$$F_{ii'} \leq MU_{ii'} \quad \forall i \in I_s^+, i' \in I_s^-,\ s \in S. \tag{11}$$

In addition, the pair of timing conditions (12) and (13) given below are to be satisfied.

(i) A run $i \in I_s^+$ supplying material s to campaign $i' \in I_s^-$ should not start later than i'.

$$C_i - \sum_{j \in J_i} L_{ij} \leq C_{i'} - \sum_{j \in J_{i'}} L_{i'j} + H(1 - U_{ii'}) \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S. \tag{12}$$

Let us consider a campaign $i' \in I_s^-$ being supplied by a pair of production runs $(i, i'') \in I_s^+$, with run i starting before and run i'' beginning later than i'. Such a campaign i' is regarded by the model as being composed by two successive campaigns, the earlier one receiving intermediate s from campaign i and the other one from both i and i''.

(ii) A run $i \in I_s^+$ supplying intermediate s to campaign $i' \in I_s^-$ should never end later than i'.

$$C_i \leq C_{i'} + H(1 - U_{ii'}) \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S \tag{13}$$

Condition (13) can be omitted if the overall consumption rate of intermediate s by any set of parallel campaigns $i' \in I_s^-$ never surpasses the production rate of the source campaign $i \in I_s^+$. In any case, constraint (13) prevents from running out of intermediate s while executing any sink campaign $i' \in I_s^-$.

4.1.10. Duration of storage tasks

The storage task for campaign $i \in I_s^+$ delivering part of its production to tank t ($W_{it} = 1$) is assumed to begin at the starting time of run i (IT_i). Moreover, it will finish as soon as the latest campaign $i' \in I_s^-$ receiving intermediate s from run i ($U_{ii'} = 1$) has been completed. Then,

$$IT_i = C_i - \sum_{j \in J_s} L_{ij} \quad \forall i \in I_s^+,\ s \in S^I, \tag{14}$$

$$CT_i \geq C_{i'} - H(1 - U_{ii'}) \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S. \tag{15}$$

Contrarily, Ierapetritou and Floudas (1998) assumed that the storage task can end at the start of the related consuming runs, e.g. a very high consuming rate by the sink campaigns $i' \in I_s^-$. Their formulation included some constraints enforcing that the end times of storage tasks should be greater than or equal to the starting time of any associated consumption task. By doing so, the storage capacity constraints are relaxed and non-feasible production schedules featuring profits even larger than the optimal one may be generated. Fortunately, the proposed MILP formulation does not rely on such an assumption.

4.1.11. Storage task sequencing constraints

The storage task for run $i' \in I_s^+$ in tank t ($W_{i't} = 1$) cannot be started until any precedence storage task i has been completed. However, no further sequencing variable for the storage stage is usually required. The reason is rather simple. If both campaigns $\{i, i'\}$ were run in the same unit $j \in J_{ii'}$ at the previous production stage and run i was accomplished before, then $X_{ii'} = 1$ and run i is also stored before in tank t. Consequently, the same sequencing variable $X_{ii'}$ can be re-used to denote the task sequencing at the next storage stage. If campaigns $\{i, i'\}$ are instead run in different units but both are still assigned to tank t and run i is stored before, then $X_{ii'}$ is made equal to one just for the storage stage since its value is meaningless for the precedence production stage. In short, whenever the runs $\{i, i'\}$ are assigned to the same tank t ($W_{it} = W_{i't} = 1$), then campaign i is stored either before ($X_{ii'} = 1$) or after ($X_{ii'} = 0$) campaign i' in tank t. If $X_{ii'}$ is equal to one, then Eq. (16) applies to sequence the storage tasks and Eq. (17) becomes redundant. The reverse situation arises if $X_{ii'} = 0$; i.e. Eq. (17) is enforced while Eq. (16) becomes redundant. In this way, it is very likely to have no further requirement of binary variables to sequence the storage tasks, thus achieving a significant saving in 0–1 variables. Additional 0–1 variables for sequencing storage tasks will be necessary only if a pair of runs $\{i, i'\}$ with $J_{ii'} = \varnothing$ can share a storage tank. Moreover, constraints (16)–(17) account for sequence-dependent transition times $\sigma_{ii'}$ between any pair of successive campaigns $\{i, i'\}$ assigned to the same tank t.

$$IT_{i'} \geq CT_i + \sigma_{ii't} - H(1 - X_{ii'}) - H(2 - W_{it} - W_{i't}) \quad \forall i, i' \in I,\ i < i',\ t \in T_{ii'}, \tag{16}$$

$$IT_i \geq CT_{i'} + \sigma_{i'it} - HX_{ii'} - H(2 - W_{it} - W_{i't}) \quad \forall i, i' \in I,\ i < i',\ t \in T_{ii'}. \tag{17}$$

Values for the starting (IT_i) and the completion time (CT_i) of the storage task for campaign i in tank t have no meaning at all unless the storage task does really exist and consequently the tank t has been assigned to run i. In fact, the storage tasks for the source campaigns $\{i, i'\}$ in tank t are sequenced by constraints (16)–(17) involving variables $\{IT_i, IT_{i'}, CT_i, CT_{i'}\}$ only if tank t has been really assigned to both runs ($W_{it} = 1$, $W_{i't} = 1$). Otherwise, CT_i or $CT_{i'}$ can take any value and therefore Eq. (15) becomes redundant.

4.1.12. Storage capacity constraints

Since the tanks for intermediates have finite capacities, then the net amount of material stored in a tank t can never exceed its capacity v_t. Assuming that the rate of production of intermediate s by campaign $i \in I_s^+$ is greater than the overall consumption rate from runs $i' \in I_s^-$ featuring $F_{ii'} > 0$, then the capacity constraints are just to be enforced at the completion time of campaign $i(C_i)$ producing s. To establish the amount of material in the tank assigned to run $i \in I_s^+$ at time C_i, it should first be known the sth-consuming runs $i' \in I_s^-$ starting after C_i. To this end, a new binary variable $Z_{ii'}$ is to be defined denoting that campaign i' starts after C_i whenever $Z_{ii'} = 1$.

4.1.12.1. Definition of the variable $Z_{ii'}$.

$$C_{i'} - \sum_{j \in J_{i'}} L_{i'j} - C_i \leq HZ_{ii'} \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S \tag{18}$$

4.1.12.2. A bound on the amount of intermediate $V_{ii'}$ transferred from $i \in I_s^+$ to $i' \in I_s^-$ at the completion time of run $i(C_i)$. As stated by Eq. (19), the amount of intermediate $V_{ii'}$ transferred from $i \in I_s^+$ to $i' \in I_s^-$ at C_i should be equal zero if the consuming campaign i' starts later than C_i ($Z_{ii'} = 1$). Moreover, $V_{ii'}$ should also be equal to zero if there is no match between runs $\{i, i'\}$. In such a case, $U_{ii'} = 0$ and, consequently, $F_{ii'} = 0$ (Eq. (20)). Otherwise, $V_{ii'}$ may be finite but it never exceed the total amount of material supplied by the production run i to the consuming run $i'(F_{ii'})$ as specified by Eq. (20). Furthermore, Eq. (21) provides a conservative bound on the value of $V_{ii'}$ in case $U_{ii'} = 1$ and run i' is still running at $C_i(Z_{ii'} = 0)$. By underestimating the value of $V_{ii'}$, the inventory of state s at time C_i and the duration of the related storage task are, in the worst case, both overestimated, thus preventing from storage infeasibilities. However, the activation of unnecessary storage tasks is not expected.

$$V_{ii'} \leq M(1 - Z_{ii'}) \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S, \tag{19}$$

$$V_{ii'} \leq F_{ii'} \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S, \tag{20}$$

$$V_{ii'} \leq \rho_{i's}\, r_{i'j}^{\min} \left(C_i - C_{i'} + \sum_{i \in Ji'} L_{i'j} \right) + MZ_{ii'} + M(1 - Y_{i'j}) \quad \forall i \in I_s^+,\ i' \in I_s^-,\ s \in S,\ j \in J_{i'}. \tag{21}$$

4.1.12.3. The amount of material s stored in tank t at the completion time of run $i \in I_s^+$ should be less than the tank capacity v_t. The LHS of constraint (22) gives the amount of material produced by run $i \in I_s^+$ not yet transferred to the matching campaigns $i' \in I_s^-$ at C_i.

$$Q_i - \sum_{i' \in I_s^-} V_{ii'} \leq \sum_{t \in T_s} v_t W_{it} \quad \forall i \in I_s^+,\ s \in S \tag{22}$$

Eq. (22) forces to assign a tank $t \in T_i$ to run i only if an inventory of intermediate produced by i is to be stored ($Q_i > \Sigma\, V_{ii'}$). In the more general case, one cannot guarantee that the rate of production will be greater than or equal to the overall consumption rate for any state s. Then, the capacity constraints must also be enforced at the starting time of every sink campaign $i' \in I_s^-$. To do that, a similar treatment should be made to establish the amount of material i in the tank assigned to run $i \in I_s^+$ at the starting time of campaign $i' \in I_s^-$, i.e. at time $(C_i - \Sigma_{j \in J_{i'}} L_{i'j})$.

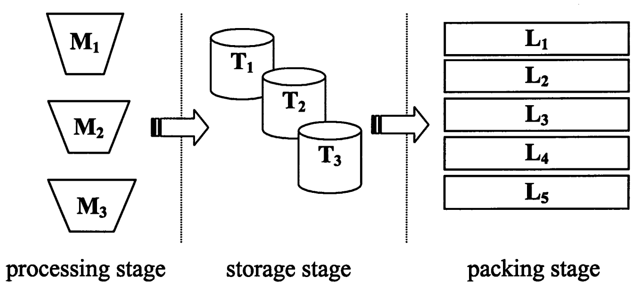

Fig. 1. Schematic representation of the plant.

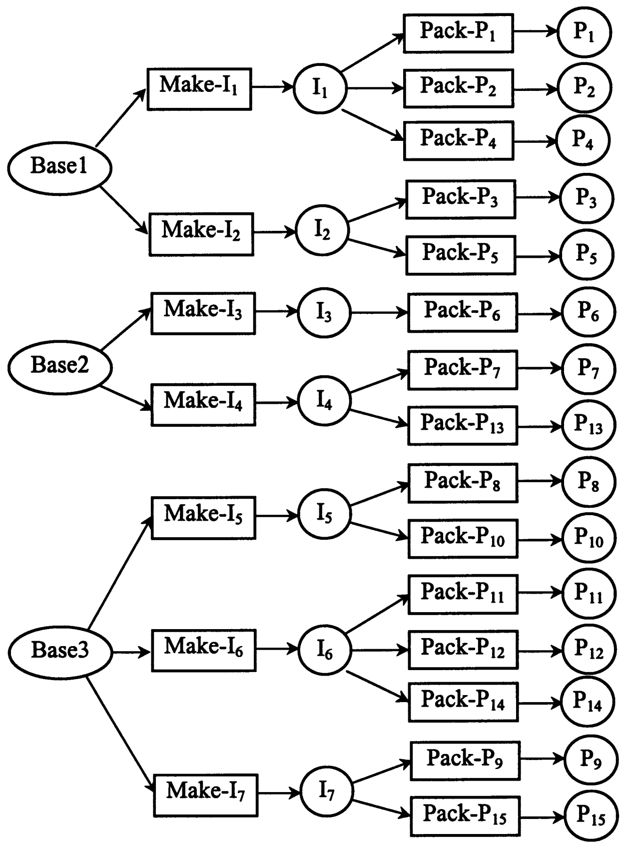

Fig. 2. State-task network representation of the plant.

4.2. Objective function

The problem objective is to maximize the economic return from production sales, whereas meeting either the constraint set (1)–(13) for the UIS case or the constraint set (1)–(22) for the FIS case.

$$\max \sum_{s \in S^P} p_s \sum_{i \in I_s^+} Q_i. \tag{23}$$

However, different scheduling targets can alternatively be considered such as the minimum makespan.

$$MK \geq C_i \quad \forall i \in I_s^+,\ s \in S^P, \tag{24}$$

$$\min \quad \mathrm{MK}. \tag{25}$$

5. Results and discussion

The proposed MILP scheduling methodology has been applied to a couple of medium-size case studies both based on an industrial fast moving consumer goods manufacturing plant (Schilling & Pantelides, 1996). Fifteen final products are manufactured in the facility following a common production sequence: mixing, storage and packing (Fig. 1). The mixing stage comprises three parallel mixers operating in a continuous mode and producing seven intermediates from three different base materials available as required. Such intermediates are then stored in three storage tanks or directly packed in five continuous packing lines. The STN representation of the plant is shown in Fig. 2. Problem data are given in Tables 1 and 2. Case I assumes an unlimited intermediate storage (UIS policy), i.e. storage capacity constraints are ignored. In turn, limited capacities of storage tanks (FIS policy) are taken into account in Case II. Assuming a single campaign per product at any available unit, the optimal schedules for Cases I and II are depicted in Figs. 3 and 4, respectively. They were found by using ILOG OPL Studio 2.1 with the embedded CPLEX Mixed-Integer Optimizer 6.5.2 release (Ilog, 1999). Additional information on the optimal schedules is shown in Tables 3 and 4.

Table 1
Equipment data

Units	Rate/capacity (ton/h or ton)	Suitability	Change-over requirements
M_1	17.00	I_1, I_2	–
M_2, M_3	12.24	I_5, I_6, I_7	–
	17.00	I_3, I_4	–
T_1,T_2,T_3	60	Store all intermediates	–
L_1	5.8333	P_2, P_3, P_7	Between {P_2, P_3} and {P_7}—1 h
L_2	2.7083	P_4, P_5, P_8, P_9	Between {P_4, P_5} and {P_8, P_9}—4 h
L_3	5.5714	P_1, P_6	Between {P_1} and {P_6}—1 h
L_4	2.2410,	P_{12}, P_{13},	Between {P_{12}, P_{13}} and {P_{14}, P_{15}}—2 h
	3.3333	P_{14}, P_{15}	
L_5	5.3571	P_{10}, P_{11}	–

Table 2
Minimum production requirements

Product	Demand (ton)
P_1	220
P_2	251
P_3	15
P_4	116
P_5	7
P_6	47
P_7	144
P_8	42.5
P_9	13.5
P_{10}	114.5
P_{11}	53
P_{12}	16.5
P_{13}	8.5
P_{14}	2.5
P_{15}	17.5

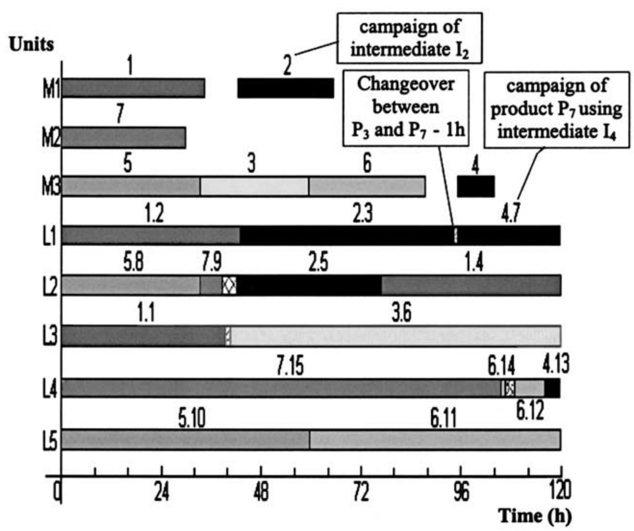

Fig. 3. Optimal schedule with UIS policy.

For the UIS case, the proposed problem formulation includes 28 sequencing variables and 10 assignment variables, i.e. a total of 38 binary variables. In addition, 44 continuous variables denoting campaign lengths (22) and completion times (22) are to be defined. Case I provides an upper bound on the feasible economic return since the packing lines can be run at full capacity.

For the FIS case, the number of binary variables rises to 84 as long as 10 storage task sequencing variables, 21 state-tank assignment variables and 15 $Z_{ii'}$-variables are to be incorporated. In both cases, the model sizes are well below those reported by other authors and the optimal solutions found are much better. Table 5 compares the size of the proposed MILP formulation, the optimal objective value and the computational requirements when applied to Cases I and II with results reported in previous work. It is observed: (a) a one-order-of magnitude reduction in binary variables and constraints; (b) an almost two-order-of magnitude reduction in continuous variables; (c) an improvement in the optimal solution found; and (d) a significant saving in CPU time requirements. Similarly to Schilling and Pantelides (1996), Ierapetritou and Floudas (1998), the LP relaxation for the FIS Case was 2724. When two campaigns for each state can be run at each unit, then the number of sequencing and assignment variables increase to 112 and 20, respectively.

Since the tank capacities are relatively small (60 tons) compared with the amounts of intermediates to be produced ranging from 152 to 587 tons, the FIS scheduling problem becomes hardly restrained by the tank limited capacities and, consequently, rather short campaigns appear as better options. Despite that, the proposed production schedule comprising a single campaign per product yields an economic return of 2670.28

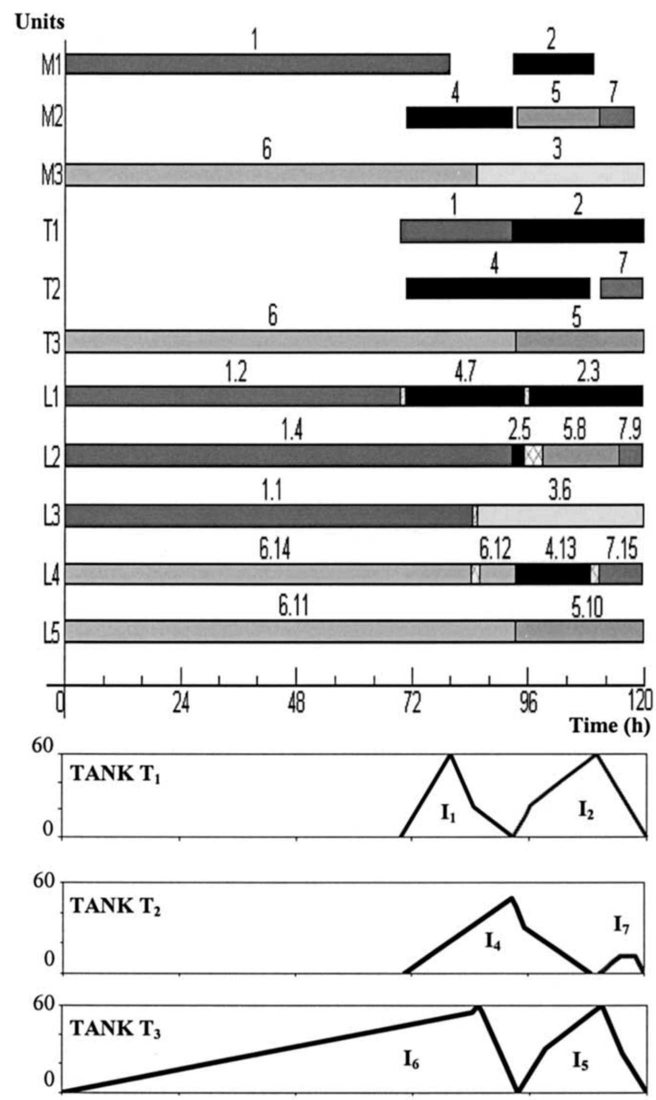

Fig. 4. Optimal schedule with FIS policy and storage profile of intermediates.

Table 3
Optimal production schedule for Case I

Unit	State	Starting time (h)	Completion time (h)	Production (ton)
Mixers				
M_1	I_1	0.00	34.53	587.00
	I_2	42.42	65.56	393.26
M_2	I_7	0.00	30.00	367.14
M_3	I_5	0.00	33.44	409.31
	I_3	33.44	59.50	443.00
	I_6	59.50	87.53	343.11
	I_4	95.31	104.28	152.50
Packing lines				
L_1	P_2	0.00	43.03	251.00
	P_3	43.03	94.31	299.16
	P_7	95.31	120.00	144.00
L_2	P_8	0.00	33.44	90.57
	P_9	33.44	38.42	13.50
	P_5	42.42	77.17	94.10
	P_4	77.17	120.00	116.00
L_3	P_1	0.00	39.49	220.00
	P_6	40.49	120.00	443.00
L_4	P_{15}	0.00	106.09	353.64
	P_{14}	106.09	106.84	2.50
	P_{12}	108.84	116.21	16.50
	P_{13}	116.21	120.00	8.50
L_5	P_{10}	0.00	59.50	318.74
	P_{11}	59.50	120.00	324.11

slightly lower than the optimal one for Case I, i.e. it decreases by only 0.93%. Though two campaigns can at most be run for producing intermediates I3–I7 at mixers M2–M3, a single one for each intermediate is merely accomplished at the optimal solution. So, there is no economic incentive to run additional campaigns.

Compared with prior approaches, the number of campaigns (22) and the number of changeovers (6) both drastically drop. Full utilization of the production capacity through longer campaigns is achieved by running parallel campaigns in the packing lines all of them consuming the intermediate being at the time produced in the mixers. This is especially true when the intermediate production rate is relatively high. Indeed, the optimal schedule includes several source and sink campaigns of a particular intermediate s starting at the same time. For instance, Fig. 4 indicates that the source campaign producing I3 in mixer M3 and the sink campaign consuming I3 to produce P6 in the packing line L3 both start at the same and no storage tank for I3 is required. Despite constraint (22) may give rise to shorter campaigns and storage bottlenecks, the proposed approach generates a feasible schedule more profitable than the best one reported by Zhang and Sargent (1998). Fig. 4 also shows the change of the intermediate inventory with time as well as the sequence of intermediates assigned to every storage tank. Maximum allowable intermediate inventory of 60 tons is reached four times over the time horizon for intermediates {I1, I2, I5, I6}, but in any case it immediately decreases.

The best schedule reported by Zhang and Sargent (1998) for the same example involved a total of 204 campaigns, 77 for the production of intermediates and 127 for final products, and 14 changeovers. Despite the runs are obviously much shorter, the maximum allowable intermediate inventory of 60 tons is reached five times and, in each case, the tank remains full during some finite interval. Therefore, constraint (22) seems to be working properly. In turn, Ierapetritou and Floudas (1998) using an improved approximation of the storage timings found an optimal solution for the FIS case that yielded a profit of 2695.25 and comprised 40 campaigns, 15 for intermediates and 25 for final products. Curiously, such a profit is even larger than the maximum one reported by the authors for the UIS case.

Table 4
Optimal production schedule for Case II

Unit	State	Starting time (h)	Completion time (h)	Production (ton)
Mixers				
M_1	I_1	0.00	79.94	1128.22
	I_2	92.74	109.71	145.00
M_2	I_4	70.66	92.73	178.23
	I_5	93.61	110.89	183.87
	I_7	110.89	118.15	43.88
M_3	I_6	0.00	85.49	798.82
	I_3	85.49	120.00	192.26
Packing lines				
L_1	P_2	0.00	69.66	406.33
	P_7	70.66	95.34	144.00
	P_3	96.34	120.00	138.00
L_2	P_4	0.00	92.74	251.16
	P_5	92.74	95.32	7.00
	P_8	99.32	115.02	42.50
	P_9	115.02	120.00	13.50
L_3	P_1	0.00	84.49	470.73
	P_6	85.49	120.00	192.26
L_4	P_{14}	0.00	84.25	280.83
	P_{12}	86.25	93.61	16.50
	P_{13}	93.61	108.89	34.23
	P_{15}	110.89	120.00	30.38
L_5	P_{11}	0.00	93.61	501.49
	P_{10}	93.61	120.00	141.37

Table 5
Comparison of results

Example	Binary vars, cont. vars, rows	Objective function	CPU time
UIS policy (*Case I*)			
Schilling & Pantelides, 1996	1042, 2746, 4981	2604	3407
Ierapetritou & Floudas, 1998	280, 1089, 2873	2689.42	540
This approach	38, 44, 140	2695.32	4.77[a]
FIS policy (*Case II*)			
Zhang & Sargent, 1998	1318, 4555, 4801	2556	1085
Ierapetritou & Floudas, 1998[b]	360, 1337, 3260	2695.25	–
This approach	84, 73, 361	2670.28	398.92[a]

[a] Seconds on a Pentium II PC (400 MHz) with ILOG/CPLEX.
[b] With improved approximation of storage timings.

6. Conclusions

A highly computationally efficient continuous-time MILP algorithmic approach to the short-term scheduling of multiproduct facilities involving continuous processes has been presented. When applied to a medium-size fast moving consumer goods manufacturing plant, it provides a better production schedule through a simpler MILP formulation in a much lower CPU time than previous approaches.

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