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Middle term optimal control problem in eutrophic lakes through advanced mathematical programming approaches

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Abstract

In this work, we address lake biorestoration through the formulation of first principles models for water quality and advanced mathematical programming techniques. Dynamic mass balances have been formulated on main phytoplankton groups and nutrients, as well as other components available in lakes and reservoirs. Main parameters in the model have been estimated in previous work [1]. The combined reduction in nutrient loading and inlake biorestoration strategies to control algae growth and reduce eutrophication have been formulated as an optimal control problem for a middle term time horizon (three years). The problem has been formulated within a simultaneous optimization framework and solving the large scale nonlinear program with an Interior Point method [2]. Numerical results give deep insight on the quantitative application of restoration strategies

Keywords: eutrophication, optimal control, dynamic optimization

1. Introduction

Most water bodies in the world are becoming increasingly eutrophic due to anthropogenic activities. Eutrophication, characterized by enrichment of plant nutrients, is associated to excessive growth of phytoplankton expressed as algal blooms and loss of biodiversity. Main point sources for eutrophication are the discharges of agricultural, industrial and urban wastewater. Much research effort has been devoted in the chemical engineering community to address effluent treatment when dealing with point sources [3, 4, 5, 6]. Non-point sources, which are mainly related to agricultural activities, are more difficult to deal with and have received little attention regarding modeling aspects. However, sustainable strategies to control eutrophication in water bodies, such as nutrient loading reduction and the application of the food-chain theory through inlake biomanipulation, requires the evaluation of the global effects on the ecosystem. Experimental analysis of this process is time-consuming and expensive. Therefore, the development of first principles-based rigorous models for water quality provides deep understanding of biogeochemical processes that take place within water bodies, which is a fundamental step when addressing lake restoration. Sagehashi et al. [7], propose a water quality simulation model to study the long-term stability of the ecological system after biomanipulation in a hypothetical water ecosystem, through simulations. Krivtsov et al. [8] and Gurkan et al. [9] have applied simulation models to study the effects of two different restoration approaches, aeration of the hypolimnion and biomanipulation, to improve water quality in different lakes. Procopkin et al. [10] have studied the effect of fish removal on cyanobacteria biomass though an eutrophication simulation model.

In this work, we address lake restoration strategies through the formulation of first principles models for water quality combined with advanced mathematical programming techniques. We have formulated dynamic mass balances on main phytoplankton groups, nutrients, DO and BOD. The application of combined reduction in external nutrient loading to the lake and inlake biomanipulation strategies to control algae growth have been formulated as an optimal control problem for a time horizon of three years, considering two optimization variables corresponding to the fraction of tributary inflows through a wetland and the rate of fish removal along the time horizon. The problem has been formulated within a simultaneous optimization framework and solving the large scale nonlinear program with an Interior Point method [2]. Numerical results provide useful information on the quantitative application of restoration strategies in the middle term, as well as dynamic behavior of the main components in the water body.

2. Mathematical model for water quality in lakes and reservoirs

The application of restoration policies to improve the water quality requires modeling and optimization of major chemical and biological processes that take place within water bodies. The present case study is Paso de las Piedras Reservoir, which is located in the south of Buenos Aires Province (Argentina) at 38° 22´ S and 61° 12´ W. It was built in 1978 to supply drinking water to Bahía Blanca (population around 450,000) and for industrial purposes at a petrochemical complex. The lake has two tributaries, which run through an important agricultural area. This water body has a coastline perimeter of 60 km and a mean depth of 8.2 m, so it can be considered as a shallow lake. The high content of phosphorus and nitrogen in Paso de las Piedras Reservoir is consequence of agricultural activities. The trophic state of this water body is currently eutrophic. In previous work [11], we have addressed the application of restoration strategies throughout a time horizon of one year as an optimal control problem. However, the middle term effect of restoration is not well documented and has had different experimental results depending on the specific water bodies reported in the literature [12]. In this work, we have extended the time horizon in our dynamic optimization model, considering sinusoidal functions for representing input variables. A detailed description of the model equations can be found in Estrada et al. [1]. We have considered averaged horizontal compositions and gradients along the water height. The partial differential equations system has been discretized into two layers and main equations are:

Total mass balances

$$\frac{dh_T}{dt} = \frac{1}{A}\sum_{k=1}^{NIN} Q_{IN_k} - \frac{1}{A}\sum_{m=1}^{NOUT} Q_{OUT_m} + Q_{rain} - Q_{evap} \tag{1}$$

Component mass balances for horizontal layers *(U: upper layer, L: lower layer: j: Cyanobacteria, Diatoms, Chlorophytes, Nitrate, Ammonium, Organic Nitrogen, Phosphate, Organic Phosphorus, Biochemical Demand of Oxygen, Dissolved Oxygen)*

Upper layer

$$\frac{dC_{Uj}}{dt} = \sum_{k=1}^{NIN} \frac{Q_{IN_{U,k}}}{V_U} C_{IN_{Ujk}} - \sum_{m=1}^{NOUT} \frac{Q_{OUT_U}}{V_U} C_{Uj} + r_{Uj} - \frac{kdA}{\Delta h_U h_U}(C_{Uj} - C_{Lj}) - \frac{C_{Uj}}{h_U}\frac{dh_U}{dt} \tag{2}$$

Lower layer

$$\frac{dC_{Lj}}{dt} = \sum_{m=1}^{NOUT} \frac{Q_{OUT_L}}{V_L} C_{Lj} + r_{Lj} + \frac{kdA}{\Delta h_L h_L}(C_{Lj} - C_{U,j}) - \frac{C_{Lj}}{h_L}\frac{dh_L}{dt} \tag{3}$$

Phytoplankton: Rate equations for phytoplankton groups take into account production and losses due to respiration, natural death, settling and grazing.

$$r_{ij} = R_{ij,growth} - R_{ij,resp} - R_{ij,death} - R_{ij,settling} - R_{ij,graz} \tag{4}$$

$$i = UL, LL;\ j = cyano, diatom, chlorophyte$$

The growth rate of the three phytoplankton groups is a function of solar radiation, water temperature and nutrients availability.

$$R_{ij,growth} = k_{i,growth} f(T)_{ij} f(I)_{ij} f(N)_{ij} C_{ij} \tag{5}$$

$$f(T)_{ij} = -\frac{(T_j - T_{opt_i})^2}{T^2_{opt_i}} + 1 \tag{6}$$

$$f(I)_{ij} = \frac{I_{oi}}{I_{opt_j}} \exp\left(1 - \frac{I_{oi}}{I_{opt_j}}\right) \tag{7}$$

$$f(N)_{ij} = \frac{C_{PO4_j}}{C_{PO4_j} + K_{P_i}} \qquad i = UL, LL;\ j = cyano, diatom, chlorophyte \tag{8}$$

Phytoplankton respiration, natural death and settling rates are given as:

$$R_{ij,resp} = k_{j,resp}\,\theta_r^{(T-20)} C_{ij} \tag{9}$$

$$R_{ij,death} = k_{j,death}\,\theta_m^{(T-20)} C_{ij} \qquad i = UL, LL;\ j = cyano, diatom, chlorophyte \tag{10}$$

$$R_{ij,settling} = k_{j,settling}\frac{C_{ij}}{h_i} \qquad i = UL, LL;\ j = cyano, diatom, chlorophyte \tag{11}$$

The herbivorous zooplankton grazing rate is:

$$R_{ij,graz} = k_{j,graz} \frac{C_{ij}}{C_{ij} + K_{graz}} C_{Zoo_i} \quad i = UL, LL;\ j = cyano, diatom, chlorophyte \tag{12}$$

Nutrients cycle in water bodies

Phosphorus cycle: State variables describing phosphorus cycle are phosphate and organic phosphorus. Phosphorus is uptaken by phytoplankton in phosphate form. As phytoplankton biomass is composed of carbon, nitrogen and phosphorus, upon death, phytoplankton increases both phosphate and organic phosphorus pool. This latter goes through a mineralization reaction to phosphate, which is then available for phytoplankton generation again.

Nitrogen cycle: Three state variables describe nitrogen cycle in the present model: ammonium, nitrate and organic nitrogen. Phytoplankton are able to uptake both ammonium and nitrate for growth. Ammonium is oxidized to nitrate and its concentration increases by organic nitrogen hydrolysis at a temperature-dependent mineralization rate. Due to internal nutrient recycle, reducing the external loading of nutrients to the lake is not enough for restoration. Detailed equations describing the described processes are presented in [1].

3. Optimal control problem and optimization algorithm

In this work, we formulate the application of inlake restoration strategies together with reduction of external loading of nutrients as an optimal control problem along a middle term horizon of three years, on a daily basis. Shapiro [13] suggested the term biomanipulation to refer to lake restoration techniques that are based on the food chain theory. This method is supported on a theory of top-down control on lakes. The basic idea is to perform zooplanktivorous fish removal or piscivorous fish stocking, or a combination of both, to keep a high grazing pressure on the phytoplankton community by the herbivore zooplankton [14]. In this work, we have considered the application of biomanipulation by fish removal, considering zooplankton concentration as the optimization variable. This variable can be later associated to the rate of fish removal, based on fish biomass data from the specific lake. The objective function is the minimization of the offset between phytoplankton concentration and a tight desired value of 0.10 mg/l throughout the time horizon.

$$min \int_0^{tf} \left(\sum_{j=phyto} C_j(t), - 0.10 \right)^2 dt \tag{13}$$

st

DAE eutrophication model

$0 \le F_{WETLAND} \le 0.5 F_{DIVISORIO} (l/d)$

$0.01 \le C_{zoo} \le 1 (mg/l)$

The dynamic optimization problem is formulated within a simultaneous dynamic optimization framework by transforming it into a large-scale nonlinear program (NLP). Both state and control variables are represented as piecewise polynomials over finite elements in time and the differential-algebraic system is discretized by collocation on finite elements. We have used an Interior point method within program IPOPT [15] with SQP techniques for solving the large-scale NLP. Extensions of this approach are described in Raghunathan et al. [16], Kameswaran and Biegler [17] and Zabala and Biegler [18].

4. Discussion of results

The DAE eutrophication model for Paso de las Piedras Reservoir has twenty one differential equations (plus an additional one for the objective function) and sixty algebraic ones, after spatial discretization into two layers. A time horizon of three years has been considered, on a daily basis. Input variables have been represented with sinusoidal functions, based on observed data for one year. The optimal control problem has two optimization variables corresponding to the fraction of inlet stream that is derived to a nearby wetland and the concentration of zooplankton in the lake to control phytoplankton growth, along the time horizon. The resulting nonlinear programming (NLP) problem for temporal discretization with eighty finite elements and two collocation points with two optimization variables has 15383 nonlinear equations. It has been solved with program IPOPT [2], which implements an Interior Point method with reduced Successive Quadratic Programming.

Numerical results show that tributary deviation through a wetland for nutrient loading reduction is required throughout the entire time horizon, at its maximum allowed flowrate (50% of the inflows, with 50% nutrient retention). Figure 1 shows cyanobacteria concentration profiles before and after biomanipulation. It can be seen that the peak in the first year is reduced to 50% of its value without restoration, but still remains above the desired concentration (0.20 mg/l). However, during the following two years the peak in concentration is below the desired value. Figure 1 also shows that zooplankton concentration (Eq. 15) is required to be at its maximum value during the first year, with small increases before cyanobacteria peaks in the following years. Figure 2 shows phosphorus concentration before and after biomanipulation. It can be seen that nutrient loading reduction requires even longer time horizons to show noticeable effects on concentration due to nutrients recycle

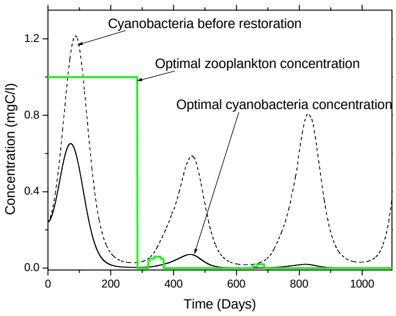

Figure 2. Comparison between cyanobacteria concentration profiles before and after restoration and optimal profile for zooplankton conc.

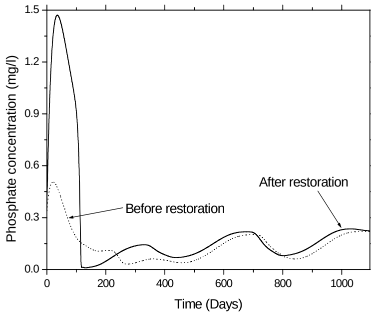

Figure 3. Main nutrient concentration profiles before and after restoration

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6. Conclusions

In this work, we have formulated an optimal control problem to plan middle term restoration within a water body and analyze the ecosystem dynamic behavior under these strategies. Process systems engineering techniques have proved to be valuable tools for addressing control of ecological systems.

7. References

[1] V. Estrada, E. R. Parodi and M. S. Diaz, Comp. & Chem. Eng., (2008a) submited.
[2] L. T. Biegler, Cervantes, A. and Waechter, A., Chem. Eng. Sci., 57 (2002) 575.
[3] M. Hostrup, P. M. Harper and R. Gani, Comp. & Chem. Eng., 23 (1999) 1395.
[4] B. Chachuat, N. Roche and M. A. Latifi, Comp. & Chem. Eng., 25 (2001) 585.
[5] F. M. San Ramón, E. Bringas, I. Ortiz and I. E. Grossmann, Comp. & Chem. Eng., 31 (2007) 456.
[6] R. Karuppiah and I. E. Grossmann, Comp. & Chem. Eng., 32 (2008) 145.
[7] M. Sagehashi, A. Sakoda and M. Susuki, Wat. Res., 35 (2001) 1675.
[8] V. Krivtsov, C. Goldspink, D. C. Sigee and E. G. Bellionger, Ecol. Model., 138 (2001) 153.
[9] Z. Gurkan, J. Zhang and S. E. Jørgensen, Ecol. Model., 197 (2006) 89.
[10] I. G. Prokopkin, V. G. Gubanov and V. G. Gladishev, Ecol. Model., 190 (2006) 419.
[11] V. Estrada, E. R. Parodi and M. S. Diaz, Comp. & Chem. Eng., (2008b) submited.
[12] E. Jeppesen, Ma. Søndergaard, Mo. Søndergaard and K. Christoffersen (eds.), Ecological studies, Springer, New York, 1997.
[13] P.L. Brezonik and J.L. Fox (eds.), Water Quality Management through Biological Control. Department of Environmental Engineering Science, Univ. Florida, Gainesville, 1975.
[14] J. Shapiro and D. I. Wright, Freshwater biol., 14 (1984) 371.
[15] A. M. Cervantes, A. Waechter, R. Tutuncu and L. T Biegler, Comp. & Chem. Eng., 24 (2000) 39.
[16] A. Raghunathan, M. S. Diaz and L. T. Biegler, Comp. & Chem. Eng., 28 (2004) 2037.
[17] S. Kameswaran and L. T. Biegler, Comp. & Chem. Eng., 30 (2006) 1560.
[18] V. Zabala and L.T. Biegler, Comp. & Chem. Eng., (2008) in press.