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A new orbit for comet C/1858 L1 (Donati)

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A new orbit for comet C/1858 L1 (Donati), based on 1036 observations in α and 971 in δ made between 7 June 1858 and 5 March 1859, is calculated using iteratively reweighted least squares. Residuals were weighted by the Welsch weighting function. The orbit represents a high eccentricity ellipse, $e = 0.996265$, with large semi-major axis, $a = 154.8612$ AU, and long period, $P = 1927.22$ yr. The residuals are relatively random, a 10.7 % chance of being random, but with a slight indication of *possible* nongravitational forces influencing the motion. The comet will not return until the year 3759, when it will pass 0.8442 AU from the Earth.

1 Introduction

Comet C/1858 L1 (Donati), which Giovanni Donati discovered in Florence on 2 June 1858 (Donati 1858), became the second brightest comet of the 19th century and aroused much popular and professional interest, resulting in over 2000 observations in both coordinates being made. The Cape Observatory obtained the last observation on 5 March 1859. The comet, therefore, was observed for over nine months.

The current orbit for this comet, in the Marsden & Williams catalog (2003) for example, is based on a thorough study that G.W. Hill performed in 1867 (Hill 1867). Given Hill's reputation as a gifted mathematician it may seem superfluous, or even impertinent, to re-examine the orbit, particularly when one realizes that Hill's analysis was meticulous. The various reference stars used are, when possible, reduced to a common system and a search made for systematic errors among the data contributed by the numerous observatories.

There exist, nevertheless, sound reasons to re-examine the orbit. Hill, like all practitioners of numerical mathematics in the 19th century, was constrained by the limitations of contemporary computing. To become tractable observations had to be formed into normal places to reduce the size of the linear system to be solved, a computational expedient no longer required or even desired. Modern statistical techniques such as tests for the randomness of the residuals and the determinacy of linear systems were nonexistent. Hill seems to have overlooked some observations, such as a possible observation made in Athens and a series made in Mussoorie, India. Finally, Hill remarks that "… there is not the slightest indication that any other force than gravity influenced the motion of the centre of gravity of the comet". Yet his own table of comparison of the observations with the computed solution shows an excess of negative values. This may, in fact, indicate that some nongravitational force is at work. The matter will be discussed later.

For these reasons I felt that a recalculation of the orbit becomes, if not a strictly necessary task, at least a reasonable one. One may then assert that the new orbit has been calculated with modern techniques. Whether it is superior or not depends on statistical tests.

2 The observations

Hill (1867) published references to all of the observations of Donati's comet that he could find. I will not repeat his list, to which I refer the reader for exact references, but should mention that he seems to have overlooked a possible observation made at the Athens observatory (Schmidt 1860). I write "possible" because Schmidt does not in fact state that the comet is Donati's but rather that he observed a large comet ("sieht man … einen grossen Cometen") that resembles Donati's comet ("… dem grossen Cometen von Donati glich er völlig"). One must verify, therefore, if this observation is in fact of Donati's comet. Hill also seems to have overlooked a series of observations made at Mussoorie, India (Tennant 1859). Nor does he include observations made at Williamstown, Australia, (Ellery 1859) because "the latitude and longitude of the place are uncertain". This comment seems strange because Ellery does in fact given the geographical coordinates of this obscure observatory, and I have included his observations. But I do agree with Hill that the Dorpat, Russia, observations are unusable because they are given in the form of instrumental measurements with no way to reduce them to right ascension (α) and declination (δ). This is unfortunate because the series is extensive. Nor could Donati's discovery observation be used (Donati 1858)

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Table 1 Observations of Donati's comet.

Observatory	Obs. in α	Obs. in δ
Williamstown, Australia	9	9
Kremsmünster, Austria	43	43
Vienna, Austria	185	162
Santiago, Chile	76	65
Copenhagen, Denmark	25	23
Cambridge, England	23	23
Durham, England	14	11
Greenwich, England	8	8
Liverpool, England	27	27
Paris, France	15	15
Altona, Germany	5	5
Berlin, Germany	25	25
Bonn,Germany	14	14
Breslau, Germany	5	5
Göttingen, Germany	22	19
Königsberg, Germany	22	22
Athens, Greece	1	1
Mussoorie, India	6	6
Batavia, Indonesia	6	6
Markree Castle, Ireland	10	10
Florence, Italy	8	8
Padua, Italy	3	3
Leiden, Netherlands	30	20
Armagh, Northern Ireland	16	16
Christiana, Norway	37	37
Pulkova, Russia	14	14
Cape of Good Hope, South Africa	234	221
Geneva, Switzerland	46	46
Ann Arbor, Mich., USA	33	33
Cambridge, Mass., USA	20	20
Old U.S. Naval Observatory, USA	54	54
Total	1036	971

because the position is only approximate. The Vienna observations are given in various volumes of the Astronomische Nachrichten as averages of individual measurements. Rather than use the averages I have opted for the individual measurements (Hornstein 1859). Some observations were made with a sextant by conscientious officers on sea duty. These observations were not even considered, with apologies to the officers concerned, because a previous study of mine (Branham 2005) shows that the sextant simply lacks precision for quality astrometry.

Altogether 2007 observations were collected, 1036 in α and 971 in δ, higher than Hill's tally of 1819, although much of the difference comes from using the individual rather than the averaged Vienna observations. Nearly 97 % of these observations were taken from the ADS data base (http://adswww.harvard.edu/). Only the Copenhagen, Markree Castle and nine of the Ann Arbor observations could not be found in the ADS. Fortunately, Hill lists all of the observations he used, including those from these observatories, which I therefore used, changing the meridian from Paris to Greenwich and α from circular to time measure.

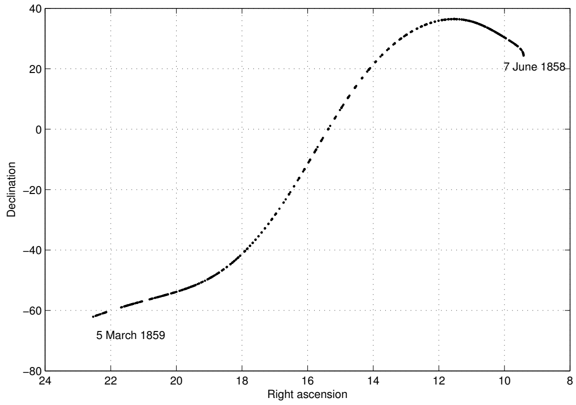

Fig. 1 The observations of Donati's comet.

Figure 1 shows the observations and Table 1 gives the contribution of each observatory to the total. Whenever a specific reference star was given to which the comet observation had been referred–this is true for the majority of the observations–its position was recalculated, with modern positions taken from the Tycho-2 catalog (Høg et al. 2000), using the algorithm in Kaplan et al. (1989). If differences in α and δ from the reference star, $\Delta\alpha$ and $\Delta\delta$, were given, they were applied, corrected for differential aberration and refraction, to the new position. If $\Delta\alpha$ and $\Delta\delta$ were not given, the differences in the positions between the older catalog and Tycho-2 were applied to the published positions of the comet; this includes, when he mentions a specific reference star, the Copenhagen, Markree Castle, and nine Ann Arbor observations from Hill's list . Observations for which no reference star is given were taken as published. Because the observations are 19th century, they were corrected for the E-terms of the aberration if the observation was a mean position. See Scott (1964) for a discussion of the E-terms.

Processing 19th century observations presents difficulties and becomes far from trivial. The observations are published in different languages, English, French, German, Italian, even Latin, do not conform to a standard format, and contain many errors. Rather than discuss these matters I refer the reader to an article of mine that goes into the details (Branham 2011). It suffices to say that the observations were reduced to the common format of: Julian Day (JD), referred to Terrestrial Time (TT), right ascension, and declination. This was necessary because some of the observers used north polar distance (NPD) instead of δ, some expressed α in degrees, minutes, and seconds rather than hours, minutes and seconds, most, with the exception of the English observers, used mean time of place rather than Greenwich for the time of the observation, and some observers recorded the time as sidereal rather than mean.

Table 2 list some errors for Donati's comet that could be identified and corrected along with reference stars previously unidentified that could be identified. The corrections are based on the precepts given for processing 19th century observations in Branham (2011). Further errors may possi-

Table 2 Errors and missing information for Donati's comet.

Observatory	Reference[1]	Date or Ref. Star No.	Error or Missing Data
Santiago	AJ, 6, 100	3 Nov. 1858 $10^{\rm h}16^{\rm m}54\overset{\rm s}{.}2$	$\Delta\alpha = -04^{\rm m}06\overset{\rm s}{.}63$
Santiago	AJ, 6, 102	38	NPD $= 144°09'49\overset{''}{.}91$
Cambridge	AN, 50, 243/244	30 Sept. 1858 $07^{\rm h}34^{\rm m}13\overset{\rm s}{.}7$	$\Delta\alpha = -00^{\rm m}19\overset{\rm s}{.}38$
Berlin	AN, 51, 65/66	29 June 1858 $10^{\rm h}48^{\rm m}16^{\rm s}$	$\Delta\delta = -22'48\overset{''}{.}8$
Breslau	AN, 50, 37/38	16 Oct. 1858 $06^{\rm h}32^{\rm m}40^{\rm s}$	$\delta = -16°08'13\overset{''}{.}0$
Göttingen	AN, 49, 235/236	16 Oct. 1858 $06^{\rm h}19^{\rm m}23\overset{\rm s}{.}4$	$\Delta\alpha = +00^{\rm m}35\overset{''}{.}41$
Göttingen	AN, 49, 237/238	h	Tycho-2 0917 01472 1
Leiden	AN, 50, 157/158	f	Tycho-2 2525 01047 1
Leiden	AN, 50, 157/158	k	Tycho-2 1996 01806 1
Leiden	AN, 50, 157/158	p misidentified	Tycho-2 5016 00787 1
Christiana	AN, 52, 277/278	All dates	Year is 1858
Christiana	AN, 52, 279/280	γ	Tycho-2 5030 00236 1
Pulkova	AN, 50, 307/308	11 Sept. 1858 $11^{\rm h}44^{\rm m}44\overset{''}{.}6$	$\alpha = +35°57'59\overset{''}{.}8$
Pulkova	AN, 50, 307/308	5 Oct. 1858 $07^{\rm h}06^{\rm m}13\overset{\rm s}{.}8$	Ref. star unidentifiable
Cape	Mem. RAS, 29, 59-83	22 Dec. 1858 $08^{\rm h}58^{\rm m}03\overset{\rm s}{.}5$	ΔNPD $= -10'31\overset{''}{.}16$
Cape	Mem. RAS, 29, 59-83	22 Dec. 1858 $09^{\rm h}28^{\rm m}16\overset{\rm s}{.}5$	ΔNPD $= -10'23\overset{''}{.}15$
Cape	Mem. RAS, 29, 59-83	78	NPD $= 144°09'49\overset{''}{.}94$
Geneva	AN, 50, 21/22	3 Oct. 1858 $07^{\rm h}14^{\rm m}13^{\rm s}$	$\delta = +24°32'22\overset{''}{.}5$
Geneva	AN, 50, 23/24	e'	Tycho-2 0346 00382 1
Cambridge (US)	AN, 51, 273/274	25 Sept. 1858 $06^{\rm h}50^{\rm m}19^{\rm s}$	$\Delta\alpha = +00^{m}28\overset{\rm s}{.}72$
Cambridge (US)	AN, 51, 277/278	k Baily	$\delta = +32°07'10\overset{''}{.}2$
Nav. Obs.	AJ, 5, 159	1	Tycho-2 1967 01030 1
Nav. Obs.	AJ, 5, 159	2	Tycho-2 1967 01059 1
Nav. Obs.	AJ, 5, 159	3	Tycho-2 2510 00211 1
Nav. Obs.	AJ, 5, 159	4	Tycho-2 2515 00054 1
Nav. Obs.	AJ, 5, 166	8 (really 5)	Tycho-2 1996 01251 1
Nav. Obs.	AJ, 5, 166	6	Unidentifiable
Nav. Obs.	AJ, 5, 166	7	Tycho-2 1472 00449 1

[1] AJ: Astronomical Journal; AN: Astronomische Nachrichten; Mem. RAS: Memoirs Royal Astron. Society.

bly exist, although they would be hard to find, because, as Hill himself states, "The typographical errors to be met with are so numerous I cannot undertake to mention them." He in fact gives no list of the errors found.

Some large errors in observed minus computed position, $(O-C)$, could not be corrected. The Athens observation has such a large $(O-C)$ that the comet is obviously not Donati's. Both the Williamstown and Mussoorie observations have such large $(O-C)$'s that something is evidently wrong. The Williamstown observations are given in altitude and azimuth, but given the sidereal time of the observation these can be converted to α and δ. According to the Williamstown observer, Ellery, the observations were corrected only for index error and for the runs of the microscope. Chauvenet (1960, Ch. 7) mentions numerous other corrections that must be applied to the altitude and azimuth instrument, which most likely explains the problems with the Williamstown observations. The Mussoorie observer, Capt. Tennant, R.E., does not specify if the time of observation is mean or sidereal or if referred to Greenwich or to place. But no combination of the possibilities renders acceptable $(O-C)$'s. The observations must, therefore, suffer from some other problem. The October Ann Arbor observations, with the exception of 22 October show $(O-C)$'s in δ between $\approx 60''$ to $\approx 70''$. I could find no cause for this behavior; it is improbable that an error of $1'$ crept into all of the suspect declinations. Most likely the observer mis-set the declination circle.

3 Ephemerides and differential corrections

The rectangular coordinates and velocities of the comet and the Earth were calculated by a program, used in numerous investigations previously, that treats the solar system as an n-body problem and takes the starting coordinates for Donati's comet from Hill's orbit as published in the Marsden and Williams catalog. The program is a 12th order Lagrangian predictor-corrector that incorporates relativity by a Schwarzschild harmonic metric. To obtain coordinates and velocities for the Earth, the moon is carried as a separate body. This means a small step-size, $0\overset{\rm d}{.}25$. To correct the comet's orbit partial derivatives are calculated by Moulton's method (Herget 1968), which integrates the partial derivatives to correct for the osculating rectangular coordinates and velocities the epoch JD 2399960.50. The rectangular coordinates, after interpolation to the moment of observation for the Earth and to the moment of observation antedated by the light time correction to allow for planetary aberration, are then converted to a unit vector that is transformed to a mean or apparent place in α and δ by application of precession, nutation, annual aberration, relativity, and so forth. The final step calculates an observed minus a computed place, $(O - C)$, in α and δ.

Various weighting schemes are possible once one has post-fit residuals from a differential correction. To assign weights to the observations, however, becomes far from trivial. 1501 of the observations are published as $\Delta\alpha$ and $\Delta\delta$ differences from a reference star, for which we can recalculate the position of the reference star and apply the differences. 320 observations give a reference star but no differences, only the observer's calculated α and δ for the comet. We can still calculate the differences between the observer's reference star and the Tycho-2 position and apply them to the comet's published position. For 186 of the observations no information is available, and we must take the observation as given. One suspects that the last class of observation should receive lower weight, but how much lower? One of the oldest weightings, Pierce's criterion (Branham 1990, pp. 79–80), establishes a cutoff for an acceptable residual. All residuals within the cutoff receive equal weight. More modern schemes usually assign higher weight to smaller residuals and zero weight to large residuals, recognizing them as errors rather than genuine but improbable residuals. Among these robust weightings are the biweight, the Talwar, and the Welsch. Branham (1990, Sect. 5.5) discusses all of these weighting possibilities. Let A represent the matrix of the equations of condition, here of size 2007×6, $\boldsymbol{d}$ the right-hand-side, $\Delta\boldsymbol{x}$ the solution for correction to the osculating rectangular coordinates and velocities $\boldsymbol{x}$, and $\boldsymbol{r}$ the vector of the residuals, $\boldsymbol{r} = \mathsf{A} \cdot \Delta\boldsymbol{x} - \boldsymbol{d}$.

To use the biweight, a weighting scheme I have used many times when working with comet orbits, double star orbits, and Galactic kinematics, one scales and individual post-fit residual r_i by the median of the absolute values of the residuals and assigns a weight wt as

$$\begin{aligned} \mathrm{wt} &= [1 - (r_i/4.685)^2]^2; \quad |r_i| \leq 4.685, \\ \mathrm{wt} &= 0; \quad |r_i| \succ 4.685. \end{aligned} \tag{1}$$

Talwar weighting is similar to Pierce's criterion, but the acceptance criterion for a residual becomes more stringent. Take once again the post-fit residual r_i. Talwar's criterion is

$$\begin{aligned} \mathrm{wt} &= 1; \quad |r_i| \leq 2.795, \\ \mathrm{wt} &= 0; \quad |r_i| > 2.795. \end{aligned} \tag{2}$$

Welsch weighting accepts all residuals, but assigns low weight to large residuals, so low as to become less than the machine ϵ for extremely large residuals,

$$\mathrm{wt} = \exp(-r_i/2.985)^2; \quad |r_i| < \infty. \tag{3}$$

The first differential correction was calculated by use of the robust L_1 norm. Because the norm is robust it becomes unnecessary to eliminate discordant observations. The residuals from this first correction then supplied the basis for further corrections based on Pierce's criterion. This criterion calculates a cutoff of 3.73 times a measure of dispersion for a discordant observation. For the first iteration I used the MAD (mean absolute deviation), the sum of the absolute values of the residuals divided by the degrees of freedom, for the measure of dispersion. Subsequent iterations employed the mean error of unit weight, $\sigma(1)$.

For the three robust weightings one can eschew a previous L_1 solution and instead use iteratively reweighted least squares (Branham 1990, Sect. 5.5). Let W be a diagonal weight matrix whose diagonal elements are the individual weights calculated from Eqs. (1)–(3). The first solution sets W to the unit matrix, calculates residuals and weights, defines a gradient $\bigtriangledown$ of the norm $\left\|\mathsf{W}^{1/2} \cdot \boldsymbol{r}\right\|_2$,

$$\bigtriangledown = (\mathsf{W}^{1/2} \cdot \mathsf{A})^{\mathrm{T}} \cdot (\mathsf{W}^{1/2} \cdot \boldsymbol{r}), \tag{4}$$

and iterates until $\bigtriangledown$ becomes less than a tolerance, which I take as the machine ϵ, $2.2{\times}10^{-16}$ for the Intel processor used for the computations.

After a solution has been calculated one should check for the randomness of the residuals. A runs test measures how often a variable, distributed about the mean, changes sign from plus to negative or negative to positive, the runs, which have a mean for n data points of $n/2 + 1$ and a variance of $n(n-2)/4(n-1)$ (Wonnacott & Wonnacott 1972, pp. 409–411). An advantage of the runs test over other tests for randomness resides in its being nonparametric, making no assumption about the normality of the data, although to actually calculate probabilities for the observed runs one does assume approximate normality. For the sake of comparison Hill's solution evinces 385 runs out of 1819 residuals, a decided lack of randomness. Hill studies the systematic differences among the observatories, which could account for the lack of randomness, and feels that such differences indeed exist, although the evidence is far from conclusive. I will discuss this matter in a later section.

Table 3 shows some statistics from the four solutions, the one based on Pierce's criterion and the three iteratively reweighted least squares solutions. The runs test is applied only to the non-zero residuals.

Table 3 Statistics for four solutions.

	Pierce	Biweight	Talwar	Welsch
$\sigma(1)$	10.″83	3.″30	3.″70	3.″23
Runs	846	786	722	933
Expected	969	846	760	969
Rejected resid.	70	316	488	70
% rejected	3.49	15.74	24.31	3.49

For a number of reasons, discussed later, I take the Welsch solution as the best of the four and base the final orbit on it. But in reality none of the solutions leads to significant changes in the osculating rectangular coordinates and velocities and any one of them could be taken as the "final" solution.

Table 4 Solution for rectangular coordinates and velocities for Donati's comet; epoch JD 2397360.5, equinox J2000.

Unknown	Value	Mean Error
x_0 (AU)	5.4862303e-001	3.3659782e-007
y_0 (AU)	–9.3318425e-002	3.3512080e-007
z_0 (AU)	2.3166005e-001	2.1177084e-007
$\dot{x}_0$ (AU d^{-1})	1.6913302e-002	1.6434447e-008
$\dot{y}_0$ (AU d^{-1})	–5.2325735e-003	1.7348924e-008
$\dot{z}_0$ (AU d^{-1})	–2.5815690e-002	9.3635375e-009
$\sigma(1)$	$3\overset{''}{.}23$	

Table 5 Covariance (diagonal and lower triangle) and correlation (upper triangle) matrices for Donati's comet.

4.82e–3	6.45e–1	1.97e–1	–8.17e–1	6.07e–1	3.33e–1
3.10e–3	4.78e–3	2.07e–1	–4.62e–1	5.80e–1	2.73e–1
5.97e–4	6.25e–4	1.91e–3	–2.56e–1	3.02e–1	4.67e–1
–1.92e–4	–1.08e–4	–3.79e–5	1.15e–5	–5.97e–1	–2.57e–1
1.51e–4	1.43e–4	4.72e–5	–7.24e–6	1.28e–5	3.57e–1
4.47e–5	3.65e–5	3.93e–5	–1.68e–6	2.47e–6	3.73e–6

4 The solution

Table 4 shows the final solution for the rectangular coordinates, x_0, y_0, z_0, and velocities, $\dot{x}_0$, $\dot{y}_0$, $\dot{z}_0$, along with their mean errors and also the repeated mean error of unit weight $\sigma(1)$ for Donati's comet. Table 5 gives the corresponding covariance and correlation matrices. The highest correlation, –81.7 %, can hardly be considered excessive, and the condition number of 58.1 for the data matrix shows that the solution is stable. Not only stable, but the low condition number for the data matrix translates into low variances for the covariance matrix and hence smaller mean errors for the unknowns than one would expect from the $\sigma(1)'$s.

Table 6 gives the orbital elements corresponding to the rectangular coordinates of Table 4: the time of perihelion passage, T_0; the period, P; the eccentricity, e; the semi-major axis, a; perihelion distance, q; the inclination, i; the node, Ω; and the argument of perihelion, ω. The calculation of the mean errors of the orbital elements proceeds via Rice's procedure (1902). Let C be the covariance matrix for the least squares solution for the rectangular coordinates and velocities. Identify the errors in a quantity such as the node Ω with the differential of the quantity, $\mathrm{d}\Omega$. Let $\boldsymbol{V}$ be the vector of the partial derivatives $\left(\partial\Omega/\partial x_0,\ \partial\Omega/\partial y_0,\ \ldots,\ \partial\Omega/\partial \dot{z}_0 \right)$. Then the error can be found from

$$(d\Omega)^2 = \sigma^2(1)\boldsymbol{V} \cdot \mathsf{C} \cdot \boldsymbol{V}^{\mathrm{T}}. \tag{5}$$

The partial derivatives in Eq. (5) are calculated from the well known expressions linking elliptical orbital elements with their rectangular counterparts. The orbit represents a high eccentricity ellipse.

The comet's period P comes from the relation

$$P = 2\pi a^{1.5}/k, \tag{6}$$

where k is the Gaussian gravitational constant.

Table 6 Elliptic orbital elements and mean errors for Donati's comet, equinox J2000.

Unknown	Value	Mean Error
T_0	JD 2399952.96486 30.46486 Sept. 1858	$0\overset{\mathrm{d}}{.}01500$
P (yr)	1927.22	3.03
a (AU)	154.8612	0.2056
e	0.996265	0.495925D–05
q (AU)	0.578472	0.495859D–03
Ω	$168\overset{\circ}{.}67479$	$0\overset{\circ}{.}62025$D–02
i	$94\overset{\circ}{.}00902$	$0\overset{\circ}{.}54205$D–02
ω	$157\overset{\circ}{.}34059$	$0\overset{\circ}{.}85864$D–02

5 Discussion

An orbit has been calculated for Donati's comet, but several lacunae must be addressed. The first asks, why use the Welsch weighting function rather than some other? Both the biweight and the Talwar weighting seem to reject too many residuals, the Talwar in particular. It seems difficult to justify excluding a quarter of the residuals. Stigler (1977), moreover, has shown that moderate trimming of the data, about 5 %, works better than extreme trimming. Although I have used the biweight assiduously and successfully in many investigations, it on occasion fails to give the best results, as occurred with the orbit of the Great Comet of 1860 (Branham 2007). For this particular comet Welsch weighting proved superior.

By looking at the expected versus the actual runs one sees that the Welsch weighting yields the best probability of the residuals being random. With Pierce's criterion there is virtually no chance, $<10^{-5}$ % with a 2-sided probability, of the residuals being random, 0.38 % with the biweight, 5.48 % with the Talwar, and 10.7 % with the Welsch. The Welsch weighting achieves this, admittedly, by assigning low weight to many of the residuals: 151 residuals represent weight greater than the machine ϵ but less than 0.01. Nevertheless, they are not discarded but receive at least some weight. Figure 2 gives the distribution of weights from the Welsch weighting and Fig. 3 the distribution of the weighted residuals versus time.

Although the residuals appear to be *relatively* random with the Welsch weighting, one should examine more thoroughly whether systematic error or non-gravitational forces may be at work. Hill seems convinced that the former are operative because he writes, "The existence of systematic error seems pretty well made out between the various observatories . . . ", but he also writes, "It would be very difficult, perhaps impossible, to arrive at a satisfactory explanation of these systematic errors . . . ." I question whether one can meaningfully look for systematic error among various observatories when there is such a disparity in the number of observations, for example 10 for Altona, Germany, and 455 for the Cape Observatory. It may be more profitable to examine the three classes of observation, those reduced to a common reference system, that of the Tycho-2 catalog with

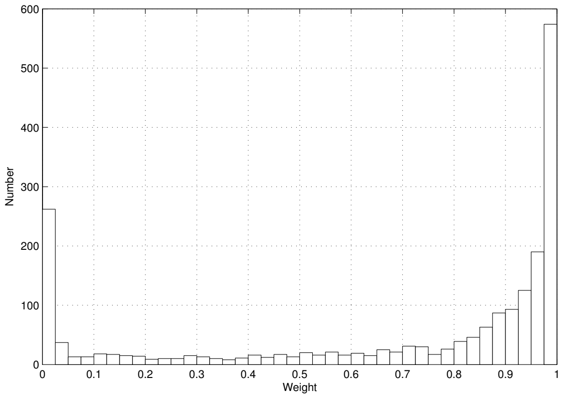

Fig. 2 Histogram of the weights.

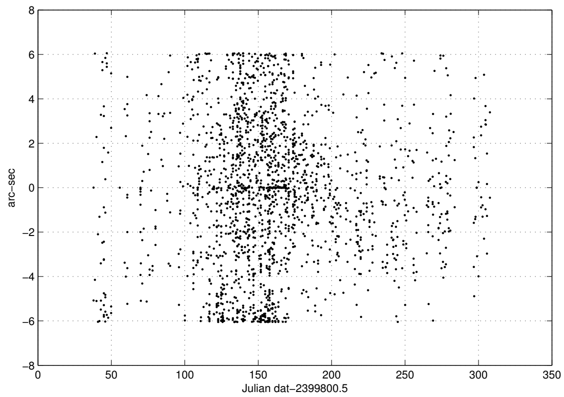

Fig. 3 Residuals for Donati's comet.

Table 7 Statistics on class of observation.

	No System	Ref. * Only	$\Delta\alpha, \Delta\delta$
Nos.	186	320	1501
$\sigma(1)$	$3''\!.68$	$3''\!.20$	$3''\!.19$
Runs (none-zero resid.)	90	130	679
Expected runs	81	158	730
Mean weight	0.48	0.80	0.70

1821 observations divided into 1501 with $\Delta\alpha$ and $\Delta\delta$ applied to a given reference star and 320 with only the reference star, versus the 186 on no system. Table 7 shows the results: $\sigma(1)$, runs from non-zero residuals, expected runs for each class of observation, and the mean weight from the Welsch weighting. It appears as if the residuals on no system have a higher dispersion and receive lower weight, but are as random or non-random as the other classes.

A more interesting comparison arises if we compare the randomness of residuals before and after perihelion passage. There are 921 non-zero residuals with 405 runs before perihelion passage and 1016 with 484 runs after. There is virtually no chance that the pre-perihelion residuals are random, but a 13.2 % chance that the post-perihelion residuals are. This indicates that nongravitational forces *may* have been at work on the comet before it reached perihelion. I thus have

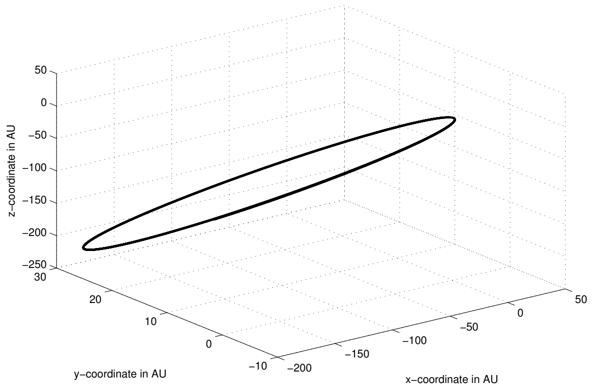

Fig. 4 Comet Donati's orbit in space.

to disagree with Hill's remark about lack of evidence for only gravity affecting the comet.

Table 8 gives Hill's orbit for this comet, referred to equinox J2000 and taken from the Marsden & Williams catalog (2003), which can be compared with the orbit in Table 6. The shape of the orbit between the two solutions agrees better than its orientation. Given that the new orbit exhibits greater randomness of the residuals, 933 runs out of an expected 969 versus 385 runs out of an expected 910, it may be considered a better representation of the true orbit. (The randomness test seems a more reliable indicator than the mean errors of both solutions because these errors are calculated differently. Hill uses the probable error of the mean computed from the normal places whereas I use the mean error of unit weight computed from the covariance matrix. When this difference is taken into account the two solutions have roughly comparable errors.)

The orbit of Donati's comet is highly elliptical, and the comet will eventually return to the inner solar system. When, and how close will it pass from the Earth? Donati's orbit was integrated forwards until JD 3140000.5. Figure 4 shows the orbit in space and Fig. 5 the distance of the comet from the Earth. The comet's closest approach to Earth will occur on 24 Feb. 3759 when it will pass 0.8442 AU from the Earth.

Table 8 Hill's elliptic orbital elements for Donati's comet.

Unknown	Value
T_0	JD 2399952.9645 30.4645 Sept. 1858
P (yr)	1950.99
a (AU)	156.1320
e	0.996295
q (AU)	0.578469
Ω	$167\overset{\circ}{.}3044$
i	$116\overset{\circ}{.}9512$
ω	$129\overset{\circ}{.}1144$

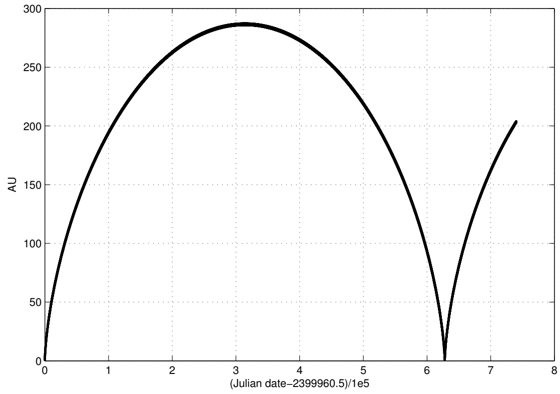

Fig. 5 Comet Donati's distance from Earth.

6 Conclusions

The new orbit for comet Donati is based on more observations than Hill used and calculates the orbit by use of iteratively reweighted least squares and the Welsch weighting function. The final orbit produces residuals that are relatively random, 10.7 % chance of being random, although a study of the pre- and post-perihelion residuals indicates that possible nongravitational forces may be at work. Reducing the observations to a common reference system, for this paper the system defined by the Tycho-2 catalog, decreases the variance of the residuals compared with those that cannot be referred to a common system, but seems to have little effect on their randomness. The final orbit represents a high eccentricity ellipse of long period. Comet Donati will not return until the year 3759, when it will pass far from the Earth, geocentric distance greater than 0.8 AU.

References

Branham, R.L., Jr. 1990, Scientific Data Analysis (Springer, New York)
Branham, R.L., Jr. 2005, Planet. Space Sci., 53, 1437
Branham, R.L., Jr. 2007, Astron. Nachr., 328, 137
Branham, R.L., Jr. 2011, Processing 19th Century Cometary Observations, in Comets: Characteristics, Composition and Orbits, ed. P.G. Melark, (Nova, New York), 123
Chauvenet, W. 1960, A Manual of Spherical and Practical Astronomy, Vol. 2 (Dover, New York)
Donati, Dr. 1858, MNRAS, 18, 271
Ellery, L.J. 1858, Astron. Nachr., 50, 7
Herget, P. 1968, AJ, 73, 737
Hill, G.W. 1867, Mem. Amer. Acad. Sci., 9, 67
Hornstein, K. 1859, Annalen Wien, 9, 158
Høg, E., Fabricius, C., Markarov, V.V., et al. 2000, The Tycho-2 Catalogue of the 2.5 Million Brightest Stars (U.S. Naval Observatory, Washington, DC)
Kaplan, G.H., Hughes, J.A., Seidelmann, P.K., & Smith, C.A. 1989, AJ, 97, 1197
Marsden, B.G., & Williams, G.V. 2003, Catalog of Cometary Orbits 2003 (Smithsonian Astrophysical Obs., Cambridge, Mass.)
Rice, H.L. 1902, AJ, 22, 149
Schmidt, J. 1860, Astron. Nachr., 53, 315
Scott, F.P. 1964, AJ, 69, 372
Stigler, S.M. 1977, Annales of Statistics, 5, 1055
Tennant, Capt. 1859, MNRAS, 19, 185
Wonnacott, T.H., & Wonnacott, R.J. 1972, Introductory Statistics, 2nd Ed. (John Wiley, New York)