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INDOOR RADON CONCENTRATION: MATHEMATICAL MODEL AND NUMERICAL SIMULATION

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Abstract:
In this work, we present a mathematical model for indoor radon concentration in a room with an air exchanger located on one of its walls. Additionally, we show some numerical tests, which suggest that radon concentration decreases as the air circulation increases inside the room. This is in line with experimental observations.

Keywords: *navier-stokes equations, indoor radon concentration, finite element method.*
2000 AMS Subject Classification: 65N99 - 35Q35 - 76D05

1 INTRODUCTION

Interest in indoor radon gas measurements has been increasing due to concerns about the effects of long-term exposure to elevated levels of radon on human health (see, for example, [7]). Radon and its decay products are classified as Group 1 carcinogens for lung cancer, making them the second leading cause of disease after smoking. It has also been considered as a possible cause of further patologies such as neurodegenerative diseases. Given these risks, international organizations such as the WHO have recommended maximum indoor radon concentration levels, and long-term projects have been undertaken to monitor and mitigate radon exposure. Typically, radon levels are high in regions with elevated uranium concentrations, since radon is part of the uranium decay chain. In Argentina, indoor radon data are scarce, and so far no epidemiological studies on radon have been conducted. Recently, the first systematic indoor radon program was initiated in the Punilla region since its geological setting indicates a significant radon potential, mainly based on the presence of uranium deposits and active faults systems. The preliminar results indicate that, indeed, dwellings located in areas affected by large active faults and settled on rather low, yet anomalous uranium-bearing rocks are exposed to high radon flow.

This work presents a mathematical model for indoor radon concentration in a room equipped with an air exchanger located on one of its walls. The model assumes that radon gas primarily enters the room through the floor, while the outdoor air, far from the ground, is free of radon. This framework is consistent with a room in a house located in Punilla, where passive detector measurements reported the highest radon concentration in the area. The air exchanger is designed to regulate air quality by reducing indoor concentrations, and consists of two concentric cylinders: the inner one supplies fresh air, while the outer cylinder extracts contaminated indoor air. The air velocity entering and leaving the room through the air exchanger can be manually adjusted, providing a potential means of controlling radon concentration.

The organization of the paper is as follows. In Section 2, we present a preliminary mathematical model for indoor radon concentration in a room with an air exchanger device as the one described above. Specifically, we focus on an isothermal steady case. Section 3 is devoted to describe the numerical method and to present some numerical tests in a 2D setting. Finally, we end the paper drawing some conclusions in Section 4. In particular, we highlight that although the parameters used in the simulations are still theoretical, our results corroborate that radon concentration decreases as air circulation increases inside the room. We plan to simulate radon concentration using real-world parameters in the near future.

2 MATHEMATICAL MODEL

Radon gas is assumed to diffuse in the air, be transported by the airflow through advection, and undergo natural radioactive decay. In this work, we focus on a steady state and assume that the air temperature inside the room remains constant. This assumption is reasonable, even with the presence of the air exchange device, provided that the incoming exterior air can be adjusted to have the same temperature as the air already present in the room; which is part of a relatively passive house. This simplification allows us to neglect the influence of temperature variations on both the indoor radon gas concentration and the air velocity (in particular, we have no air density changes). Thus, we will model the concentration of radon gas using an advection-diffusion-reaction equation. Furthermore, since we neglect any temperature variations, the airflow velocity can be determined by solving the incompressible Navier-Stokes equations. This leads to the following governing equations, which, for simplicity, we present directly in dimensionless form:

$$\left\{\begin{array}{rl} (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \frac{1}{\mathrm{Re}}\Delta\boldsymbol{v} + \nabla p & = 0 \quad \text{in} \quad \Omega, \\ \nabla\cdot\boldsymbol{v} & = 0 \quad \text{in} \quad \Omega, \\ \boldsymbol{v}\cdot\nabla c - \frac{1}{A}\Delta c + \frac{1}{B}\lambda c & = 0 \quad \text{in} \quad \Omega, \end{array}\right. \tag{1}$$

for fluid velocity $\boldsymbol{v}$, fluid kinematic pressure p, and radon gas concentration c (recall that the air density is assumed to be constant). Here, $\Omega \subset \mathbb{R}^N$, $N = 2$ or $N = 3$, is an open and bounded set with a Lipschitz boundary $\Gamma = \partial\Omega$. In addition, the dimensionless parameters Re, A, and B are given by

$$\mathrm{Re} = \frac{V_* L_*}{\nu}, \qquad A = \frac{V_* L_*}{\delta}, \qquad B = \frac{V_*}{L_*\lambda},$$

where L_* is a lenght associated with the domain Ω (for example, the height of the room), V_* is calculated as L_*/τ_* with τ_* the radon half-life, $\nu > 0$ is the air kinematic viscosity, $\delta > 0$ is the diffusivity of radon, and $\lambda > 0$ is the constant of decay of radon. In order to describe the boundary conditions, we introduce a boundary partition $\{\Gamma_i, \Gamma_o, \Gamma_f, \Gamma_w\}$ consisting on mutually disjoint relative open subsets of Γ with positive Lebesgue $(N-1)$-measure such that the union of their closures gives Γ. The boundary parts Γ_i and Γ_o represent the inner and outer cylinders of the air exchanger device, respectively; Γ_f represents the room floor, and Γ_w stands for the room walls and roof. The boundary conditions for the air velocity are as follows,

$$\boldsymbol{v} = 0 \;\text{ on }\; (\Gamma_f \cup \Gamma_w)\,, \quad \boldsymbol{v} = \boldsymbol{v}_i \;\text{ on }\; \Gamma_i, \quad \boldsymbol{v} = \boldsymbol{v}_o \;\text{ on }\; \Gamma_o. \tag{2}$$

The first condition enforces a no-slip boundary at the fluid-solid interface, ensuring that the air velocity is zero relative to the solid surface. The second and third conditions specify the inflow and outflow velocities, respectively, which are driven by the air exchange device. Here, the velocity profiles $\boldsymbol{v}_i$ (inflow) and $\boldsymbol{v}_o$ (outflow) are assumed to be given. The boundary conditions for the radon gas concentration are,

$$c = c_f \;\text{ on }\; \Gamma_f, \quad c = 0 \;\text{ on }\; \Gamma_i, \quad \frac{\partial c}{\partial \mathbf{n}} = 0 \;\text{ on }\; \Gamma_w \cup \Gamma_o. \tag{3}$$

The first condition defines a radon gas source located on the floor, where c_f represents the known concentration of radon at the source. The second condition specifies that the air entering the room through the inner cylinder is radon-free. Finally, the third condition ensures that there is no radon flux through the room walls (recall that $\boldsymbol{v} = 0$ on the walls) as well as that the radon flux at the outlet is exclusively driven by advection. It is worth mentioning here that the only part of the room boundary in contact with the rocky soil is the floor. Additionally, field data shows that radon flux is negligible outdoors.

3 Numerical simulations

To perform a numerical simulation of our problem, we will use the finite element method (FEM). We denote by $H^1(\Omega)$ the classical Sobolev space $H^1(\Omega) = \{v \in L^2(\Omega) : D^\alpha v \in L^2(\Omega), |\alpha| = 1\}$, and $L^2_0(\Omega) = \{q \in L^2(\Omega) : \int_\Omega q = 0\}$. The weak formulation of (1)-(3) is given by: find $\boldsymbol{v} \in (H^1(\Omega))^N$, $q \in L^2_0(\Omega)$ and $c \in H^1(\Omega)$, with $\boldsymbol{v} = \boldsymbol{v}_o$ in Γ_o, $\boldsymbol{v} = \boldsymbol{v}_i$ in Γ_i and $c = c_f$ in Γ_f such that

$$\begin{aligned} \int_\Omega (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\cdot\boldsymbol{w} - \frac{1}{\mathrm{Re}}\int_\Omega \nabla\boldsymbol{v} : \nabla\boldsymbol{w} - \int_\Omega (\nabla\cdot\boldsymbol{w})p &= 0 \quad \forall \boldsymbol{w} \in (H^1_0(\Omega))^N, \\ \int_\Omega q\nabla\cdot\boldsymbol{v} &= 0 \quad \forall q \in L^2_0(\Omega), \\ \int_\Omega (\boldsymbol{v}\cdot\nabla c)\phi + \frac{1}{A}\int_\Omega \nabla c\cdot\nabla\phi + \frac{1}{B}\int_\Omega c\phi &= 0 \quad \forall \phi \in H^1(\Omega), \end{aligned} \tag{4}$$

where $(H_0^1(\Omega))^N = \{\boldsymbol{w} \in (H^1(\Omega))^N : \boldsymbol{w} = 0 \text{ on } \Gamma\}$. The first two equations of (4) correspond to the weak formulation of the stationary Navier-Stokes equations with non-homogeneous boundary data. The existence and uniqueness of a weak solution are well-known, provided that the Reynolds number Re and the boundary data $\boldsymbol{v}_d$ are sufficiently small, with $\boldsymbol{v}_d$ being regular enough; see for example [1]. The existence and uniqueness of a weak solution of advection-reaction-diffusion equations, as the third equation of the formulation (4), are studied, for example, in [9].

For the numerical simulation, we take a 2D section of the room, namely $\Omega \subset \mathbb{R}^2$, whose boundary includes the vertical axis of the air exchanger; see Figure 1. We take, for example, $L_* = 2.4\ m$ (room height), $\nu = 15.52 \times 10^{-6}\ m^2/s$ (viscosity at 25°C [3]), $V_* = 7.31 \times 10^{-6}\ m^2/s$ ($V_* = L_*/\tau_*$, with $\tau_* = 328320\ s$), $\delta = 1 \times 10^{-5}\ m^2/s$ and $\lambda = 0.1812 \times day$ (see [4]). Thus, the parameters in the dimensionless model are given by $\text{Re} = 1.13041$, $A = 1.7544$, $B = 1.4504$. For the boundary data we must take into account that in the dimensionless model $\boldsymbol{v}_i = \frac{\boldsymbol{v}_i^*}{V_*}$, $\boldsymbol{v}_o = \frac{\boldsymbol{v}_o^*}{V_*}$ and $c_f = \frac{c_f^*}{C_*}$, where $\boldsymbol{v}_i^*, \boldsymbol{v}_o^*$ and c_f^* are the data in the dimensional model, with $C_* = 300\ Bq/m^3$ (radon reference level for Argentina [2]) and $c_f^* = 1100\ Bq/m^3$ (radon concentration on the room floor).

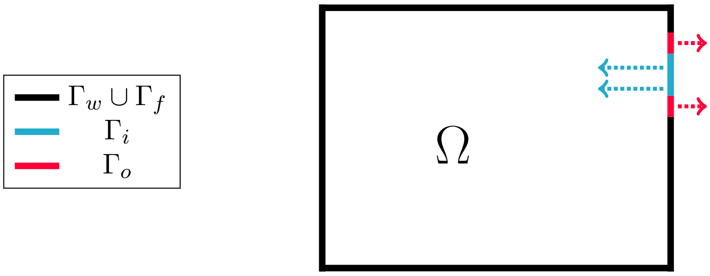
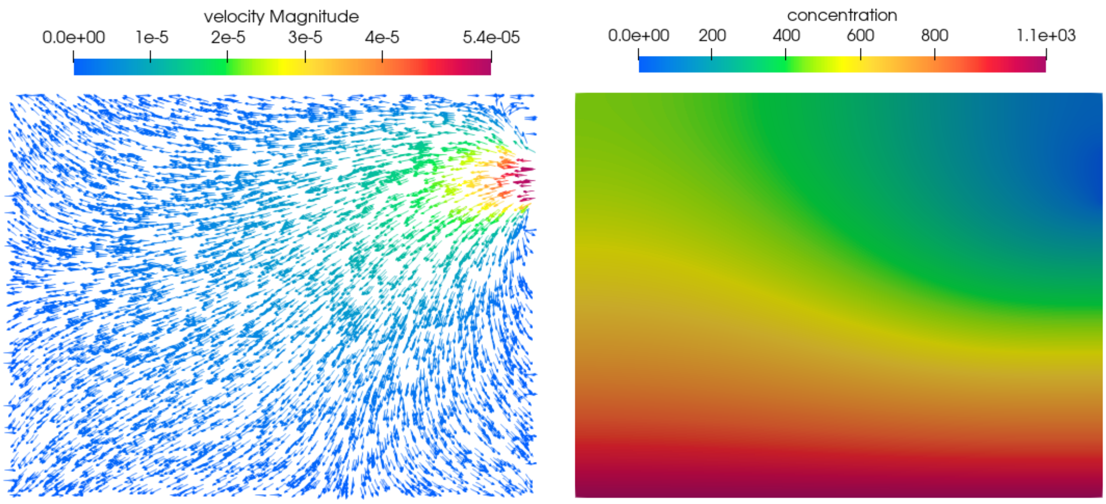


Figure 1: 2D room section

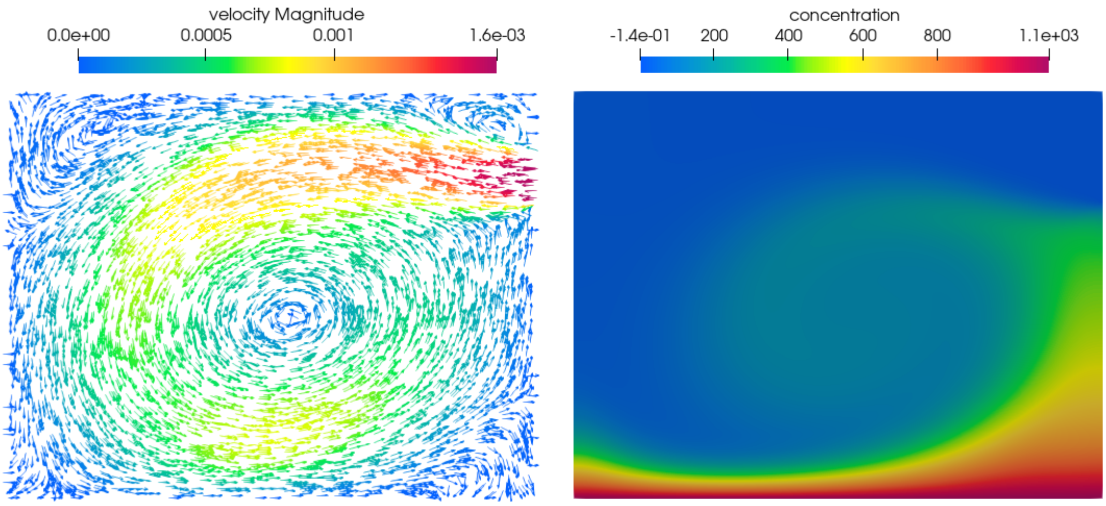
In this work we include two simulations, which have been obtained using the well-known P_2P_1 Taylor-Hood elements for the Navier-Stokes equation and P_2 elements for the advection-reaction-diffusion equations, to observe the effect of changing air flow. In a first instance, we take a boundary data $\boldsymbol{v}_i^* = 5.3 \times 10^{-5}\ m/s$ and $\boldsymbol{v}_o^* = 4.2 \times 10^{-5}\ m/s$ (figure 2). Then, in the second example, we increase the magnitudes taking $\boldsymbol{v}_i^* = 1.6 \times 10^{-3}\ m/s$ and $\boldsymbol{v}_o^* = 1.2 \times 10^{-3}\ m/s$ (figure 3).

The implementation is based on the *NGSolve* code [5].



(a) Velocity field (b) Gas concentration (Bq/m^3)

Figure 2: $\boldsymbol{v}_i^* = 5.3 \times 10^{-5}\ m/s$, $\boldsymbol{v}_o^* = 4.2 \times 10^{-5}\ m/s$.



(a) Velocity field (b) Gas concentration (Bq/m^3)

Figure 3: $\boldsymbol{v}_i^* = 1.6 \times 10^{-3}\ m/s$, $\boldsymbol{v}_o^* = 1.2 \times 10^{-3}\ m/s$.

4 Conclusions

We present a mathematical model for indoor radon concentration and two numerical simulations. In the first simulation, we observe that the air circulation is poor (Figure 2a), resulting in insufficient dissipation of radon gas concentration (Figure 2b). In the second simulation, where the air flow in the inlet and outlet is higher, the air circulation improves (Figure 3a), which seems to lower the gas concentrations inside the room (Figure 3b), as expected. As future work, we will compare the numerical results with measurements made at a specific location, in order to evaluate alternative solutions for dwellings recording high radon concentrations.

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