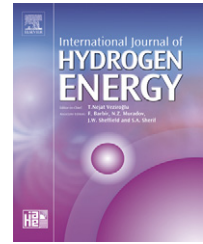


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## Software tool for horizontal-axis wind turbine simulation

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### ABSTRACT

The main problem of a wind turbine generator design project is the design of the right blades capable of satisfying the specific energy requirement of an electric system with optimum performance.

Once the blade has been designed for optimum operation at a particular rotor angular speed, it is necessary to determine the overall performance of the rotor under the range of wind speed that it will encounter.

A software tool that simulates low-power, horizontal-axis wind turbines was developed for this purpose. With this program, the user can calculate the rotor power output for any combination of wind and rotor speeds, with definite blade shape and airfoil characteristics. The software also provides information about distribution of forces along the blade span, for different operational conditions.

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## 1. Introduction

Let the wind turbine be represented by a uniform area “actuator disc”  $A$ . From the conservation of linear momentum for a one-dimensional, incompressible, time-invariant flow, the thrust  $F$ , which is the force of the wind on the wind turbine, is equal and opposite to the change in momentum of air stream. The power output,  $P$ , is equal to the thrust times the velocity  $v$  at the disc [1]:

$$P = Fv \quad (1)$$

Since  $F$  depends on blade shape and airfoil characteristics, this simple formula takes a great deal of calculus. So, the determination of the complete performance characteristic of a rotor, i.e., the manner in which the power varies over the range of wind and rotor speed that it will encounter, requires an iterative solution.

## 2. Aerodynamics of horizontal-axis wind turbines

The performance of a wind turbine can be characterized by the manner in which power, torque and thrust vary with wind speed [2]. The power determines the amount of energy captured by the rotor. The torque developed determines the size of the gear box. The rotor thrust has great influence on the structural design of the tower.

Power, torque and thrust are determined by the aerodynamic forces generated by the mean wind  $U_\infty$  (see Fig. 1). The resultant forces on a blade section at a radius  $r$  are resolved into two forces [1–3]: the lift force  $L$ , defined to be perpendicular to the direction of an effective, or relative, wind  $W$ , and the drag force  $D$ , defined to be parallel to the direction of  $W$  (Fig. 1). For an angular velocity  $\Omega$ , the relative wind is the vector sum of the wind velocity at the rotor,  $U_\infty(1 - a_r)$ , and the net tangential flow velocity experienced by the blade

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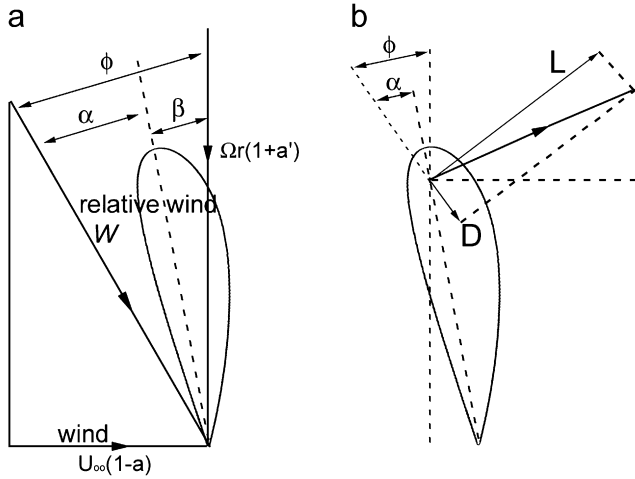


Fig. 1 – Blade element velocities (a) and forces (b).

element due to rotation,  $u = (1 + a'_r)\Omega r$ . Thus, the resultant relative velocity  $W$  on the blade is

$$W = \sqrt{U_\infty^2(1 - a_r)^2 + (\Omega r)^2(1 + a'_r)^2} \quad (2)$$

which acts at an angle  $\phi$  to the plane of rotation;  $a_r$  and  $a'_r$  are the axial and angular inductor factors for a radius  $r$  [1–4]. The angle of attack,  $\alpha$ , is the angle between the chord line and the relative wind  $W$ , and it is given by  $\alpha = \phi - \beta$ , where  $\beta$  is the pitch angle.

The aerodynamic lift and drag forces on the span-wise elements of radius  $r$  and length  $dr$  of the several blades of a wind turbine rotor are responsible for the rate of change of axial and angular momentum of all of the air which passes through the annulus swept by the blade elements. In addition, the force on the blade elements caused by the drop in pressure associated with the rotational velocity in the wake must also be provided by the aerodynamic lift and drag [1,2].

Momentum theory and blade element theory, combined into a strip theory, enable calculation of the performance characteristics of an annular section of the rotor. The characteristics for the entire rotor are then obtained by integrating, or adding, the values obtained for each of the annular sections [1,2,4].

### 3. Blade element theory

The lift and drag forces ( $L$  and  $D$ ) on the blades of a wind turbine of  $N$  blades of tip radius  $R$  each with chord  $c$  and set pitch angle  $\beta$  can be expressed as a function of lift and drag coefficients  $C_D$  and  $C_L$ , obtained from the airfoil characteristic curves  $C_D(\alpha)$  and  $C_L(\alpha)$  at a certain value for  $\alpha$  [2,5]:

$$D = 0.5\rho W^2 N c C_D \quad (3)$$

$$L = 0.5\rho W^2 N c C_L \quad (4)$$

where  $\rho$  is the air density.

The incremental force  $dq_r$ , tangential to the circle sweep of the rotor (which is the force creating useful torque) and the incremental force  $df_r$  normal to the plane of rotation (which contributes to the thrust), on an annular section of radius  $r$

and thickness  $dr$ , are expressed as follows [2,4]:

$$dq_r = 0.5\rho W^2 N c (C_L \sin \phi - C_D \cos \phi) r dr \quad (5)$$

$$df_r = 0.5\rho W^2 N c (C_L \cos \phi + C_D \sin \phi) dr \quad (6)$$

The complete rotor, therefore, develops a total torque  $Q$ :

$$Q = \int_{R_i}^R q_r dr \quad (7)$$

where  $R$  is the tip radius and  $R_i$  is the radius at the hub of the blade.

The total axial, or normal, force  $F$  is

$$F = \int_{R_i}^R f_r dr \quad (8)$$

## 4. Momentum theory

The forces on a wind turbine blade and the flow conditions at the blades can be derived by considering conservation of momentum since force is the rate of change of momentum [1].

The application of the conservation of linear momentum to the actual rotor area  $A_D$  is an expression for the thrust [4,7]:

$$F = 2\rho A_D U_\infty^2 a_r (1 - a_r) \quad (9)$$

Similarly, from the conservation of angular momentum, the torque  $Q$  imparted to the blades is [4,7]

$$Q = \pi(R^4 - R_i^4)(1 - a_r)a'_r U_\infty \Omega \quad (10)$$

The actual rotor area  $A_D$  is

$$A_D = \pi(R^2 - R_i^2) \quad (11)$$

## 5. Blade element momentum theory

The forces and moments derived from the momentum theory and the blade element theory must be equal. Equating these, the flow conditions for the analyses of a turbine design performance can be derived [1,2].

Equating Eqs. (7) and (10) for the rotor torque:

$$a'_r(1 - a_r) = \frac{NcR^3}{2\pi(R^4 - R_i^2)} \frac{U_\infty}{\Omega R} \times \int_{R_i}^R \frac{W^2}{U_\infty^2} (C_L \sin \phi - C_D \cos \phi) \frac{r dr}{R} \quad (12)$$

Since the thrust is the same force as the normal force, Eqs. (8) and (9) can be equated for the axial force:

$$a_r(1 - a_r) = \frac{NcR}{2\pi(R^4 - R_i^2)} \times \int_{R_i}^R \frac{W^2}{U_\infty^2} (C_L \cos \phi + C_D \sin \phi) \frac{r dr}{R} \quad (13)$$

The flow induction factors  $a_r$  and  $a'_r$  remain constant only inside each differential  $dr$ , so Eqs. (12) and (13) become:

$$a'_r(1 - a_r) = Q^* = \frac{NcW^2}{8\pi\Omega r^2 U_\infty} (C_L \sin \phi - C_D \cos \phi) \quad (14)$$

$$a_r(1 - a_r) = F^* = \frac{NcW^2}{8\pi\Omega r U_\infty^2} (C_L \cos \phi + C_D \sin \phi) \quad (15)$$

From Fig. 1:

$$\sin \phi = \frac{U_\infty}{W} (1 - a_r) \quad (16)$$

$$\cos \phi = \frac{\Omega r}{W} (1 + a_r') \quad (17)$$

The power contribution from each annulus is

$$dP = \Omega dQ \quad (18)$$

The total power of the rotor is

$$P = \int_{R_i}^R \Omega dQ \quad (19)$$

It can be seen from Eqs. (9), (10) and (19) that the calculation of the forces and the power developed by the rotor requires the knowledge of the flow induction factors  $a_r$  and  $a_r'$ , which are obtained by solving Eqs. (14) and (15). The solution is carried out iteratively [2,4], because of the non-linearity of the airfoil characteristics with the angle of attack.

## 6. Computational method

For a known blade shape, known airfoil characteristic curves  $C_D(x)$  and  $C_L(x)$ , and known average wind speed, all of them input by the user, ZEUS SIMULADOR [6] obtains the flow induction coefficients for each  $dr$  along the blade span by an iterative process [4], and then calculates how rotor power, rotor efficiency, torque and thrust vary with rotational speed.

By repeating the calculation for different wind velocities, the program computes the family of curves [7] that represent the rotor behavior in different situations.

The software also provides information about distribution of forces along the blade span, for any operating point (defined by a chosen pair of values of rotor power and angular speed) belonging to any performance curve, selected among the family of curves (that is, for a selected wind speed).

The program can also plot the possible maximum values of rotor power and rotor efficiency that can be taken for different rotational (or wind) speed of the wind turbine under simulation.

## 7. General aspects of the visual interface of the software [6]

The visual interface has four windows. The first is destined for the input of the airfoil characteristic curves  $C_D$  and  $C_L$ . The second displays the input profile and the corresponding curves. The third is for the input of the tables that describe the blade shape. In the last window, the user can configure the simulation as follows.

### 7.1. Simulation configuration window and results display

This window has three principal sectors (Fig. 2): “Parámetros Básicos” (basic parameters), “Simulación Paramétrica” (parametric simulation), “Configuración General” (general configuration).

The *Parámetros Básicos* sector is destined for the input of: wind speed (“*Velocidad del Viento*”) and number of blades (“*Cantidad de Palas*”).

In the sector destined for general configuration (“*Configuración General*” sector), the user establishes the variables for the  $x$ -axis (“*Ver en el Eje Inferior*” box in Fig. 2) and for the  $y$ -axis



Fig. 2 – Configuration window.

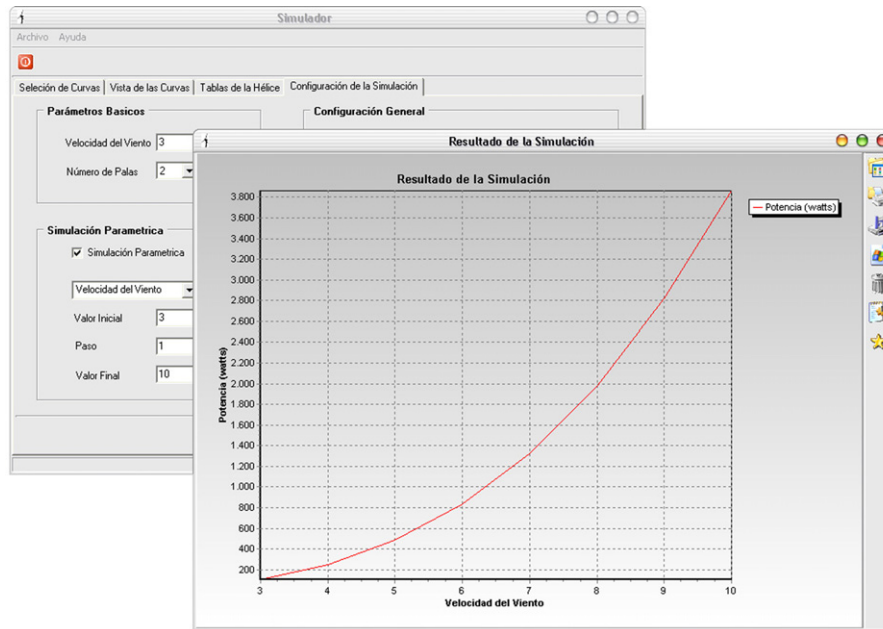


Fig. 3 – Maximum rotor power outputs for different wind speeds.

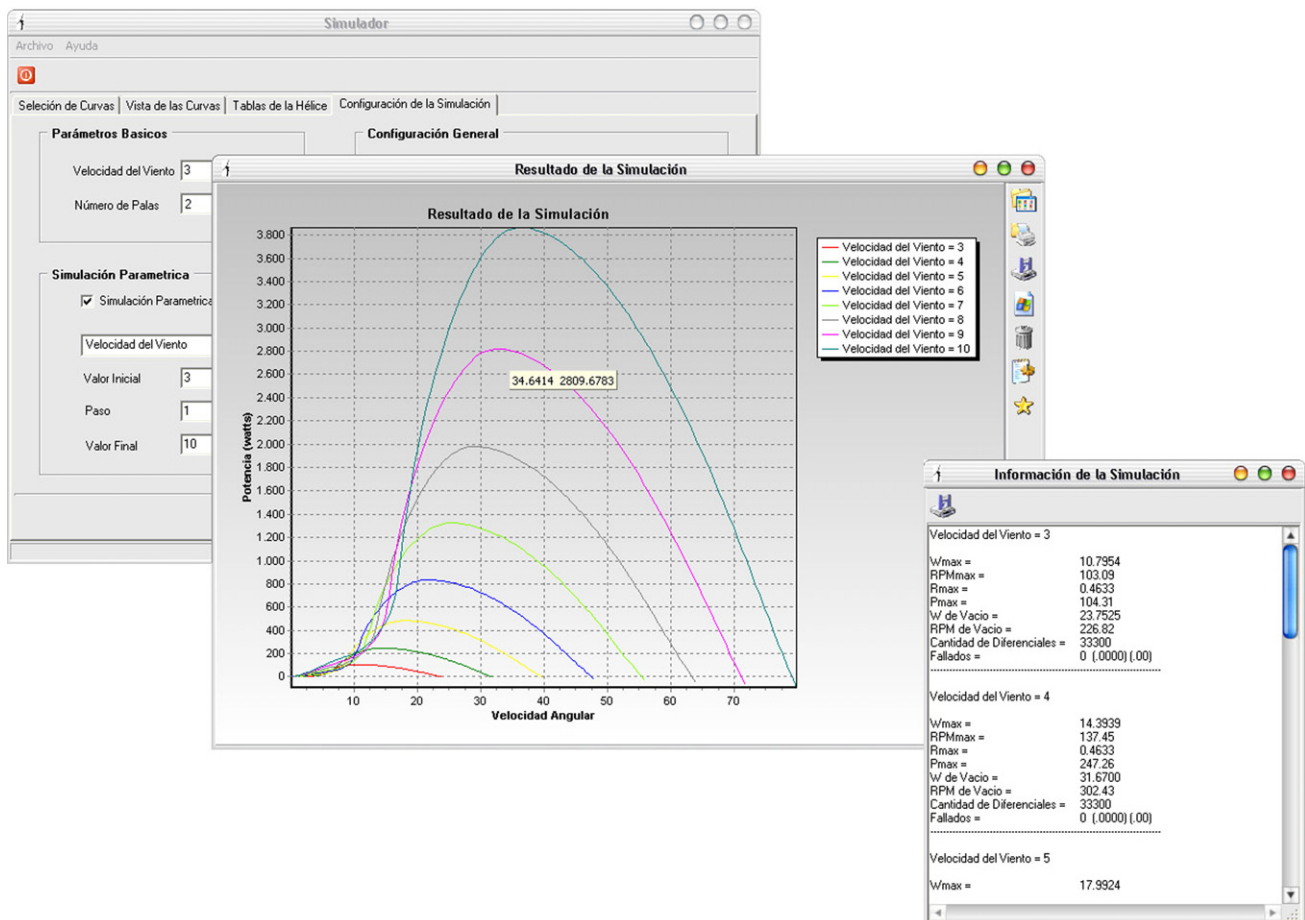


Fig. 4 – Parametric simulation results for different wind speeds.

("Ver en el Eje Izquierdo" box). By clicking inside "Trabajar con los Máximos" box, the program plots the possible maximum values of the y-axis variable (i.e., rotor power or rotor

efficiency) that can be taken for different values of the x-axis (i.e., rotational (or wind) speed) of the wind turbine under simulation (see Fig. 3).

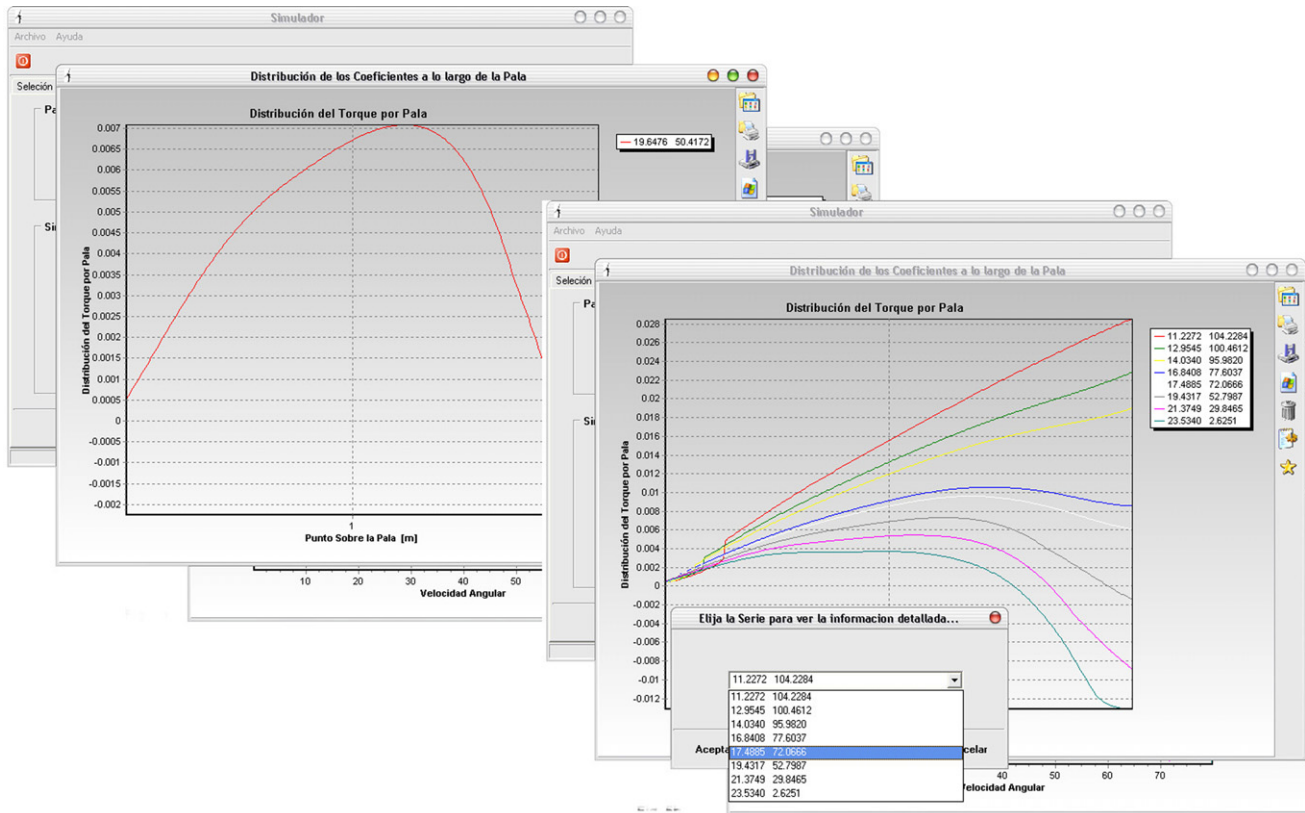


Fig. 5 – Distribution of the torque along the blade span.

In the “Simulación Paramétrica” sector, the user can choose that variable parameter for which the family of curves is to be plotted (see Fig. 4). The possible variable parameters are (see Fig. 2): blade width at the tip radius (“Ancho Arriba”), blade width at the hub of the blade (“Ancho Abajo”), pitch angle at those same points (“Pitch Arriba” and “Pitch Abajo”), tip radius  $R$  (“Largo”), radius at the hube of the blade  $R_i$  (“Radio Inferior”), wind speed (“Velocidad del Viento”) and number of blades (“Cantidad de Palas”).

By choosing an operating point on one of the curves of Fig. 4 (for example, 50.4172 W of rotor power and 19.6476 rpm, for 3 m/s wind speed) the software plots the distribution of the torque along the blade span (see Fig. 5, left). By the selection of different operating points for each wind speed curve of Fig. 4, the family of curves of Fig. 5 (right) can be obtained.

## 8. Conclusions

Several commercial wind turbines were simulated, using manufacturers data. Performance results were similar to those offered in the corresponding catalogues for operational conditions.

Other working conditions were simulated in order to compare the performance of commercial wind turbines for a specific electric system energy requirement.

The first point to notice is that the maximum values of rotor power were much smaller than the corresponding Betz limit

[2,3,5,8]. The discrepancy is caused by drag and tip losses that reduce efficiency at low values of rotational and wind speed.

From program tests, one can conclude that the numerical method carried out by the software can represent the behavior of commercial wind turbine blades.

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