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Testing Extended General Relativity with Galactic Sizes and Compact Gravitational Sources

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Abstract

Considering a five-dimensional (5D) Riemannian spacetime with a particular stationary Ricci-flat metric, we have applied the recently developed Extended General Relativistic theory to calculate the lower limit for the size of some relevant galaxies: the Milky Way, Andromeda and M87. Our results are in very good agreement with observations.

1 Introduction, basic equations and motivation

The standard 4D General Relativity and its Newtonian weak-field limit fail at describing the observed phenomenology when it is applied to cosmic structure on galactic and larger scales. To reconcile the theory with observations we need to assume that $\sim 85\,\%$ of the mass is seen only through its observational effect and that $\sim 74\,\%$ of the energy content of the universe is due to either to an arbitrary cosmological constant or to a not well defined dark energy fluid. The cosmological constant problem appears to be so serious as the dark matter problem. The Einstein equations admit the presence of an arbitrary constant Λ. Since observations indicate $\Lambda > 0$, the dark energy fluid has negative pressure. Current observations suggest $\omega = -1$ at all probed epochs[1], so models more sophisticated than a simple Λ could seem in principle unnecessary.

In a previous paper[2] we developed a extended general relativistic formalism from which we obtained an effective 4D static and spherically symmetric metric which give us ordinary gravitational solutions on small (planetary and astrophysical) scales, but repulsive (anti gravitational) forces on very large (cosmological) scales with $\omega = -1$. Our approach is an unified manner to describe dark energy, dark matter and ordinary matter in the framework of the induced matter theory[3]. In this letter we extend our calculations to study the lower limit for the size of some galaxies (the Milky Way, Andromeda and Messier 87) of interest.

1.1 5D massive test particles dynamics

In a previous work[2] we have considered a 5D extension of General Relativity such that the effective 4D gravitational dynamics has a vacuum dominated, $\omega = -1$, equation of state. The starting 5D Ricci-flat metric g_{ab}, there considered is determined by the line element[2, 4]

$$dS^2 = \left(\frac{\psi}{\psi_0}\right)^2 \left[c^2 f(r) dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + sin^2(\theta)\, d\phi^2\right)\right] - d\psi^2, \qquad (1)$$

where $f(r) = 1 - (2G\zeta\psi_0/(rc^2))[1 + c^2r^3/(2G\zeta\psi_0^3)]$ is a dimensionless function, $\{t, r, \theta, \phi\}$ are the usual local spacetime spherical coordinates employed in general relativity and ψ is the space-like extra dimension that following the approach of the induced matter theory, will be considered as non-compact. Furthermore, the space-like coordinates ψ and r have length units, meanwhile θ and ϕ are angular coordinates, t is a time-like coordinate and c denotes the speed of light. We shall consider that ψ_0 is an arbitrary constant with length units and the constant parameter ζ has units of $(mass)(length)^{-1}$.

For a massive test particle outside of a spherically symmetric compact object in 5D with exterior metric given by (1) the 5D Lagrangian can be written as

$$
{}^{(5)}L = \frac{1}{2} g_{ab} U^a U^b = \frac{1}{2}\left(\frac{\psi}{\psi_0}\right)^2 \left[c^2 f(r)\left(U^t\right)^2 - \frac{\left(U^r\right)^2}{f(r)} - r^2\left(U^\theta\right)^2 - r^2 sin^2\theta \left(U^\phi\right)^2\right] - \frac{1}{2}\left(U^\psi\right)^2 . \tag{2}
$$

We shall take $\theta = \pi/2$. Since t and ϕ are cyclic coordinates, hence their associated constants of motion p_t and p_ϕ, are

$$
\begin{aligned}
p_t \equiv \frac{\partial\, {}^{(5)}L}{\partial U^t} &= c^2 \left(\frac{\psi}{\psi_0}\right)^2 f(r) U^t, & (3)\\
p_\phi \equiv \frac{\partial\, {}^{(5)}L}{\partial U^\phi} &= -\left(\frac{\psi}{\psi_0}\right)^2 r^2 U^\phi. & (4)
\end{aligned}
$$

Using the constants of motion given by (3) and (4), we can express the five-velocity condition as follows:

$$
\left(\frac{\psi_0}{\psi}\right)^2 \frac{p_t^2}{c^2 f(r)} - \left(\frac{\psi}{\psi_0}\right)^2 \frac{\left(U^r\right)^2}{f(r)} - \frac{p_\phi^2}{r^2}\left(\frac{\psi_0}{\psi}\right)^2 - \left(U^\psi\right)^2 = c^2. \tag{5}
$$

After rearranging some terms and using the expression for $f(r)$, the equation (5) can be written as

$$
\frac{1}{2}\left(U^r\right)^2 + \frac{1}{2}\left(\frac{\psi_0}{\psi}\right)^2 \left(U^\psi\right)^2 + V_{eff}(r) = E. \tag{6}
$$

If we identify the energy E as

$$
E = \frac{1}{2}\left(\frac{\psi_0}{\psi}\right)^4 (p_t^2 c^{-2} + p_\phi^2 \psi_0^{-2}) - \frac{c^2}{2}\left(\frac{\psi_0}{\psi}\right)^2 . \tag{7}
$$

the effective 5D potential $V_{eff}(r)$ results to be

$$
\begin{aligned}
V_{eff}(r) &= -\left(\frac{\psi_0}{\psi}\right)^2 \frac{G\zeta\psi_0}{r} + \left(\frac{\psi_0}{\psi}\right)^4 \left[\frac{p_\phi^2}{2r^2} - \frac{G\zeta\psi_0 p_\phi^2}{c^2 r^3}\right] \\
&- \frac{1}{2}\left(\frac{\psi_0}{\psi}\right)^2 \left[\left(U^\psi\right)^2 \left(\frac{2G\zeta\psi_0}{c^2 r} - \frac{r^2}{\psi_0^2}\right) - \left(\frac{rc}{\psi_0}\right)^2\right]. \qquad (8)
\end{aligned}
$$

However, we are interested in the study of this potential for massive test particles on static foliations $\psi = \psi_0 = c/H_0$, such that the dynamics evolves on an effective 4D manifold Σ_0. From the point of view of an relativistic observer, this implies that $U^\psi = 0$.

1.2 Physics on the 4D manifold Σ_0

When we take a foliation $\{\Sigma_0 : \psi = \psi_0\}$ on (1), we obtain the induced metric given by the 4D line element

$$dS^2_{ind} = c^2 f(r) dt^2 - \frac{dr^2}{f(r)} - r^2 \left[d\theta^2 + sin^2(\theta)\, d\phi^2\right], \tag{9}$$

which is known as the Schwarzschild-de Sitter metric. From the relativistic point of view, observers that are on Σ_0 move with $U^\psi = 0$. We assume that the induced matter on Σ_0 can be globally described by a 4D energy momentum tensor of a perfect fluid $T_{\alpha\beta} = (\rho c^2 + P) U_\alpha U_\beta - P g_{\alpha\beta}$, where $\rho(t, r)$ and $P(t, r)$ are respectively the energy density and pressure of the induced matter, such that

$$P = -\rho c^2 = -\frac{3c^4}{8\pi G} \frac{1}{\psi_0^2}, \tag{10}$$

which corresponds to a vacuum equation of state. We associate the energy density of induced matter ρ to a mass density of a sphere of physical mass $m \equiv \zeta\psi_0$ and radius r_0. If we do that, it follows that m and the radius r_0 of such a sphere are related by the expression $\zeta = r_0^3/(2G\psi_0^3)$, such that $G\zeta = \sqrt{3}/9$ and there is only a single Schwarzschild radius. In this case the Schwarzschild radius is $r_{Sch} = \psi_0/\sqrt{3} \geq r_0$. When it is greater than the radius of the sphere of parameter ζ, the compact object has properties very close to those of a black hole on distances $1 \gg r/\psi_0 > r_{Sch}/\psi_0$. This condition holds when $G\zeta \leq 1/(2\sqrt{27}) \simeq 0.096225$. For $G\zeta > \sqrt{3}/9$ one obtains $f(r) < 0$ and there is not Schwarzschild radius. When $G\zeta \leq \sqrt{3}/9$ there are two Schwarzschild radius, an interior r_{S_i} and an exterior one r_{S_e}, such that by definition $f(r_{S_i}) = f(r_{S_e}) = 0$. In this paper we shall focus in this last possibility which is relevant for astrophysical scales. We shall assume that we live on the 4D hypersurface $\Sigma_{H_0} : \psi_0 = cH_0^{-1}$, H_0 being $H_0 = 73\,\frac{km}{sec} Mpc^{-1}$ the present day Hubble constant.

When one takes $U^\psi = 0$, the induced potential $V_{ind}(r)$ on the hypersurface Σ_0 is given by

$$V_{ind}(r) = -\frac{Gm}{r} + \frac{p_\phi^2}{2r^2} - \frac{Gm}{c^2}\frac{p_\phi^2}{r^3} - \frac{c^2}{2}\left(\frac{r}{\psi_0}\right)^2, \tag{11}$$

where $m = \zeta\psi_0$ is the effective 4D physical mass. The confining force $\Phi^\psi = \epsilon/\psi_0$ is perpendicular to the penta-velocities U^μ on all the hypersurface Σ_0, so that the system is conservative on Σ_0. Hence Φ^ψ cannot be interpreted as a fifth force.

The first two terms in the right hand side of (11) correspond to the classical potential, the third term is the usual relativistic contribution and the last term is a new contribution coming from the 5D metric solution (1). The acceleration associated to the induced potential (11) reads

$$a = -\frac{Gm}{r^2} + \frac{p_\phi^2}{r^3} - \frac{3Gm}{c^2}\frac{p_\phi^2}{r^4} + \frac{rc^2}{\psi_0^2}. \tag{12}$$

The condition for circular motion of the test particle $(dV_{ind}/dr) = 0$ acquires the form

$$r^5 - \frac{Gm}{c^2}\psi_0^2 r^2 + \frac{p_\phi^2 \psi_0^2}{c^2} r - \frac{3Gm}{c^4} p_\phi^2 \psi_0^2 = 0. \tag{13}$$

By expressing the equation (6) as a function of the angular coordinate ϕ (indeed assuming $1/u = r = r(\phi)$), we obtain, after make the derivative with respect to ϕ, the orbit equation on 4D

$$\frac{d^2u}{d\phi^2} + u + c^2 p_\phi^{-2} \psi_0^{-2} u^{-3} = \left(\frac{Gm}{c^2}\right) p_\phi^{-2} + \left(\frac{3Gm}{c^2}\right) u^2. \tag{14}$$

This equation is almost the same that the one usually obtained in the 4D general theory of relativity for an exterior Schwarzschild metric with the exception of the third term on the left hand side. This new term could be interpreted as a new contribution coming in this case from the extra coordinate.

1.3 Motivation

It is well known that galaxies are finite in size. However, from the gravitational point of view this fact cannot be explained in a satisfactory manner. From the observational point of view it is considered that the limit of one galaxy is where the angular velocity of matter around the center is zero: $\dot{\phi} = 0$. At this radius, the squared momentum p_ϕ^2

$$p_\phi^2 = \frac{c^2}{(c/H_0)^2} \frac{(Gm(c/H_0)^2 - r^3 c^2)}{(c^2 r - 3Gm)}, \tag{15}$$

becomes zero. Hence, we shall consider some examples of galaxies to determine their size once we know the masses of the BH which are in the center of each one of these galaxies. This radius will be that for which $p_\phi(r_{size}) = 0$ in eq. (15):

$$r_{size} = \left(\frac{Gm}{H_0^2}\right)^{1/3}. \tag{16}$$

However, this radius should be only a lower limit for the galactic size, because in our calculation we are neglecting the galactic mass outside the BH.

In in this letter, we shall deal with size of galaxies in the present day, so that we shall consider that (c/H_0) is the present day Hubble horizon. However, for very distant galaxies one should take into account the horizon of the galaxy at the moment of the signal emission.

2 Some examples

In this letter we are interested to study galaxies to estimate its size once we know the compact object [or black hole (BH)] which is in its center. This kind of galaxies has spiral arms which evolves on the plane perpendicular to the angular moment.

2.1 Milky Way

The immediate case to be studied is our galaxy (or Milky Way), which has in its center a BH of a mass $\sim 4.1 \times 10^6\ M_\odot$[5]. It is agreed that the Milky Way is a barred spiral galaxy, with observations suggesting that it is a spiral galaxy. The stellar disk of the Milky Way galaxy is approximately $10^5\, ly$ in diameter, and is considered to be, on average, about $10^3\, ly$ thick. It is estimated to contain at least 200 billion stars and possibly up to 400 billion stars, the exact figure depending on the number of very low-mass, or dwarf stars, which are hard to detect, especially more than $300\, ly$ from the Sun, and so current estimates of the total number remain highly uncertain, though often speculated to be around 250 billion. In the figure (1) we plot the evolution of p_ϕ^2 as a function of the galactic radius. It is obvious from the graphic that the lower limit of the radius of our galaxy is close to $5 \times 10^4\, ly$, in agreement with observations.

The Galactic Halo extends outward, but is limited in size by the orbits of two Milky Way satellites, the Large and the Small Magellanic Clouds, whose distance is at about $1.8 \times 10^5\ ly$ [6]. In our model it corresponds with a galactic mass of $2 \times 10^8\ M_\odot$, which is very close to the Milky Way mass.

2.2 Andromeda

Another case of interest is the Andromeda galaxy. It is also known as Messier 31, **M31** (or **NGC 224**). Andromeda is the nearest spiral galaxy to the Milky Way. **M31** was the second galaxy in which stellar dynamics revealed the presence of a supermassive black hole[7]. Axisymmetric dynamical models implied

BH masses of $(1-10)\times 10^7\,M_\odot$. The smallest masses were given by disk models, and the largest were given by spherical models. Hubble Space Telescope (HST) spectroscopy reveals a dark mass (presumed black hole) located at the center of this cluster with an estimated mass of $1.4\pm^{0.7}_{0.3}\times 10^8\ M_\odot$[8]. In the figure (2) we show the evolution of p_ϕ^2 for three different BH mass values ($3.0\times 10^7\,M_\odot$ with thick line, $6.2\times 10^7\,M_\odot$ with thin line and $1.0\times 10^8\,M_\odot$ with points), as a function of the minimum galactic radius. From the graphic we infer that the radius of Andromeda is greater than $1.2\times 10^5\,ly$ for a BH mass greater than $6.2\times 10^7\,M_\odot$. Since the observations agree with a minimum galactic radius close to $1.2\times 10^5\,ly$, we infer that the BH mass in the core of Andromeda would have a mass close to $6.2\times 10^7\,M_\odot$.

2.3 Messier 87

A very interesting example is the galaxy Messier 87 (or **M87**). At the core is a supermassive BH with mass estimated in the range $(2.4-6.5)\times 10^9\ M_\odot$[9]. **M87** forms the primary component of an active galactic nucleus that is a strong source of multiwavelength radiation, particularly radio waves. A jet of energetic plasma originates at the core and extends out at least 5000 ly[10]. In the figure (3) it is shown that its bound radius r_{M87} is really bigger than other galaxies, r_{M87} being in the range $(4.05\times 10^5 < r_{M87} < 5.65\times 10^5)\,ly$.

3 Final Comments

We have calculated the lower radius for three different galaxies (the Milky Way, Andromeda and Messier 87) using the estimation for the BH masses in their core of these ones. Our results agree very good with observations. In view of our calculations we conclude that the mass of the BH in the Milky Way should be close to $\sim 4.1\times 10^6\ M_\odot$ for a minimum radius close to $r_{MW}\simeq 4.9\times 10^4\,ly$. Moreover, taking into account that our galaxy is limited in size by the orbits of two Milky Way satellites (whose distance is at about 1.8×10^5), we conclude that our galaxy mass should take a mass of at least $2\times 10^8\ M_\odot$.

In the case of Andromeda, we conclude that the mass of the BH in its center should be close to $r_A\simeq 6.2\times 10^7\ M_\odot$ in order to obtain a minimum galactic size close to $r_A\simeq 1.2\times 10^5\,ly$. Finally, for the range of masses $(2.4-6.5)\times 10^9\ M_\odot$, our calculations conclude that the bound radius r_{M87}, of Messier 87 should be in the range $(4.05\times 10^5 < r_{M87} < 5.65\times 10^5)\,ly$.

Overall, we have shown how the here adopted theory of extended General Relativity implies the existence of the radius r_{size} for a given gravitational source. Beyond this radius the gravitational force becomes repulsive. As discussed above for the three example cases, this radius is yet compatible with the observed size of the source.

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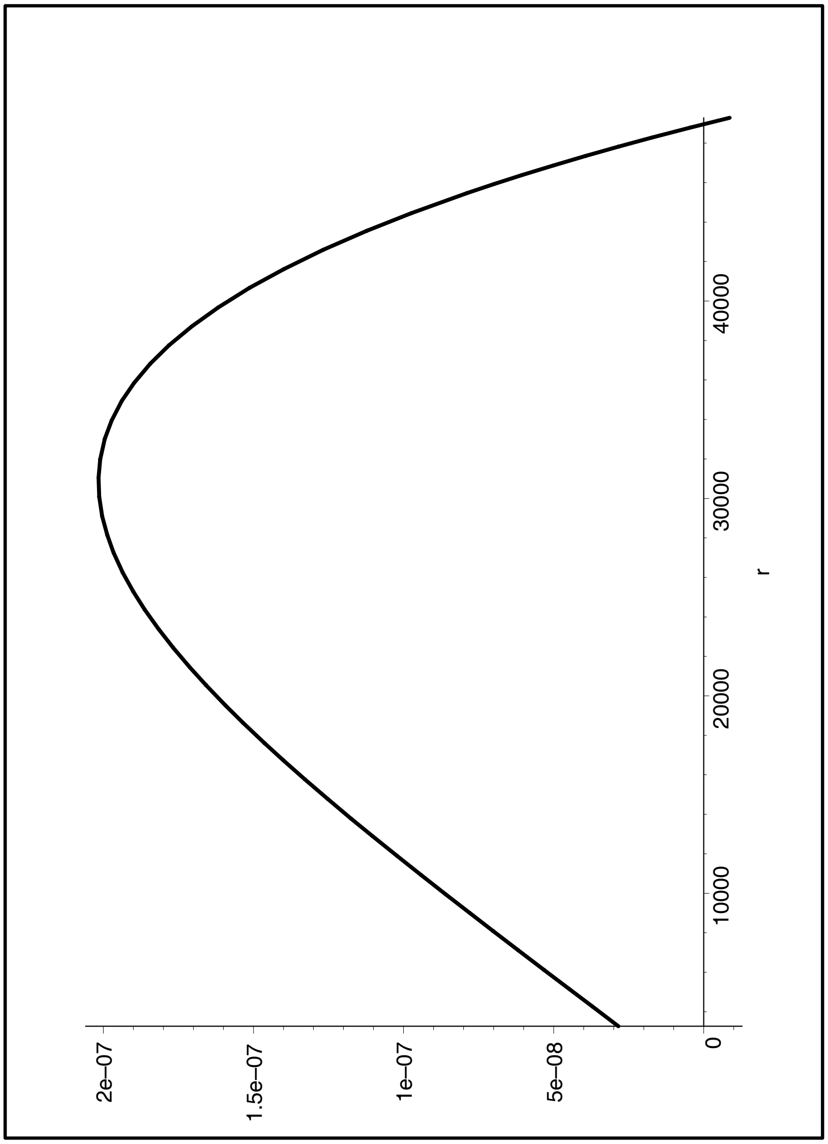

Figure 1: p_ϕ^2 as a function of the Milky Way galactic radius r_{MW} [in light years (ly)] for a BH mass $4.1 \times 10^6\, M_\odot$.

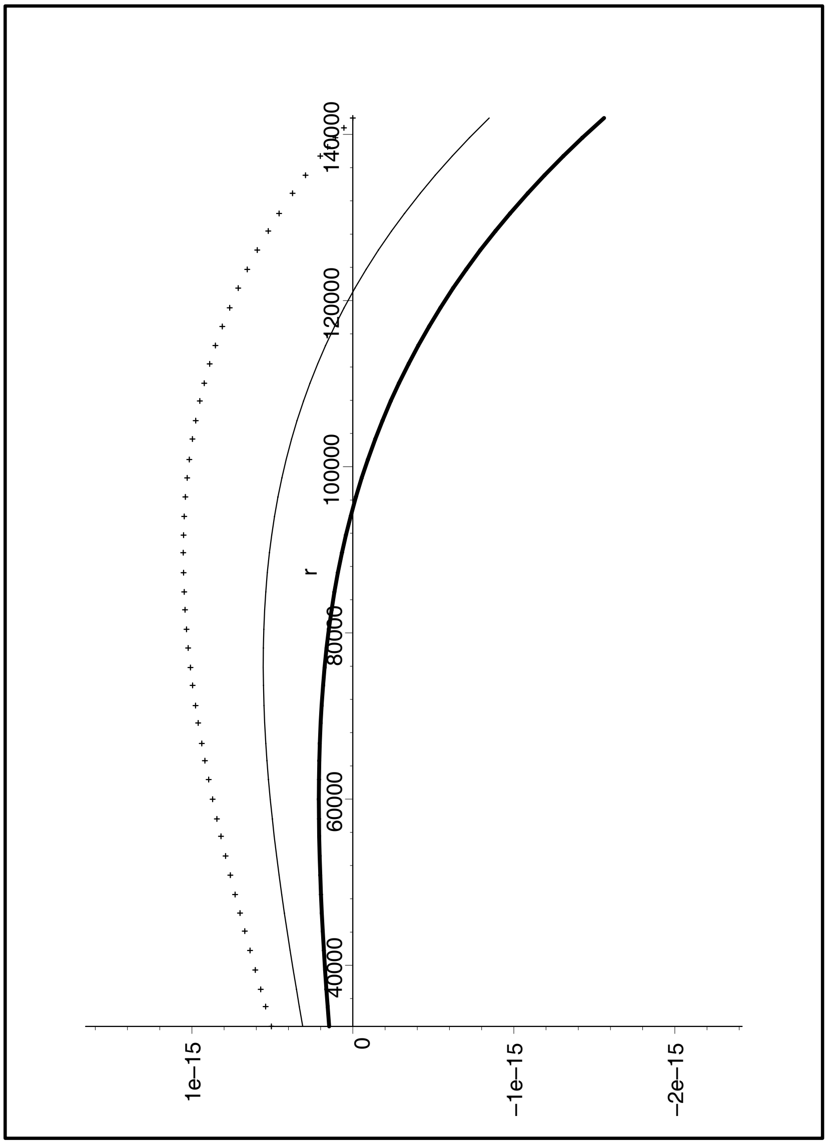

Figure 2: p_ϕ^2 as a function of the Andromeda galactic radius r_A [in light years (ly)] for three different BH masses: $3.0 \times 10^7\, M_\odot$ with thick continuous line, $6.2 \times 10^7\, M_\odot$ with thin continuous line and $1.0 \times 10^8\, M_\odot$ with points.

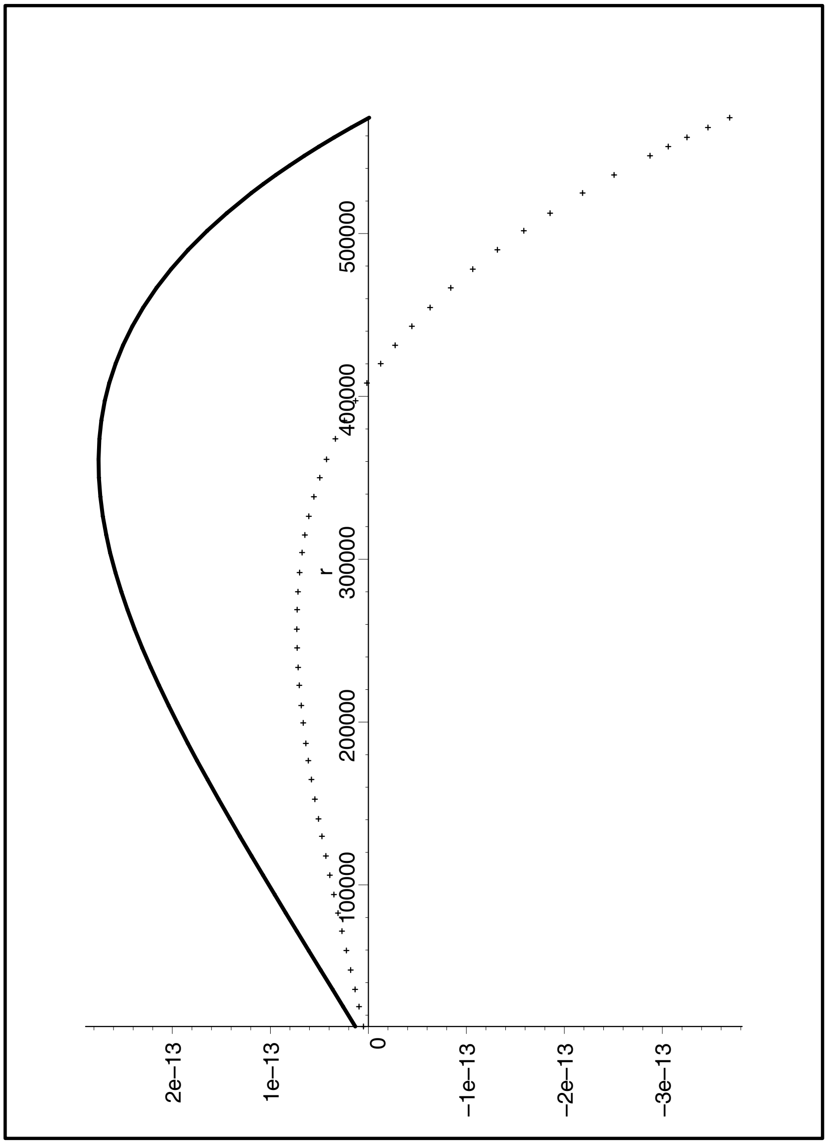

Figure 3: p_ϕ^2 as a function of the M87 galactic radius r_{M87} [in light years (ly)] for two different BH masses: $2.4 \times 10^9\, M_\odot$ with points and $6.0 \times 10^9\, M_\odot$ with continuous line.

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