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Quantum origin of pre-big-bang collapse from induced matter theory of gravity

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ABSTRACT

We revisit a collapsing pre-big-bang model of the universe to study with detail the non-perturbative quantum dynamics of the dispersal scalar field whose dynamics becomes from the dynamical foliation of test massless scalar field φ on a 5D Riemann-flat metric, such that the extra space-like coordinate is noncompact. The important result here obtained is that the evolution of the system, which is described thorough the equation of state has the unique origin in the quantum contributions of the effective 4D scalar field $\tilde{\varphi}$.

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1. Introduction

The five-dimensional model is the simplest extension of General Relativity (GR), and is widely regarded as the low-energy limit of models with higher dimensions (such as 10D supersymmetry and 11D supergravity). Modern versions of 5D GR abandon the cylinder and compactification conditions used in original Kaluza–Klein (KK) theories, which caused problems with the cosmological constant and the masses of particles, and consider a large extra dimension. In particular, the Induced Matter Theory (IMT) is based on the assumption that ordinary matter and physical fields that we can observe in our 4D universe can be geometrically induced from a 5D Ricci-flat metric with a space-like noncompact extra dimension on which we define a physical vacuum [1,2]. The Campbell–Magaard theorem [3–7] serves as a ladder to go between manifolds whose dimensionality differs by one. This theorem, which is valid in any number of dimensions, implies that every solution of the 4D Einstein equations with arbitrary energy momentum tensor can be embedded, at least locally, in a solution of the 5D Einstein field equations in vacuum. Because of this, the stress-energy may be a 4D manifestation of the embedding geometry. Physically, the background metric there employed describes a 5D extension of an usual de Sitter spacetime. By making a static foliation on the space-like extra coordinate, it is possible to obtain an effective 4D universe that suffered an exponential accelerated expansion driven by a scalar (inflaton) field with an equation of state close to a vacuum dominated one [9–12]. The most conservative assumption is that the energy density $\rho = P/\omega$ is due to a cosmological pa-

rameter which is constant and the equation of state is given by a constant $\omega = -1$, describing a vacuum dominated universe with pressure P and energy density ρ . On the other hand, exists a kind of exotic fluids that may be framed in theories with matter fields that violate the weak energy condition [13], such that $\omega < -1$. These models were called phantom cosmologies, and their study represents a currently active area of research in theoretical cosmology [14,15].

On the other hand, the spherically symmetric collapse of a massless scalar field has been of much interest towards understanding the dynamical evolutions in general relativity. A remarkable finding of some numerical investigations is the demonstration of criticality in gravitational collapse. Specifically, it was found that for a range of values of the parameter characterizing the solution, black hole forms and there was a critical value of the parameter beyond which the solutions are such that the scalar field disperses without forming any black hole. However, this result has been obtained mainly through numerical studies and a proper theoretical understanding of this phenomenon is still lacking (see e.g. [16] and the references therein). In order to study the dynamics of a massless scalar field φ on a 5D vacuum, we consider the canonical metric

$$ds^2 = g_{\mu\nu}(y^\sigma, \psi) dy^\mu dy^\nu - d\psi^2. \quad (1)$$

Here the 5D coordinates are orthogonal: $y \equiv \{y^a\}$.¹ The geodesic equations for a relativistic observer are

$$\frac{dU^a}{ds} + \Gamma_{bc}^a U^b U^c = 0, \quad (2)$$

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¹ Greek letters run from 0 to 3, and Latin letters run from 0 to 4.

where $U^a = \frac{dy^a}{ds}$ are the velocities and Γ_{bc}^a are the connections of (1). Now we consider a parametrization $\psi(x^\alpha)$, where $x \equiv \{x^\alpha\}$ are an orthogonal system of coordinates, such that the effective line element (1), now can be written as

$$dS^2 = h_{\alpha\beta} dx^\alpha dx^\beta. \quad (3)$$

It is very important to notice that S will be an invariant, so that derivatives with respect to S will be the same on 5D or 4D. In other words, in this Letter we shall consider spacetime lengths that remain unaltered when we move on an effective 4D spacetime.

1.1. Einstein equations for dynamical foliations from a 5D vacuum state

Now we consider the Einstein equations on the 5D canonical metric like (1)

$$G_{ab} = -8\pi G T_{ab}, \quad (4)$$

where the Einstein tensor is given by $G_{ab} = \mathcal{R}_{ab} - \frac{1}{2}g_{ab}\mathcal{R}$ and \mathcal{R}_{ab} is the Ricci tensor, such that the scalar of curvature is $\mathcal{R} = g_{ab}\mathcal{R}^{ab}$. Because we are considering a 5D Ricci-flat metric, the Einstein tensor and the Ricci scalar will be null. Using the transformations previously introduced, we obtain that

$$\bar{G}_{\alpha\beta} = \bar{\mathcal{R}}_{\alpha\beta} - \frac{1}{2}h_{\alpha\beta}\bar{\mathcal{R}} = -8\pi G \bar{T}_{\alpha\beta}, \quad (5)$$

where we have used respectively the transformations

$$\bar{\mathcal{R}}_{\alpha\beta} = e^a_\alpha e^b_\beta \mathcal{R}_{ab}, \quad (6)$$

$$\bar{\mathcal{R}} = h_{\alpha\beta} \bar{\mathcal{R}}^{\alpha\beta}, \quad (7)$$

$$\bar{T}_{\alpha\beta} = e^a_\alpha e^b_\beta T_{ab}, \quad (8)$$

for the effective 4D Ricci tensor, the scalar of curvature and the energy-momentum tensor.

1.2. Energy-momentum tensor

We consider a quantum massless scalar field $\varphi(y^a)$ on the metric (1). In order to make a complete description for the dynamics of the scalar field, we shall consider its energy momentum tensor. In order to describe a true 5D physical vacuum we shall consider that the field is massless and there is absence of interaction on the 5D Ricci-flat manifold, so that

$$T^a_b = \Pi^a \Pi_b - g^a_b \mathcal{L}[\varphi, \varphi, c], \quad (9)$$

where $\mathcal{L}[\varphi, \varphi, c] = \frac{1}{2}\varphi^a \varphi_a$ is the Lagrangian density for a free and massless scalar field on (1) and the canonical momentum is $\Pi^a = \frac{\partial \mathcal{L}}{\partial \varphi_{,a}}$. Notice that we are not considering interactions on the 5D vacuum, because it is related to a physical vacuum in the sense that the Einstein tensor is zero: $G^a_b = 0$.

1.3. Dynamics of the scalar field for a dynamical foliation

We are interested to study how is the effective 4D dynamics obtained from a dynamic foliation of a 5D Ricci-flat canonical metric. We consider a classical massless scalar field $\varphi(y^a)$ on the metric (1). The effective 4D energy momentum tensor will be

$$\bar{T}_{\alpha\beta} = e^a_\alpha e^b_\beta T_{ab}|_{\psi(x^\alpha)}. \quad (10)$$

In other words, using the fact that \mathcal{L} is an invariant it is easy to demonstrate that

$$\bar{T}^\alpha_\beta = \bar{\Pi}^\alpha \bar{\Pi}_\beta - h^\alpha_\beta \bar{\mathcal{L}}, \quad (11)$$

where \mathcal{L} is an invariant of the theory: $\mathcal{L} = \frac{1}{2}\varphi_{,a}\varphi_{,a} = \frac{1}{2}(e^a_\alpha \bar{\varphi}_{,\alpha}) \times (\bar{e}^\beta_a \bar{\varphi}_{,\beta})$. The equation of motion for the scalar field $\bar{\varphi}$ becomes from $\bar{\nabla}_\alpha \bar{T}^\alpha_\beta = 0$,² so that one obtains

$$h^{\mu\nu} \bar{\nabla}_\nu \bar{\varphi}_{,\mu} = 0, \quad (12)$$

that describes the dynamics of $\bar{\varphi}(x^\alpha)$ on the effective 4D hypersurface (3). Notice that in the dynamics of $\bar{\varphi}$, which is described by Eq. (12), it is absent any kind of interaction. This is because the dynamical foliations as we are studied in this Letter describe a dispersal system [8].

In a previous Letter [17] we have studied the gravitational collapse of the universe which is driven by a massless dispersal scalar field. The system was studied from a 5D Riemann-flat canonical metric, on which we make a dynamical foliation on the extra space-like dimension. The asymptotic universe there obtained, which is absent of singularities, results to be finite in size and energy density, which tends to zero for asymptotic large times, so that the asymptotic equation of state becomes $\omega|_{t \rightarrow \infty} \rightarrow -\infty$. This is because the pressure is negative (opposes the collapse) along all the contraction and its asymptotic value tends to zero, but more slowly than does the energy density. In this Letter we shall revisit a collapsing system, but from a different 5D metric, with the aim to study with detail the non-perturbative quantum dynamics of the dispersal scalar field.

2. An example: pre-big-bang collapsing universe

We consider the 5D canonical extended de Sitter Riemann-flat metric [18]

$$dS^2 = \left(\frac{\psi}{\psi_0}\right)^2 [dt^2 - e^{-2\psi_0^{-1}t} dr^2] - d\psi^2, \quad (13)$$

such that $dr^2 = dx^i \delta_{ij} dx^j$. The relevant nonzero connections are

$$\begin{aligned} \Gamma_{ii}^0 &= -\frac{1}{\psi_0} e^{-2\psi_0^{-1}t}, & \Gamma_{\alpha 4}^\alpha &= \frac{1}{\psi}, \\ \Gamma_{i0}^i &= -\frac{1}{\psi_0}, & \Gamma_{00}^4 &= \frac{\psi}{\psi_0^2}. \end{aligned} \quad (14)$$

Since the metric (13) is Riemann-flat (and therefore Ricci-flat), hence it is suitable to describe a 5D vacuum ($G_{ab} = 0$) in the framework of the IMT of gravity. With this aim we shall consider the 5D action

$$\mathcal{I} = \int d^4x d\psi \sqrt{|g|} \left(\frac{\mathcal{R}}{16\pi G} + \frac{1}{2} g^{ab} \varphi_{,a} \varphi_{,b} \right), \quad (15)$$

where g is the determinant of the covariant metric tensor g_{ab} : $g = (\frac{\psi}{\psi_0})^8 e^{-6\psi_0^{-1}t}$.

2.1. Effective 4D dynamics of φ

The effective 4D spacetime being described by the line element

$$dS^2 = \left[\frac{\psi^2(t)}{\psi_0^2} - \dot{\psi}^2 \right] dt^2 - \frac{\psi^2(t)}{\psi_0^2} e^{-2\psi_0^{-1}t} dR^2, \quad (16)$$

where the dot denotes the derivative with respect to t and ψ_0 is some constant. In order to consider t as a cosmic time, one must require that

$$\frac{\psi^2(t)}{\psi_0^2} - \dot{\psi}^2 = 1, \quad (17)$$

² Here, $\bar{\nabla}_\alpha$ denotes the covariant derivative on the effective 4D hypersurface, with respect to the Christoffel connections $\bar{\Gamma}^\alpha_{\beta\gamma}$.

so that the foliation is described by

$$\psi(t) = \psi_0 \cosh(t/\psi_0) \rightarrow \dot{\psi}(t) = \sinh(t/\psi_0). \quad (18)$$

Finally, the metric (16), for a foliation (18) is described by

$$dS^2 = dt^2 - \cosh(t/\psi_0)^2 e^{2\psi_0^{-1}t} dR^2, \quad (19)$$

which describes an 3D (flat) spatially isotropic universe which is collapsing with a scale factor $a(t) = \cosh(t/\psi_0)$, $e^{-\psi_0^{-1}t}$, a Hubble parameter $H(t) = \frac{\dot{a}}{a}$ and a deceleration parameter $q = -\frac{\ddot{a}}{a^2}$ given by (for $H_0 = 1/\psi_0$)

$$H(t) = H_0 [\tanh(H_0 t) - 1], \quad (20)$$

$$q(t) = -\frac{2 \cosh(H_0 t)}{\cosh(H_0 t) - \sinh(H_0 t)}. \quad (21)$$

Notice that $\dot{H} > 0$ and $a(t)|_{t \rightarrow \infty} = 1/2$, such that the asymptotic size of the universe is finite. Furthermore the late time asymptotic derivative the Hubble parameter and the deceleration parameter, are

$$\dot{H}(t)|_{t \rightarrow \infty} \rightarrow 0, \quad (22)$$

$$q(t)|_{t \rightarrow \infty} \rightarrow -\infty, \quad (23)$$

which means that the universe describes a collapse with asymptotic Minkowski spacetime.

2.2. Einstein's equations

On the other hand, the relevant components of the Einstein tensor in Cartesian coordinates, are

$$\bar{G}^0_0 = -\frac{3H_0^2}{\cosh^2(H_0 t)} [\cosh(H_0 t) - \sinh(H_0 t)]^2, \quad (24)$$

$$\bar{G}^i_j = -\frac{H_0^2}{\cosh^2(H_0 t)} [\cosh(H_0 t) - \sinh(H_0 t)] \times [5 \cosh(H_0 t) - \sinh(H_0 t)] \delta^i_j, \quad (25)$$

so that, using the fact that the Einstein equations are respectively $G^0_0 = -8\pi G \rho$ and $G^x_x = G^y_y = G^z_z = 8\pi G P$, we obtain the equation of state for the universe

$$\frac{P}{\rho} = \omega(t) = -\frac{1}{3} \frac{[5 \cosh(H_0 t) - \sinh(H_0 t)]}{[\cosh(H_0 t) - \sinh(H_0 t)]}. \quad (26)$$

Notice that ω always remains with negative values $\omega(t) < -1$, and evolves from $-5/3$ to $-\infty$, for large asymptotic times. The effective 4D scalar curvature

$$\bar{\mathcal{R}} = \frac{6H_0^2}{\cosh^2(H_0 t)} [\cosh(H_0 t) - \sinh(H_0 t)] \times [3 \cosh(H_0 t) - \sinh(H_0 t)], \quad (27)$$

decreases with the time and has a null asymptotic value $\bar{\mathcal{R}}|_{t \rightarrow \infty} \rightarrow 0$.

The expectation values for the energy density and the pressure, written in terms of the scalar field $\varphi(t, \vec{r}, \psi(t)) \equiv \bar{\varphi}(t, \vec{r})$, are

$$\begin{aligned} \bar{\rho} &= \langle 0 | \bar{T}^0_0 | 0 \rangle \\ &= \left\langle \frac{\psi_0^2}{\psi^2(t)} \left[\frac{1}{2} \dot{\varphi}^2 + \frac{1}{2a^2(t)} (\vec{\nabla} \varphi)^2 \right] + \frac{1}{2} \left(\frac{\partial \varphi}{\partial \psi} \right)^2 \right\rangle_{\psi(t)} \\ &= \left\langle \frac{1}{2} \dot{\varphi}^2 + \frac{1}{2a^2(t)} (\vec{\nabla} \bar{\varphi})^2 \right\rangle, \end{aligned} \quad (28)$$

$$\begin{aligned} \bar{P} &= -\langle 0 | \bar{T}^i_j | 0 \rangle \\ &= -\delta^i_j \left\langle \frac{\psi_0^2}{\psi^2(t)} \left[\frac{1}{2} \dot{\varphi}^2 - \frac{1}{6a^2(t)} (\vec{\nabla} \varphi)^2 \right] - \frac{1}{2} \left(\frac{\partial \varphi}{\partial \psi} \right)^2 \right\rangle_{\psi(t)} \\ &= -\delta^i_j \left\langle \frac{1}{2} \dot{\varphi}^2 - \frac{1}{6a^2(t)} (\vec{\nabla} \bar{\varphi})^2 \right\rangle. \end{aligned} \quad (29)$$

Here, the notation $\langle 0 | \dots | 0 \rangle$ denotes the quantum expectation value calculated on a 4D vacuum state. Because we are considering a spatially isotropic and homogeneous background, we shall consider an averaging value with respect to a Gaussian distribution on a Euclidean 3D volume.

3. Field dynamics and vacuum

The effective 4D equation of motion for $\bar{\varphi}$, is

$$\ddot{\bar{\varphi}} - \frac{e^{2H_0 t}}{\cosh^2(H_0 t)} \bar{\nabla}_r^2 \bar{\varphi} + 3H_0 [\tanh(H_0 t) - 1] \dot{\bar{\varphi}} = 0, \quad (30)$$

which in the limit of $t \rightarrow \infty$ tends to an equation of motion for a massless scalar field on a asymptotic Minkowski spacetime: $\ddot{\bar{\varphi}} - \bar{\nabla}_r^2 \bar{\varphi} = 0$.

Using Eqs. (24) and (25) joined with Eqs. (28) and (29), we obtain from the effective 4D Einstein equations (5) the following relevant non-perturbative expressions for the expectation values of squared scalar field $\bar{\varphi}$:

$$\langle (\vec{\nabla} \bar{\varphi})^2 \rangle = -\frac{3H_0^2}{4\pi G} e^{-2H_0 t} [\cosh^2(H_0 t) - \sinh^2(H_0 t)], \quad (31)$$

$$\langle (\dot{\bar{\varphi}})^2 \rangle = -\frac{3H_0^2}{2\pi G} [1 - \tanh(H_0 t)], \quad (32)$$

that has the asymptotic large time limits

$$\langle (\vec{\nabla} \bar{\varphi})^2 \rangle_{t \rightarrow \infty} \rightarrow 0, \quad (33)$$

$$\langle (\dot{\bar{\varphi}})^2 \rangle_{t \rightarrow \infty} \rightarrow 0. \quad (34)$$

3.1. The modes

If we redefine the modes $\chi_k(t) = a^{3/2} \xi_k(t)$, we obtain the equation of motion for $\chi_k(t)$

$$\ddot{\chi}_k(t) + \left[\frac{k^2}{a^2} - \left(\frac{9}{4} H^2(t) + \frac{3}{2} \dot{H}(t) \right) \right] \chi_k(t) = 0, \quad (35)$$

which has the general solution

$$\begin{aligned} \chi_k(t) &= e^{3H_0 t} e^{\frac{1}{2} \ln(1+e^{2H_0 t})} \left\{ A_k e^{-\frac{1}{2} \sqrt{1-(k/H_0)^2} \ln(1+e^{2H_0 t})} \right. \\ &\quad \times {}_2F_1[a_1, b_1, [c_1]; 1 + e^{2H_0 t}] \\ &\quad + B_k e^{\frac{1}{2} \sqrt{1-(k/H_0)^2} \ln(1+e^{2H_0 t})} \\ &\quad \left. \times {}_2F_1[a_2, b_2, [c_2]; 1 + e^{2H_0 t}] \right\}, \end{aligned} \quad (36)$$

such that ${}_2F_1[[a, b], [c]; x(t)]$ is the Gaussian hypergeometric function with argument $x(t) = 1 + e^{2H_0 t}$, and

$$a_1 = 2 + i \frac{k}{H_0} - \sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 2, \quad (37)$$

$$b_1 = 2 - i \frac{k}{H_0} - \sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 2 \left(1 - i \frac{k}{H_0} \right), \quad (38)$$

$$c_1 = 1 - 2\sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 1 - 2i \frac{k}{H_0}, \quad (39)$$

$$a_2 = 2 + i \frac{k}{H_0} + \sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 2 \left(1 + i \frac{k}{H_0} \right), \quad (40)$$

$$b_2 = 2 - i \frac{k}{H_0} + \sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 2, \quad (41)$$

$$c_2 = 1 + 2\sqrt{1 - (k/H_0)^2} \Big|_{k \gg H_0} \simeq 1 + 2i \frac{k}{H_0}. \quad (42)$$

In this UV limit the hypergeometric functions take the asymptotic expressions

$${}_2F_1[a_1, b_1, [c_1]; e^{2H_0 t}] \Big|_{UV} \simeq \frac{[1 + 2ik/H_0]}{[1 - 2ik/H_0]} e^{-4H_0 t}, \quad (43)$$

$${}_2F_1[a_2, b_2, [c_2]; e^{2H_0 t}] \Big|_{UV} \simeq \frac{[1 - 2ik/H_0]}{[1 + 2ik/H_0]} e^{-4H_0 t}, \quad (44)$$

so that the large times asymptotic UV redefined modes $\chi_k(t)$ can be written as

$$\begin{aligned} \chi_k(t) \Big|_{H_0 t \gg 1, k/H_0 \gg 1} \\ \simeq \frac{A_k e^{-ikt} [1 + 2ik/H_0]^2 + B_k e^{ikt} [1 - 2ik/H_0]^2}{[1 + 4(k/H_0)^2]}, \end{aligned} \quad (45)$$

where A_k and B_k are constants to be determined by normalization of the modes on the effective 4D hypersurface: $\chi_k \dot{\chi}_k^* - \dot{\chi}_k \chi_k^* = i$. From this condition we obtain that $\chi_k \chi_k^* = \frac{1}{2k}$, so that if we choose $B_k = 0$, we obtain that

$$A_k = -\frac{1}{\sqrt{2k}} \frac{[1 + 4(k/H_0)^2]}{[1 + 2ik/H_0]^2},$$

and the normalized modes χ_k on the UV sector become

$$\chi_k(t) \Big|_{H_0 t \gg 1, k/H_0 \gg 1} \simeq -\frac{1}{\sqrt{2k}} e^{-ikt}. \quad (46)$$

The final solution for the redefined modes are

$$\begin{aligned} \chi_k(t) = -\frac{e^{3H_0 t} e^{\frac{1}{2}[1 + \sqrt{1 - (k/H_0)^2}] \ln(1 + e^{2H_0 t})} [1 + 4(k/H_0)^2]}{\sqrt{2k} [1 + 2ik/H_0]^2} \\ \times {}_2F_1[a_2, b_2, [c_2]; 1 + e^{2H_0 t}]. \end{aligned} \quad (47)$$

Using the fact that $\langle (\vec{\nabla} \bar{\varphi})^2 \rangle = -\frac{1}{2\pi^2 a^3} \int_0^{k_0(t)} k^4 (\chi_k \chi_k^*) dk$, with the expression (31), we obtain that the maximum time-dependent wave-number, $k_0(t)$, is

$$\begin{aligned} k_0(t) = [12\pi H_0^2 M_p^2]^{1/4} e^{-\frac{5H_0}{4} t} \cosh^{3/4}(H_0 t) \\ \times [\cosh^2(H_0 t) - \sinh^2(H_0 t)]^{1/4}, \end{aligned} \quad (48)$$

which tends to zero as $t \rightarrow \infty$.

3.2. Classical and quantum contributions

We consider the background solution of the field, which is given by the zero mode ($k = 0$) solution $\chi_0(t)$ of the differential equation (35). A particular solution for this equation is

$$\chi_0(t) = \frac{C}{[1 + \tanh(H_0 t)]^{3/2}}, \quad (49)$$

such that $\xi_0(t) = a^{-3/2} \chi_0(t)$ comply with $\dot{\xi}_0(t) = 0$. Such that solution provide us the background (classical) contribution for the

field $\bar{\varphi}$. Therefore, the results (31) and (32), for $\langle (\vec{\nabla} \bar{\varphi})^2 \rangle$ and $\langle (\dot{\bar{\varphi}})^2 \rangle$, respectively, has the unique origin in the quantum contributions of the field. This result is valid along all the collapse and has been calculated exactly without using any approximation.

4. Final comments

The idea that our universe is a 4D space-time embedded in a higher-dimension has been a topic of increased interest in several branches of physics, and in particular, in cosmology. This idea has generated a new kind of cosmological models that includes quintessential expansion. In particular, theories on which is considered only one extra dimension have become quite popular in the scientific community. Among these theories are counted the braneworld scenarios [19], the Induced Matter (IM) theory [20–24] and all noncompact Kaluza–Klein theories. The approach here considered is inspired in the IM, where 4D sources appear as induced by one extended extra dimension, meaning, by extended, that the fifth dimension is considered noncompact. However, we have studied the case where the foliation on the fifth extra coordinate is dynamical, so that the resultant effective 4D scalar field $\bar{\varphi}(x^\alpha)$ that describes the collapsing system on the effective 4D hypersurface (19) is dispersal. The important result here obtained relies in that the origin of $\langle (\vec{\nabla} \bar{\varphi})^2 \rangle$ and $\langle (\dot{\bar{\varphi}})^2 \rangle$, is in the quantum contributions of the field. This result is valid along all the collapse and has been calculated exactly without using any approximation.

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