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While the interaction forces between two electric and two magnetic dipoles are formally identical, their interaction energies differ because in addition to mechanical work, the magnetic energy includes the electrical work needed to keep the dipole moments unaltered. This energy difference appears to contradict a calculation based on the integrals of the squares of the electric and magnetic fields since the electric and magnetic dipole fields have precisely the same geometry.
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I. INTRODUCTION

It is well known that, except for the obvious differences in the coefficients, the electric field of an electric dipole is formally identical to the magnetic field of a magnetic dipole. Furthermore, the interaction forces between electric and magnetic dipoles are also formally identical.[1–4] However, the interaction energy of a system made of two electric dipoles does not coincide with the energy of a similar system made of two magnetic dipoles. The reason is because the mechanical work needed to bring an electric dipole from infinity to its final position is only equal to the corresponding work in the magnetic case if both magnetic dipole moments are kept fixed during the entire process; in order to accomplish this, additional electrical work is required. Therefore, the total energies of the two interacting systems must be different, and this appears to contradict the fact that the electric and magnetic fields of these systems have exactly the same geometry and, therefore, the electrostatic and magnetostatic energies calculated as integrals of the squares of the fields should have the same value. In what follows we will take a closer look at this situation and resolve this apparent paradox.

II. THE PARADOX

Consider two electric dipoles: $\vec{p}$ located at position $\vec{r}$ and $\vec{p}'$ located at the origin. The electric force acting on dipole $\vec{p}$ is

$$\vec{F}_E = (\vec{p}\cdot\vec{\nabla})\vec{E} = \frac{3}{4\pi\varepsilon_0}\left[\frac{\vec{r}\cdot\vec{p}'}{r^5}\vec{p} + \frac{\vec{r}\cdot\vec{p}}{r^5}\vec{p}' + \left(\frac{\vec{p}\cdot\vec{p}'}{r^5} - 5\frac{\vec{r}\cdot\vec{p}\,\vec{r}\cdot\vec{p}'}{r^7}\right)\vec{r}\right]. \tag{1}$$

An analogous expression exists for the magnetic force between two magnetic dipoles $\vec{m}$ (at position $\vec{r}$) and $\vec{m}'$ (at the origin):

$$\vec{F}_M = (\vec{m}\cdot\vec{\nabla})\vec{B} = \frac{3\mu_0}{4\pi}\left[\frac{\vec{r}\cdot\vec{m}'}{r^5}\vec{m} + \frac{\vec{r}\cdot\vec{m}}{r^5}\vec{m}' + \left(\frac{\vec{m}\cdot\vec{m}'}{r^5} - 5\frac{\vec{r}\cdot\vec{m}\,\vec{r}\cdot\vec{m}'}{r^7}\right)\vec{r}\right]. \tag{2}$$

The use of the first equality in Eq. (2) is justified because in the system considered the sources of either dipole are not present at the position of the other.[5–11]

The forces in Eqs. (1) and (2) become identical if the magnitudes of the dipole moments are related by

$$m = cp, \tag{3}$$

where

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{4}$$

is the speed of light. For the remainder of this paper we will assume that Eq. (3) holds.

In the simplest case of two identical dipoles located so that they are facing each other, one at the origin and the other at an arbitrary point on the x-axis, the forces are repulsive and have magnitudes

$$F_E = \frac{3}{4\pi\varepsilon_0}\frac{2p^2}{x^4}, \tag{5}$$

$$F_M = \frac{3\mu_0}{4\pi}\frac{2m^2}{x^4}. \tag{6}$$

Therefore, the mechanical work required to move the right-hand dipole from $x \to \infty$ to a finite x value is

$$W_E^m = -\frac{3}{4\pi\varepsilon_0}2p^2\int_\infty^x \frac{dx}{x^4} = \frac{1}{4\pi\varepsilon_0}\frac{2p^2}{x^3}, \tag{7}$$

$$W_M^m = -\frac{3\mu_0}{4\pi}2m^2\int_\infty^x \frac{dx}{x^4} = \frac{\mu_0}{4\pi}\frac{2m^2}{x^3}. \tag{8}$$

These expressions make it possible to write down the total electric energy of the final configuration as

$$W_E^t = 2W_E^c + W_E^m = 2W_E^c + \frac{1}{4\pi\varepsilon_0}\frac{2p^2}{x^3}, \tag{9}$$

where W_E^c is the energy needed to create each electric dipole. As for the magnetic case, the total magnetic energy of the final configuration is

$$W_M^t = 2W_M^c + W_M^m + 2W_M^j = 2W_M^c + \frac{\mu_0}{4\pi}\frac{2m^2}{x^3} + 2W_M^j, \tag{10}$$

where W_M^c is the energy needed to create each magnetic dipole and W_M^j is the energy needed to maintain a constant current density for each magnetic dipole while $\vec{m}$ moves from $x \to \infty$ to its final position.

It should also be possible to calculate the energy of these two configurations by integrating the square of the electric and magnetic fields over all space:[12–15]

$$W_E^t = \frac{\varepsilon_0}{2}\int E^2 dV, \tag{11}$$

$$W_M^t = \frac{1}{2\mu_0}\int B^2 dV. \tag{12}$$

However, because the (electric) field of an electric dipole has exactly the same geometry as the (magnetic) field of a magnetic dipole,[16] these two expressions should have exactly the same value. This raises the question, why does the term $2W_M^j$, present in Eq. (10), not appear in the latter calculation based on electromagnetic field energies?

III. RESOLUTION

In order to answer this question it is necessary to analyze in more detail how the electric and magnetic dipoles are made. Among the infinite number of charge and current density distributions that can be used to represent electric and magnetic dipoles, we will use the simplest models that provide analytical solutions for the electric and magnetic fields everywhere in space.

Consider a thin, conducting spherical shell of radius R that is inserted into a uniform electric field $\vec{E} = E_0\,\hat{e}_z$. This electric field induces a surface charge density $\sigma(\theta)$ on the sphere that produces a uniform field inside, which has the same magnitude but opposite direction to the applied field (thereby shielding the external field), and a dipolar field outside. The total internal and external fields, written using spherical coordinates, are given by

$$\vec{E}_i^t = 0, \tag{13}$$

$$\vec{E}_e^t = E_0(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta) + C\frac{2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta}{r^3}, \tag{14}$$

where C is a constant. The coefficient C and the surface charge density $\sigma(\theta)$ can be determined using the following boundary conditions: (1) continuity of the tangential component of the electric field and (2) discontinuity of the radial component of the electric displacement due to the surface charge density. Applying these boundary conditions gives

$$C = R^3 E_0, \tag{15}$$

$$\sigma(\theta) = 3\varepsilon_0 E_0 \cos\theta. \tag{16}$$

These results and the superposition principle show that a possible model for an electric dipole $\vec{p}$ is a spherical surface of radius R with surface charge density

$$\sigma(\theta) = \frac{3p}{4\pi R^3}\cos\theta. \tag{17}$$

The external field of this configuration coincides with the field of a point dipole

$$\vec{E}_e = \frac{1}{4\pi\varepsilon_0} p \frac{2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta}{r^3}, \tag{18}$$

while the internal field is uniform

$$\vec{E}_i = -\frac{1}{4\pi\varepsilon_0} p \frac{\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta}{R^3}. \tag{19}$$

Now consider a thin, perfectly conducting spherical shell of radius R that is inserted into a uniform magnetic field $\vec{B} = B_0\hat{e}_z$. Similar to the electric field situation, this magnetic field induces a surface current density $\vec{j}(\theta)$ on the sphere that produces a uniform field inside, which has the same magnitude but opposite direction to the applied field (to maintain a magnetic flux of zero), and a dipolar field outside. The total internal and external fields are given by

$$\vec{B}_i^t = 0, \tag{20}$$

$$\vec{B}_e^t = B_0(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta) + D\frac{2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta}{r^3}, \tag{21}$$

where D is a constant. The coefficient D and the surface current density $\vec{j}(\theta)$ can be determined by the following boundary conditions: (1) continuity of the radial component of the magnetic field and (2) discontinuity of the tangential component of the magnetic field intensity due to the surface current density. Applying these boundary conditions leads to

$$D = -\frac{R^3 B_0}{2}, \tag{22}$$

$$\vec{j}(\theta) = \frac{B_0}{2\mu_0}\sin\theta\,\hat{e}_\varphi. \tag{23}$$

Once again we find that a possible model for a magnetic dipole $\vec{m}$ is a spherical surface of radius R with surface current density

$$\vec{j}(\theta) = \frac{3m}{4\pi R^3}\sin\theta\;\hat{e}_\varphi. \tag{24}$$

The external field of this configuration coincides with the field of a point dipole

$$\vec{B}_e = \frac{\mu_0}{4\pi} m \frac{2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta}{r^3}, \tag{25}$$

while the internal field is uniform

$$\vec{B}_i = \frac{\mu_0}{4\pi} 2m \frac{\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta}{R^3}. \tag{26}$$

Note that, except for the obvious dimensional differences, the external electric and magnetic fields [Eqs. (18) and (25)] are identical, while the internal fields [Eqs. (19) and (26)] are not; the magnetic field is twice as large as, and points in the opposite direction to, the electric field (see Fig. 1). (These conclusions are in agreement with Refs. 11, 17, and 18).

The energy to create each of these electric and magnetic dipoles can be calculated by integrating the square of the electric and magnetic fields over all space, giving

$$W_E^c = \frac{\varepsilon_0}{2}\frac{4\pi R^3}{3}|\vec{E}_i|^2 + \frac{\varepsilon_0}{2}\int_R^\infty\int_0^\pi |\vec{E}_e|^2 2\pi r^2 \sin\theta\, d\theta\, dr$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p^2}{R^3}\left(\frac{1}{6}+\frac{1}{3}\right) = \frac{1}{4\pi\varepsilon_0}\frac{p^2}{2R^3}, \tag{27}$$

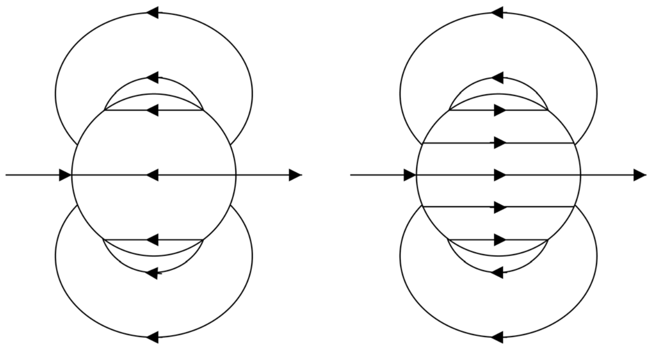

Fig. 1. Internal and external fields of the proposed electric (left) and a magnetic (right) dipole models.

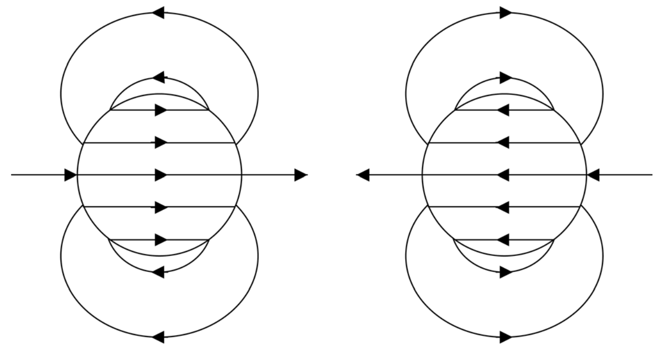

Fig. 3. Two interacting magnetic dipoles. Notice that the internal field of each dipole is *decreased* by the external field of the other.

$$W_M^c = \frac{1}{2\mu_0}\frac{4\pi R^3}{3}|\vec{B}_i|^2 + \frac{1}{2\mu_0}\int_R^\infty \int_0^\pi |\vec{B}_e|^2 2\pi r^2 \sin\theta \, d\theta \, dr$$
$$= \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(\frac{2}{3}+\frac{1}{3}\right) = \frac{\mu_0}{4\pi}\frac{m^2}{R^3}. \quad (28)$$

Note that the creation energy for the magnetic dipole is twice as large as that of the electric dipole. Furthermore, this difference corresponds entirely to the internal field, which comprises an energy that is four times larger for the magnetic dipole compared to the electric dipole.

Combining Eq. (27) with Eq. (7) for the mechanical work leads to an expression for the total electric energy of the two interacting electric dipoles in their final configuration (see Fig. 2),

$$W_E^t = 2W_E^c + W_E^m = \frac{1}{4\pi\varepsilon_0}\frac{p^2}{R^3}\left(1+\frac{2R^3}{x^3}\right). \quad (29)$$

As for the magnetic case, the total magnetic energy of the final configuration, obtained using Eqs. (8) and (28), is given by (see Fig. 3)

$$W_M^t = 2W_M^c + W_M^m + 2W_M^j = \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(2+\frac{2R^3}{x^3}\right) + 2W_M^j. \quad (30)$$

The expression for W_M^j, corresponding to the energy needed to maintain a fixed surface current density for each magnetic dipole during the displacement of dipole $\vec{m}$, can now be calculated dividing each dipole into a series of differential circular circuits with radius $R\sin\theta$ and current $dI = j(\theta)R d\theta$. In view of Faraday's law, if the magnetic flux across each of these circuits changes by an amount $\delta\phi$, an electrical work $\delta W_M^j = dI\,\delta\phi$ is required in order to prevent the current from changing [see Ref. 3, second equation on page 212]. Using Eqs. (24) and (25), we then obtain

$$W_M^j = -\int_0^\pi \frac{3m}{4\pi R^3}\sin\theta\, R\frac{\mu_0}{4\pi}\frac{2m}{x^3}\pi(R\sin\theta)^2 d\theta$$
$$= -\frac{\mu_0}{4\pi}\frac{2m^2}{x^3}, \quad (31)$$

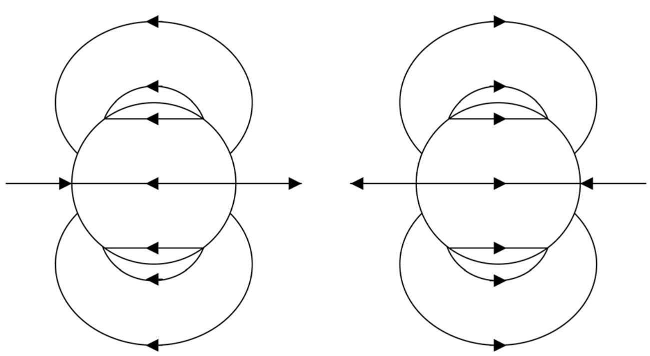

Fig. 2. Two interacting electric dipoles. Notice that the internal field of each dipole is *increased* by the external field of the other.

where we have assumed that $x \gg R$ so that the external field of one dipole at the position of the other is (approximately) uniform. Combining Eqs. (30) and (31) leads to

$$W_M^t = 2W_M^c + W_M^m + 2W_M^j = \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(2-\frac{2R^3}{x^3}\right), \quad (32)$$

which shows that the interaction part of the total energy has the same value in the electric and magnetic cases [Eqs. (29) and (32)] *except for the sign*.[11,19,20]

The total energy of the final configuration of two interacting electric dipoles can also be calculated by integrating the electric field squared over all space, giving

$$W_E^t = 2\frac{\varepsilon_0}{2}\frac{4\pi}{3}R^3\left[\left(\frac{1}{4\pi\varepsilon_0}\frac{p}{R^3}\right)^2 + 2\frac{1}{4\pi\varepsilon_0}\frac{p}{R^3}\frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}\right] + W_{Eext}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p^2}{R^3}\left(\frac{1}{3}+\frac{4R^3}{3x^3}\right) + W_{Eext}. \quad (33)$$

In writing the first equality it was assumed that $x \gg R$ so that the external field of one dipole at the position of the other one is (approximately) uniform and much smaller than the internal field. The last term in this expression, W_{Eext}, represents the electric field energy located in the space outside the dipoles. While this term cannot be easily integrated, its value can be deduced by equating Eqs. (29) and (33), leaving us with

$$W_{Eext} = \frac{1}{4\pi\varepsilon_0}\frac{p^2}{R^3}\left(\frac{2}{3}+\frac{2R^3}{3x^3}\right). \quad (34)$$

Comparing Eqs. (33) and (34) shows that one third of the interaction energy is stored in the field outside the electric dipoles and, therefore, two thirds is stored in the field inside the electric dipoles.

The magnetic energy of the final configuration of two interacting magnetic dipoles can similarly be written (again assuming $x \gg R$) as

$$W_M^t = 2\frac{1}{2\mu_0}\frac{4\pi}{3}R^3\left[\left(\frac{\mu_0}{4\pi}\frac{2m}{R^3}\right)^2 - 2\frac{\mu_0}{4\pi}\frac{2m}{R^3}\frac{\mu_0}{4\pi}\frac{2m}{x^3}\right] + W_{Mext}$$
$$= \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(\frac{4}{3} - \frac{8R^3}{3x^3}\right) + W_{Mext}. \quad (35)$$

The magnetic field energy located in the space outside the dipolar spheres W_{Mext} must be identical to the corresponding electric energy [Eq. (34)] because the external dipolar fields have exactly the same form. Therefore

$$W_{Mext} = \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(\frac{2}{3} + \frac{2R^3}{3x^3}\right), \quad (36)$$

and when included in Eq. (35) we get

$$W_M^t = \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(2 - \frac{2R^3}{x^3}\right), \quad (37)$$

which coincides with Eq. (32) as expected. Once again we see that the total magnetic interaction energy has the same value but opposite sign compared to the electric case [Eq. (29)].[11]

Therefore, the answer to the question formulated in the preceding section is that the assertion that the fields of electric and magnetic dipoles have exactly the same geometry is only true for their far fields. The "internal" fields are completely different, both in magnitude and direction. Because of this difference, the field energy of two interacting dipoles calculated over all space, including the volume occupied by the dipoles themselves, is not the same in the electric and magnetic cases.

IV. CONCLUSION

While the external fields of electric and magnetic dipoles have exactly the same form, their internal fields are very different—the magnetic field is twice as large as the electric field and the two fields have opposite directions (Fig. 1). Because of the field intensity difference, the internal field energy of a magnetic dipole is four times larger than that of an electric dipole, and this gives rise to a total field energy that is two times larger [Eqs. (27) and (28)].

Because of the field direction difference, the original field inside either of the two interacting electric dipoles (Fig. 2) has the same direction as the external field of the other dipole. Consequently, the total internal field is increased. On the contrary, in the magnetic case (Fig. 3), the external field of one dipole is in the opposite direction to the internal field of the other, so the total internal field decreases. This qualitative difference leads to the interaction energy terms for electric and magnetic dipoles having opposite signs [Eqs. (29) and (37)].

As already noted, our results for the internal fields of the electric and magnetic dipoles, Eqs. (19) and (26), are consistent with Refs. 11, 17, and 18. In these works, however, the interaction energy is only calculated considering the external dipolar field, as if the internal field contribution can be neglected for point dipoles. On the contrary, we show that the relative contribution of the internal energy is independent of the dipole size and is the origin of the sign difference between the electric and magnetic interaction energies. This difference leads to the usual statement that while the electric force is equal to the negative gradient of the total energy, the magnetic force is equal to the negative gradient of the *potential* energy or the positive gradient of the *total* energy.[11,21–24]

While the presented results were deduced using particularly simple models for the electric and magnetic dipoles, the main conclusions are completely general. This can be verified using Eqs. (4.18) and (5.62) in Ref. 3, which demonstrates that the integral of the electric (magnetic) field of any charge (current) distribution used to represent an electric (magnetic) dipole is:

$$\int_V \vec{E}(\vec{r})dV = -\frac{\vec{p}}{3\varepsilon_0}, \quad (38)$$

$$\int_V \vec{B}(\vec{r})dV = \frac{2\mu_0\vec{m}}{3}, \quad (39)$$

where V is a sphere of radius R that includes all the charges (currents) that make the dipole. Equation (38), together with Eq. (34), makes it possible to rewrite Eq. (33) for the general case as

$$W_E^t = 2\frac{\varepsilon_0}{2}\int_V\left[|\vec{E}(\vec{r})|^2 + 2\vec{E}(\vec{r})\cdot\frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}\right]dV + W_{Eext}$$
$$= 2\frac{\varepsilon_0}{2}\int_V|\vec{E}(\vec{r})|^2dV + \frac{1}{4\pi\varepsilon_0}\frac{p^2}{R^3}\left(\frac{2}{3} + \frac{2R^3}{x^3}\right). \quad (40)$$

The first term on the right-hand-side of the last equality represents the part of the creation energy that depends on the particular charge distribution of the two dipoles. The second term corresponds to the remaining part, which has a general form since it is associated with the external field of the dipoles. Finally, the third term represents the interaction energy, which coincides with the corresponding term in Eq. (29).

Proceeding in a similar way, Eq. (39), together with Eq. (36), makes it possible to rewrite Eq. (35) for any current distribution of the magnetic dipoles

$$W_M^t = 2\frac{1}{2\mu_0}\int_V\left[|\vec{B}(\vec{r})|^2 + 2\vec{B}(\vec{r})\cdot\frac{\mu_0}{4\pi}\frac{2\vec{m}}{x^3}\right]dV + W_{Mext}$$
$$= \frac{1}{\mu_0}\int_V|\vec{B}(\vec{r})|^2dV + \frac{\mu_0}{4\pi}\frac{m^2}{R^3}\left(\frac{2}{3} - \frac{2R^3}{x^3}\right). \quad (41)$$

Again, the interaction energy term coincides with the corresponding term in Eq. (37). Therefore, the expressions obtained for the dipole-dipole interaction energies for electric and magnetic dipoles, as well as the conclusion that these energies have the same value but opposite sign in the electric and magnetic cases, are completely general.

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[22]See Eqs. 6.34 and 12.20 in Ref. 1.
[23]See Eq. 26.43 in Ref. 14.
[24]See Eqs. 9.11 and 16.21 in Ref. 15.

Optical Pyrometer

The instrument allows remote sensing of the temperature of a black body in the 1400°F to 3400°F range. The body may be a pot of molten metal or the hot bed of coals under a steam boiler. Inside the tube is a lamp whose temperature is controlled by a rheostat and a battery in the box that is slung by a strap around the user's neck. The tube (part of a stereoscopic hand viewer that was common at the time) is pointed toward the hot body, and the temperature of the filament adjusted until it is of the same brightness as the body under test. The instrument was made by Leeds & Northup in 1930, and cost $175. It is in the Greenslade Collection. Note: "Pyrometer" is often used to denote an early thermal expansion apparatus. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)