Interpretation misunderstandings about elementary Quantum Mechanics

Confusiones de interpretación en Mecánica Cuántica elemental

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Abstract

Quantum Mechanics is a fundamental physical theory about atomic-scale processes. It was built between 1920 and 1940 by the most distinguished physicists of that time. The accordance between its formal predictions and experimental results is remarkable. The physical interpretation of its mathematical constructs, however, raised unprecedented controversies. Ontological, semantic, and epistemic vagueness abound in the orthodox interpretations and have resulted in serious misunderstandings that are often repeated in textbooks and elsewhere. In this work, we identify, criticize, and clarify the most spread ones.

Resumen

La Mecánica Cuántica es una teoría de física fundamental que modela procesos a escalas atómicas. La teoría fue formulada entre los años 1920 y 1940. El acuerdo entre las predicciones de su formalismo matemático y los resultados experimentales es notable. Sin embargo, las interpretaciones físicas de sus constructos originaron controversias sin precedentes en la historia de la Física. Las imprecisiones ontológicas, semánticas y epistémicas de las distintas interpretaciones han ocasionado que se repitan y propaguen graves malentendidos que obstaculizan la investigación básica. En este trabajo identificaremos, criticaremos y aclararemos filosóficamente algunas de estas confusiones, con énfasis en las más básicas y difundidas.

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1. Introduction

Modeling atomic-scale physical systems with classical laws yields inaccurate results. This fact was noticed in the first half of the XX century with phenomena such as black body radiation, the photoelectric effect, Stern-Gerlach deflections, and the Comptom effect. These novel experiments, along with remarkable theoretical work, led to the formulation of Quantum Mechanics (QM). The list of physicist that contributed is long and includes illustrious names: Max Planck, Niels Bohr, Werner Heisenberg, Albert Einstein, Erwin Schrödinger, Max Born, John von Neumann, Wolfgang Pauli, David Hilbert, Paul Dirac, and Pascual Jordan among many others.

The predictions made with the theory were confirmed by several experimental measurements and the mathematical formalism was rapidly accepted by most physicist of the time. Nevertheless, the interpretation of its constructs was far from clear and raised serious controversies. From this intellectual conflict, numerous interpretations of QM emerged. To name just a few: the Copenhagen interpretation, the de Broglie–Bohm theory, interpretations based on Quantum Logic, Time-Symmetric theories, the Many-Worlds interpretation, statistical interpretations, and realistic ones (for a review of QM interpretations and their historical context, see Jammer 1974)

The most accepted and spread interpretation of QM is the Copenhagen interpretation, proposed by Niels Bohr and Werner Heisenberg in 1927. The popularity of this view resides on its practical usefulness. The Copenhagen interpretation is, however, implicitly influenced by subjective and pragmatic philosophy.

In this work, we briefly describe a realistic interpretation of QM. Then, we discuss some widely used statements that are usually heard in QM elementary courses and even in academic discussions. We hope to clarify some misunderstandings about interpretation issues related to the deterministic character of the theory, the so-called uncertainty principles, the wave function "collapse", the Einstein-Podolsky-Rosen (EPR) paradox, and its ontology. The main reference of the article is Bunge (1967).

2. Realistic approach

We present a short description of the formalism of QM with a realistic perspective. We do not intend to be exhaustive, but rather to focus on key postulates for understanding the theory. We include some equations using bra-ket notation in state space, but actually we are interested on their semantics.

The referents of QM are particular physical systems called *quantum systems*. This statement may sound trivial, but is crucial for properly understanding the theory. We discuss it in Section 3.5.

The states of a quantum system are represented by non-unique unit vectors $|\psi>$ in some Hilbert space $\mathcal{H}$, known as the state space, with a defined inner product. The state space $\mathcal{H}$ is isomorphic to $L_2(\mathbb{R}^{3N})$, the set of square integrable functions on configuration space of the system. For this reason, it is usual to associate square integrable normalized functions on configuration space $\psi(\bar{p})$ to quantum states. The latter function is called *wave function* and is a fundamental tool for calculating quantum system properties.

Then, unlike classical theories, quantum states are represented by vectors in a space where a summation operation is defined. This fact and the linearity of QM dynamic equations imply that the Principle of Superposition holds at the level of states. Consider, for instance, the wave functions $|\psi_+>, |\psi_->$ that represent the states of an electron with its spin up, and down, respectively. Then, the state:

$$|\psi> = \tfrac{3}{5}|\psi_+> + \tfrac{4}{5}|\psi_->,$$

represents a plausible quantum system that is a superposition of the spin-up and spin-down electron. For empirical confirmation of such counterintuitive feature of QM called quantum entanglement, see Schlosshauer (2007), p. 21.

The values of quantum system's properties can be calculated with self-adjoint operators $\hat{\mathcal{A}}(t) \colon \mathcal{H} \to \mathcal{H}$, acting on the corresponding states. But, unlike classical systems, quantum systems may not have precise values for its properties. Instead, we can calculate the average $<\hat{\mathcal{A}}>$ of a certain property by:

$$<\hat{\mathcal{A}}> = <\psi|\hat{\mathcal{A}}|\psi>.$$

The spread $\Delta_\psi \hat{\mathcal{A}}$ of the average being:

$$\Delta_\psi \hat{\mathcal{A}}^{\,2} = <\hat{\mathcal{A}}>^2 - <\hat{\mathcal{A}}^2>.$$

If the spread $\Delta_\psi \hat{\mathcal{A}}$ of a certain property of a quantum state $|\psi_k>$ is null, then the property takes a precise value λ_k. The corresponding state $|\psi_k>$ is called eigenstate of the operator $\hat{\mathcal{A}}$, λ_k its eigenvalue, and they satisfy:

$$\hat{\mathcal{A}}\,|\psi_k> = \lambda_k\,|\psi_k>.$$

Under certain conditions, the values λ_k may constitute a countable set, i.e. may be quantized. This is another peculiar and contrastable feature of QM.

Because of the Superposition Principle, quantum states are not exclusive. For example, given an eigenstate $|\psi_k>$ of certain self-adjoint operator $\hat{\mathcal{A}}(t)$, the *propensity* p_k of any quantum state $|\psi>$ to take the value λ_k is:

$$p_k = |<\psi|\psi_k>|^2,$$

where $0 < p_k < 1$ (see Popper, 1959).

Finally, QM has an evolution equation that describes how properties change with time. The equation reads:

$$\frac{d\hat{\mathcal{A}}}{dt} = \frac{i}{\hbar}\left(\hat{\mathcal{H}}\hat{\mathcal{A}} - \hat{\mathcal{A}}\hat{\mathcal{H}}\right) + \frac{\partial \hat{\mathcal{A}}}{\partial t},$$

where $\hat{\mathcal{H}}$ denotes a particular operator called Hamiltonian of the system.

We have specified the referents of QM, how to calculate their properties, and their dynamical equation. This short explanation suffices to clarify some usual misconceptions about quantum systems. For a detailed realistic formalism see Perez Bergliaffa, et al. (1993, 1996).

3. Interpretation misunderstandings

3.1. Determinism

It is usually argued that: "QM *is not deterministic as Classical Mechanics because it cannot predict precisely the properties of its referents*". The latter argument is misleading and needs clarification. The subsection is based mostly on Earman (1986).

Determinism is a polysemic word. We find numerous definitions in the literature but none of them refers to the accuracy of predictions. Instead, they refer to dynamical properties of the theory.

QM dynamic equation is invariant under time reversal if we accept that time reversal operation is given by $[\psi(\bar{p})]^R \rightarrow \psi^*(\bar{p})$ and $\hat{\mathcal{A}}(-t) \rightarrow \hat{\mathcal{A}}^*(t)$, where * denotes complex conjugation.

On the other hand, QM evolution equation admits unique solutions under precise initial conditions. Classical theories do not posses this attribute because of possible disturbances coming from spatial infinity with unbounded velocity. QM forbids the invasion from spatial infinity because, in order to keep the wave function normalized, $\lim_{x \rightarrow \pm\infty} \psi(\bar{x}) = 0$.

Moreover, the normalization of the wave function entails stability under variations of the initial conditions. This is not the general case in Classical Mechanics.

In summary, determinism does not refer to predictability, but to time evolution features. In this sense, we find that QM is even more deterministic than Classical Mechanics.

It is true that QM does not predict precise values for the properties of the system. However, that is not a problem of QM dynamics, but of its ontology. Quantum systems do not have precise values for its properties, that's why we cannot predict them.

3.2. Uncertainty relations

Under the misleading name of uncertainty principles, it is stated that: *"Given a quantum system, we cannot measure simultaneously and with arbitrary precision the values of properties whose associated operators do not commute[1]".* The main reference of the subsection is Bunge (1967).

First, the mentioned statement is based on a theorem; it is not a principle. The theorem reads: *"Let $\hat{\mathcal{A}}, \hat{\mathcal{B}}, \hat{\mathcal{C}}$ be self-adjoint operators such that $[\hat{\mathcal{A}}, \hat{\mathcal{B}}] = i\hat{\mathcal{C}}$, then $\Delta_\psi\hat{\mathcal{A}}\Delta_\psi\hat{\mathcal{B}} \geq |\langle\mathcal{C}\rangle_\psi|/2$, for any quantum state ψ".*

The theorem does not make any reference to measurements processes, neither to unavoidable disturbances caused by our interaction with the system, nor to our incapability to measure precise quantum properties. It makes reference to spreads of quantum systems averaged properties. In short, ontological variance of properties is distinct to epistemic uncertainty.

Furthermore, it is usually argued that time and energy satisfy an uncertainty relation of the form: $\Delta_\psi E \Delta_\psi t \geq \hbar/2$. This relation is alien to QM formalism simply because the parameter time does not have an associated operator and, therefore, does not apply to the mentioned theorem (see Bunge, 1970). The correct time dependent uncertainty relation between the energy E and a self-adjoint operator $\hat{\mathcal{A}}$ is (Messiah, 1981):

$$\frac{\Delta_\psi\hat{\mathcal{A}}}{|d\langle\hat{\mathcal{A}}\rangle_\psi|/dt} \, \Delta_\psi E \geq \frac{\hbar}{2}.$$

3.3. The wave function collapse

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[1] Two operators $\hat{\mathcal{A}}, \hat{\mathcal{B}}$ do not commute if, and only if, $i\hat{\mathcal{C}} = [\hat{\mathcal{A}}, \hat{\mathcal{B}}] = \hat{\mathcal{A}}\hat{\mathcal{B}} - \hat{\mathcal{B}}\hat{\mathcal{A}} \neq 0$.

The most controversial postulate of the Copenhagen interpretation is the Projection Postulate: *"If the measurement of a property $\hat{\mathcal{A}}$ of a quantum system $|\psi>$ gives the result λ_k, where λ_k is an eigenvalue of $\hat{\mathcal{A}}$, the quantum state $|\psi>$ immediately changes to the corresponding eigenstate $|\psi_k>$ ".*

QM is a fundamental physical theory and it models the behavior of its referents, i.e. quantum systems. There is no place in its formalism for observers, human measurements, or classical entities.

Understanding the quantum-classical interaction is essential, and the Projection Postulate contributes on that sense, but it does not belong to QM. It should be a consequence of a theory, yet inexistent, that formally models quantum systems, classical apparatus, and their interactions.

The Projection Postulate is a valuable pragmatic statement, but should be replaced by a formal model of the quantum-classical interaction. Moreover, strictly speaking mathematical objects such as functions in a Hilbert space cannot collapse. Only the physical systems can change their state or "collapse".

3.4. The Einstein-Podolsky-Rosen paradox

Einstein, Podolsky and Rosen (1935) argued that QM formalism was either incomplete, or unrealistic. The argument is based on a thought experiment about an entangled quantum state, whose components are arbitrary distant from each other. The apparent paradox resides on instantaneous effects over one of the components, produced by the local interaction of the other with an observer.

Thanks to the leading work of Bell (1964), the ideas of EPR could be tested by experience. Several experiments were made, the latest by Hensen, et al. (2015), and the results are clear: QM formalism is not incomplete.

The work of EPR and the mentioned experimental results seem to imply that QM is inconsistent with realism. However, the previous reasoning is based on a questionable hypothesis: locality. We choose to maintain a realistic interpretation of QM, willing to accept non-local effects of entangled systems.

Setting aside locality is polemical, but the reader should notice that we are not proposing some action at distance between a general class of physical systems. We are accepting possible non-local correlations of highly manipulated entangled systems.

In short, we propose that QM may manifest non-local effects in order to preserve systemic features (see Perez Bergliaffa, et al 1996).

3.5. Ontology

Ontological questions in elementary courses on QM are often answered with the so-called wave–particle duality: *"depending on the experimental set up, particles may behave as waves, or waves as particles".*

This proposition was stated by Louis de Broglie before the formalization of QM and was fundamental in an heuristic sense. However, we emphasize that the referents of QM are not particles, nor waves, not even the wave function. The referents of QM are quantum systems, *per se*.

In other words, quantum systems are not compound by particles nor waves. The latter are classical concepts that do not belong to QM. Classical analogies are heuristically essential for the making of a theory, but they have no place in its final postulates.

The ontology based on quantum systems may not satisfy our curiosity about the components of the quantum world. In that case, we should proceed to study Quantum Field Theory, a deeper theory that models quantum systems as particular physical fields.

3. Conclusions

QM is an extraordinary fundamental physical theory. However, it has been victim of imprecise, subjective and vague interpretations. In this article we show a realistic approach of QM and criticize several misleading propositions that are usually heard around.

Inattention on interpretational issues may not affect experimental predictions. However, they engender confusing statements that obscure the theory and hinder further theoretical developments.

From an ethical perspective, vague and confusing statements enhance the action of pseudoscience: QM has been applied to New Age culture, telepathy, pseudo-medicine, the existence of God, mind-body dualism, and other forms of non-sense.

Precise and formal philosophy is essential for interpreting counterintuitive modern physical theories.

Bibliografía

Ankersmit, F. (1983), *Narrative Logic. A Semantic Analysis of the Historian's Language*, The Hague-Boston-London: Martinus Nijhoff Publishers.

Bell, J. S. (1964), "On the Einstein-Podolsky-Rosen paradox", *Physics* 1, 195-200

Bunge, M.A. (1967), *Foundations of physics*, Springer tracts in natural philosophy.

Bunge, M.A. (1970), "The so-called fourth indeterminacy relation", Can. J. Phys. 48, 1410.

Earman, J. (1986), *A primer on determinism*, Springer.

Einstein, A., Podolsky, B., Rosen, N. (1935), "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?", *Phys. Rev.* 47, 10, 777-780.

Hensen, B., et al (2015), "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres", *Nature* 526, 682-686.

Jammer, M. (1974), *The philosophy of quantum mechanics: the interpretations of quantum mechanics in historical perspective*, John Wiley & Sons, Inc.

Messiah, A. (1981), *Quantum Mechanics*, North-Holland.

Perez Bergliaffa, S.E., Vucetich, H., Romero, G.E. (1993), "Axiomatic Foundations of Nonrelativistic Quantum Mechanics: A Realistic Approach", *Int. J. Theor. Phys.* 32, 1507.

Perez Bergliaffa, S.E., Romero, G.E., Vucetich, H. (1996), "Axiomatic foundations of quantum mechanics revisited: The case for systems", *Int. J. Theor. Phys.* 35, 1805-1819.

Popper, K.R. (1959), "The Propensity Interpretation of Probability", *The British Journal for the Philosophy of Science* 10, 37, 25-42.

Schlosshauer, M.A. (2007), *Decoherence and the quantum-to-classical transition*, Berlin: Springer.