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Effects of different objective functions on optimal decision variables: a study using modified complex method to optimize hamburger cooking

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Abstract

Hamburger patties are prepared from ground beef and cooked to obtain a safe product before consumption. Cooking process eliminates microbial hazards and results in certain quality changes (e.g., cooking loss, textural changes). All these changes can be used as an objective function to achieve an optimum cooking process, but their effects on decision variable (e.g. process temperature profiles) of the optimization should be known. The use of different objective functions (minimization of cooking losses, hardness, chewiness, and shear to work) was compared to see their effects on plate temperature profiles for double-sided contact cooking. Modified Complex Method was applied as the optimization procedure. Lower and higher limits of grill temperatures (177–220°C) were explicit constraints while lethality and temperature at the patties center ($F_0 \geqslant 15$ s; $T_c \geqslant 71$°C) were implicit constraints. The objective functions and implicit constraints were determined using a previously developed numerical heat transfer simulation model. Constant temperature profiles (decision variables) for different objective functions at different processing times (121 and 130 s) were determined. Same decision variables were found regarding the different objective functions (198.3°C and 184.1°C) for the given processing times.

Keywords: Optimization; Complex method; Hamburger patties; Cook losses; Textural changes

1. Introduction

Hamburger patties are prepared from ground beef and cooked to obtain a safe product before consumption. Undercooked patties have been linked to outbreaks of food-borne illnesses especially caused by *Escherichia coli* O157:H7 (Hague et al., 1994; Ahmed, Conner, & Huffman, 1995). Therefore, USDA recommends that cooking processes for patties' center temperatures to at least 71°C resulting in a center lethality value of 15 s with respect to *Escherichia coli* O157:H7, respectively (USDA, 1998). There are different methods of preparing meat patties for consumption, such as deep fat frying, infrared radiation, convection heating, double-sided contact cooking (Zorrilla & Singh, 2000). Double-sided contact cooking is very common in restaurants, and measurement of temperatures in patties is not an easy process. Therefore, mathematical models to predict temperature changes and microbiological thermal death kinetics would be useful. Some publications in the literature have addressed these issues before (Dagerskog, 1979a,b; Ikediela, Correia, Fenton & Ben-Abdallah, 1996; Pan 1998; Zorrilla & Singh, 2000).

Cooking process, while avoiding microbial hazards, also results in certain changes of quality attributes of the patties (e.g., cooking loss, changes in texture and sensory attributes). Prediction of these changes by a mathematical model in addition to the temperature would also be useful, and these can be used for further

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optimization or sensory-related prediction purposes as observed by Zorrilla, Rovedo and Singh (2000). Zorrilla and Singh (2000) developed an explicit numerical mathematical model to predict the temperature change in frozen meat patties during double sided cooking and applied this model to predict inactivation kinetics of microorganisms and textural and quality attributes (cooking loss, hardness, chewiness, work to shear, etc.) of patties after cooking (Zorrilla et al., 2000).

Since inactivation kinetics of microorganisms (regarding safety) and quality factors show different temperature sensitivities, it is possible to achieve an optimal process regarding quality and safety where process temperature change is a decision variable and minimization of cooking loss or some textural changes depending on the sensory results is an objective function. Constraints of this problem can be lower and higher limits of the decision variable, as well as the coldest point lethality and temperature obtained at the end of the process. As seen, the theoretical basis for an optimization problem is the combination of the time–temperature distribution inside the product (established by heat and mass transfer) and kinetics of microbial and nutrient destruction or other quality attributes (Holdsworth, 1985).

Optimization may be defined as the ultimate goal of decisions either to minimize the effort required or to maximize the desired benefit, or the use of specific methods to determine the most cost-effective and efficient solution to find the maximum or minimum value of a function, or choosing the best alternative among the others for an efficient solution to a problem or design for a process (Rao, 1996; Edgar, Himmelblau, & Lasdon, 2001). Any optimization problem has three parts: an objective function to compare the alternatives, a set of constraints to accomplish (e.g., lower and higher limits of the processing temperature and/or accumulated coldest point lethality during processing) and decision variables (process temperatures) to create the alternatives. Thermal processing of foods may be attributed as a problem where the optimization problem is, as stated by Banga, Pan and Singh (2001), a dynamic optimization problem. The term "dynamic" comes from the ordinary and partial differential equations describing the process with appropriate initial and boundary conditions. Dynamic optimization problems are also called optimal control problems since only the initial state of the system is considered to compute the optimal control. The general optimal control problem may be stated as finding the control variable to maximize (or minimize) the objective function subjecting to the equality and/or inequality constraints and upper and lower bounds for control variables and constraints (Banga et al., 2001). These problems are especially difficult to solve because of the non-linear and distributed nature of the system dynamics and the existence of explicit and implicit constraints on both control variable and objective function (Banga et al., 2001). There have been numerous methods suggested for this kind of problem and accumulated knowledge in the literature (Gallardo & Casares, 1991; Noronha, Hendrickx, Suys, & Tobback, 1993; Banga & Seider, 1996; Kazmierczak, 1996; Noronha, Loey, Hendrickx, & Tobback, 1996a,b; Terajima & Nonaka, 1996; Banga, Perez-Martin, Banga, Alonso & Singh, 1997; Chalabi, Van Willigenburg, & Van Straten, 1999; Banga et al., 2001; Balsa-Canto, Alonso & Banga, 2002; Erdogdu & Balaban, 2002). As seen in the literature, the maximum principle of Pontryagin has been used by many authors to solve the thermal processing optimization problems. However, this approach may be very difficult due to presence of constraints and other complexities since it is based on the solution of additional necessary conditions; therefore several alternative methods have been proposed. Simultaneous (also called direct; and maximum principle of Pontryagin is referred as an indirect method) strategies are one of these proposed methods. (Banga et al., 1997; Balsa et al., 2002, and Summanwar, Jayaraman, Kulkarni, Kusumakar, & Rajesh, 2002). Biegler, Cervantes, and Wachter (2002) summarized the advantages and disadvantages of these methods. Most important advantages include that they fully discretize the control variable and constraints and directly couple the solution of differential equation system. On the other hand, there are a number of open questions related to convergence and stability problems of these methods (Biegler et al., 2002).

Choosing the objective function, control variable, and constraints are important steps in the application of any method used since the global optimum might be difficult to achieve due to the insensitivity of the objective function to the control variable. Convergence difficulties may also appear due to the high-linear and/or discontinuous nature of the thermal processing systems (Banga et al., 2001). Variable process temperatures have been suggested as alternatives in the food sterilization processes. It was reported that the benefits may include the improved nutrient nutrition, reduced heat damage to the food surface, lower energy costs, and shorter process times (Durance, Dou, & Mazza, 1997). Because a large number of variable process temperature profiles are possible for a given product, selection of an optimum process, when the process temperature profiles were chosen to be the decision variable, can be easily found with an optimization search technique (Durance et al., 1997). Zorrilla, Banga & Singh (2002) computed optimal operating conditions to minimize cooking loss of hamburger patties for double-sided cooking. They used a stochastic optimization algorithm developed and explained in detail by Banga et al. (1991); Banga & Seider (1996) and Banga et al. (1997).

Even though all the empirical equations for different quality attributes could be used as objective functions, Zorrilla et al. (2002) used only cooking loss, may be since it is an economically important parameter, to minimize while ensuring the center lethality with respect to *Escherichia coli* O157:H7 and center temperature. They reported some improvements in the cooking loss when variable process temperatures and longer processing times were applied. However, effects of these parameters on the change of process temperature (decision variable), uniqueness and reproducibility of the optimization method on the results with respect to both decision variable and objective function were not mentioned.

Therefore, the objectives of this research were to use the different textural changes (hardness, chewiness, and shear to work) as well as the cooking loss as the objective function for different processing times to observe their effects on the decision variable (plate temperature profiles) and to determine if the results obtained were unique and reproducible.

2. Materials and methods

To accomplish the given objectives, a modified algorithm of Complex Method (Erdogdu & Balaban, 2002) for different thermal processing conditions was used. The algorithm used the previously developed mathematical model (Zorrilla & Singh, 2000) to predict the implicit constraints (center lethality and center temperature of patties), the applied objective functions (minimization of cooking loss, hardness, chewiness, and work to shear values), the effects of different objective functions on calculated decision variable (grill temperature profile) and to determine the uniqueness and reproducibility of the results.

2.1. Mathematical model

During cooking of patties, after they are placed on the grills at high temperature, generally higher than 160°C, heat starts penetrating into the patties leading to some certain physical and chemical changes; such as, fat and ice start melting, temperature near the patty surface exceeds 100°C, and water evaporates leading to a crust formation as a result of dehydration and browning reactions.

Since there is a solid–liquid interphase (during melting) and a liquid–vapor interphase (during evaporation), this problem may be defined as a multi-phase moving-boundary problem (Vijayan & Singh, 1997; Zorrilla & Singh, 2000). For the solid–liquid interphase part, the problem can be treated as a thawing problem (heat conduction problem with a phase change) and solved by enthalpy formulation as proposed by Mannepperuma and Singh (1988). In addition to this part, when the evaporation temperature is reached, another moving interphase appears, separating core and crust regions, as proposed by Farkas, Singh, and Rumsey (1996). The temperature at this interphase is the boiling temperature of water. Vijayan and Singh (1997) developed a mathematical model by enthalpy formulation to predict heat transfer in the crust region during immersion frying of frozen foods. Using this previous information and developed mathematical models based on enthalpy formulation, the models developed by Zorrilla and Singh (2000) and Zorrilla et al. (2000) to predict transient temperature change, microbial inactivation kinetics and textural changes in the patties were adapted into the optimization algorithm and used in this study. The details of these models, as explained by them, are given below.

The assumptions for double sided cooking of hamburgers were:

- hamburger patty was an infinite slab of constant thickness L (Fig. 1),
- one-dimensional conduction heat transfer,
- thermal properties (thermal conductivity and enthalpy) were as a function of time (t), temperature (°C) and location inside the patty.

According to these assumptions, the governing equation, and the initial and boundary conditions for the core region was:

$$\frac{\partial H(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x,T)\cdot\frac{\partial T(H(x,t),x)}{\partial x}\right) \quad L_1(t)<x<L_2(t). \tag{1}$$

The initial condition

$$T(x,0) = T_i \;\; t=0;\;\; L_1(t)<x<L_2(t). \tag{2}$$

The boundary conditions

$$\begin{aligned} T(L_1(t),t) &= T_b \text{ where } L_1(0)=0,\\ T(L_2(t),t) &= T_b \text{ where } L_2(0)=L, \end{aligned} \tag{3}$$

where H was enthalpy, k was thermal conductivity of the core region, $L_1(t)$ and $L_2(t)$ were positions (Fig. 1) separating the core and crust regions, associated with the evaporation interphase, T_b was boiling temperature of water at the evaporation interphase, and T_i was the uniform initial temperature of the patty.

Two additional conditions were employed to determine the positions of moving interphases (Vijayan & Singh, 1997):

$$-k_{crust}(x,T)\frac{\partial T(H(x,t),x)}{\partial x} + k(x,T)\frac{\partial T(H(x,t),x)}{\partial x} = \lambda_v \rho m \frac{dL_i(t)}{dt}, \quad x = L_i(t);\; t>0;\; i=1,2, \tag{4}$$

where k_{crust} was the thermal conductivity of the crust, λ_v was latent heat of water, ρ was density, and m was the

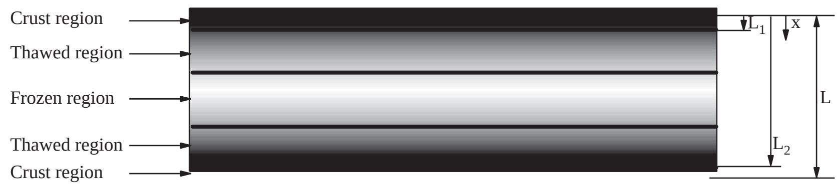

Fig. 1. A view of hamburger patty undergoing cooking (Adapted from Zorrilla & Singh, 2000).

decimal moisture content. The boundary conditionsbetweenthecrust and the heating plates by equating the heat flux from the plates and the conductive heat flux toward the crust were:

$$-k_{\mathrm{crust}}(x,T)\frac{\partial T(H(x,t),x)}{\partial x} = h_{\mathrm{p}_1}(t)\big(T_{\mathrm{p}_1}(t) - T(H(x,t),x)\big), \quad (x=0, t>0),$$
$$-k_{\mathrm{crust}}(x,T)\frac{\partial T(H(x,t),x)}{\partial x} = h_{\mathrm{p}2}(t)\big(T_{\mathrm{p}2}(t) - T(H(x,t),x)\big), \quad (x=L, t>0), \tag{5}$$

where $h_{\mathrm{p}_1}(t)$ and $h_{\mathrm{p}_2}(t)$ were contact heat transfer coefficients (both including convection and radiation on the boundary) (Wichchukit, Zorrilla, & Singh, 2001), and $T_{\mathrm{p}_1}(t)$ and $T_{\mathrm{p}_2}(t)$ were plate temperatures as a function of time. Since the crust thickness was small compared to the whole patty thickness, the temperature change could be assumed to be linear and approximated as follows (Vijayan & Singh, 1997):

$$\frac{\partial T(H(x,t),x)}{\partial x} = \frac{T_{\mathrm{b}} - T(H(x,t),x)}{L_1(t)}, \quad (x=0),$$
$$\frac{\partial T(H(x,t),x)}{\partial x} = \frac{T_{\mathrm{b}} - T(H(x,t),x)}{L - L_2(t)}, \quad (x=L). \tag{6}$$

Zorrilla & Singh (2000) obtained the solution for the above given equations numerically using an explicit finite difference method and validated the model's results with experiments. Then, the model was used to determine the microbial inactivation kinetics inside the patty with the given microorganism and processing conditions.

Zorrilla et al. (2000) gave the correlation equations to predict cooking loss, hardness, chewiness, work to shear and other textural attributes of hamburger patties for double sided cooking. These changes were determined as a function of volume (V) average temperature (T_{avg}) and gap thickness (L) of plates in double-sided contact cooking method using the above explained mathematical model (Table 1). T_{avg} was given with the following equation (Zorrilla et al., 2000):

$$T_{\mathrm{avg}}\frac{\int\int_V\int T(H(x,t),x)\cdot \mathrm{d}V}{\int\int_V\int \mathrm{d}V} = \frac{\int_0^L T(H(x,t),x)\mathrm{d}x}{L}. \tag{7}$$

The center lethality value for the given microorganism's (*Escherichia coli* O157:H7) inactivation kinetics was determined using Eq. (8):

$$F_0 = \int_0^t 10^{(T_{\mathrm{c}}(t)-T_{\mathrm{ref}})/z}\,\mathrm{d}t, \tag{8}$$

where F_0(s) was center lethality, T_{ref} (=68.3°C) was reference temperature, and z (=7.38°C) was the temperature change needed to reduce the D-value of the microorganism by one log- cycle.

2.2. Optimization algorithm

The Complex Method for constrained nonlinear optimization was first presented by Box (1965). Umeda, Shindo and Ichikawa (1972) used this method to solve variational problems with state-variable inequality constraints and demonstrated its applicability. Kazmierczak (1996) gave an example of a pest management problem to illustrate that the Complex Method was a very efficient optimization approach and mathematically very simple 10 compared to some other methods. Erdogdu and Balaban (2002) modified this method and showed that it allowed the incorporation of different constraints on microbial sterility and final temperature at the coldest point as implicit constraints in thermal processing problems. This method was applied with the following general steps (further details were given in Erdogdu and Balaban, 2002):

Step 1: The decision variable (plate temperature) was randomized between the highest and lowest range (explicit constraints) to establish the initial vertex (the decision variable represented the vertex). Then, the objective function and the implicit constraints were calculated with respect to the randomized decision variable.

Step 2: The violations of any explicit and implicit constraints were checked.

Step 3: If violation of any constraint was met, the required cautions to correct this issue was applied until the violation was corrected, and all the implicit constraints were satisfied. The decision variable was generally the parameter (with processing time if necessary) to correct the violations.

Step 4: After the initial complex (consisting of 3-vertices for a constant grill temperature process) was constituted applying steps 1–3, the conventional Complex Method was started. The vertices were moved (reflection, extension and retraction) based on their objective function values. In all these steps, when a violation of any implicit constraint occurred, the retraction 11/contraction/shrinkage processes were applied until the violation was satisfied.

Table 1
The correlation equations for different quality attributes of hamburger patties for double-sided contact cooking

Quality attribute (z)	Predictive equation	r^2
Cooking loss (%)	$= -122.25 + 4.56 \times 10^{-5}\, T_{\text{avg}}^3 + 54.59 \ln(L)$	0.90
Hardness (N)	$= -55.89 + 0.58\, T_{\text{avg}} + 3.73\, L$	0.55
Chewiness (N mm)	$= -260.78 + 2.65\, T_{\text{avg}} + 0.12/L$	0.54
Work to shear (N mm)	$= -2468.19 + 35.03\, T_{\text{avg}} + 210.84\, L$	0.74

Adapted from Zorrilla et al. (2000).

Step 5: Stopping criterion for the algorithm was the last step. Generally, the difference between the resulting objective functions of the best and worst vertices (each decision variable represented a vertex) was very small (<0.001), meaning that the algorithm could not move the vertices to result in any improvement in the objective function, the optimization procedure stopped, giving the best vertex as the optimum decision variable resulting in the optimum objective function value while satisfying the given implicit constraints.

2.3. The optimization procedure

The mathematical models to predict the temperature changes, lethality calculations, and the given objective functions as a function of volume average temperature of patties developed by Zorrilla and Singh (2000) and Zorrilla et al. (2000) were adapted into the modified Complex Method and used in the optimization calculations. The optimization procedure started with finding a randomized grill temperature profile. It was assumed that the upper and lower heating plates had the same constant temperature profiles during double-sided contact cooking. Then, the mathematical model was applied to predict the temperature change in the hamburger patty, volume average temperature, resulting center lethality, resulting center temperature and the value of objective function for a given process time. Thermophysical properties of the patties were determined based on the composition (24% fat, 60% moisture and 16% protein) of the patties. Zorrilla and Singh (2000) reported that the following thermophysical properties of the unfrozen state: density, 1056.7 kg/m^3; heat capacity, 3268 J/kg K; thermal conductivity, 0.416 W/m K, and initial freezing point, −1°C. The heat transfer coefficient values for the top and bottom surfaces were a function of time and location on the patty surface, as reported by Wichchukit et al (2001). Then, it continued with the standard algorithm of the Complex Method (reflection, expansion, retraction, shrinkage, etc.) to improve the predicted value of the objective function as explained above.

The variables used in the optimization algorithm were as follows:

- Processing times: 121 and 130 s,
- Gap thickness between the cooking plates: 9.65 mm,
- Initial temperature of hamburger patties: −22°C,
- Decision variable: constant plate temperature profiles; $T_{p_1} = T_{p_2}$,
- The explicit constraints for the decision variable were the lower (T_{L}) and higher (T_{H}) limits of the plate temperatures:

$$T_L \leq T(t) \leq T_{\text{H}}, \tag{9}$$

where $T_{\text{L}} = 177°\text{C}$ and $T_{\text{H}} = 220°\text{C}$
- The implicit constraints for the optimization were the center temperature (T_{c}) and center lethality (F_0) achieved at the end of the cooking process:

$$T_c \geq 71°\text{C and } F_0 \geq 15\text{ s} \tag{10}$$

- The objective functions (Table 1) used were to optimize (minimize) the:

- cooking loss (%),
- hardness (N),
- chewiness (N mm), and
- shear to work (N mm) of the patties after double-sided cooking.

All calculations were performed using a computer program written in Microsoft Visual Basic V. 6.0 by Erdogdu (2000).

3. Results and discussion

The optimization algorithm to minimize the above given objective functions (Table 1) for different processing times and different objective functions was run independently 10 times to see the uniqueness and reproducibility of the method to find the grill temperature profiles. This also showed the effect of starting the procedure with randomly calculated initial complex of plate temperatures. Tables 2 and 3 show the average and standard deviation results of 10 different runs for decision variable, reached center F_0 value, and minimized objective function value for 121 and 130 s, respectively. As seen in Tables 2 and 3, use of different objective functions resulted in the same decision variable for the same processing time (around 198.3°C for 121 s and 184.1°C for 130 s) giving very similar objective function values. Lethality value for the short time process was 15 s. It increased almost 4 times (62 s) with the 9 s difference in the processing time even though the optimal profile was 14°C lower, and there were no changes in objective function values as a result of different temperature sensitivities of safety and quality factors. In all runs, the second implicit constraint, center temperature was always higher than 71°C (71.5°C and 73.8°C). Figs. 2 and 3 show the center temperature change with the explicit constraints and critical temperature of 71°C for both processes.

Very low standard deviations (Tables 2 and 3) of different runs prove the uniqueness and reproducibility of the method for and with respect to the different objective functions. These small standard deviations are the result of randomly calculated starting complex at each run and the accumulation of truncation errors in the finite explicit numerical procedures. These procedures were used to predict the implicit constraints and the objective function. The results obtained for cooking loss also matched with the study of Zorrilla et al. (2002). The matching results of these two research studies show that two different optimization methods gave the same results for the same processing conditions of hamburger patties.

Erdogdu (2000) specifically emphasized the uniqueness and reproducibility issues in optimization of thermal processing systems, and they concluded that the optimization was reproducible and results in unique results regarding the objective function but not the decision variable. In this study, decision variables were assumed to be constant (not a function of time). This may be the main reason for obtaining both unique and reproducible results with respect to both different objective functions and decision variable. Zorrilla et al.

Table 2
Optimum decision variables, resulting objective function and implicit constraint values (lethality)for 121 s processing time

Objective function	Decision variable (°C)	Minimized objective function value	F_0 (s)
Cooking loss (%)	198.3±0.11	33.8±0.03	15.1
Hardness (N)	198.2±0.04	31.8±0.003	15.0
Chewiness (N mm)	198.3±0.11	138.8±0.08	15.2
Work to shear (N mm)	198.3±0.13	2691.1±1.24	15.2

Table 3
Optimum decision variables, resulting objective function and implicit constraint values (lethality)for 130 s processing time

Objective function	Decision variable (°C)	Minimized objective function value	F_0 (s)
Cooking loss (%)	184.0±0.02	33.4±0.005	61.9
Hardness (N)	184.2±0.40	31.6±0.06	63.8
Chewiness (N mm)	184.1±0.12	138.0±0.08	62.3
Work to shear (N mm)	184.2±0.26	2680.5±2.39	63.2

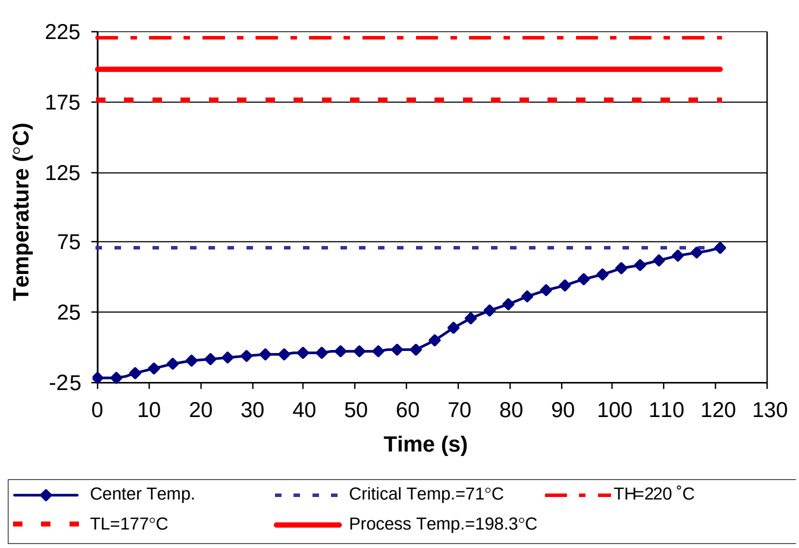

Fig. 2. Change of center temperature of patties with respect to grill temperature (Resulting $F_0 = 15$ s, $T_c = 71.5$°C) with respect to the process temperature of 198.3°C.

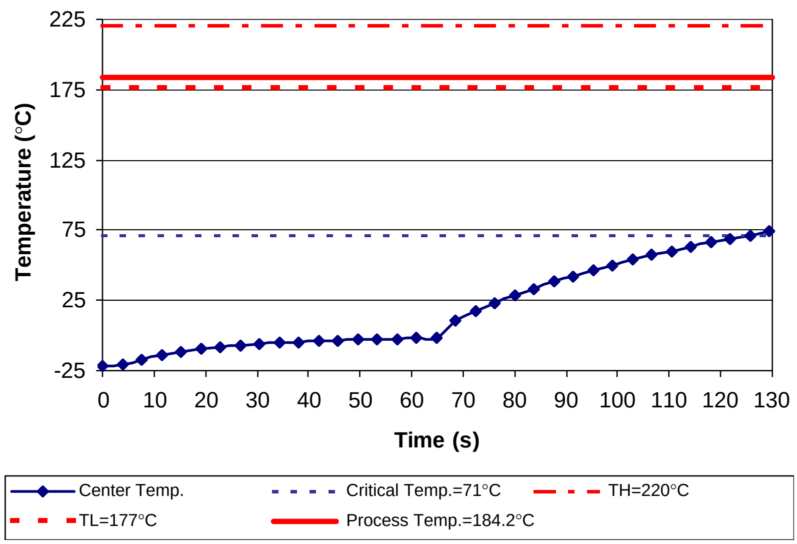

Fig. 3. Change of center temperature of patties with respect to grill temperature (Resulting $F_0 = 60.2$ s, $T_c = 73.8°C$) with respect to the process temperature of 184.2°C.

(2002) also used the processing time as a decision variable. This approach resulted in some improvements (less than 5%) in the cooking losses. Similar improvements were obtained by Erdogdu (2000) in different thermal processing problems. The algorithm in this study was not applied to see the effects of time dependent grill temperature profiles on the improvements of objective functions because the objective was to see the uniqueness and reproducibility of the method at the given processing conditions and the effects of different objective functions on the optimization.

4. Conclusions and recommendations

The results showed that different objective functions resulted in very similar decision variable (grill temperature profiles) satisfying the implicit constraints and minimizing the objective functions. A further attempt to completely observe the effects of different objective functions would be to increase processing time and to use a combination of objective functions converting the problem into a multi optimization problem and to correlate the textural results with sensory tests. Correlating the result with sensory tests would be the most important part of the study since the texture results do not mean anything by themselves.

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