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Predictive control applied to heat-exchanger networks

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Abstract

This paper discusses the online optimization and control of a heat-exchanger network (HEN) through a two-level control structure. The low level is a constrained model predictive control (MPC) and the high level is a supervisory online optimiser. Since MPC is a multivariable control technique capable of handling control-input constraints, it is neither necessary to define a variable-pairing approach nor to include individual loop-protections to avoid close-loop saturations. The proposed MPC algorithm uses an approximate linear model of the system to perform the output predictions and to account for the constraints. On the other hand, the supervisory program, based on a rigorous model, computes desired values to key manipulated variables of MPC, leading to minimum utility consumption. The coordination between the supervisory program and MPC is achieved through the definition of an extended cost-function that enables the controller to drive the system to the optimal operating condition. The proposed method was successfully tested by rigorous simulation of a typical HEN of the process industry.

Keywords: Model predictive control; Heat-exchanger networks; On-line optimization; Minimum utility consumption; Extended cost-function

1. Introduction

Large chemical processes usually require energy recovery systems to maintain a competitive operation. A heat-exchanger network (HEN) usually plays an important role in these process systems, where the thermal outlet condition of several process streams must be controlled without reducing heat integration. For HEN systems, the condition of maximum heat integration is achieved through minimum utility consumption, which becomes a complementary goal to the usual requirement of satisfying the temperature targets.

The eighties have shown a myriad of methods to the analysis and design of tailor-made HEN systems for a variety of process plants. Shortly after the process community started addressing this problem, the difficulties in finding proper control structures aroused as a demanding research subject, particularly because hard constraints on manipulated variables emerge as a natural and frequent part of the control problem. Several articles focused on the control of HEN systems, and provided useful procedures to define the appropriate control structure. Important preliminary contributions can be found in Marselle et al. [1], Beautyman and Cornish [2], Calandranis and Stephanopoulos [3], Huang and Fan [4] and Mathisen et al. [5]. More recently, Aguilera and Marchetti [6] proposed a method for on-line optimization and control of HEN systems. They also discussed the degrees of freedom of the system with regard to the steady-state optimization. Glemmestad et al. [7] presented an alternative approach to the optimal operation of HEN systems based on the on-line optimization of the steady-state and a fixed control structure, which is selected offline. Lately, Giovanini and Marchetti [8] have shown that low-level Distributed Control Systems (DCS) are also capable of handling HEN control problems when a flexible control-loop structure is provided. However, they indicated some limitations of the DCS to reach the most convenient operation point.

In practice, any constraint on a manipulated input, which is known in advance, leads process operators to take actions to keep the system away from the uncontrolled condition. These actions reduce the operation window and typically hold the process at less efficient operating conditions. Lately, many of the process control articles dealing with constraints in manipulated variables investigate the solution to the general constrained-control problem through a combination of predictive control and

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optimization structures. Model Predictive Control (MPC), also known as Moving Horizon Control or Receding Horizon Control, has been successfully applied in the process industry due to its ability to handle input and output constraints optimally. This is a control strategy that was first empirically implemented in chemical process industries, and is currently recognized as a consolidated process control technology. It basically uses a dynamic model of the process and performs a constrained on-line optimization to determine the optimal sequence of future control moves. The first control move is implemented and the calculations are repeated at the next sampling step. Excellent reviews on MPC applications and comparison of commercial MPC controllers are available in Maciejowski [9] and Qin and Badgwell [10]. Despite the large amount of articles dealing with applications to a variety of process systems, the use of MPC in HEN systems has not been extensively evaluated yet; to the authors knowledge, only Gremm et al. [11] have reported the application of MPC to a laboratory set-up composed of two heat exchanger units.

As mentioned before, the control of HEN systems presents several challenging features spanning from multivariable interactions and moderate nonlinearities to well-defined operation objectives associated to heat integration and utility consumption. Thus, the objective of this work is to develop an efficient extension of MPC that will be capable of dealing with most of the control problems presented by HEN systems. In particular, it is desired to satisfy the final temperature targets of the process streams while achieving minimum utility expenses. Here, we show that, taking advantage of the flexibility of the traditional MPC, it is possible to modify the objective function, and achieve satisfactory control performance with lower utility consumption. It is also presented a condition that guarantees the convergence of the undisturbed system to the desired steady-state condition without loss of the stable operation.

This paper is organized as follows: Section 2 describes the main ideas supporting the HEN steady-state optimization presented by Aguilera and Marchetti [6], and shows how to formulate the problem for a centralized multivariable control structure like MPC. Section 3 resumes main MPC concepts and equations using a linear model of the process. It is also discussed the conditions to reach convergence to the optimal operation point. In Section 4, a description of the network application example and its degrees of freedom are first discussed. Then, alternative driving variables in the hierarchical structure as well as robustness to errors in the model of the supervisory program are analysed. Section 5 provides simulation results and shows the robustness of the proposed approach to plant non-linearity and model mismatch at the supervisory level. The concluding remarks are given in Section 6.

2. Minimum operation cost

2.1. Basic concepts related to the optimal operating condition

The main reason for using a HEN system is the need to recover as much energy as possible from high-temperature process streams and transferring the energy to cold-process streams. In this way, important savings in utilities like steam, fuels or cold water are obtained. However, it is not a less important objective of the HEN system to achieve the proper thermal conditioning of process streams going to neighbour process facilities. For this reason, the control system must be capable of not only permitting the HEN system to reach the point of minimum utility consumption, but also of driving the final process-stream temperatures to their setpoints.

Two main categories of heat-exchanger units in a HEN must be distinguished: those actually producing heat recovery (in this article referred to as “heat exchangers” or “E units”) and those that complement the required task of reaching all the temperature targets (referred as “services” or “S units”). The use of service units in a HEN system is frequently quantified by the sum of the heat duties of these units. However, the weighted sum of utility flow rates provides a better cost function to represent the network condition at any time González and Marchetti [12].

The control variables in a HEN system are of three kinds: (a) process stream bypasses around heat exchangers, (b) utility stream flow rates in service units and (c) splits of process streams. For a given HEN system (stream paths and stream matches), the number of possible control variables or control inputs is well defined. If the HEN system has more inputs than outlet temperatures to be controlled, there will be a set of different control positions that satisfy the same outlet set points. These operation possibilities may exist in different levels of heat integration, and they certainly represent a flexibility to satisfy different load requirements under control constraints.

The approach taken in this paper assumes a hierarchical structure where a real-time optimization (RTO) is performed in supervisory level. The role of the supervisory layer is to determine the optimal or desired steady-state inputs that minimize the total service cost of the system. This layer consists on a non-linear optimization problem that takes into account, as a set of constraints, the non-linear model of the HEN system and the output set points that are the desired output temperatures of the process streams. On the other hand, the role of the MPC layer is to drive the outputs to their set points, and to drive the inputs to the optimal steady-state values obtained in the supervisory layer.

2.2. The optimal-operation problem formulation

Once the ranges of outlet process streams and input conditions are fixed, most HEN systems have some degrees of freedom that can be used for optimization. The possibilities basically depend on the number of independent free outlet streams, including all the utility streams. According to Aguilera and Marchetti [6], an easy way to determine if the network has enough degrees of freedom for optimization is to confirm that $f=n^{\mathrm{o}}+s-1>0$, where n^{o} is the number of process streams without temperature targets and s is the number of services available in the network. When this condition is satisfied, there are different input combinations that can satisfy the same outlet specifications. Hence, the optimal operation of the HEN system implies the availability of a strategy to face the following two problems: (i) how to determine which of the input combinations causes the lowest service cost and (ii) how to dynamically guide the process to the optimal point. The first problem refers to a steady-state optimal problem while the second one addresses the dynamic nature of the system. Initially, we formulate the first problem.

When the desired operating condition corresponds to the minimum operation cost, the purpose of the optimization problem is the minimization of the total utility consumption. Then, assuming the HEN system has *ne* exchanger units and *s* service units, and H and C represent the sets of hot and cold process streams, respectively, then the optimal operating condition is obtained from the solution of the following problem:

$$\min_{w_{c_i}, w_{h_j}} \left\{ \sum_i c_{c_i} w_{c_i} + \sum_j c_{h_j} w_{h_j} \right\} \quad i \in H,\ j \in C, \tag{1}$$

subject to

$$-\sum_{z \in K_i} q_z - q_{c_i}(w_{c_i}) = Q_i, \quad i \in H, \tag{2}$$

$$\sum_{z \in K_j} q_z + q_{h_j}(w_{h_j}) = Q_j, \quad j \in C, \tag{3}$$

$$q_z \le e_z^{\rm o} L_z^{\rm o} \quad z \in \{1, ne + s\}, \tag{4}$$

$$-q_z \le 0 \quad z \in \{1, ne + s\}, \tag{5}$$

$$q_{c_i}(w_{c_i}) = e_{c_i}(w_{c_i}) L_{c_i}(w_{c_i}), \quad i \in H, \tag{6}$$

$$q_{h_j}(w_{h_j}) = e_{h_j}(w_{h_j}) L_{h_j}, \quad j \in C. \tag{7}$$

In these equations, w_{c_i} stands for the flow rate of the cold utility in the service unit located on the hot stream i, w_{h_j} is the flow rate of the hot utility in the service unit located on the cold stream j, c_{c_i} and c_{h_j} are the cold and hot utility cost per unit weight, respectively. Eqs. (2) and (3) define the tasks $Q_i = w_i c_i (T_i^{\rm out} - T_i^{\rm in})$ and $Q_j = w_j c_j (T_j^{\rm out} - T_j^{\rm in})$ to be performed on the hot process stream i and on the cold process stream j, respectively. K_i and K_j stand for sets of heat duties q_z. Variables q_{c_i} and q_{h_j} are the final service heat duties that accomplish the required tasks. Eq. (4) uses the efficiency e_z and variable L_z to establish the maximum amount of heat to be exchanged at the generic unit z. Furthermore, Eqs. (6) and (7) are non-linear equations (where $z = c_i$ and $z = h_j$, respectively) that are included in the optimization problem to allow for the computation of w_{c_i} and w_{h_j}, which are needed by the performance cost, and because they frequently define the commanding variables to MPC. Here, the problem defined by Eqs. (1)–(7) is supposed to be solved within the same sampling period as the model predictive controller.

3. Model based control

3.1. The control problem formulation

The basic MPC formulation consists of the on-line computation of the future control moves that minimize the predicted future error along the prediction horizon p, subject to constraints on both, inputs and outputs variables [13]. The MPC optimization problem can be written as:

$$\begin{aligned}
&\min_{\Delta u} \{ \|e^0 - A\Delta u\|_Q^2 + \|\Delta u\|_S^2 \} \\
&\text{subject to :} \\
&e^0 = r - y^0 \\
&\Delta u_{\min} \le \Delta u(k + j) \le \Delta u_{\max} \\
&u_{\min} \le u(k + j) \le u_{\max}, \quad j = 0, 1, \cdots, m - 1 \\
&\Delta u = [\Delta u(k)^{\rm T} \quad \Delta u(k + 1)^{\rm T} \cdots \Delta u(k + m - 1)^{\rm T}]^{\rm T} \\
&\Delta u(k + j) = u(k + j) - u(k + j - 1)
\end{aligned} \tag{8}$$

where r is the output set point, y^0 the output trajectory assuming that no future control actions are introduced into system, $\Delta u(k+m+i) = 0$, $i \ge 0$, A the dynamic matrix of the system, *nu* the number of inputs, *ny* the number of outputs and weight matrices $Q \in \mathbb{R}^{p\cdot ny \times p\cdot ny}$ and $S \in \mathbb{R}^{m\cdot nu \times m\cdot nu}$ are assumed positive definite.

The solution to this problem consists of *nu* sequences (control trajectories) of m control moves and is designated Δu^*. The first component of each control sequence is applied to the system at time k, and the optimization problem (8) is repeated at the next sampling time $k+1$.

In order to use the MPC strategy to control HEN systems, it is proposed to include an extra term in the cost function of problem (8) to account for the utility cost. As mentioned before, the controller purpose is to reach the output set points, but also to guide the process to an optimal condition from the point of view of utilities consumption. The approach presented in this paper proposes a two level structure, in which the optimal operating point is calculated at the supervisory level, and the result translated into desired values for selected manipulated inputs of the MPC level. These desired input values are included in the cost function of the control problem as a new term that penalizes the distance that the system is from the desired condition. Hence, the MPC optimization problem is reformulated as follows:

$$\begin{aligned}
&\min_{\Delta u} V_k = \{ \|e^0 - A\Delta u\|_Q^2 + \|u(k + m - 1) - u_{\rm opt}\|_R^2 \\
&\qquad + \|\Delta u\|_S^2 \} \\
&\text{subject to :} \\
&e^0 = r - y^0 \\
&\Delta u_{\min} \le \Delta u(k + j) \le \Delta u_{\max} \\
&u_{\min} \le u(k + j) \le u_{\max}, \quad j = 0, 1, \cdots, m - 1
\end{aligned} \tag{9}$$

where $u_{\rm opt} \in \mathbb{R}^{nu}$ refers to the optimal input values obtained at the supervisory level. Matrix $R \in \mathbb{R}^{nu \times nu}$ weights the deviation of the control input value at the end of the control horizon from the desired optimum. Penalizing only the future final value attempts to separate the effect of the optimising action of the supervisor from the more immediate MPC closed-loop corrections. In this way, the original MPC objective function is modified the least possible and the activity of each control level tends to be decoupled. In other words, the NLP supervisory layer looks for steady-state input values in agreement with minimum utility consumption, while the MPC accounts for transient performance and offset free controlled outputs.

The analysis in Section 3.2 and the simulation results in Section 5 provide some additional insight into this particular problem.

3.2. Convergence to the optimal steady state

HEN systems are usually open loop stable. Thus, increasing the prediction horizon tends to guarantee the convergence of the closed-loop optimization problem defined in (8) [9]. Since an extra cost term is added to the cost function of problem (8) to achieve the minimum utility goal, a specific analysis concerning the necessary conditions to drive the system to the desired operation point is required. To clarify this point, suppose that the output horizon p is large enough, such that the output reaches the steady state at the end of this horizon. As the gap between the input steady state and its optimal value, would remain constant at sampling step $k+1$ if the following control sequence is used:

$$(\Delta u^*)_{k+1}^{\mathrm{T}} = [\Delta u(k+1)^* \ \Delta u(k+2)^* \ \dots \ \Delta u(k+m-1)^* \ 0]^{\mathrm{T}}$$

then, the optimal cost, at time $k+1$ will satisfy

$$V_{k+1}^* \le V_k^* + \|e(k+1/k)\|_Q^2 + \|\Delta u(k/k)\|_S^2 \tag{10}$$

Because there are enough degrees of freedom in the HEN system, the fact that the outputs will reach their set points at the end of the output horizon does not assure that the inputs will reach their optimal values. In this way, only the convergence of the outputs to the set points is guaranteed, and a permanent error/offset between control inputs and their optimal values may appear. In order to avoid this offset and the consequent excess of utility consumption, it is desirable to adopt a weight R such that the MPC efficiently drives the control inputs to their optimal values without spoiling the dynamic response. Note, at this point, that the non-linear model used in the supervisory layer is different from the HEN simulator used in the simulation. Following Odloak [14], the theorem below provides a sufficient condition for the convergence of the inputs to the desired steady state.

Theorem 1. *Assume a stable HEN system with more inputs* (nu) *than outputs* (ny), *and let $\mathcal{F}$ be a set that includes all possible steady-state gain matrices G of the system. Furthermore, assume that the controller obtained in* (9) *is stable, and define the augmented matrix* $\tilde{G} = \begin{bmatrix} G \\ I_d \end{bmatrix}$ *where I_d defines the inputs that receive targets from the supervisory program.*

$$\textit{For instance, } I_d = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \underbrace{0 \ \ \cdots \ \ 1}_{nu-ny} & & & \cdots & 0 \end{bmatrix}$$

corresponds to the inclusion of targets for the first $nu-ny$ inputs. If matrix $\tilde{G}$ is full rank for all $G \in \mathcal{F}$, then, there is a matrix R such that the control law produced by the solution of problem (9) *drives the selected inputs to their desired optimal values.*

Proof. Suppose that for a large enough time $\bar{k}$, the controlled system tends to an equilibrium point defined by:

$$\begin{aligned} y &= r \\ u &\ne u_{\text{opt}} \end{aligned} . \tag{11}$$

In other words, assume that the system goes to an operating point where there is a permanent offset $\delta_{\bar{k}}^u$ between the actual input and the optimal input value. Since at this steady state $\Delta u_{\bar{k}} = 0$, and assuming that there is no offset in the controlled outputs, the cost function in the optimization problem (9) becomes

$$V_{\bar{k}} = \|\delta_{\bar{k}}^u\|_R^2 \tag{12}$$

where

$$\delta_{\bar{k}}^u = \begin{bmatrix} u_1(\bar{k}) - u_{1,\text{opt}} \\ \vdots \\ u_d(\bar{k}) - u_{d,\text{opt}} \end{bmatrix}, \quad nu - ny \le d \le nu.$$

Let us now search for a virtual control move that would take the system to its optimal operating point. Assuming for simplicity that the first $d = nu - ny$ control inputs are arbitrarily selected to receive targets, and that the control horizon is reduced to $m = 1$, the control increment must satisfy

$$\underbrace{\begin{bmatrix} & & G & & \\ 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \underbrace{0 \ \ \cdots \ \ 1}_{d} & & & \cdots & 0 \end{bmatrix}}_{\tilde{G}} u(\bar{k}) = \underbrace{\begin{bmatrix} \hat{r} \\ u_{1,\text{opt}} \\ \vdots \\ u_{d,\text{opt}} \end{bmatrix}}_{\tilde{r}}$$

$$\Rightarrow \tilde{G}u(\bar{k}-1) + \tilde{G}\Delta u(\bar{k}) = \tilde{r}, \tag{13}$$

Because by hypothesis the square matrix $\tilde{G} \in \mathbb{R}^{nu \times nu}$ is of full rank for all $G \in \mathcal{F}$, it is possible to compute its inverse and to solve Eq. (13) determining the following input increment

$$\Delta u(\bar{k}) = \tilde{G}^{-1}[\tilde{r} - \tilde{G}u(\bar{k}-1)] = \tilde{G}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \delta_{\bar{k}}^u \end{bmatrix} = \tilde{G}^{-1} C \delta_{\bar{k}}^u \tag{14}$$

where

$$C = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{nu \times d}$$

Now, substituting (14) into the cost function at time $\bar{k}$, we have

$$\bar{V}_{\bar{k}} = \left\| \underbrace{e^0}_{=0} - A\tilde{G}^{-1}C\delta^u_{\bar{k}} \right\|^2_Q + \underbrace{||u_{\bar{k}} - u_{\text{opt}}||^2_R}_{=0} + ||\tilde{G}^{-1}C\delta^u_{\bar{k}}||^2_S$$

$$= (A\tilde{G}^{-1}C\delta^u_{\bar{k}})^{\mathrm{T}} Q(A\tilde{G}^{-1}C\delta^u_{\bar{k}}) + (\tilde{G}^{-1}C\delta^u_{\bar{k}})^{\mathrm{T}} S(\tilde{G}^{-1}C\delta^u_{\bar{k}})$$

$$= \delta^{u^{\mathrm{T}}}_{\bar{k}} \underbrace{(C^{\mathrm{T}}\tilde{G}^{-1^{\mathrm{T}}}A^{\mathrm{T}}QA\tilde{G}^{-1}C + C^{\mathrm{T}}\tilde{G}^{-1^{\mathrm{T}}}S\tilde{G}^{-1}C)}_{R_{\min}}\delta^u_{\bar{k}}$$

$$= ||\delta^u_{\bar{k}}||^2_{R_{\min}} \quad (15)$$

where e^0 is equal to zero because the system is assumed at steady-state and with null offset. If the supervisory program is able to computes the optimal input values, which is always possible if $\tilde{G}$ is of full rank, then the proposed objective function of problem (9) will tend to zero, because $\bar{V}_{\bar{k}} < V_{\bar{k}}$, if the weight matrix R satisfies

$$R_{\min} < R \quad (16)$$

For those system inputs that were selected to have optimal targets, inequality (16) becomes a sufficient condition for the convergence of these inputs to their optimal values, and the theorem is proved. If the condition specified in (16) is satisfied, then the QP related to the control optimization problem will have a positive definite Hessian matrix. Consequently, this optimization problem will be convex and will have a unique solution that will be equal to zero only when the predicted error on the system outputs and selected inputs becomes equal to zero. □

This result produces a simple tuning sequence that defines the parameters in the MPC cost function. Once the commanding manipulated variables are selected (matrix C), the values of Q and S are searched to produce good closed-loop dynamic performance. Finally, R is chosen such that (16) is satisfied.

Note that, in case of penalizing the complete control horizon instead of only the final input value, the cost relation (10) would include a potentially conflicting third term

$$V^*_{k+1} \leq V^*_k + \|e(k+1/k)\|^2_Q + \|\Delta u(k/k)\|^2_S + \left\|u(k/k) - u_{\text{opt}}\right\|^2_R.$$

In this case, the MPC behaviour might be somehow distorted since both, the supervisory program and MPC will attempt to drive common control inputs aiming at different objectives. Then, the resulting system performance may be deteriorated. A preliminary analysis indicates that the loss of performance may become important in the presence of load disturbances that are not accounted for in the NLP supervisory program. When this happens, the NLP optimization, which works as an open-loop controller, might tend to maintain the manipulated variables at the same position, while MPC needs to take control actions to reject the disturbance.

Finally, it is worth remembering that if every exchanger unit provides a manipulated variable and all the process streams have temperature targets, the lowest value ($nu - ny$) for dimension d corresponds to the degrees of freedom proposed by Marselle et al. [1]. Though they do not necessarily imply degrees of freedom for energy integration, they certainly tell about network flexibility to reach the temperature targets. In most of the cases, using values of d larger than $nu - ny$ results in over specification of the desired operating point condition, but this argument seems insufficient to remove this alternative from consideration. This raises some questions about the right number d of control inputs that should be included in the cost function, and asks for a rational procedure for selecting these variables from the nu available in the system. Preliminary answers to these questions are presented in the following sections.

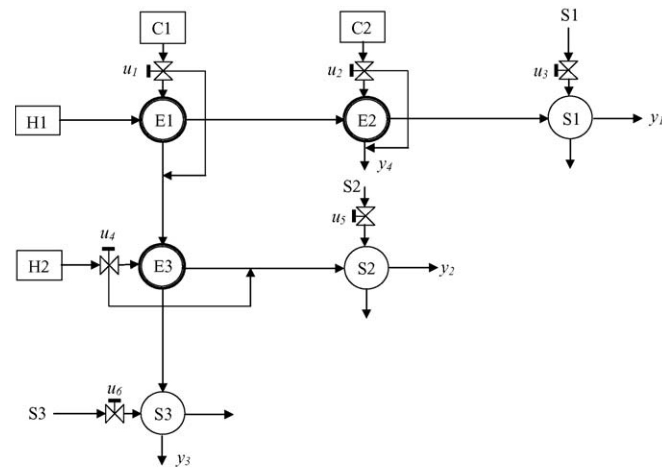

Fig. 1. Heat-exchanger network example.

4. Application example

4.1. The heat-exchanger network used in this work

The problem to be solved concerns the control and optimization of the heat-exchanger network shown in Fig. 1, which is composed of three recovery exchangers and three service units. There are four process streams that have to receive a proper thermal conditioning and three utility streams that help to reach the desired temperatures. Thus, the complete system has six input or manipulated variables (three bypasses and three utility flow rates, u) and four outputs to be controlled (process stream temperatures, y). The objective is to reach a satisfactory control quality, which includes reasonable disturbance rejections, rapid tracking for set-point changes, and minimum utility consumption at any operating point.

For the particular network in Fig. 1, the connection structure can be translated into a stationary gain matrix G that has the form

$$G = \begin{bmatrix} G^{11} & G^{12} & G^{13} & 0 & 0 & 0 \\ G^{21} & 0 & 0 & G^{24} & G^{25} & 0 \\ G^{31} & 0 & 0 & G^{34} & 0 & G^{36} \\ G^{41} & G^{42} & 0 & 0 & 0 & 0 \end{bmatrix}.$$

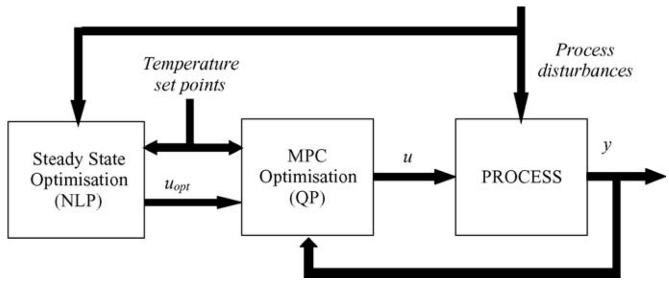

Fig. 2. General structure of the proposed control system.

In this matrix, the element G^{ij} stands for the stationary gain between the output y_i and the input u_j.

4.2. Coordination between supervisory level and MPC

The general control structure assumed here is depicted in Fig. 2, where the steady-state optimization and the MPC layers are represented, along with the variables used to connect them. Following the ideas presented before, it is necessary to define the inputs through which the supervisory level passes the commands to the MPC with extended cost function. The smallest number of commanding inputs that can be selected is given by the degrees of freedom of the HEN system that are available for steady-state optimization. However, adopting a small number of inputs gives a large number of possible combinations. As any valid set of commanding inputs must generate a full rank $\tilde{G}$ matrix for all $G \in \mathcal{F}$, many of these combinations can be eliminated, as they do not satisfy the condition for convergence of the inputs to the desired target. For instance, in the network of Fig. 1, the estimated system-gain matrix at the nominal operation point is:

$$G = \begin{bmatrix} 14.09 & 11.90 & -8.38 & 0 & 0 & 0 \\ -18.03 & 0 & 0 & 14.65 & -70.71 & 0 \\ -25.20 & 0 & 0 & -7.59 & 0 & 23.69 \\ 13.53 & -17.77 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and, if the first and the second control inputs are selected, the resulting matrix $\tilde{G}$ is of rank 5. This means that $\tilde{G}^{-1}$ cannot be computed and the implication is that when these inputs are selected, it is not possible to guide all the manipulated variables to their optimal values. In other words, sending the manipulated variables u_1 and u_2 to the desired positions is not sufficient to define the complete set of inputs, even though the outputs are at their set points.

In order to explore the possible control structures, let us consider the results in Table 1. This table shows all the different combinations of two control inputs in this HEN system that might be selected as commanding inputs to MPC. Note that any pair combining the first three control inputs (u_1, u_2, u_3) yields a singular matrix $\tilde{G}$. Furthermore, even using these three variables together, we fail to produce a fully determined system, namely the corresponding matrix $\tilde{G}$ has also rank 5. This feature is clearly related to the network structure, which implies in the existence of several zero entries in matrix $\tilde{G}$.

Table 1
Rank of the augmented $\tilde{G}$ matrix for different combinations of two control inputs

Control inputs included in $\tilde{G}$	Rank ($\tilde{G}$)
u_1u_2	5
u_1u_3	5
u_1u_4	6
u_1u_5	6
u_1u_6	6
u_2u_3	5
Any other pair	6

Table 2
Stream conditions for the nominal operation point

Stream	T^{in} (°C)	T^{out} (°C)	$w^{max}c$ (kW/°C)
H_1	90	40	50
H_2	130	100	20
C_1	30	80	40
C_2	20	40	40
S_1	15	–	35
S_2	30	–	30
S_3	200	–	10

The definition of which inputs are to be used as commanding variables in the MPC is obtained through the selection of the non-zero entries in matrix R. However, the selected structure must be validated by the full rank of the corresponding matrix $\tilde{G}$. If this condition is satisfied, the corresponding R_{min}, defined in (15) does exist, and any R large enough should be sufficient to guarantee the convergence of all the inputs to the optimal values. However, as several sets of inputs are theoretically capable of driving the system to the optimal point, additional guidelines describing the effect of R on the system behaviour would be beneficial. The next section is devoted to test the effect of R on the performance of the proposed MPC controller.

5. Simulation results

5.1. Main features of the HEN simulations

An interactive dynamic simulator of HEN systems developed at INTEC has been used for testing the proposed control structure. Correa [15] presented the main features of this simulator, which is based on a non-linear model of shell-and-tubes heat exchangers previously reported by Correa and Marchetti [16]. Table 2 shows the nominal conditions for the inlet and outlet streams, while Table 3 gives the effective heat-exchanger areas used to simulate the network in Fig. 1. Table 4 shows the adopted main parameters of the MPC.

Table 3
Effective heat transfer areas UA (kW/°C) for the heat exchangers in Fig. 1

E1	E2	E3	S1	S2	S3
80	50	20	30	20	10

Table 4
MPC parameters

Control horizon (m)	Prediction horizon (p)	Δu Weight (diagonal matrix Q)	Output weight (diagonal matrix Q)	Sampling time (T)
2	15	20	5	3

For the HEN system, the service cost that quantifies the operating expense may be defined as:

$$J_{\text{service cost}} = \sum_{i \in H} c_{c_i} w_{c_i} + \sum_{j \in C} c_{h_j} w_{h_j}. \tag{17}$$

For the network represented in Fig. 1, the above cost becomes

$$J_{\text{service cost}} = c_3 w_3^{\max} u_3 + c_5 w_5^{\max} u_5 + c_6 w_6^{\max} u_6 \tag{18}$$

where u stands for fraction of valve opening.

Figs. 3–6, show the dynamic responses of the HEN system when the following sequence of changes is introduced into the system: starting from the nominal conditions, the set point of $T_{C_1}^{\text{out}}$ is changed from 80 to 70 °C, next, the set point of $T_{C_2}^{\text{out}}$ is changed from 40 to 45 °C and finally, the set point of $T_{H_2}^{\text{out}}$ is changed from 100 to 90 °C. The optimal steady states, for these operating conditions, are indicated in Sequence 1 of Table 5 as fractions of maximum utility flow rates in service units (S), or as bypass fractions in exchanger units (E).

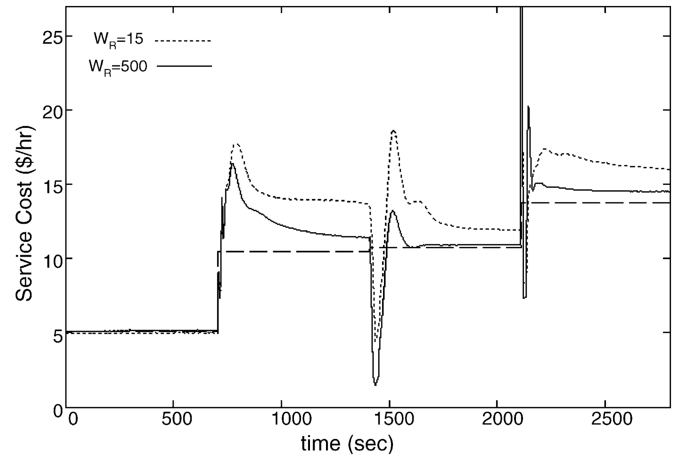

Fig. 4. Service costs with different weight matrices R.

Finally, an additional test is designed to evaluate the performance achieved by the proposed MPC for disturbance rejection when the following sequence of load changes is introduced into the system: first, the inlet temperature $T_{H_1}^{\text{in}}$ is changed from 90 to 80 °C, next, the temperature $T_{H_2}^{\text{in}}$ of stream H_2 is changed from 130 to 140 °C and, finally, $T_{H_2}^{\text{in}}$ is changed from 140 to

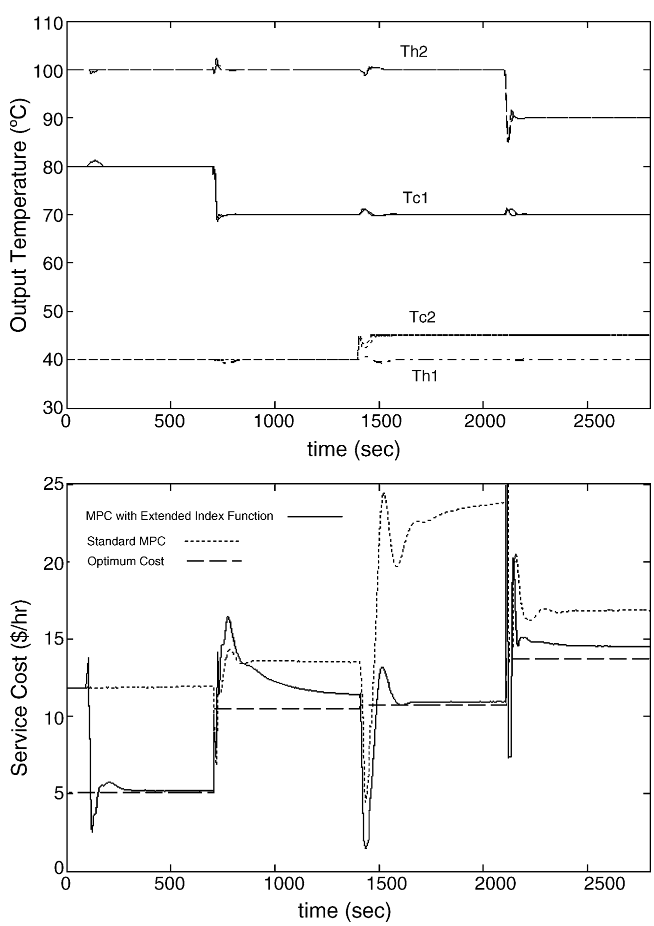

Fig. 3. Output temperature and service cost responses with and without input penalization.

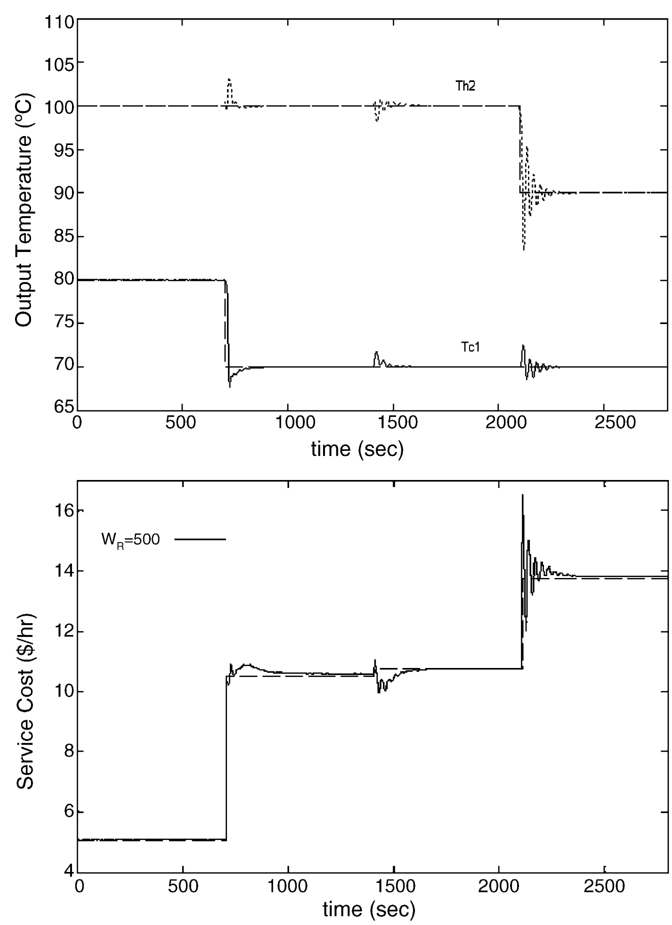

Fig. 5. System responses with targets to u_3, u_5 and u_6 and penalizing the inputs at all time instants in the control horizon.

Table 5
Fractions of valve opening obtained by the NLP optimization

Case	u_1 (opt)	u_2 (opt)	u_3 (opt)	u_4 (opt)	u_5 (opt)	u_6 (opt)	Utility expenses ($/h)
Nominal	0	0.1463	0.0590	0.6739	0.1001	0	4.950
Sequence 1							
$T_{C_1}^{out} = 70\,°C$	0.0687	0.2010	0.1060	1	0.2260	0	10.278
$T_{C_2}^{out} = 45\,°C$	0.3741	0	0.2041	0.7737	0.1200	0	10.335
$T_{H_2}^{out} = 90\,°C$	0.3741	0	0.2041	0.7737	0.2200	0	13.335
Sequence 2							
$T_{H_1}^{in} = 80\,°C$	0.2550	0.0705	0	0.4354	0	0.1608	1.608
$T_{H_2}^{in} = 140\,°C$	0.2550	0.0705	0	0.2067	0	0	0
$T_{H_2}^{in} = 150\,°C$	0.2550	0.0705	0	0.3935	0.0834	0	2.502

150 °C. The optimal steady-state flow rate fractions related to these inlet conditions are indicated in Sequence 2 of Table 5. The simulation results corresponding to this case are shown in Figs. 7 and 8.

5.2. Adopting all the service inputs as commanding variables

In order to highlight the effect of including or not the supervisory level, we simulate the HEN system with the standard MPC and with the MPC with the additional term that considers the utility expenses. With regard to the selection of the commanding inputs, an attractive preliminary choice is to use targets for all the service inputs u_3, u_5 and u_6 ($d=3$), as all of them are included in the service cost that is optimised in the supervisory program and they yield a matrix $\tilde{G}$ of full rank 6. Notice that when working with this set of inputs, matrix R takes the form:

$$R = W_R \text{diag}(0\,0\,1\,0\,1\,1), \quad W_R \in \mathbb{R}$$

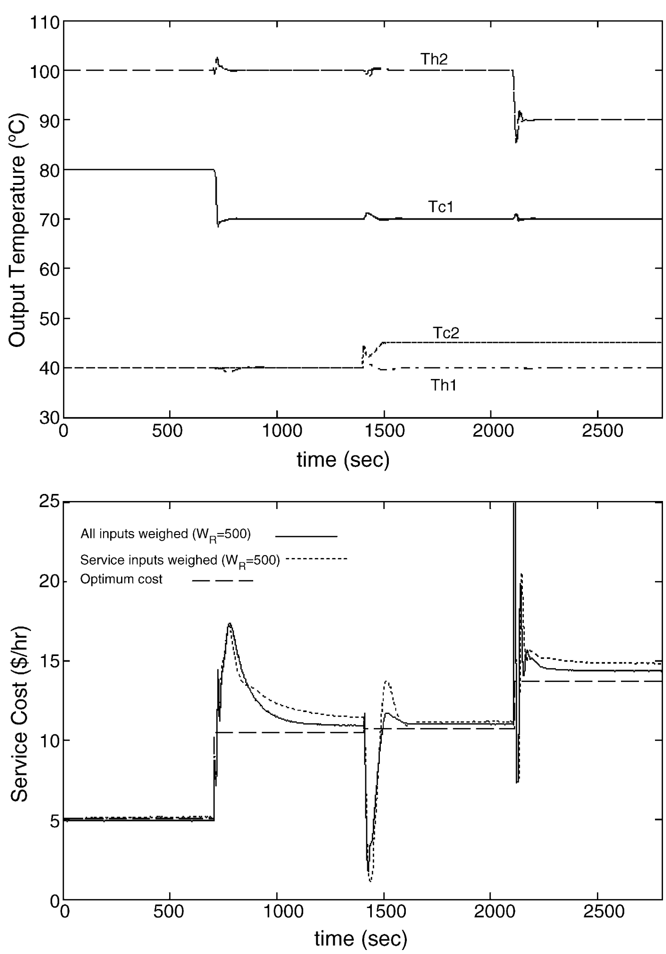

Fig. 6. System responses when the global heat-transfer coefficient of E1 is decreased.

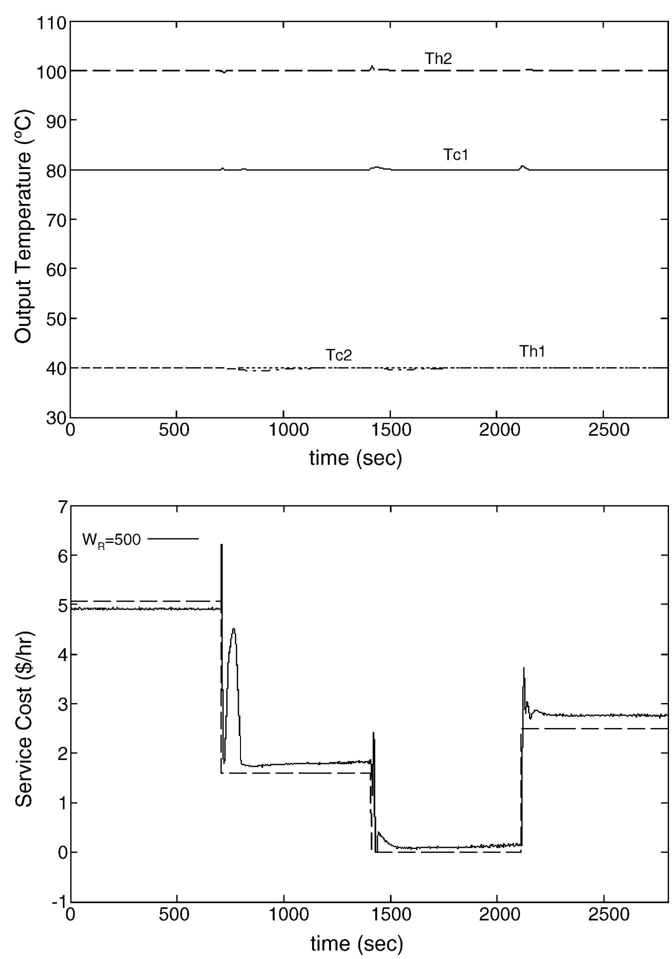

Fig. 7. System responses for a load change with targets to u_1 and u_4.

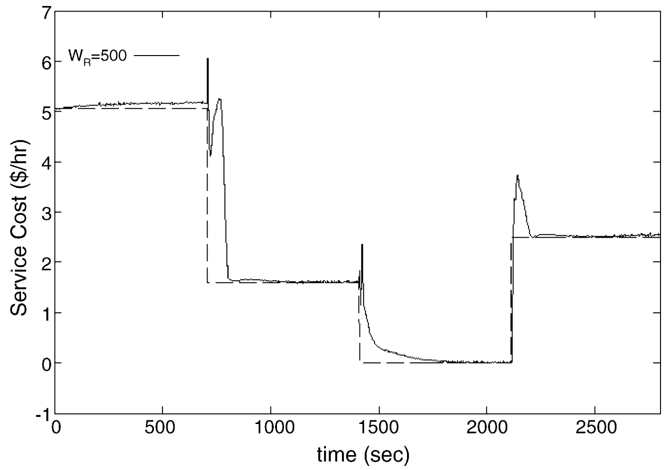

Fig. 8. System responses for a load change with targets to u_3, u_5 and u_6.

The responses shown in Fig. 3 start from an operating point that satisfies all the process temperature targets, and at time $t = 100$, the control is switched to the MPC with the new utility term in the cost function. Fig. 3 shows that MPC with extended cost drives the operating cost down towards the minimum, and finally achieves a significantly lower value of the cost defined in (18) when compared with the standard MPC. Furthermore, we can see that the dynamic responses of the system output are nearly unmodified when the utility term is included in the cost function. This means that, at the optimal steady state, the services flow rates are substantially different from the services flow rates at the starting steady state. These results confirm that there are many steady-state control values that satisfy the temperature targets, and demonstrate the ability of the combined supervisor-MPC controller to guide the system to the optimal steady state.

The simulation results in Fig. 4 compare the responses of the utilities cost for different values of weight W_R in matrix R with a fixed structure. The simulations are performed with $W_R = 15$ and $W_R = 500$. It is not shown, but the output temperature responses are basically undistinguishable for the two cases, Fig. 4 shows a tendency of the cost defined in (18) to settle down far from the desired values, when $W_R = 15$, because the utilities flow rates do not reach their optimal values. This behavior may be attributed to the non-convexity of the cost V_k when R is not large enough. On the other hand, with $W_R = 500$, a much better approach to the optimised conditions can be observed in the full-line response. The remaining offset in the input responses can be attributed to the differences between the model used by the supervisor program and the non-linear simulator. This interpretation is supported by further observations, which show that increasing even further the value of W_R would cause no improvement on the manipulated variable end positions. It is also important to emphasize that it is mainly the weight matrix S that determines the controller speed of reaction to disturbances or set point changes.

Notice that the alternative strategy of embedding the optimization of the steady state cost into the MPC optimization problem would eliminate the mismatch between the predicted optimal cost and the cost value that is really obtained. However, that would be equivalent to solve the supervisory optimization problem with additional constraints. In other words, we would be eliminating the offset by moving u_{op} instead of $u(t)$, and there is no guarantee that the eventual operating cost will be better than with the proposed approach.

Another argument in favor of the two-level approach is that the inclusion of the rigorous model of the system into the controller turns the controller optimization problem much more complicated and possibly non-convex. This makes, the solution of the MPC problem more time consuming and may cause convergence problems. If a feasible solution is not found within the sampling period of the MPC, the process unit will remain in open loop and the control strategy becomes inefficient and unsafe. In the two-level approach, the same convergence problem may occur, but in this case it is concentrated only in the optimization level. The MPC that lies in the lower level will remain in closed loop, although the targets may not be the optimal ones. Thus, with the two-level approach the operators may feel more confident than with the integrated approach.

5.3. Steady-state model mismatch and control horizon penalization

When reaching temperature targets takes priority over achieving minimum utility expenses, it is desirable that inaccuracies in the desired input values do not originate offset in the controlled outputs. At this point, we should note that the optimal input conditions are calculated based on an approximated non-linear model (Eqs. (2)–(7)) while the behavior of the true plant is represented in the simulator by a rigorous and more complex model. The closer the non-linear model utilized in the steady-state optimization is from the rigorous simulation model, the more consistent will be the targets and set points sent to the control system. Furthermore, the fact that a very high value of W_R does not deteriorate the closed-loop output responses, in spite of steady-state model mismatch, is a consequence of penalizing only the last control action in the control horizon. This leaves MPC free to use the remaining control actions to satisfy dynamic requirements and set point targets. This sort of decoupling between steady-state optimization and MPC becomes more accentuated as the length of the control horizon m increases, and turns into flexibility to adjust the weighting matrices. On the contrary, if the utilities penalization includes all the control actions in the control horizon, the output performance may deteriorate. Fig. 5 shows some temperature responses obtained when this strategy is implemented. Even though the utilities cost is guided quite rapidly to the minimal value, a poor dynamic output response is obtained in comparison with those in Fig. 3. These results strengthen the proposed approach that penalizes only the final value of the input in the control horizon to define the extended cost.

5.4. Adopting all the inputs as commanding variables

Another simple and appealing possibility is to adopt the optimal final positions of all the control inputs as commanding variables to the MPC. If the desired steady-state values are consistent with the actual plant, the resulting over specification of the desired steady state should not affect the performance of the control system. However, when model mismatch is present, a conflict may arise between the desired output values and the input targets. It is important to verify the extension of this conflict for the most common source of model mismatch in HEN systems.

To simulate the model mismatch problem, the global heat-transfer coefficient of heat exchanger E1 was decreased by 25%. This intends to represent the effect of fouling, which is not taken into account by the model adopted in the supervisory layer. Surprisingly, this model uncertainty does not cause a major impact on the control system. Fig. 6 shows one of the best performances in terms of output responses, with all the controlled temperatures following their set points very closely. These responses correspond to the two cases: (a) when only the utility flows u_3, u_5 and u_6 have commanding targets, (b) when all the inputs have commanding targets. Three main aspects can be pointed out: (i) the dynamic performances of the system outputs in the two cases (full commanding and partial commanding) are similar, (ii) the strategy in which the supervisory program sets steady state targets to all the inputs results in a better approach to the optimal operation cost, (iii) model mismatch introduced by the presence of fouling tends to increase the gap between the desired and the actual utilities cost.

5.5. Disturbance rejection with the minimum number of input targets

The selection of which control inputs of the HEN system will have explicit targets, aims in this case at reducing the undesirable interaction between the supervisory program and MPC. For this purpose, we select the manipulated variables u_1 and u_4 that affect key heat exchangers of the simulated HEN system, and are capable of distributing heat duties to different parts of the network. Thus, the desired steady state operating condition of the network is now defined in terms of the preconditioning units E1 and E3 that are not located at the end of any process stream like the other units. Since the control variables u_3, u_5 and u_6 are now free to be used exclusively for output tracking, it might be expected that MPC would provide a slightly better output response. In other words, the expected consequence of this configuration is a lower interaction between the regulation task and the steady-state optimization. This is confirmed by the satisfactory responses shown in Fig. 7, which was obtained for the regulator operation case, by weighting the final value in the control-horizon of u_1 and u_4, with $W_R = 500$. As the control system guides these two essential inputs directly to the values indicated by the supervisory program, the whole process goes to the desired operating condition quite rapidly, following very closely the output set points. However, a persistent offset in the service cost is observed in Fig. 7, even though the commanding inputs u_1 and u_4 reach the desired values. This is certainly related to the uncertainty in the model of the supervisor program, because no direct information about the desired service-variable positions is passed to the MPC optimization problem. The observed performance in Fig. 7 is partially explained by the complementary results shown in Fig. 8. These responses were obtained for the same sequence of input changes as in Fig. 7, but this time the service inputs u_3, u_5 and u_6 are again used with commanding targets.

Hence, two alternative sets of commanding variables are appropriate to produce an acceptable gap between the actual service expense and the minimum obtained by the NLP solution of the supervisory layer: (1) a set containing all the available control inputs; (2) a set containing only the variables directly involved in the steady-state cost function; i.e., the inputs associated with service units. Even though these alternatives do not show any deterioration of output tracking responses, they may exhibit some utility cost offset due to steady-state model inaccuracies. In this regard, the first alternative seems to be more reliable; if the supervisory program yields slightly biased results, the final input positions would tend to satisfy the minimum squared error criterion. This provides with certain robustness to the connection with MPC, since any failure will be shared among all the control inputs.

6. Conclusions

A standard linear predictive control with constraints in the manipulated variables has been successfully applied to a realistic non-linear simulator of a heat-exchanger network (HEN). In order to drive the HEN steady-state operation to minimum utility cost condition, an extra cost term has been included into the MPC formulation without deteriorating the dynamic performance.

The complete control system structure includes two different levels: a supervisor-optimizer program (NLP) and a low-level multivariable predictive controller (MPC). The upper level is exclusively devoted to determine an optimal steady state such that the minimum service cost is achieved. The lower level takes care of temperature targets and drives the system towards the optimal steady state while focusing on the dynamic performance. The results obtained in this work show that penalizing only the last control action in the control horizon provides a perceptible decoupling between steady-state optimization and MPC, with a significant improvement in the dynamic performance.

The supervisory level optimizes the operation expenses and set desired input targets to a new cost term in the MPC objective function. This work demonstrates that if this cost term is weighted adequately, convergence to the desired input values is obtained. However, model inaccuracies may cause displacement of the operating point from the optimal condition. In this regard, the simulation results show the convenience of setting targets to all the available control inputs. Setting a large number of inputs provides robustness to track the HEN optimal condition since the model mismatch is shared among all the inputs yielding an average operating condition, which is usually close to the desired one.

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