Infinite Horizon MPC of a heat-exchanger network

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Abstract

A method to control a heat-exchanger network (HEN) is presented where both, the control objective and the economic objective are taken into account. It is assumed that we have a two-level structure in which the steady state economic optimisation is performed in the upper level and the MPC controller is used to enforce the optimal operating point defined by the economic layer. Since HENs are multivariable highly interactive systems, which integrate large energy consumers of a process plant, stability of the proposed structure becomes one of the issues of the HEN control problem. In this approach, integration is achieved through the definition of an extended cost-function that provides the controller with the ability of driving the system to optimal conditions. The proposed MPC algorithm uses a linear state space model that is particularly suitable to the development proposed in this work. The method is exemplified with the simulation of a HEN system that shows the typical characteristics of an industrial system.

Keywords*: model predictive control, heat-exchanger networks, minimum utility consumption, stable control*

Introduction

Large chemical plants are intensive energy consumers and usually require energy recovery systems to maintain the operation at a competitive economic level. To help achieve this goal, heat-exchanger networks (HENs) usually play an important role mainly in oil refineries and petrochemical plants where a large number of hot process streams that leave the process can be used to increase the thermal energy of cold streams that are fed to the process. A complicating factor in this scenario is that the thermal outlet condition of several process streams must be controlled close to desired values associated to products specifications, environmental restrictions and safety constraints without reducing the operation efficiency. Consequently, the usual requirement of satisfying the control targets must be accomplished without reducing heat integration, or almost equivalently, maintaining low utility consumption.

In the last two decades, the design of a proper control structure for the HEN system has become a subject to academic research, mainly after it was realized that hard constraints in the manipulated inputs plays an important part in the control problem. Most works that appeared in the literature focused on the design problem of the HEN control system, producing several procedures to define the appropriate control structure. Some initial contributions related to this topic can be found in Marselle et al. (1982), Beautyman and Cornish (1984), Calandranis and Stephanopoulos (1988), Huang and Fan (1992) and Mathisen et al. (1992). More recently, Aguilera and Marchetti (1998) studied the integration of on-line optimisation and control of HENs. In their study, it was emphasized the necessity of defining the degrees of freedom of the HEN system with regard to the steady-state optimisation. Glemmestad et al. (1999) propose an approach for the optimal operation of HENs in which a periodic steady-state optimisation is integrated to a fixed control structure. In the practical field, Giovanini and Marchetti (2003) have shown that low-level Distributed Control Systems (DCS) are capable of handling some HEN control structures, but in general, there are some limitations to reaching the most convenient operation point.

For those continuous systems that exhibit constraints in the manipulated inputs, using MPC seems a natural alternative to pursue the economic optimal operating point. Model Predictive Control (MPC), also known as Moving Horizon Control or Receding Horizon Control, has been extensively applied in the process industry due to its ability to handle input constraints for highly interacting systems as the HEN system. The controller uses a dynamic model of the process to predict the output trajectories and performs a constrained on-line optimisation to determine the optimal future input sequence. The first control move is injected in the real plant and the procedure is repeated in the next sampling time. Detailed overviews of MPC and comparisons of commercial MPC controllers can be found in Maciejowski (2002) and Qin and Badgwell (2003).

A simple method to obtain a stable MPC is to include in the control optimization problem, a terminal state constraint (Keerthi and Gilbert, 1988, Mayne and Michalska, 1990). This state constraint assures that, at the end of the output prediction horizon, the state will lie at the origin. This method is easy to codify but it is difficult to implement in practice for systems that show unmeasured persistent output disturbances and systems that have more outputs than inputs. In such cases, the optimization problem, which is solved by MPC, may easily become infeasible at normal operating conditions. Some authors (Michalska and Mayne, 1993, Scokaert et al. 1999) have proposed an alternative approach, in which the terminal state is expanded to a terminal set around the origin. This terminal set is such that it is positive invariant when in closed loop with the Linear Quadratic Regulator. In this method, the region in which the controller is feasible still depends on the control horizon and on the magnitude of the disturbances that enter the system.

Another method to obtain a stable MPC is to consider an infinite prediction horizon. Rawlings & Muske (1993) have shown that for linear stable systems with constraints in the inputs and states, the infinite horizon MPC is stable independent of the other tuning parameters. Rodrigues & Odloak (2003) and Odloak (2004) extended the infinite horizon approach to systems with model uncertainty and unknown steady state. The method was also extended to non-square systems where the number of outputs is larger than the number of inputs. The new approach is based on the softening of the terminal state constraint. Here, the method of Odloak (2004) is further extended to include an economic objective in the MPC optimization problem, in such a way that a nominally stable MPC for the HEN system is obtained.

Although a large number of articles focusing on applications of MPC to a variety of process systems can be found in the control literature, the application of MPC to HEN systems has not been fully evaluated. There is no justification for this lack of interest of the control community in the control of HEN systems. These systems present challenging operating features, which combine heat integration and low utility consumption with the dynamic regulation of the temperature of several process streams.

This paper is organized as follows: Section 2 describes the HEN steady-state optimisation as presented by Aguilera and Marchetti (1998) and discusses how to integrate the economic optimisation into the multivariable control structure of MPC. Section 3 presents the development of the infinite horizon MPC associated with the state space model form that is used here. It is also included a discussion on how to extend the controller of Odloak (2004) to be applied to the HEN system. In Section 4, a description of the network application example and its degrees of freedom are discussed. Section 5 provides simulation results with and without the insertion of set points to the manipulated variables. Finally, in Section 6 the paper is concluded.

2. The optimal operating point of the HEN system

The main purpose of the HEN system is to recover as much energy as possible from high-temperature process streams and to transfer this energy to cold-process streams. The benefits are savings in steam and fuels. However, the HEN system has to provide the proper thermal conditioning of some of the process streams involved in the heat transfer network. This means that a control system must be included in order to drive the exit process-stream temperatures to the desired values in the presence of external disturbances and input constraints, The control system should be designed in such a way that it can lead the HEN system to the point of minimum utility consumption.

There are two classes of heat-exchanger units in a HEN system: those in which energy is exchanged between process streams (referred here as “heat exchangers” or “E units”) and those in which heat is exchanged between a process stream and a utility stream as steam or water (referred here as “services” or “S units”). It is desirable that the global task executed by the services be minimized in order to achieve the highest energy integration. The weighted sum of utility flowrates is then used as an index function to represent the network in terms of energy recovery efficiency.

The usual manipulated variables in a HEN system are the flowrates at bypasses around heat exchangers, the flowrates of utility streams in service units and the splits of process streams. Usually, the HEN system has more control inputs than outlet temperatures to be controlled and so, the set of input values satisfying the output targets is not unique. The possible operation points may result in different levels of heat integration and utilities consumption. Hence, the optimal operation of HEN systems requires the development of a strategy that in a first step determines the inputs that produce the lowest service-cost, and in a second step defines how to dynamically guide the process towards this optimal point. Thus, the strategy involves two separate optimization problems whose variables are not exactly the same. The first problem is a steady-state optimization problem wherein the objective function can be written as

$$\min_{w_{c_i}, w_{h_j}} \left\{ \sum_i c_{c_i} w_{c_i} + \sum_j c_{h_j} w_{h_j} \right\} \qquad i \in H, j \in C, \tag{1}$$

where H and C are the sets of hot and cold streams respectively, w_{ci} stands for the cold-utility flowrate of the service unit related to hot stream i , w_{hj} is the hot-utility flowrate of the service unit related to the cold stream j, c_{ci} and c_{hj} are the utility costs.

Associated with the above objective we have several equality and inequality constraints defining the heat exchanger network model, constraints in heat duties, valves opening bounds, flow limits, temperature targets, etc. A complete definition of this problem can be found in Aguilera and Marchetti (1998).

Assuming that the structure of heat exchanger network is fixed, the solution to the above problem is a set of optimal values for flowrates of process streams and utilities. If the controller of the HEN system can steer these variables to the desired optimal values while maintaining the outlet temperatures at their targets, then the operation of the HEN system will be optimal. One type of controller that is a natural candidate to this kind of assignment is MPC, and in the next section we discuss the extension of a stable MPC of the control literature to the control of HEN systems.

3. The infinite horizon MPC applied to the HEN system

A simple strategy to obtain a stable MPC is to consider an infinite prediction horizon as proposed by Rawlings and Muske (1993), who designated this controller as IHMPC. The controller developed by these authors focused only on the regulator case. IHMPC was recently extended to the output tracking case in the presence of unmeasured disturbances or unknown steady state. Rodrigues & Odloak (2003) proposed an approach in which the output prediction is treated as a continuous function of time. Odloak (2004) presented another version of the same controller for the case wherein the prediction time is discretized. Here, we will extend the method of Odloak (2004) to control a system like the HEN system where the economic objective imposes targets to the inputs.

The MPC strategy consists in obtaining a control sequence that minimizes, at each sample step, the predicted future error along the prediction horizon, subject to constraints on the amplitude of the control input and input move. From this control sequence, only the first component is implemented in the real system. At the next sample step the whole procedure is repeated.

For stable systems with *nu* inputs and *ny* outputs, assuming that the poles relating any input u_i to any output y_j are non-repeated, a state space model that is suitable to the implementation of IHMPC can be represented in the following form:

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} + \begin{bmatrix} B^s \\ B^d \end{bmatrix} \Delta u(k) \tag{2}$$

$$y(k) = \begin{bmatrix} I_{ny} & \Psi \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} \tag{3}$$

where

$$x^s = \begin{bmatrix} x_1 & \cdots & x_{ny} \end{bmatrix}^T,\ x^s \in \mathbb{R}^{ny},\ x^d = \begin{bmatrix} x_{ny+1} & x_{ny+2} & \cdots & x_{ny+nd} \end{bmatrix}^T,\ x^d \in \mathbb{C}^{nd},\ \ F \in \mathbb{C}^{nd \times nd}$$

$$\Psi = \begin{bmatrix} \Phi & & 0 \\ & \ddots & \\ 0 & & \Phi \end{bmatrix}, \qquad \Psi \in \mathbb{R}^{ny \times nd}, \qquad \Phi = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}, \quad \Phi \in \mathbb{R}^{nu\, na}$$

In the state equation (2), the state components x^s correspond to the output steady state and components x^d correspond to the stable modes of the system. When the system approaches steady state these components tend to zero. F is a diagonal matrix with components of the form $e^{r_i T}$ where r_i is a pole of the system and T is the sampling period. The system has nd stable poles. Matrix B^s is the gain matrix of the system. To build up matrix Φ, it is assumed that na is the number of poles associated to any input u_i and any output y_j.

The extended IHMPC cost function can be written as:

$$V_k = \sum_{j=0}^{\infty} (e(k+j)-\delta_k)^T Q(e(k+j)-\delta_k) + \sum_{j=0}^{m-1} \Delta u(k+j)^T R \Delta u(k+j) + \delta_k{}^T S \delta_k \tag{4}$$

where $e(k+j) = y(k+j) - r$, $y(k+j)$ is the output prediction, k is the present sampling time, r is the output reference, m is the control horizon and $\delta_k \in \mathbb{R}^{ny}$ is a vector of slack variables. $Q \in \mathbb{R}^{ny \times ny}$, $R \in \mathbb{R}^{nu \times nu}$ and $S \in \mathbb{R}^{ny \times ny}$ are assumed positive definite. The slack variables allow the controller to be applied to the cases in which there are not enough degrees of freedom to zero the error at steady state on all the system outputs.
It can be shown that the control objective defined in (4) will be bounded only if

$$x^s(k+m) - \delta_k - r = 0 \tag{5}$$

With such constraint, the expression of the control cost becomes

$$\begin{aligned} V_k = &\sum_{j=0}^{m} \left(e(k+j)-\delta_k\right)^T Q\left(e(k+j)-\delta_k\right) + x^d(k+m)^T \bar{Q} x^d(k+m) \\ &+ \sum_{j=0}^{m\text{-}1} \Delta u(k+j)^T R \Delta u(k+j) + \delta_k^T S \delta_k \end{aligned} \tag{6}$$

where $\bar{Q} \in \mathbb{R}^{ny \times ny}$ is such that

$$\bar{Q} - F^T \bar{Q} F = F^T \Psi^T Q \Psi F \tag{7}$$

Using model equations (2) and (3) to represent the output prediction as a function of the future control actions and the current state, the control objective represented in (6) can be written as follows:

$$V_k = \begin{bmatrix} \Delta u_k^T & \delta_k^T \end{bmatrix} H \begin{bmatrix} \Delta u_k \\ \delta_k \end{bmatrix} + 2 c_f^T \begin{bmatrix} \Delta u_k \\ \delta_k \end{bmatrix} + c \tag{8}$$

where

$$H=\begin{bmatrix}\left(B_m^s+\Psi_1F_u\right)^T Q_1\left(B_m^s+\Psi_1F_u\right)+F_u^TQ_2F_u+R_1 & -\left(B_m^s+\Psi_1F_u\right)^T Q_1\bar{I}\\ -\bar{I}^TQ_1\left(B_m^s+\Psi_1F_u\right) & S+\bar{I}^TQ_1\bar{I}+Q\end{bmatrix} \tag{9}$$

$$c_f=\begin{bmatrix}\left(B_m^s+\Psi_1F_u\right)^T Q_1\left(\bar{I}e^s(k)+\Psi_1F_xx^d(k)\right)+F_u^TQ_2\left(F_xx^d(k)\right)\\ -\bar{I}^TQ_1\left(\bar{I}e^s(k)+\Psi_1F_xx^d(k)\right)-Q\,e(k)\end{bmatrix} \tag{10}$$

$$c=e(k)^TQe(k)+\left(\bar{I}e^s(k)+\Psi_1F_xx^d(k)\right)^T Q_1\left(\bar{I}e^s(k)+\Psi_1F_xx^d(k)\right)+\left(F_xx^d(k)\right)^T Q_2\left(F_xx^d(k)\right) \tag{11}$$

$$\bar{I}=\begin{bmatrix}I_{ny}\\ \vdots\\ I_{ny}\end{bmatrix},\quad B_m^s=\begin{bmatrix}B^s & \cdots & 0\\ \vdots & \ddots & \vdots\\ B^s & \cdots & B^s\end{bmatrix},\quad \Delta u_k=\begin{bmatrix}\Delta u(k)\\ \vdots\\ \Delta u(k+m-1)\end{bmatrix}$$

$$F_x=\begin{bmatrix}F\\ F^2\\ \vdots\\ F^m\end{bmatrix},\qquad F_u=\begin{bmatrix}B^d & 0 & \cdots & 0\\ FB^d & B^d & \ddots & \vdots\\ \vdots & \vdots & \ddots & \vdots\\ F^{m-1}B^d & F^{m-2}B^d & \cdots & B^d\end{bmatrix}$$

$Q_1=diag[\overbrace{Q \ \cdots \ Q}^{m}]$, $Q_2=diag[\overbrace{0 \ \cdots \ 0 \ \bar{Q}}^{m}]$, $R_1=diag[\overbrace{R \ \cdots \ R}^{m}]$ and

$\Psi_1=diag[\overbrace{\Psi \ \cdots \ \Psi}^{m}]$

$e^s(k)=x^s(k)-r$

Analogously, the constraint represented in (5) can be written as follows:

$$e^s(k)+\tilde{B}^s\,\Delta u_k-\delta_k=0 \tag{12}$$

where $\tilde{B}^s=\begin{bmatrix}B^s & \cdots & B^s\end{bmatrix}$

Finally, the control optimization problem of the infinite horizon MPC proposed by Odloak (2004) can be formulated as:

Problem P1

$$\min_{\Delta u_k,\delta_k} \quad V_k \tag{13}$$

subject to equations (8) and (12) and the following inequalities:

$$
\begin{aligned}
&-\Delta u^{\max} \le \Delta u(k+j) \le \Delta u^{\max} \\
&\Delta u(k+j) = 0; \quad j \ge m \\
&u^{\min} \le u(k-1) + \sum_{i=0}^{j} \Delta u(k+i) \le u^{\max}; \; j = 0,1,\cdots,m-1
\end{aligned}
\tag{14}
$$

It can be proved that if the system remains controllable along the trajectory from the present state to the steady state corresponding to the desired output reference, then the control law, resulting from the solution of Problem P1 at successive time steps, will drive the system output asymptotically to the desired reference. If the system is not controllable at the desired output reference, the closed loop with the controller obtained from Problem P1 is still stable for open loop stable systems, but the controller will not be able to drive the outputs to the reference values.

However, driving the HEN system outputs to their desired values is not sufficient to achieve the required energy integration. As mentioned before, the control general purpose is not only to reach the output set points, but also to guide the process to an optimal condition from the point of view of energy integration. The approach presented in this paper assumes that the optimal operating point has been calculated at a supervisor level, and the results translated into desired values for a selected set of manipulated inputs. Then, to include the economic objective into the control objective, the cost function defined in Eq. (6) has to be extended with new terms that penalize the distance between the predicted control actions and the desired target. Hence, the extended IHMPC cost function is written as follows:

$$
\begin{aligned}
V_{k,u} = &\sum_{j=0}^{\infty} \left(e(k+j) - \delta_k\right)^T Q \left(e(k+j) - \delta_k\right) + \sum_{j=0}^{\infty} \left(e_u(k+j) - \delta_{k,u}\right)^T Q_u \left(e_u(k+j) - \delta_{k,u}\right) + \\
&+ \sum_{j=0}^{m-1} \Delta u(k+j)^T R \, \Delta u(k+j) + \delta_k{}^T S \delta_k + \delta_{k,u}{}^T S_u \delta_{k,u}
\end{aligned}
\tag{15}
$$

where

$e_u(k+j) = u(k+j) - r_u$, r_u is the vector of desired values for the system inputs

$\delta_{k,u}$ is a vector of slack variables related to the inputs, which have economic targets

Q_u and S_u are positive weighting matrices of appropriate dimensions

To deal with the new control objective function of the IHMPC, it is convenient to redefine the state space model as follows

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \\ u(k) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & I_{nu} \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \\ u(k-1) \end{bmatrix} + \begin{bmatrix} B^s \\ B^d \\ I_{nu} \end{bmatrix} \Delta u(k) \tag{16}$$

$$\begin{bmatrix} y(k) \\ y_u(k) \end{bmatrix} = \begin{bmatrix} I_{ny} & \Psi & 0 \\ 0 & 0 & I_{nu} \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \\ u(k-1) \end{bmatrix} \tag{17}$$

In order to force the extended objective function defined in (15) to be bounded, the constraint represented in Eq. (12) must be satisfied, and a new constraint related to state x^u have to be imposed also. This new constraint is similar to the constraint represented in Eq. (12) and has the following form:

$$e^u(k-1) + \tilde{B}^u \Delta u_k - \delta_{k,u} = 0 \tag{18}$$

where

$$e^u(k-1) = u(k-1) - r_u \,, \quad \tilde{B}^u = \begin{bmatrix} I_{nu} & \cdots & I_{nu} \end{bmatrix}$$

With this extended state space model defined in Eqs. (16) and (17), the control cost defined in Eq. (15) can be written as follows

$$V_{k,u} = \begin{bmatrix} \Delta u_k^T & \delta_k^T & \delta_{k,u}^T \end{bmatrix} H_u \begin{bmatrix} \Delta u_k \\ \delta_k \\ \delta_{k,u} \end{bmatrix} + 2c_{f,u}^T \begin{bmatrix} \Delta u_k \\ \delta_k \\ \delta_{k,u} \end{bmatrix} + c_u \tag{19}$$

where

$$H_u = \begin{bmatrix} H_{11} & -\left(B_m^s + \Psi_1 F_u\right)^T Q_1 \bar{I} & -B_m^{u\,T} Q_{u,1} \bar{I}^u \\ -\bar{I}^T Q_1 \left(B_m^s + \Psi_1 F_u\right) & S + \bar{I}^T Q_1 \bar{I} + Q & 0 \\ -\bar{I}^{u\,T} Q_{u,1} B_m^u & 0 & \bar{I}^{u\,T} Q_{u,1} \bar{I}^u \end{bmatrix}$$

$$H_{11} = \left(B_m^s + \Psi_1 F_u\right)^T Q_1 \left(B_m^s + \Psi_1 F_u\right) + F_u^T Q_2 F_u + B_m^{u\,T} Q_{u,1} B_m^u + R_1$$

$$c_{f,u} = \begin{bmatrix} \left(B_m^s + \Psi_1 F_u\right)^T Q_1 \left(\bar{I} e^s(k) + \Psi_1 F_x x^d(k)\right) + F_u^T Q_2 \left(F_x x^d(k)\right) + B_m^{u\,T} Q_{u,1} \bar{I}^u e^u(k) \\ -\bar{I}^T Q_1 \left(\bar{I} e^s(k) + \Psi_1 F_x x^d(k)\right) - Q\, e(k) \\ -\bar{I}^{u\,T} Q_{u,1} \bar{I}^u e^u(k) \end{bmatrix}$$

$$c_u = e(k)^T Q e(k) + \left(\bar{I} e^s(k) + \Psi_1 F_x x^d(k)\right)^T Q_1 \left(\bar{I} e^s(k) + \Psi_1 F_x x^d(k)\right) + \\ + \left(F_x x^d(k)\right)^T Q_2 \left(F_x x^d(k)\right) + e^u(k)^T \bar{I}^{u\,T} Q \bar{I}^u e^u(k)$$

$$\bar{I}^u = \begin{bmatrix} I_{nu} \\ \vdots \\ I_{nu} \end{bmatrix}, \quad B_m^u \triangleq \begin{bmatrix} I_{nu} & \cdots & I_{nu} \\ \vdots & \ddots & \vdots \\ I_{nu} & \cdots & I_{nu} \end{bmatrix}, \quad Q_{u,1} = diag[\overbrace{Q_u \quad \cdots \quad Q_u}^{m}]$$

Thus, the IHMPC control problem adapted to the HEN system, in which targets to the manipulated inputs are defined by economics, can be formulated as follows:

Problem P2

$$\min_{\Delta u_k, \delta_k, \delta_{k,u}} V_{k,u} \tag{20}$$

subject to constraints defined in (19), (14), (12) and (18).

It can be shown that Problem P2 is always feasible and the control law produced by the solution of this problem stabilizes the closed loop system. However, the complete definition of the targets to the manipulated inputs is a subject that needs more attention, mainly when the performance of the closed loop system is concerned. For instance, the number of input variables, which have targets to be reached, may affect not only the speed of convergence to the optimal steady state, but it may also affects the dynamic performance of the system.

Including targets for all the input variables may render a too rigid system with possible offset in the controlled outputs. On the other side, including targets for too few inputs, or inadequate inputs, may not produce the expected benefits. Another important point is the selection of weighting matrix Q_u that is an additional tuning parameter of the extended IHMPC. This parameter plays an important role in the controller performance and must be carefully selected to obtain a satisfactory performance of the HEN control system.

We should observe that depending on the degrees of freedom of the HEN system, the outputs may reach the desired set points, but this does not imply that the inputs have reached their optimal value. The inclusion of input targets tends to prevent this problem as long as an adequate number of inputs are chosen to receive targets from the economic optimisation layer. A simple criterion to verify if a given set of inputs can be selected to receive targets without introducing offsets in the controlled outputs is provided by González et al. (2004). For instance, if we want to send targets for the first d inputs of the HEN system, the following matrix should be full rank:

$$G_D = \begin{bmatrix} B^s \\ I_d \end{bmatrix}, \qquad I_d = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \underbrace{0 & \cdots & 1}_{d} & \cdots & 0 \end{bmatrix},$$

where B^s is the process-gain matrix. It can be shown that if this condition is satisfied, then the extended IHMPC can drive the controlled outputs and the selected inputs to their desired optimal values.

4. The HEN system

The HEN system studied here is represented schematically in Figure 1. It is a small system with only three recovery exchangers and three service units. There are two hot process streams and two cold process streams that take part of the heat exchange process. We have also three utility streams that can be used to help reaching the desired outlet temperatures. As it is shown in Fig. 1, this HEN system has six manipulated inputs (three bypasses and three utility flow rates) and four outputs to be controlled (process stream temperatures). The objective is to obtain a control strategy that performs well for disturbance rejection and output tracking, while minimizing the amount of utilities expended at any operating condition.

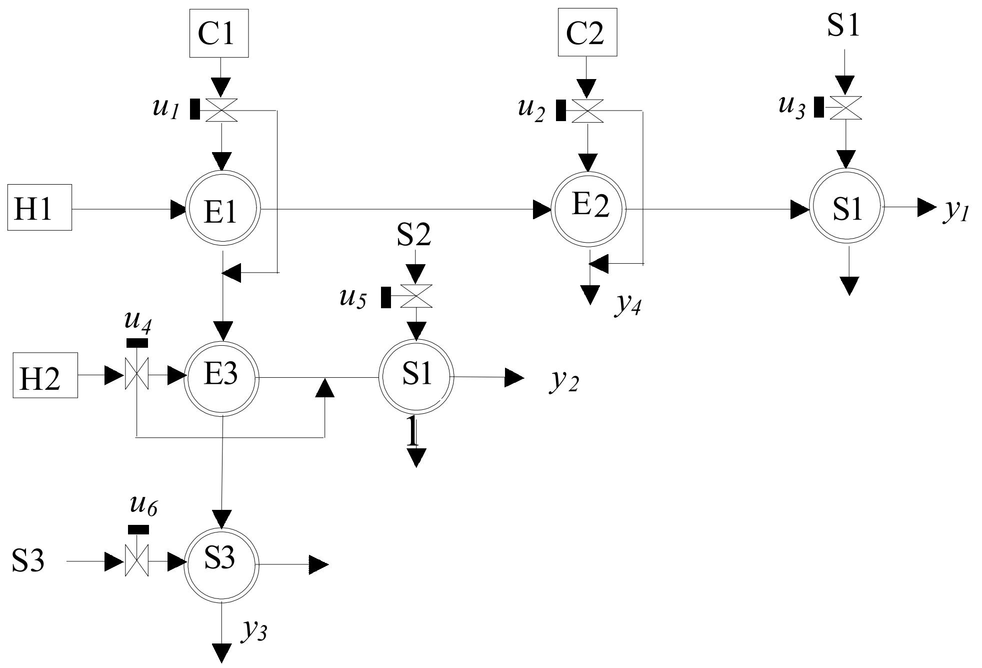

Figure 1. Schematic representation of the HEN system

In the study presented here, the HEN system is simulated through a rigorous nonlinear model and the controller uses a linearized state space model for state and output prediction. For the system represented in Fig. 1, the connection structure translates into a transfer function model that has the following form:

$$G(s)=\begin{bmatrix} \frac{-0.1870s+0.0068}{s^2+0.0279s+0.0005} & \frac{-0.2681s+0.0243}{s^2+0.0726s+0.0019} & \frac{-0.1462s-0.0223}{s^2+0.1058s+0.0043} & 0 & 0 & 0 \\ \frac{0.6443s-0.0923}{s^2+0.1696s+0.0054} & 0 & 0 & \frac{-1.6799s+0.5050}{s^2+0.6830s+0.0382} & \frac{-0.7948s-0.1299}{s^2+0.0862s+0.0018} & 0 \\ \frac{1.4335s-0.2796}{s^2+0.1096s+0.0117} & 0 & 0 & \frac{0.2553s-0.0727}{s^2+0.2377s+0.0101} & 0 & \frac{0.8848s+0.0023}{s^2+0.0313s+0.0001} \\ \frac{-0.0694s+0.0130}{s^2+0.0511s+0.0010} & \frac{-118.476s-5.0140}{s^2+2.6870s+0.2586} & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general control structure is represented in Figure 2, which shows the integration of the two-optimisation stages and the variables used to connect them. An important remaining question about this structure is the selection of the input variables that will be used to interface the economic optimisation to the control dynamic optimisation. As there are differences between the HEN true nonlinear model and the linear model on which the IHMPC is based, using the exact number of inputs that reduces the degrees of freedom to zero may not be the best policy. If there are negligible differences between the model gain matrix B^s and the actual plant gain, and matrix G_D is full rank, then the convergence of the all the inputs and outputs to the desired values is ensured. However, in actual HEN systems these conditions are not always fulfilled and it is necessary to determine by trial and error which input variables should be commanded by the economic optimisation level. For the network represented in Figure 2, the gain matrix of the linearized system corresponding to the nominal operation point is:

$$B^s = \begin{bmatrix} 14.0849 & 11.8967 & -8.3808 & 0 & 0 & 0 \\ -18.0248 & 0 & 0 & 14.6536 & -70.7115 & 0 \\ -25.1961 & 0 & 0 & -7.5913 & 0 & 23.6918 \\ 13.5323 & -17.7732 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Table 1 shows all the different combinations of two control inputs available in this HEN system that might be selected to receive set points from the economic layer. We observe that any combination of the first three control inputs yields a rank deficient matrix G_D, showing they cannot be used to drive the system to the optimal operation point. However, we see that there are many pairs of inputs that can be used to drive the HEN system to the optimal operating point.

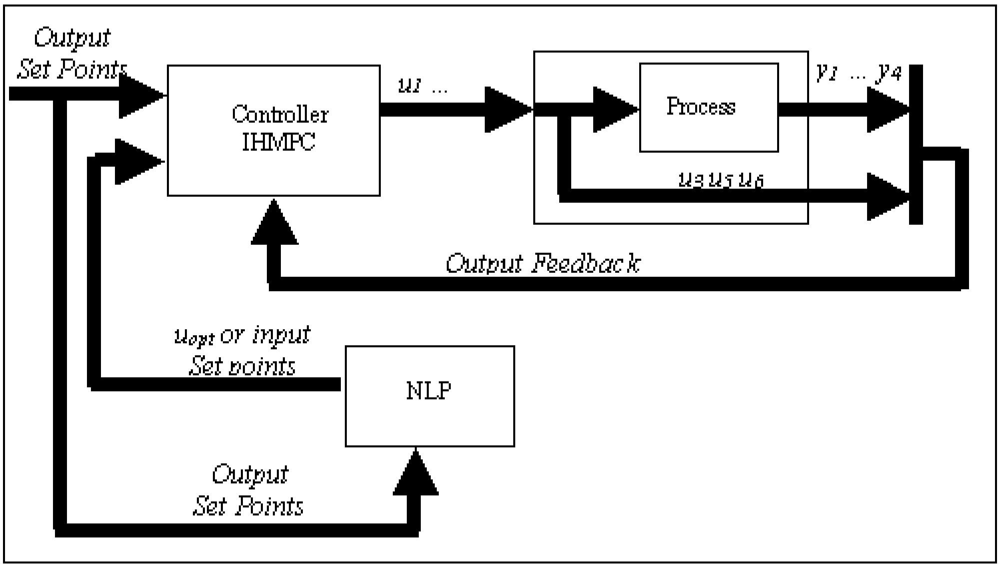

Figure 2. Controller and optimiser structure of the HEN system

Control inputs with set points	Rank of G_D
$u_1\ u_2$	5
$u_1\ u_3$	5
$u_1\ u_4$	6
$u_1\ u_5$	6
$u_1\ u_6$	6
$u_2\ u_3$	5
$u_2\ u_4$	6
$u_2\ u_5$	6
$u_2\ u_6$	6
$u_3\ u_4$	6
$u_3\ u_5$	6
$u_3\ u_6$	6
$u_4\ u_5$	6
$u_4\ u_6$	6
$u_5\ u_6$	6

Table 1: Rank of G_D matrix for all combinations of two inputs with set points

5. Simulation results

In this section, to represent the true HEN system shown in Fig.1, it is used a simulator based on a nonlinear model of shell-and-tubes heat exchangers developed by Correa and Marchetti (1987). As discussed in the previous sections, for the HEN system, not only the dynamic performance of the control system is important, but also the cost associated with the resulting operating condition must be taken into account. Thus, in this section, we study the operation of the HEN system depicted in Fig. 1, with the extended IHMPC, considering or not the inclusion of economic optimization term in the control objective function. This corresponds to the inclusion or not of desired values for an appropriate set of manipulated variables. To quantify the economic efficiency of the HEN system represented in Fig.1, we use the utility cost that is defined as follows:

$$J_{\text{utility cost}} = c_1 u_3 + c_2 u_5 + c_3 u_6 \,. \qquad (19)$$

Figures 3a to 3c show the dynamic responses of the HEN system operating with IHMPC without the economic term in the controller cost function. This means that the controller manipulates the six inputs indicated in Fig. 1 (u_1 to u_6) to control the outlet temperatures of the four process streams. We consider the output tracking operation obtained when the following sequence of changes is introduced into the system: after stabilizing at nominal conditions, the set point of the outlet temperature of process stream C_1, which corresponds to controlled variable y_3 is changed from 80 to 70 C; next, the set point of the process stream C_2 (controlled variable y_4) is changed from 40 to 45 C; and finally, the set point of the outlet temperature of stream H_2 (controlled variable y_2) is moved from 100 to 90 C. The optimal steady states corresponding to these operating conditions are indicated in Table 2, as fractions of maximum utility flowrates in utility units, or as bypass fractions in heat exchangers (E). According to the notation adopted in the extended IHMPC, the controller tuning parameters are the following:

$T = 3$, $m = 2$, $Q = \text{diag}(5\ 1\ 1.5\ 2)$, $R = 5000 I_{nu}$, $S = 500 I_{ny}$, $\Delta u_{\max} = 0.15\ [1\ 1\ 1\ 1\ 1\ 1]^T$.

case	*u_1 (opt)*	*u_2 (opt)*	*u_3 (opt)*	*u_4 (opt)*	*u_5 (opt)*	*u_6 (opt)*	*Utility Cost (\$/Kg)*
Nominal	0	0.1463	0.0590	0.6739	0.1001	0	4.9500
$T_{c1}^{out} = 70°C$	0.0687	0.2010	0.1060	1	0.2260	0	10.2780
$T_{c2}^{out} = 45°C$	0.3741	0	0.2041	0.7737	0.1200	0	10.3353
$T_{h2}^{out} = 90°C$	0.3741	0	0.2041	0.7737	0.2200	0	13.3353

Table 2. Fractions of valve openings obtained by the NLP optimization for different steady states.

It is clear that in this case, parameters Q_u and S_u, which are related to the economic objective, are both equal to zero. Figure 3a shows that the controller can follow the set point without major difficulties. Fig. 3b shows that the controller uses all the six manipulated inputs to force the outputs to follow their set points. It is also clear that the control inputs tend to a steady state that is substantially different from the optimal steady state shown in Table 2. This is translated into a significant departure from the optimal utilities consumption as can be seen in Fig. 3c, which shows that the obtained utilities cost can be as high as twice the optimal cost. Tuning IHMPC to improve the economic performance can reduce the gap between the optimal utilities consumption and the utilities consumption obtained with the controller. Figures 4a to 4c show the system responses when the slack variables weight is reduced to

S=20I_{ny}, while the other controller parameters remained the same as in the previous case. Fig.4c shows that in this case, the economic behavior is better than in the previous case as true cost is only about 50% higher than the optimal cost. However, the dynamic response of the system is significantly worse than in the previous case as shown Fig. 4a wherein we can see that the response of output y_4 is much slower than in the previous case while the response of y_2 is too much oscillatory.

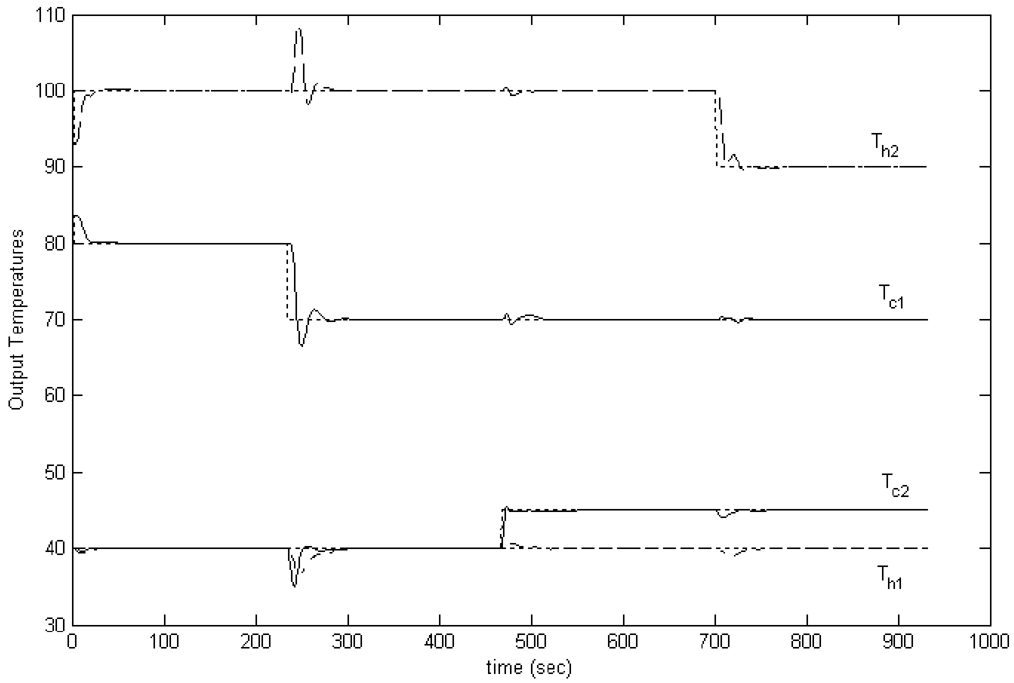

Figure 3a. Controlled outputs of the HEN system without economic objective

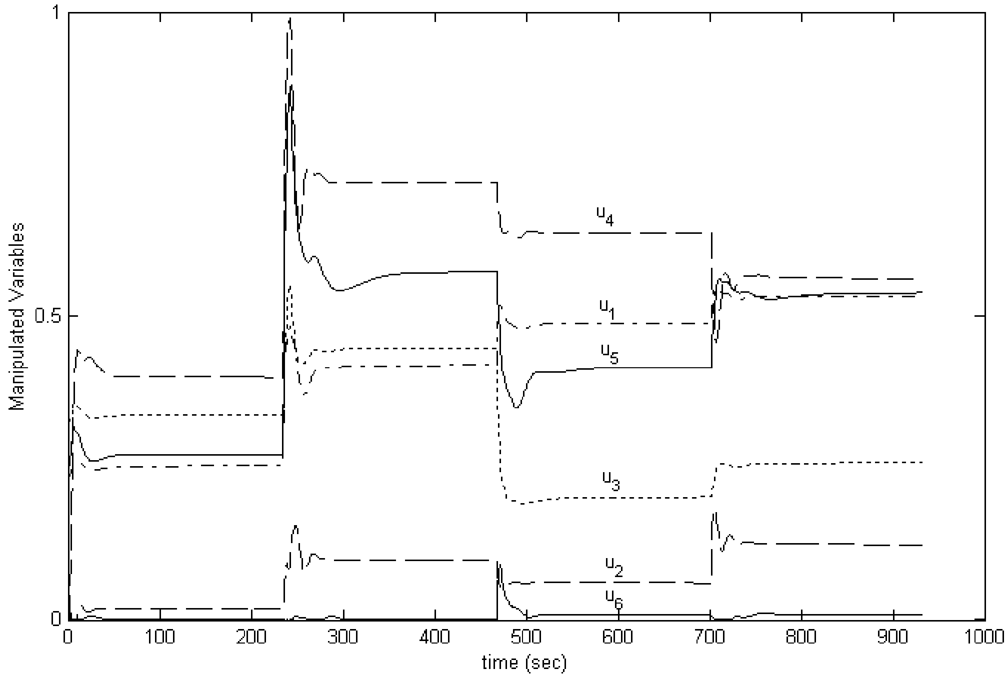

Figure 3b. Manipulated inputs of the HEN system without economic objective

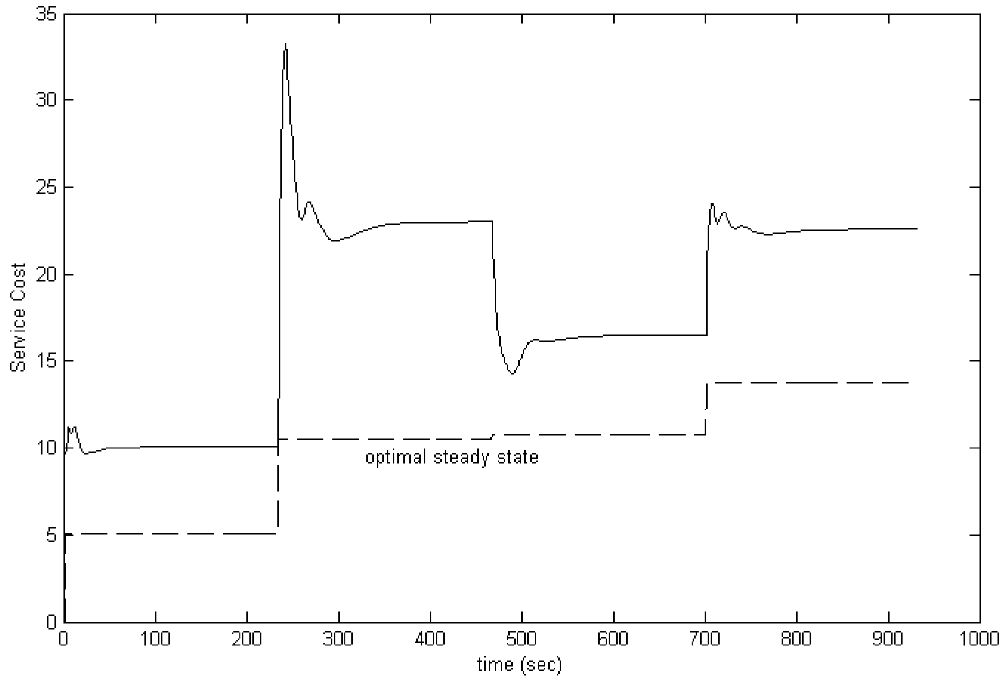

Figure 3c. Utilities cost for the HEN system without economic objective

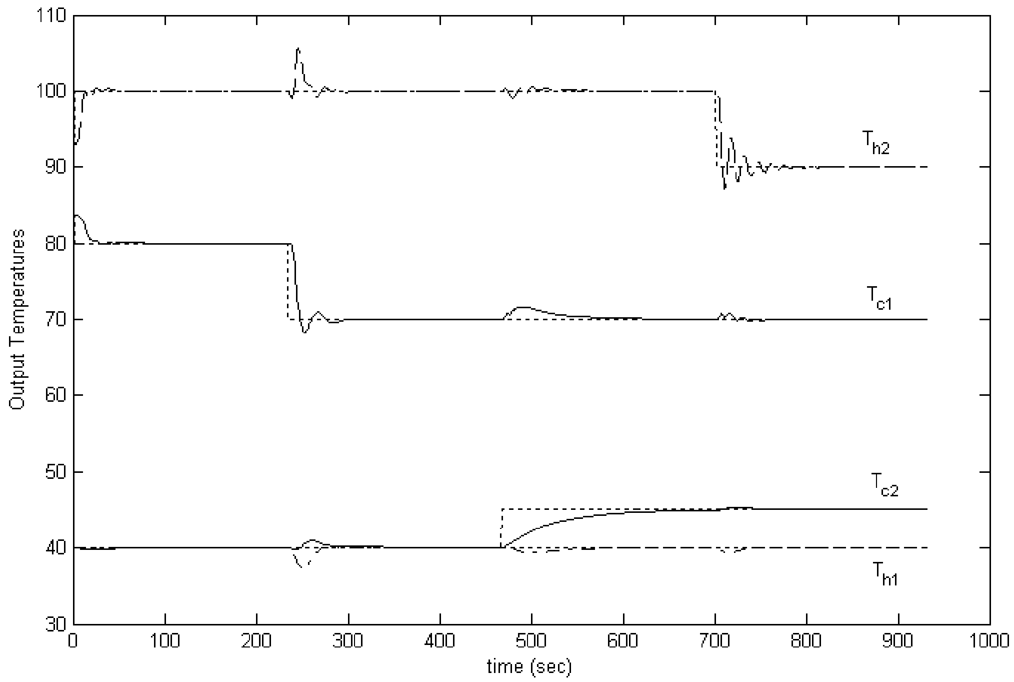

Figure 4a. Controlled outputs of the HEN system without economic objective and $S = 20I_{ny}$

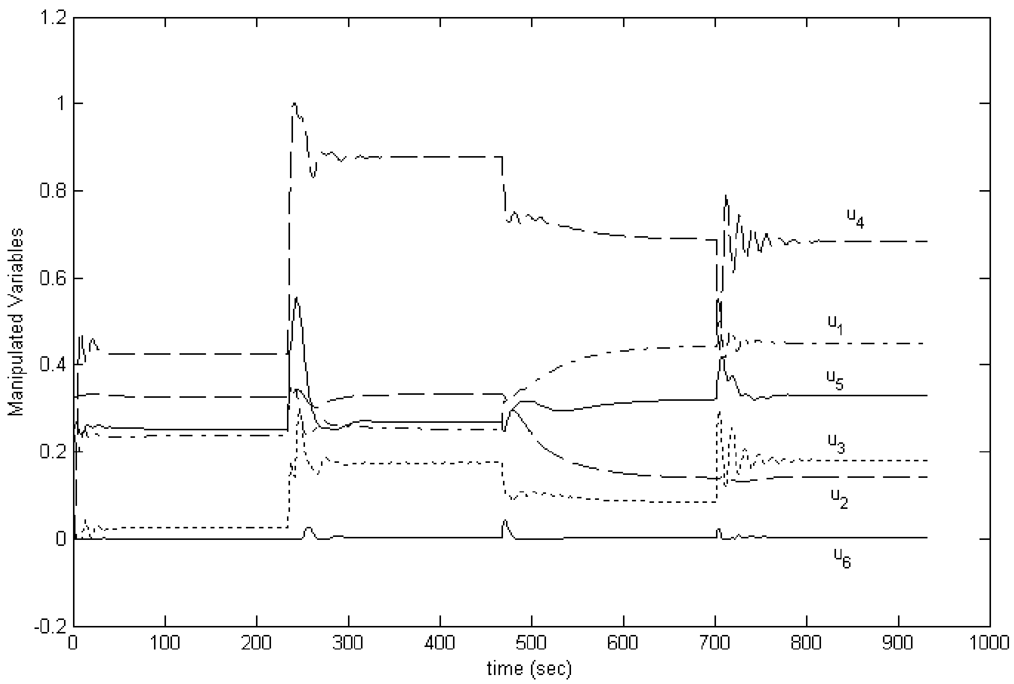

Figure 4b. Manipulated inputs of the HEN system without economic objective and $S = 20I_{ny}$

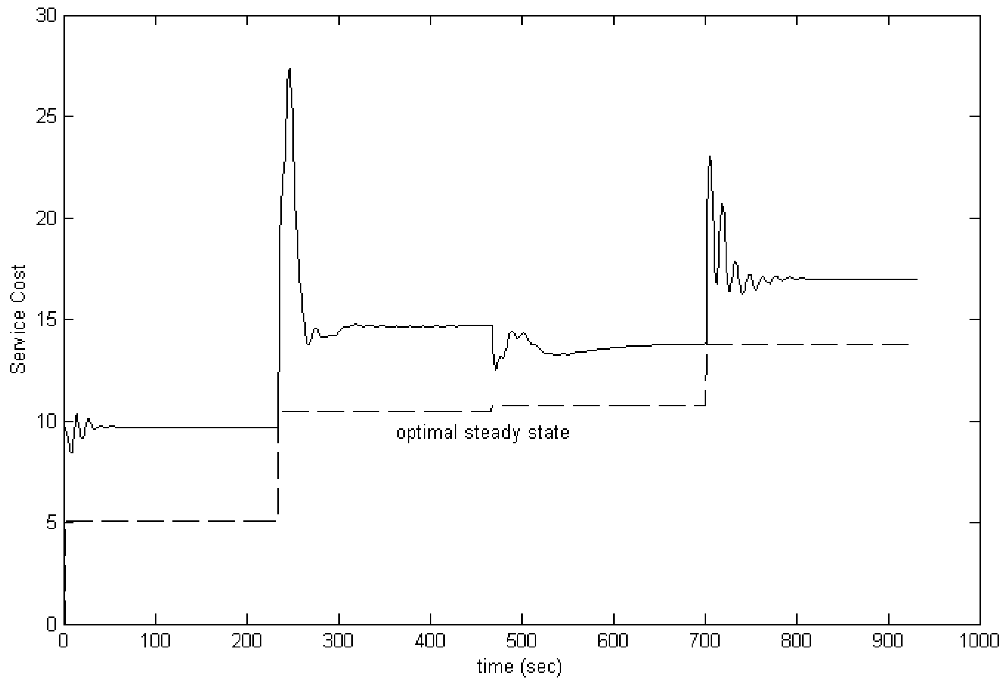

Figure 4c. Utilities cost for the HEN system without economic objective and $S = 20I_{ny}$

Next, we observe the dynamic responses of the HEN system with the IHMPC extended with the economic objective. In the case simulated here, we assume that inputs u_3, u_5 and u_6 receive optimal set points from the economic optimization level. Observe that, as shown in Table 1, considering set points to only two of these inputs is sufficient to guarantee that matrix G_D has rank 6 and consequently to assure that the controller will guide the HEN system to the optimal operating point. Here, we consider set points to three of the manipulated inputs and consequently the system becomes over specified. If the model is perfect and the set points to the inputs are consistent, there are no consequences to the control problem as the controller is capable of reducing the control cost to zero and stability of the closed loop system is assured. If the set points to the three manipulated inputs are not consistent with the optimal steady state, off sets in the controlled and manipulated variables will develop. However, there will be no consequences to the stability of the closed loop, because it can be proved that with the presence of the slack variables δ_k and $\delta_{k,u}$ in the cost function, the cost will be nonincreasing and stability is preserved.

Assuming the same output-tracking scenario described above, Figs. 5a to 5c show the dynamic responses of the HEN system with the extended IHMPC with the following tuning parameters:

$T = 3$, $m = 2$, $Q = \text{diag}(5\ 1\ 1.5\ 2)$, $Q_u = \text{diag}(0\ 0\ 1\ 0\ 1\ 1)$, $R = 5000I_{nu}$, $S = 5000I_{ny}$,

$S_u = 5000\text{diag}(0\ 0\ 1\ 0\ 1\ 1)$, $\Delta u_{\max} = 0.15\ [1\ 1\ 1\ 1\ 1\ 1]^T$.

Comparing Figs. 5a and 3a, we observe that the inclusion of the economic objective in the infinite horizon MPC causes a small deterioration on the responses of the controlled outputs. However this loss of performance is acceptable if we compare Figs. 5c and 3c that show the utilities cost in both cases. The extended IHMPC approaches very closely the optimal operation at steady state and the period of time that the HEN system departs from the optimal operation during transient conditions is acceptable. From Fig. 5b, we observe that all the inputs also approach closely to their optimal values showing that the system nonlinearity is mild.

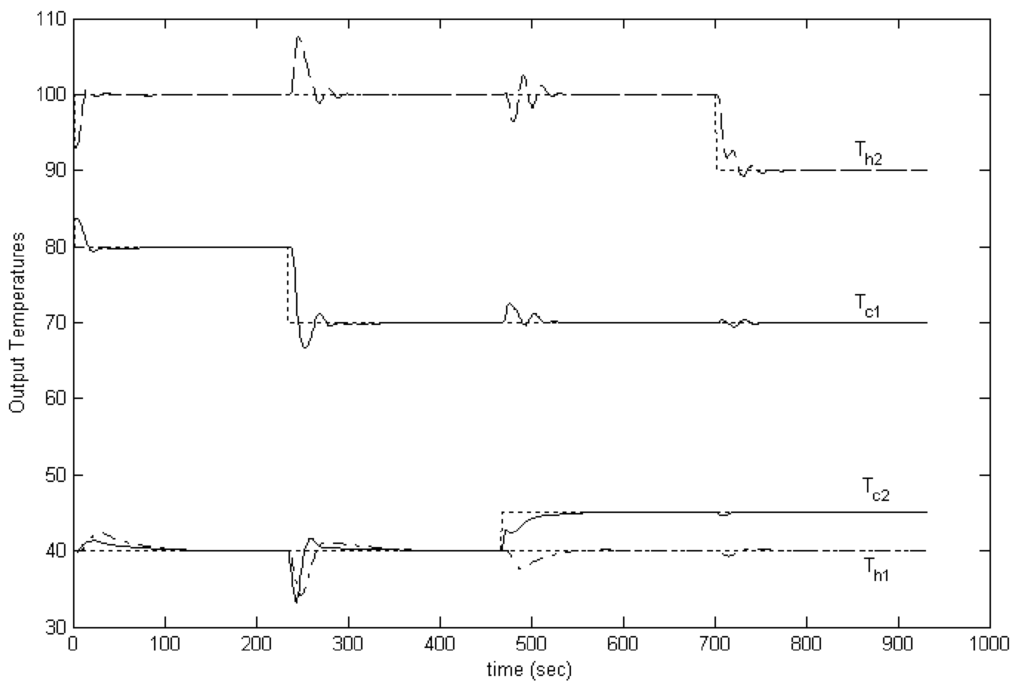

Figure 5a. Controlled outputs of the HEN system with economic objective

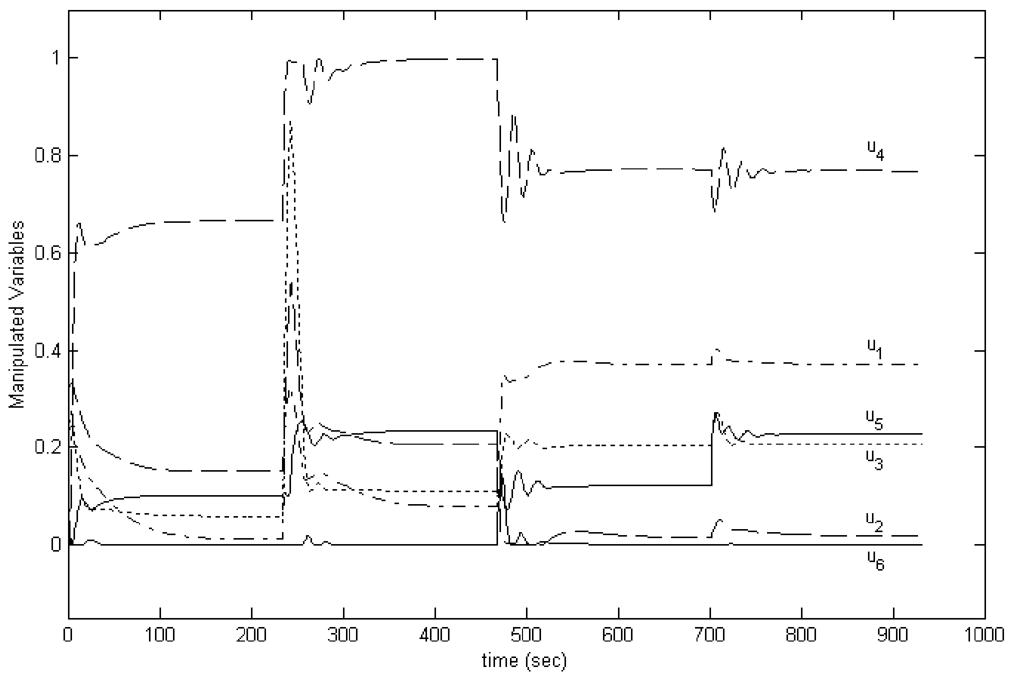

Figure 5b. Manipulated inputs of the HEN system with economic objective

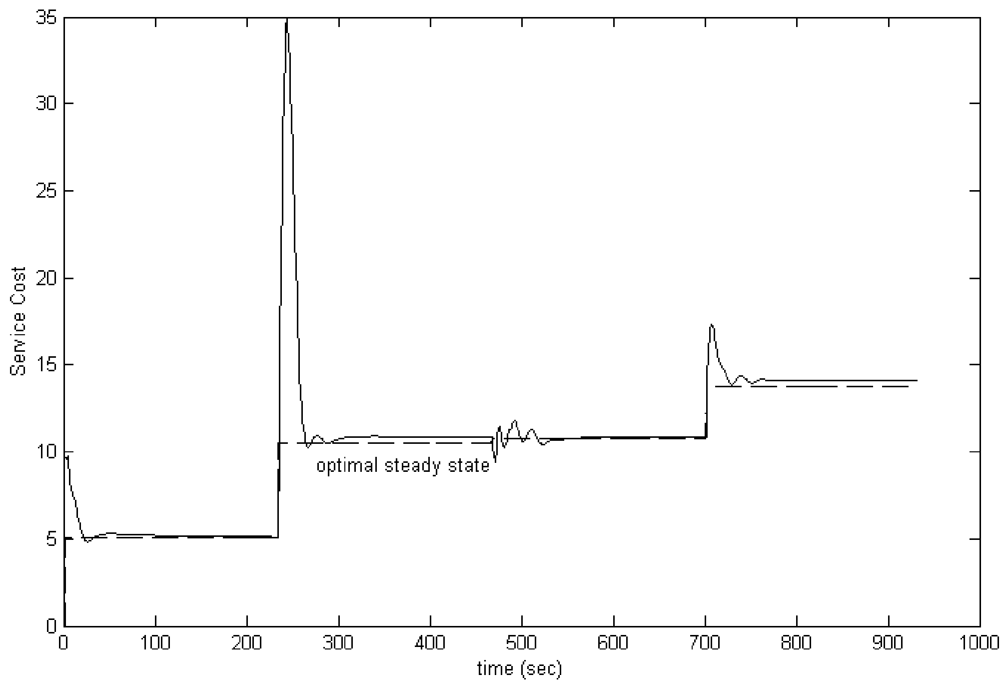

Figure 5c. Utilities cost for the HEN system with economic objective

6. Conclusion

In this work, it has been shown how to extend a nominally stable linear MPC to a HEN system in which the minimization of the utilities cost is one of the major challenges. This was achieved by extending the objective function (control cost) of an infinite horizon MPC developed in a previous work, with the inclusion of an economic term. This additional term assumes that the same inputs can be treated simultaneously as controlled outputs and manipulated inputs. To prevent the control cost to become unbounded new slack variables were introduced into the dynamic optimization problem. With this approach, the new controller inherited the already proved stabilizing properties of an existing controller (Odloak, 2004).

Tuning the new controller is a little more complicated than tuning a conventional MPC, as the extended IHMPC has a larger number of tuning parameters than the conventional MPC. A correct balance between dynamic performance of the output variables and the speed of approach of the inputs to their optimal values is a key point in the tuning procedure. However the additional tuning effort is not prohibitive in practical terms and the proposed approach seems quite promising.

The proposed controller was tested in a HEN system of small dimension, but that has the main characteristics of a typical industrial system. For the output tracking operation, in which the optimal operating conditions were changed significantly, the new controller performed quite well with a reasonable dynamic performance and an economic yield that surpassed 90% of the maximum benefit.

Acknowledgments

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