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A straightforward method for tuning of Lyapunov-based controllers in semi-active vibration control applications

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A B S T R A C T

Lyapunov-based control is an attractive strategy for semi-active vibration control as it has a mathematical basis ensuring stability in the sense of Lyapunov and great flexibility in the design. Unfortunately, that flexibility complicates the controller tuning since it involves the construction of a weighting matrix, which is usually done by trial-and-error.

In this work, a straightforward (closed form) method to construct such a matrix is proposed. The proposed method is based on penalizing vibrational modes according to their contributions to the response in the uncontrolled case. For this purpose, a new concept of Generalized Modal Contribution Factor is developed. This takes into account the following: spatial distribution of the excitation, knowledge of the frequency content of the excitation, and control objective.

The capability of the proposed tuning method is demonstrated through a numerical example.

1. Introduction

The use of passive systems is the simplest and most reliable approach for vibration control of structures. However, in some cases, the performance of passive systems is not good enough, even when they are perfectly optimized. Moreover, passive systems are neither adaptive to changes in the magnitude, the frequency content, and the spatial distribution of the excitation nor to changes in the control objective. For its part, active vibration control is very attractive since it does not have those limitations of passive systems. Unfortunately, it has some inherent problems, e.g. risk of instability due to spillover, and heavy power demands [1]. However, it is worth noting that recent works have addressed the issues of stability [2], and power demand [3], through H_∞ norm and Linear Matrix Inequality methods.

An interesting alternative is semi-active vibration control. The semi-active approach consists in modifying, in real time, parameters (e.g. inertia, stiffness, or more commonly damping) of special control devices linked to the structure. In general, semi-active systems are stable, low-power consuming, and at the same time more adaptive and better performing than passive systems [1].

Several control strategies have been developed and applied to semi-active control systems. Some of these strategies are heuristic approaches, such as sky-hook and ground-hook control laws [4,1]. They are appropriated for use in systems having very few degrees of freedom (DOFs), such as suspension systems (e.g. [5]). Other control strategies have a more sophisticated mathematical background which makes them also suitable for structures with many DOFs. A comparative study of many of these strategies can be found in Jansen and Dyke [6].

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Among control strategies, Lyapunov-based control is attractive since it has (1) a mathematical basis which ensures stability in the sense of Lyapunov; and (2) great flexibility in their design, which enables the achievement of sophisticated control objectives. Unfortunately, the optimal tuning of Lyapunov-based controllers is not straightforward because a $2n \times 2n$ weighting matrix ($\mathbf{Q}$), where n is the number of DOFs of the controlled structure, must be constructed and the effectiveness of the control algorithm depends on it [1].

Even though there is no standard procedure to obtain $\mathbf{Q}$ [6], there are some approaches. For instance, Ge et al. [7] studied a genetic-algorithm optimization procedure to tune and optimize the performance of a Lyapunov-based robust controller for a single-link flexible robot. Kuehn and Stalford [8] proposed a procedure referred to as "state penalties" consisting in the construction of a diagonal matrix in which the diagonal elements are selected by trial-and-error through simulation tests (ensuring the positive-definiteness of $\mathbf{Q}$). Another technique used by Kuehn and Stalford [8] is "modal penalties" which is similar to the previous one but in modal coordinates. All of these methods have two clear drawbacks: (1) they are essentially trial-and-error methods, and therefore very time-consuming and (2) the execution of trials needs a record of the excitation.

In this work, a method for straightforward construction of an appropriate matrix $\mathbf{Q}$ is presented, avoiding those drawbacks. This method takes into account the following: (1) spatial distribution of the excitation; (2) knowledge of the frequency content of the excitation; and (3) control objective specified as a particular response, which must be written as a linear combination of the states. The method is based on the calculation of the relative contributions of modes to the response in the uncontrolled case, and the penalization of those modes in the weighting matrix $\mathbf{Q}$ according to their contributions. For this purpose, a new concept of Generalized Modal Contribution Factor (GMCF) is first presented and then applied to the construction of $\mathbf{Q}$. It is important to highlight that GMCFs, and therefore the matrix $\mathbf{Q}$, are calculated in closed form.

Finally, through a numerical study, the capability of the proposed method to construct a matrix $\mathbf{Q}$ reaching nearly the same effectiveness than the best option among all the "modal penalties" trials is demonstrated, without needing previous simulations.

2. Generalized Modal Contribution Factor (GMCF)

The concept of Modal Contribution Factor (MCF), which quantifies the contribution of a particular mode to the response of a structure under ground-motion excitation, is used in Earthquake Engineering [9]. MCF does not take account of the frequency content of the excitation, and it is, in principle, restricted to ground-motion excitation type. MCF and Modal Participation Factor have other interpretations in Linear Systems Theory, for instance, the paper of Hashlamoun et al. [10] takes account of the initial conditions of a system without excitation.

The GMCF introduced in this work quantifies the contribution of a particular mode to a specific response of the uncontrolled structure for a known excitation (frequency content and spatial distribution) considering it as a stationary Gaussian process.

Consider a n-DOF structure as a linear system whose dynamics is governed by the following vector equation of motion:

$$\mathbf{M_s}\ddot{\mathbf{q}}_\mathbf{s} + \mathbf{C_s}\dot{\mathbf{q}}_\mathbf{s} + \mathbf{K_s}\mathbf{q_s} = \mathbf{f_{es}}, \tag{1}$$

in which $\mathbf{M_s}$, $\mathbf{C_s}$, $\mathbf{K_s}$ are the mass, damping and stiffness matrices, respectively, $\mathbf{q_s}$ is the vector of displacements in geometric coordinates, and $\mathbf{f_{es}}$ is the vector of external disturbances. The state-space form of Eq. (1) can be written as

$$\begin{cases} \dot{\mathbf{x}}_\mathbf{s} = \mathbf{A_s}\mathbf{x_s} + \mathbf{B_{us}}u \\ y = \mathbf{C_{sy}}\mathbf{x_s} \end{cases}, \tag{2}$$

where $\mathbf{x_s}$ is the state vector, $\mathbf{A_s}$ is the state matrix, being

$$\mathbf{x_s} = \begin{bmatrix} \mathbf{q_s} \\ \dot{\mathbf{q}}_\mathbf{s} \end{bmatrix}, \quad \mathbf{A_s} = \begin{bmatrix} \mathbf{0}_{n\times n} & \mathbf{I}_{n\times n} \\ -\mathbf{M_s}^{-1}\mathbf{K_s} & -\mathbf{M_s}^{-1}\mathbf{C_s} \end{bmatrix}, \tag{3}$$

y is the scalar structural response sought to be controlled, which in turn is specified through the row matrix $\mathbf{C_{sy}}$, and u is a scalar excitation such that

$$\mathbf{B_{us}}u = \begin{bmatrix} \mathbf{0}_{n\times n} \\ \mathbf{M_s}^{-1} \end{bmatrix}\mathbf{f_{es}}. \tag{4}$$

The matrices $\mathbf{I}_{n\times n}$ and $\mathbf{0}_{n\times n}$ are identity and zero matrices of order n, respectively.

The statistical attributes of excitations can be specified through their Power Spectral Density (PSD) functions (for instance, from a compatible design response spectrum [11]), which can be obtained as accurately as desired from the output of some linear filter excited with stationary unit white-noise [12]. Then, the frequency content of the excitation u is expressed as a zero-mean white-noise signal w filtered as follows:

$$\begin{cases} \dot{\mathbf{x}}_\mathbf{n} = \mathbf{A_n}\mathbf{x_n} + \mathbf{B_{wn}}w \\ u = \mathbf{C_{nu}}\mathbf{x_n} + \mathbf{D_{wu}}w \end{cases}, \tag{5}$$

$\mathbf{x_n}$ being the state vector of the filter.

The relations between the PSD function $S_u(\omega)$ of the excitation u and the matrices $\mathbf{A_n}$, $\mathbf{B_{wn}}$, $\mathbf{C_{nu}}$ and $\mathbf{D_{wu}}$ defining the filter are summarized below:

$$S_u(\omega)=|H(\omega)|^2,$$
$$H(\omega)=H(s)|_{s=\mathrm{i}\omega},$$
$$H(s)=\mathbf{C_{nu}}(s\mathbf{I}_p-\mathbf{A_n})^{-1}\mathbf{B_{wn}}+\mathbf{D_{wu}}, \tag{6}$$

where ω is the angular frequency, $H(s)$ is the transfer function of the filter, $\mathrm{i}=\sqrt{-1}$, s is the Laplace variable, and p is the order of the filter.

Both subsystems, the uncontrolled structure (Eq. (2)) and the filter (Eq. (5)), are coupled as follows:

$$\begin{cases}\dot{\mathbf{x}}_{\mathbf{sn}}=\mathbf{A_{sn}}\mathbf{x_{sn}}+\mathbf{B_{wsn}}w\\ y=\mathbf{C_{sny}}\mathbf{x_{sn}}\end{cases}, \tag{7}$$

in which $\mathbf{x_{sn}}$ is the augmented state vector defined as

$$\mathbf{x_{sn}}=\begin{bmatrix}\mathbf{x_s}\\ \mathbf{x_n}\end{bmatrix}, \tag{8}$$

and

$$\mathbf{A_{sn}}=\begin{bmatrix}\mathbf{A_s} & \mathbf{B_{us}}\mathbf{C_{nu}}\\ \mathbf{0}_{p\times 2n} & \mathbf{A_n}\end{bmatrix},\quad \mathbf{B_{wsn}}=\begin{bmatrix}\mathbf{B_{us}}\mathbf{D_{wu}}\\ \mathbf{B_{wn}}\end{bmatrix},\quad \mathbf{C_{sny}}=\mathbf{C_{sy}}[\mathbf{I}_{2n}\ \ \mathbf{0}_{2n\times p}]. \tag{9}$$

The steady-state covariance matrix of the augmented state vector $\mathbf{x_{sn}}$ is given by

$$\mathbb{E}(\mathbf{x_{sn}}\mathbf{x_{sn}}^\top)=\mathbf{X_{sn}}, \tag{10}$$

where $\mathbb{E}$ stands for the expectation operator. And, since the coupled system (Eq. (7)) is considered to be excited by a stationary white-noise signal w, $\mathbf{X_{sn}}$ is the unique symmetric solution of the linear Lyapunov equation [12]

$$\mathbf{A_{sn}}\mathbf{X_{sn}}+\mathbf{X_{sn}}\mathbf{A_{sn}}^\top+\mathbf{B_{wsn}}W\mathbf{B_{wsn}}^\top=\mathbf{0}, \tag{11}$$

in which W is a scalar that defines the noise intensity of w. For simplicity, and without loss of generality, it is considered that $W=1$. It is important to highlight that Eq. (11) has a closed-form solution [12].

The variance Y of the structural output y as a function of the covariance matrix $\mathbf{X_s}$ is

$$Y=\mathbf{C_{sy}}\mathbf{X_s}\mathbf{C_{sy}}^\top, \tag{12}$$

in which

$$\mathbf{X_s}=[\mathbf{I}_{2n}\ \ \mathbf{0}_{2n\times p}]\mathbf{X_{sn}}\begin{bmatrix}\mathbf{I}_{2n}\\ \mathbf{0}_{p\times 2n}\end{bmatrix}. \tag{13}$$

Since the purpose of this development is to express Y as a sum of modal contributions, the state vector is expressed in modal coordinates as

$$\mathbf{x_s}=\mathbf{E_s}\mathbf{z_s}, \tag{14}$$

in which $\mathbf{E_s}$ is the modal matrix containing the $2n$ column eigenvectors $\mathbf{e}_{\mathbf{s}i}$ of $\mathbf{A_s}$ with the following normalization:

$$\|\mathbf{e}_{\mathbf{s}i}\|_2=1\quad \text{for } i=1,\ldots,2n, \tag{15}$$

and $\mathbf{z_s}$ is the vector of modal coordinates.

Denoting the covariance matrix of $\mathbf{z_s}$ as $\mathbf{Z_s}$ is found that

$$\mathbf{X_s}=\mathbf{E_s}\mathbf{Z_s}\mathbf{E_s^*}, \tag{16}$$

where the superscript $*$ is the complex conjugate operator.

Substituting Eq. (16) into (12) yields

$$Y=\mathbf{C_{sy}}\mathbf{E_s}\mathbf{Z_s}\mathbf{E_s^*}\mathbf{C_{sy}}^\top, \tag{17}$$

where $\mathbf{Z_s}$ can be calculated from Eq. (16) as

$$\mathbf{Z_s}=\mathbf{E_s}^{-1}\mathbf{X_s}(\mathbf{E_s}^{-1})^*. \tag{18}$$

Note that the diagonal elements of $\mathbf{Z_s}$ are the mean square values of the modal responses. In order to separate those modal responses in the calculation of Y, Eq. (17) is rewritten as

$$Y=\sum_{i=1}^{2n}\{[\mathbf{C_{sy}}\mathbf{E_s}]_i[\mathbf{Z_s}]_{i,i}([\mathbf{C_{sy}}\mathbf{E_s}]_i)^*\}+\text{cross}-\text{terms}. \tag{19}$$

Neglecting the cross terms, the GMCFs are defined from each (normalized) term in the sum of Eq. (19), i.e.

$$\mathrm{GMCF}_i = \frac{1}{Y}\|\mathbf{C_{sy}}\mathbf{e_s}_i\|_2^2[\mathbf{Z_s}]_{i,i}. \tag{20}$$

The factor GMCF_i is a dimensionless real number which quantifies the mean square contribution of the mode i to the structural output y defined by Eq. (2); for an excitation u characterized by its PSD S_u defined by Eqs. (6), and considering that it is applied to the structure according to (4).

Note that, similar to MCF developed in [9], GMCFs are independent of how modes are normalized; see Appendix A.

3. Controlled system

Several types of variable dampers can be used in semi-active control. However, in order to demonstrate the proposed method through an example, in this work the n-DOF structure system stated by Eq. (2) is considered to be controlled by means of m semi-active friction dampers.

3.1. Modeling of semi-active friction dampers

Each friction damper consists of three friction pads in a sandwich configuration in which the central pad moves relative to the external pads. To induce friction at the interfaces between the pads, a normal force is applied by a piezoelectric stack actuator.

After taking some constitutive and constructive considerations, the normal force n_f can be obtained as [13]

$$n_f = n_0 + n_{0\Delta}\nu, \tag{21}$$

in which n_0 is a pre-load force applied to ensure a minimum contact between the surfaces of the pads; $n_{0\Delta}$ is a modulation term which depends on the piezoelectric stack actuator characteristics, the maximal voltage applied, and the stiffness of the device itself; and ν is a dimensionless command signal. The pre-load value n_0 and the range of variation of ν must ensure that the condition $n_f \geq 0$ is met.

On the basis of the above considerations, it is stated that the normal force of the j-th friction damper can be instantaneously adjusted within the following range:

$$n_{\min j} < n_j < n_{\max j}, \tag{22}$$

with $n_{\min j} \geq 0$, for $j = 1, \dots, m$.

Assuming steady-state motion, with operating conditions such that the velocity of the moving pad is much higher than the Stribeck velocity, and if the viscous damping is neglected with respect to the friction damping; the friction force f_{cj} which is exerted by the j-th damper can be expressed as [14]

$$f_{cj} = \mu_j n_j \,\mathrm{sign}(\dot{q}_{rj}) \quad \text{for } j = 1, \dots, m; \tag{23}$$

in which $\dot{q}_{rj}$ is the relative velocity between the friction pads, and $\mathrm{sign}(\bullet)$ is the sign function defined as

$$\mathrm{sign}(\tau) = \begin{cases} -1 & \text{if } \tau < 0 \\ 0 & \text{if } \tau = 0 \\ 1 & \text{if } \tau > 0 \end{cases} \tag{24}$$

Eq. (23) matches with the classical Coulomb's friction law; $\mu_j = 2\mu_{mj}$ being the total dynamic friction coefficient corresponding to the contact surfaces of the j-th damper (μ_{mj} is the dynamic friction coefficient of the surface materials). Note that, according to Dupont and Stokes [15], more sophisticated friction models would conduce to Lyapunov-based control strategies similar to those developed from the classical Coulomb's model.

In this work the friction dampers are considered as rigidly coupled to the structure, therefore the control forces applied to the structure are equal to the friction forces stated in Eq. (23).

3.2. Equations of motion of the controlled system

Incorporating the control forces of semi-active friction dampers in Eq. (1) yields

$$\mathbf{M_s}\ddot{\mathbf{q}}_\mathbf{s} + \mathbf{C_s}\dot{\mathbf{q}}_\mathbf{s} + \mathbf{K_s}\mathbf{q_s} = \mathbf{f_{es}} + \mathbf{B}'_{\mathbf{cs}}\mathbf{f_c}, \tag{25}$$

in which $\mathbf{B}'_{\mathbf{cs}}$ is a matrix containing the directing cosines of the braces of the friction dampers and $\mathbf{f_c}$ is a row vector containing the control forces defined by Eq. (23).

The state-space form of Eq. (25) is

$$\begin{cases} \dot{\mathbf{x}}_\mathbf{s} = \mathbf{A_s}\mathbf{x_s} + \mathbf{B_{us}}u + \mathbf{B_{cs}}\mathbf{f_c} \\ y = \mathbf{C_{sy}}\mathbf{x_s} \end{cases}, \tag{26}$$

in which

$$\mathbf{B_{cs}} = \begin{bmatrix} \mathbf{0}_n \\ \mathbf{M_s}^{-1} \end{bmatrix} \mathbf{B}'_{\mathbf{cs}}. \tag{27}$$

For convenience, the vector $\mathbf{f_c}$ is written as follows:

$$\mathbf{f_c} = \boldsymbol{\mu}\mathbf{N}\,\mathrm{sign}(\mathbf{B_{sc}x_s}), \tag{28}$$

in which $\mathbf{N}$ and $\boldsymbol{\mu}$ are the diagonal matrices containing the normal forces n_j and the friction coefficients μ_j of the dampers, respectively; and

$$\mathbf{B_{sc}} = -\mathbf{B}'^{\top}_{\mathbf{cs}}[\mathbf{0}_n \ \ \mathbf{I}_n]. \tag{29}$$

3.3. Lyapunov-based controller

In this section, a Lyapunov-based control law is developed considering the system represented by Eq. (26). The development is based on the work of Kuehn and Stalford [8].

The energy in the structural dynamic system represented by Eq. (26) is quantified by a quadratic Lyapunov function candidate of the form:

$$V = \mathbf{x_s}^{\top}\mathbf{P}\mathbf{x_s}, \tag{30}$$

in which $\mathbf{P}$ is a symmetric positive-definite weighting matrix which is the solution of the Lyapunov equation (34). The rate of change of energy in the structure can be stated as the first derivative of the function V with respect to time:

$$\dot{V} = \mathbf{x_s}^{\top}\mathbf{P}\dot{\mathbf{x}}_\mathbf{s} + \dot{\mathbf{x}}_\mathbf{s}^{\top}\mathbf{P}\mathbf{x_s}. \tag{31}$$

Substituting the state equation (26) into (32) results in

$$\dot{V} = \mathbf{x_s}^{\top}(\mathbf{A_s}^{\top}\mathbf{P} + \mathbf{P}\mathbf{A_s})\mathbf{x_s} + 2\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}\mathbf{f_c}. \tag{32}$$

Note that the external disturbance u has been neglected according to the direct Lyapunov control method [1].

The first derivative of the Lyapunov function (32) can be rewritten as

$$\dot{V} = -\mathbf{x_s}^{\top}\mathbf{Q}\mathbf{x_s} + 2\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}\mathbf{f_c}. \tag{33}$$

where $\mathbf{Q}$ is defined as

$$\mathbf{Q} = -(\mathbf{A_s}^{\top}\mathbf{P} + \mathbf{P}\mathbf{A_s}). \tag{34}$$

At this point, $\mathbf{P}$ is determined by solving Eq. (34) for some specified real positive-definite matrix $\mathbf{Q}$. For $\mathbf{A_s}$ Hurwitz and $\mathbf{Q}$ positive definite, there is a unique positive-definite solution $\mathbf{P}$ to the Lyapunov equation (34) [16]. This value of $\mathbf{P}$ is used below to determine the control law. The matrix $\mathbf{Q}$ can be designed by using modal or state penalties and genetic algorithms, trial-and-error methods, etc. However, in Section 4.1, a novel straightforward method is proposed in order to construct $\mathbf{Q}$.

Since $\mathbf{Q}$ is considered to be positive definite, Eq. (33) can be rewritten as

$$\dot{V} < 2\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}\mathbf{f_c}. \tag{35}$$

In *quickest descent* control [8], the goal is to make $\dot{V}$ as negative as possible, and non-positive at all times if Lyapunov stability is required. Considering Eq. (28), the following control law can be stated:

$$n_j = \begin{cases} n_{\min j} & \text{if } [\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}]_j[\mathbf{B_{sc}x_s}]_j \ge 0 \\ n_{\max j} & \text{if } [\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}]_j[\mathbf{B_{sc}x_s}]_j < 0 \end{cases} \quad \text{for } j = 1, \ldots, m. \tag{36}$$

This control law needs $\mathbf{A_s}$ and $\mathbf{B_{cs}}$ to form a controllable pair in order to work properly.

It must be pointed out that, in real world applications, sensing the full state vector can be impractical and therefore an state observer must be used.

3.4. Stability analysis

Substituting Eq. (28) into (35) yields

$$\dot{V} < 2\mathbf{x_s}^{\top}\mathbf{P}\mathbf{B_{cs}}\boldsymbol{\mu}\mathbf{N}\,\mathrm{sign}(\mathbf{B_{sc}x_s}) = \dot{V}_c. \tag{37}$$

Recalling that $\boldsymbol{\mu}$, $\mathbf{N}$ and $\mathbf{P}$ are positive-definite matrices, the sign of $\dot{V}_c$ is found to be

$$\mathrm{sign}(\dot{V}_c) = \mathrm{sign}(\mathbf{x_s}^{\top}\mathbf{B_{cs}}\mathbf{B_{sc}}\mathbf{x_s}), \tag{38}$$

and it can be easily proven from (27) and (29) that the product $\mathbf{B}_{cs}\mathbf{B}_{sc}$ is negative semi-definite, which leads to

$$\dot{V}_c \le 0. \tag{39}$$

Then,

$$\dot{V} < 0. \tag{40}$$

This ensures the stability of the controlled system in the sense of Lyapunov [16], which is the most important feature of Lyapunov-based controllers.

4. Tuning of Lyapunov-based controller

In this section, three methods to construct matrix $\mathbf{Q}$, and thus tuning the Lyapunov-based controller, are outlined.

The first method is the one proposed in this work. It penalizes all the modes according to their contribution to the response of the uncontrolled case and therefore it is denoted as Generalized Modal Contribution Factor Tuning (GMCF-T).

The last two methods are presented just because they are used as baseline to assess GMCF-T.

Modal Penalties Tuning (MP-T) is a trial-and-error method which consists in penalizing each mode heavily one at a time and simulating the system performance for each case. The MP-T options (trials) are labeled as MP-T1, when the first two complex conjugated modes are the most penalized; MP-T2, when the following two complex conjugated modes are the most penalized; and so on.

A Control Objective Tuning (CO-T) method consists simply in penalizing the control objective more than the states. This third method is presented just to show that a bad matrix $\mathbf{Q}$ can be found if only the control objective is taken into account.

4.1. Generalized Modal Contribution Factor Tuning (GMCF-T)

In order to construct the matrix $\mathbf{Q}$, it is proposed to penalize each squared modal response proportionally to a factor Γ_i as follows:

$$\mathbf{x_s}^\top \mathbf{Q} \mathbf{x_s} = \sum_{i=1}^{2n} \{\Gamma_i \| [\mathbf{E_s}^{-1}]_{i,\bullet} \mathbf{x_s} \|_2^2\}, \tag{41}$$

which leads to

$$\mathbf{Q} = \sum_{i=1}^{2n} \{\Gamma_i ([\mathbf{E_s}^{-1}]_{i,\bullet})^* [\mathbf{E_s}^{-1}]_{i,\bullet}\}, \tag{42}$$

with

$$\Gamma_i = \mathrm{GMCF}_i \tag{43}$$

calculated by Eq. (20). In Eqs. (41) and (42), $[\mathbf{E_s}^{-1}]_{i,\bullet}$ is the i-th row of the matrix $\mathbf{E_s}^{-1}$.

This method yields a real positive-definite matrix since the factors Γ_i have the following properties:

$$\Gamma_{2i} = \Gamma_{2i-1} \quad \text{for } i = 1, \ldots, n, \tag{44}$$

and

$$\Gamma_i > 0 \quad \text{for } i = 1, \ldots, 2n. \tag{45}$$

The rationale of this method relies on the assumption that the mode shapes of the structure remain almost invariant after the installation of the control system. This should be verified, e.g. through a Modal Assurance Criterion [17], as made in the example in Section 5.5.

4.2. Modal Penalties Tuning (MP-T)

This is the method referred to as "modal penalties" by Kuehn and Stalford [8]. The matrix $\mathbf{Q}$ is constructed by using Eq. (42), but the factors Γ_i are selected as follows. To penalize mode i and its complex conjugated $i+1$ heavily, $\Gamma_i = \Gamma_{i+1} = 100$ and the remaining factors are set to 1, thus ensuring that

$$\Gamma_i > 0 \quad \text{for } i = 1, \ldots, 2n. \tag{46}$$

This also yields a real positive-definite matrix $\mathbf{Q}$, as a direct Lyapunov control method requires [1]. Note that this method needs to test by simulations each option of $\mathbf{Q}$ in order to identify the best among them.

4.3. Control Objective Tuning (CO-T)

This method consists in penalizing the control objective more heavily than the states, i.e.

$$\mathbf{x_s}^\top \mathbf{Q} \mathbf{x_s} = 100 y^2 + \|\mathbf{x_s}\|_2^2, \tag{47}$$

which leads to

$$\mathbf{Q} = 100\mathbf{C}_{\mathbf{sy}}^{\top}\mathbf{C}_{\mathbf{sy}} + \mathbf{I}_{2n} \tag{48}$$

also being a real and positive-definite matrix.

5. A demonstration example: Vibration control of a plane frame structure

A numerical example, whose aim is to demonstrate the performance of GMCF-T over MP-T and CO-T, is outlined below.

5.1. Controlled structure

The structure to be controlled and the placement of semi-active dampers were taken from the work of Jansen and Dyke [6]. It is a 6-story plane frame structure with semi-active dampers in the first two stories; see Fig. 1. The magnetorheological dampers of that work were replaced with two semi-active friction dampers, installed on diagonal braces, with $\mu_1 = \mu_2 = 0.5$, and $\phi_{b1} = \phi_{b2} = 0.5\pi$ rad. The maximal normal forces ($n_{\max 1} = n_{\max 2}$) were set to the passive optimum normal force value, and the minimal forces ($n_{\min 1} = n_{\min 2}$) were set to 1 percent of that value. The optimum passive (OP) case was also used as baseline in order to assess the studied cases of semi-active control.

5.2. Excitation

Two types of spatial distribution of the excitation were considered: (1) ground acceleration, e.g. due to earthquakes; and (2) direct force, e.g. due to unbalanced rotating machines. These are typical excitation cases in real-life structures.

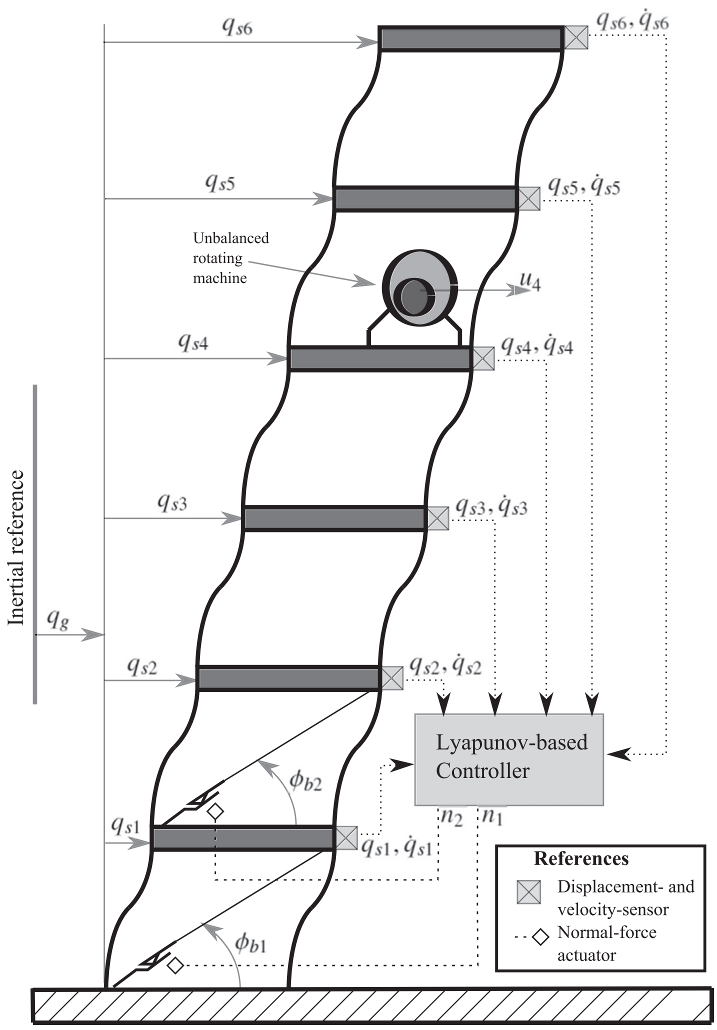

Fig. 1. Schematic diagram of a 6-DOF structural system with semi-active friction dampers and a feedback control system.

In order to give insight into the concept of GMCF and demonstrate the advantages of GMCF-T, two types of excitation spectrum were considered: (1) white noise and (2) strong frequency content around the second natural frequency (ω_{02}) of the structure.

Accordingly, four different combinations of excitations were considered. The first two cases were ground acceleration; first with white noise (WN_1), and then with a strong frequency content around ω_{02} (PSD_1). The other two cases were a force applied directly to the fourth story; first with white noise (WN_2), and then with a strong frequency content around ω_{02} (PSD_2).

To generate the excitation signals u of the system stated in (26), a Gaussian white-noise of unit intensity and 100 s duration was used as input w of the filter defined by Eq. (5). Magnitude of excitations, adjusted through the filter transfer function gains, was chosen in order to reach a peak displacement of 0.02 m in the sixth story of the uncontrolled structure.

5.2.1. Ground motion

For the ground-motion cases, the external disturbances were stated as

$$\mathbf{f_{es}} = -\ddot{q}_g \mathbf{M_s} \mathbf{1}_n, \tag{49}$$

in which $\ddot{q}_g = u$ (the filter output) is the support acceleration, and $\mathbf{1}_n$ is a column vector of ones.

For the white-noise case, labeled as WN_1, the transfer function of the filter defined by Eq. (5) was set to

$$H(s) = 2. \tag{50}$$

For the case with strong frequency content around ω_{02}, labeled as PSD_1, an earthquake-based PSD was considered through the following transfer function:

$$H(s) = H_K \frac{\frac{2\zeta_1}{\omega_1}s + 1}{\frac{1}{\omega_1^2}s^2 + \frac{2\zeta_1}{\omega_1}s + 1} \frac{\frac{1}{\omega_2^2}s^2}{\frac{1}{\omega_2^2}s^2 + \frac{2\zeta_2}{\omega_2}s + 1} \tag{51}$$

which is the Kanai–Tajimi filter modified by Clough and Penzien [18]. Its parameters were chosen to be $\omega_1 = 15.6$ rad s^{-1}, $\zeta_1 = 0.6$, as suggested by Kanai for firm soil; $\omega_2 = 10$ rad s^{-1}, $\zeta_2 = 0.6$, in order to localize the PSD peak near ω_{02}; and $H_K = 2.2$.

5.2.2. Force applied directly to the fourth story

For the cases of force applied directly to the fourth story, the external disturbances were stated as

$$\mathbf{f_{es}} = [0 \;\; 0 \;\; 0 \;\; u_4 \;\; 0 \;\; 0]^\top, \tag{52}$$

in which $u_4 = u$ (the filter output).

For the white-noise case, labeled as WN_2, the transfer function of the filter defined by Eq. (5) was set as

$$H(s) = 226. \tag{53}$$

For the case with a strong frequency content around ω_{02}, labeled as PSD_2, the transfer function of the filter defined by Eq. (5) was set equal to that defined by Eq. (51); but using the following parameters: $\omega_1 = 15$ rad s^{-1}, $\zeta_1 = 0.1$, $\omega_2 = 10$ rad s^{-1}, $\zeta_2 = 0.1$, which yield an almost harmonic excitation as those produced by unbalanced rotating machines; and $H_K = 140$.

5.3. Control objectives

The proposed method (GMCF-T) works with any control objective which can be written as a linear combination of the states. In order to test the outlined methods, in this work, the problem of mitigating the relative oscillations between sensitive instruments placed in the stories 2, 5 and 6 of the structure is considered. Taking into account that problem, two arbitrary control objectives were selected as examples.

The first control objective consists in minimizing the relative displacement between the sixth and second stories ($y = x_6 - x_2$), which leads to

$$\mathbf{C_{sy}} = [0 \;\; -1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; \mathbf{0}_{1\times 6}]. \tag{54}$$

The other control objective consists in minimizing the relative displacement between the two last stories ($y = x_6 - x_5$), which leads to

$$\mathbf{C_{sy}} = [0 \;\; 0 \;\; 0 \;\; 0 \;\; -1 \;\; 1 \;\; \mathbf{0}_{1\times 6}]. \tag{55}$$

5.4. Performance indexes

Two performance indexes were defined in order to assess the effectiveness of the considered strategies: J_{rms}, which indicates the ratio between RMS structure responses of controlled and uncontrolled cases; and J_{peak}, which indicates the ratio between peak structure responses of the controlled and uncontrolled cases. They are defined as

$$J_{\mathrm{rms}} = \sqrt{\frac{\int_0^T |y_c(t)|^2 \, \mathrm{d}t}{\int_0^T |y_{nc}(t)|^2 \, \mathrm{d}t}}, \tag{56}$$

and

$$J_{\mathrm{peak}} = \frac{\max_{0 \le t \le T} |y_c(t)|}{\max_{0 \le t \le T} |y_{nc}(t)|}, \tag{57}$$

where t is the current simulation time, $T = 100$ s is the time duration of the simulation, and y_c and y_{nc} are the responses y in the controlled and uncontrolled cases, respectively.

5.5. Results and discussion

The GMCFs used to tune the Lyapunov-based controllers with control objectives $y = x_6 - x_2$ and $y = x_6 - x_5$ are shown in Tables 1 and 2, respectively. Note that, mainly for the white-noise excitation cases, the sum of all the GMCFs is approximately equal to one, which implies that the cross-terms in Eq. (19) can actually be neglected. It is worth noting that, despite some GMCFs appear as 0 in Tables 1 and 2, they are very small but not zero.

From Tables 1 and 2, it is noted that the contribution of modes to the response depends not only on the structure and excitation (frequency content and spatial distribution) but also on the particular response which is being studied. For example, for ground motion excitation, there is almost no difference between WN_1 and PSD_1 excitations when the studied response is $y = x_6 - x_2$ (despite the strong difference in frequency content). However, there is a small difference when the response is $y = x_6 - x_5$.

In the case of white noise force applied to the fourth story, that difference is even larger. Specifically, when the selected control objective is $y = x_6 - x_2$, the first mode is the most contributory (Table 1); but when the control objective is $y = x_6 - x_5$, the most contributory mode is the third one (Table 2). The capability of GMCF for capturing such a difference makes GMCF-T effective in generating a good matrix **Q** even for sophisticated control objectives.

Several simulations were carried out in the time domain by numerical integration of the coupled equations (26), (28), and (36). Figs. 2 and 3 show the results of such simulations in terms of the performance indexes defined in Section 5.4 for optimal passive (OP) case and semi-active (MP-T, CO-T and GMCF-T) cases. It is important to note that GMCF-T has almost the same effectiveness than the best option of MP-T in six of the eight studied cases, in terms of J_{rms}; and in seven cases, in terms of J_{peak}. However, GMCF-T does not need simulation results in order to construct matrix **Q**, while MP-T needs as many simulations as modes the structure has (in this example, 6), in order to identify the best option among all MP-T trials.

Table 1
GMCFs for the four different excitation cases considering the control objective $y = x_6 - x_2$.

GMCF	Ground motion		Force applied to the fourth story	
	WN_1	PSD_1	WN_2	PSD_2
$\Gamma_1 = \Gamma_2$	0.474	0.475	0.471	0.016
$\Gamma_3 = \Gamma_4$	0.025	0.024	0.020	0.545
$\Gamma_5 = \Gamma_6$	0.000	0.000	0.000	0.000
$\Gamma_7 = \Gamma_8$	0.000	0.000	0.000	0.000
$\Gamma_9 = \Gamma_{10}$	0.000	0.000	0.005	0.000
$\Gamma_{11} = \Gamma_{12}$	0.000	0.000	0.001	0.000
$\sum_{i=1}^{12} \Gamma_i$	1.000	1.001	1.000	1.124

Table 2
GMCFs for the four different excitation cases considering the control objective $y = x_6 - x_5$.

GMCF	Ground motion		Force applied to the fourth story	
	WN_1	PSD_1	WN_2	PSD_2
$\Gamma_1 = \Gamma_2$	0.355	0.384	0.058	0.003
$\Gamma_3 = \Gamma_4$	0.095	0.100	0.013	0.599
$\Gamma_5 = \Gamma_6$	0.035	0.012	0.277	0.151
$\Gamma_7 = \Gamma_8$	0.011	0.002	0.052	0.009
$\Gamma_9 = \Gamma_{10}$	0.002	0.000	0.061	0.006
$\Gamma_{11} = \Gamma_{12}$	0.000	0.000	0.035	0.003
$\sum_{i=1}^{12} \Gamma_i$	1.000	0.999	0.998	1.548

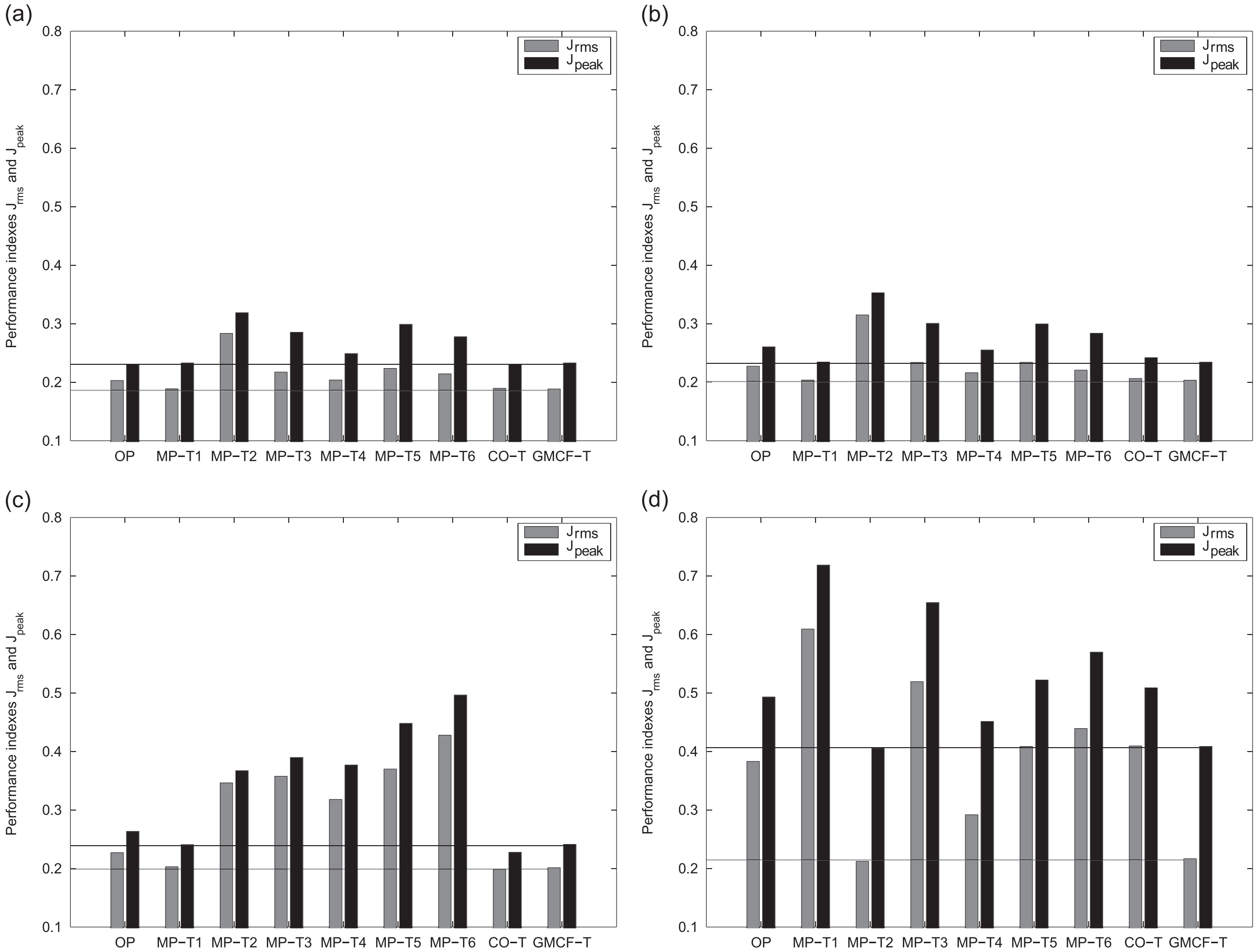

Fig. 2. Performance indexes results for control objective $y = x_6 - x_2$; with excitations: (a) WN_1 seismic, (b) PSD_1 seismic, (c) WN_2 force applied directly to the fourth story, and (d) PSD_2 force applied directly to the fourth story.

Table 1 shows that, for control objective $y = x_6 - x_2$ with ground motion excitation (in both cases, WN_1 and PSD_1), and in the case of WN_2 force applied directly to the fourth story, first mode is the most contributory to the response of the uncontrolled structure. Fig. 2a, b and c show that, precisely for those excitation cases, MP-T1 is the best option among all MP-T trials (in terms of both J_{peak} and J_{rms}). This fact fits with the basis of the proposed method, which is to tune the controller by penalizing modes according to their contribution to the response of uncontrolled structure. In these three excitation cases, GMCF-T generates matrices which penalize the first mode more heavily (according to Table 1). Then, GMCF-T and MP-T1 have nearly the same effectiveness, as is shown in Fig. 2a–c.

Analogously, for control objective $y = x_6 - x_2$, in the case of PSD_2 force applied directly to the fourth story, the most contributory mode in the uncontrolled structure is the second mode, as is shown in Table 1. Fig. 2d shows that the best option among MP-T trials is MP-T2. Then, since GMCF-T generates a matrix which penalizes the second mode more heavily (according to Table 1), its effectiveness is nearly the same as that of MP-T2; which is also shown in Fig. 2d.

From Fig. 2d it is also evident how inappropriate can be to construct the matrix $\mathbf{Q}$ only from the control objective (CO-T). In this case, CO-T generates a weighting matrix similar to that generated by MP-1, because the control objective ($y = x_6 - x_2$) is roughly similar to the shape of the first mode. Then, since the first mode has small contribution to the response (fourth column in Table 1), CO-T has poor performance. It can be concluded that, in order to obtain a good matrix $\mathbf{Q}$, it is necessary to consider not only the control objective but also excitation and structure characteristics.

In the two cases of seismic excitation and control objective $y = x_6 - x_5$ (Fig. 3a and b), GMCF-T has poor performance in making a good tuning of the controller in terms of J_{rms}, as compared to the best MP-T trial. However in these cases, for this particular example, no option of tuning of $\mathbf{Q}$ outperforms OP significantly. Therefore, a semi-active approach should not be used in such cases.

In the case of control objective $y = x_6 - x_5$, for the cases of force applied directly to the fourth story with both frequency contents, WN_2 and PSD_2 (Fig. 3c and d, respectively), the best option among MP-T trials matches with the most contributory mode (third and second modes, respectively); compare Fig. 3c and d with Table 2. Unlike the cases with control objective $y = x_6 - x_2$, in these cases, the effectiveness of GMCF-T is not the same as that of the best option among MP-T trials. Actually, in one case it is better (Fig. 3c), and in the other case is slightly worse (Fig. 3d). This is because two or more modes

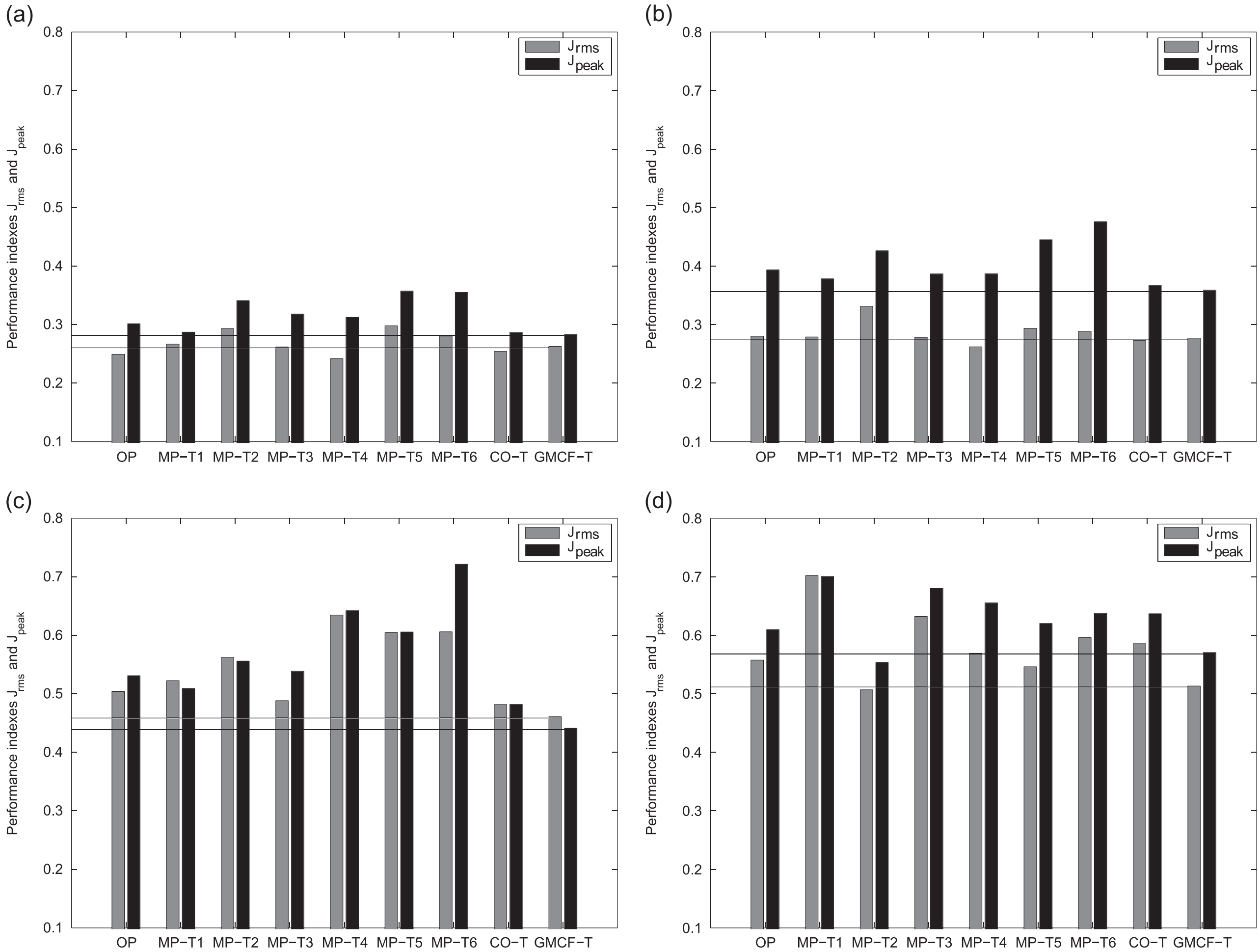

Fig. 3. Performance indexes results for control objective $y = x_6 - x_5$; with excitations: (a) WN_1 seismic, (b) PSD_1 seismic, (c) WN_2 force applied directly to the fourth story, and (d) PSD_2 force applied directly to the fourth story.

contribute significantly to the response of the uncontrolled structure (see Table 2), and GMCF-T makes a combination of two or more MP-T options, respectively. Whatever the case may be, GMCF-T has good performance in tuning the controller.

As stated in Section 4.1, Modal Assurance Criterion (MAC) [17] was used to verify the assumption "mode shapes are almost invariant after the installation of the control system". An equivalent linear system was obtained from the simulation results of the nonlinear system by the least square method [19, p. 188]. The MAC index was used to compare the most contributory mode (in terms of GMCF) of the uncontrolled system to the closest mode of the linear equivalent model. It was found that in the six cases in which GMCF-T was effective in constructing a good matrix **Q** (Figs. 2a–d, 3c and d), the MAC index was between 71 and 98 percent. Whereas, for the two cases in which GMCF-T had poor performance (Fig. 3a and b), the MAC was less than 68 percent.

If other control objectives are set (e.g. drifts at all stories), or if excitation has different spectra (e.g. strong frequency content around the first natural frequency of the structure); in general, the proposed method (GMCF-T) has a similar performance in tuning the controller as MP-T.

The trial-and-error method "state penalties" (presented in [8]) was also tested in this research, but all the results were worse than MP-T, and thus they are not shown.

6. Conclusions

The development of Lyapunov-based controllers in general has the challenge of constructing a weighting matrix. In this work, the concept of Generalized Modal Contribution Factor (GMCF) has been developed in order to construct such a matrix.

Through a numerical example it was shown that the proposed method for tuning the Lyapunov-based controller (GMCF-T) yields weighting matrices which reach the same results than the best among all the "Modal Penalties" Tuning (MP-T) options, with no need of its expensive nonlinear simulations. This is the main contribution of this work. Moreover, in some cases the effectiveness was much better than that of other methods such as, for instance, simply penalizing much heavily the first mode of the structure (MP-T1), penalizing only the control objective (CO-T) or using "state penalties".

The method gave poor results only in the cases in which the semi-active approach itself was no better performing than (optimized) passive control and when the modal shapes change greatly after the installation of the control system. This change can be predicted by using the Modal Assurance Criterion (MAC), as it was demonstrated.

Without loss of generality, the method proposed in this work could be applied to other control strategies in order to construct a weighing matrix, as for instance linear quadratic regulators.

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Appendix A. Independence of GMCF on the norm of the eigenvectors

In this appendix it is shown that Γ_i is independent of the normalization of $\mathbf{e_s}_i$.

First, a useful property of the inverse of an invertible matrix is obtained. Consider the invertible matrix $\mathbf{E_s}$ written as a block matrix containing column vectors $\mathbf{e_s}_i$ for $i = 1, \dots, 2n$, i.e.

$$\mathbf{E_s} = [\mathbf{e}_{\mathbf{s}_1} \ \dots \ \mathbf{e_s}_i \ \dots \ \mathbf{e}_{\mathbf{s}_{2n}}]; \tag{A.1}$$

and consider its inverse $\mathbf{E_s}^{-1}$ expressed as a block matrix containing row vectors $\mathbf{e}'_{\mathbf{s}_i}$ for $i = 1, \dots, 2n$, i.e.

$$\mathbf{E_s}^{-1} = \begin{bmatrix} \mathbf{e}'_{\mathbf{s}_i} \\ \vdots \\ \mathbf{e}'_{\mathbf{s}_i} \\ \vdots \\ \mathbf{e}'_{\mathbf{s}_{2n}} \end{bmatrix}. \tag{A.2}$$

From $\mathbf{E_s}^{-1}\mathbf{E_s} = \mathbf{I}$, it follows that

$$\mathbf{e}'_{\mathbf{s}_i}\mathbf{e_s}_i = 1. \tag{A.3}$$

Now consider another invertible matrix $\mathbf{E_s}_\alpha$ constructed from $\mathbf{E_s}$ scaling its i-th column by a non-zero real scalar α, i.e.

$$\mathbf{E_s}_\alpha = [\mathbf{e}_{\mathbf{s}_1} \ \dots \ \alpha\mathbf{e_s}_i \ \dots \ \mathbf{e}_{\mathbf{s}_{2n}}]; \tag{A.4}$$

and consider its inverse $\mathbf{E}_{\mathbf{s}_\alpha}^{-1}$ written as block matrix containing row vectors $\mathbf{d}_i$ for $i = 1, \dots, 2n$, i.e.

$$\mathbf{E}_{\mathbf{s}_\alpha}^{-1} = \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_i \\ \vdots \\ \mathbf{d}_{2n} \end{bmatrix}. \tag{A.5}$$

Since $\mathbf{E}_{\mathbf{s}_\alpha}^{-1}\mathbf{E_s}_\alpha = \mathbf{I}$, it follows that

$$\mathbf{d}_i(\alpha\mathbf{e_s}_i) = 1. \tag{A.6}$$

Then, equaling Eqs. (A.3) and (A.6), and post-multiplying both sides of the resulting equality by the Moore–Penrose pseudoinverse of $\mathbf{e_s}_i$, i.e. $(\mathbf{e_s}_i^*(\mathbf{e_s}_i\mathbf{e_s}_i^*)^{-1})$, lead to

$$\mathbf{d}_i = \frac{1}{\alpha}\mathbf{e}'_{\mathbf{s}_i}. \tag{A.7}$$

The inverse of $(\mathbf{e_s}_i\mathbf{e_s}_i^*)$ exists if and only if $\mathbf{e_s}_i$ is not the null vector. Note that this is always true because $\mathbf{E_s}$ is an invertible matrix.

Now consider two GMCFs, Γ_i and $\Gamma_{\alpha i}$, which are calculated for the mode i from the modal matrices $\mathbf{E_s}$ and $\mathbf{E_s}_\alpha$, respectively. From Eq. (20), they can be expressed as

$$\Gamma_i = \frac{1}{Y}\|\mathbf{C_{sy}}\mathbf{e_s}_i\|_2^2[\mathbf{E_s}^{-1}\mathbf{X_s}\mathbf{E_s}^{-*}]_{i,i} = \frac{1}{Y}\|\mathbf{C_{sy}}\mathbf{e_s}_i\|_2^2\mathbf{e}'_{\mathbf{s}_i}\mathbf{X_s}(\mathbf{e}'_{\mathbf{s}_i})^*; \tag{A.8}$$

and recalling Eq. (A.7),

$$\Gamma_{\alpha i} = \frac{1}{Y}\|\mathbf{C_{sy}}\alpha\mathbf{e_s}_i\|_2^2[\mathbf{E}_{\mathbf{s}_\alpha}^{-1}\mathbf{X_s}\mathbf{E}_{\mathbf{s}_\alpha}^{-*}]_{i,i} = \frac{1}{Y}\|\mathbf{C_{sy}}\alpha\mathbf{e_s}_i\|_2^2\frac{1}{\alpha}\mathbf{e}'_{\mathbf{s}_i}\mathbf{X_s}\left(\frac{1}{\alpha}\mathbf{e}'_{\mathbf{s}_i}\right)^*. \tag{A.9}$$

This leads to

$$\Gamma_i = \Gamma_{\alpha i}, \tag{A.10}$$

showing that the value of Γ_i is independent of the scaling (and therefore, on the normalization) of the corresponding eigenvector.

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