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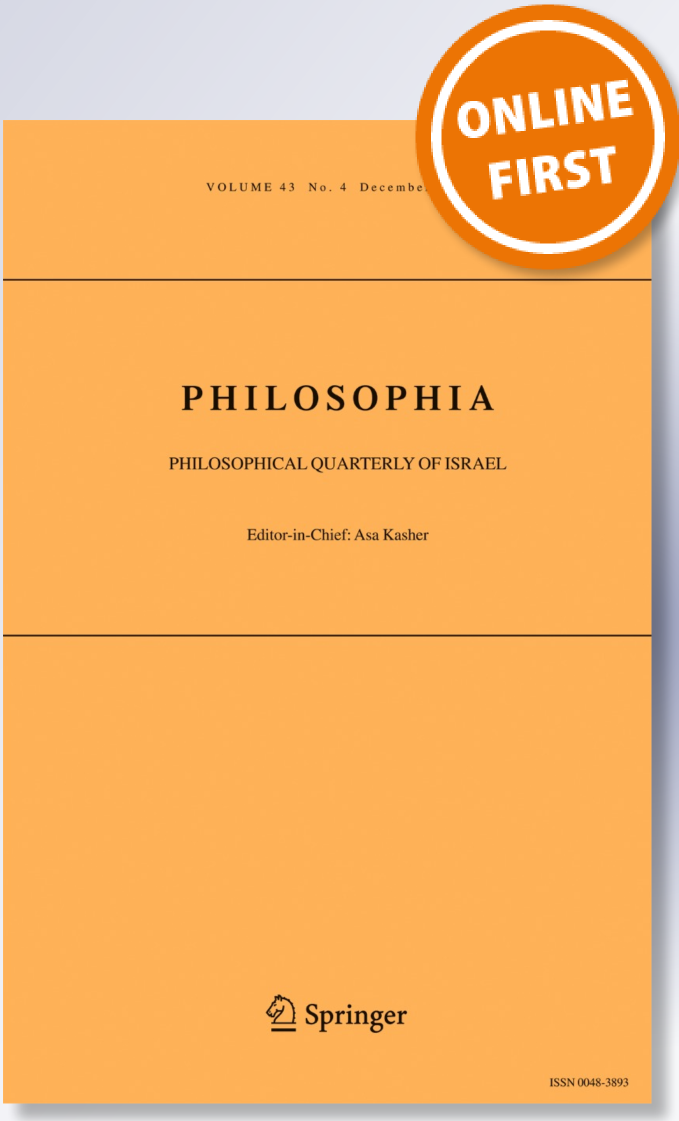

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A Formal Ontological Theory Based on Timeless Events

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Abstract I offer a formal ontological theory where the basic building blocks of the world are timeless events. The composition of events results in processes. Spacetime emerges as the system of all events. Things are construed as bundles of processes. I maintain that such a view is in accord with General Relativity and offers interesting prospects for the foundations of classical and quantum gravity.

Keywords Events · Processes · Formal ontology · Spacetime

> The invisible structure is greater than the visible.
> *Heraclitus, Fr. B54.*

Introduction

In his *Cratylus*, Plato attributed to Heraclitus the doctrine that change is basic and that "all things are in flux" (DK 22A6).[1] I have argued elsewhere that there is nothing in the extant fragments of Heraclitus that may compel us to think that he denied substance and material things (Romero 2012). A pure event ontology is more in accord

[1] See also Aristotle: "[Plato] as a young man became familiar with Cratylus and the Heraclitean doctrines that all sensible things are always flowing (undergoing Heraclitean flux)" DK 65A3 (The notation refers to the doxography in H. Diels and W. Kranz, *Die Fragmente der Vorsokratiker*, 6th ed., Berlin, 1951.)

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with the spirit of some minor Socratic schools, such as the Cyrenaics; see, e.g., the book by Ziloli (2013). Regarding Cratylus himself, little is known beyond what Plato included in his dialog.

The preeminence of Plato and Aristotle during Late Antiquity and the Middle Ages led to a loss of interest in event-based metaphysics in the West. The idea that events are prior to things, nevertheless, has been strong in the East through Buddhism. Both the Theravada and Mahayana traditions of Buddhism emphasise the importance of the *Paticca-Samuppada* ('dependent origination') and *Anicca* ('impermanence') as ultimate features of reality. For all major Buddhist schools the world and the self are a manifold of processes and happenings without a stable essence or intrinsic nature. These processes, for the Buddhist, are causally conditioned and dependent on other events. The whole world is an inter-dependent storm of events that, here and there, cluster giving the illusion of stability and delivering the delusion of being.

In modern Western philosophy, processes regained centrality in the work of Leibniz: his *monads*, which he considered to be the basic constituents of the world, are not atoms but "centres of force", i.e. units of change or activity that compose the processes that form the world. In the early twentieth century two prominent philosophers advocated for an event ontology: Russell (1914) and Whitehead (1920, 1929). Both formulated programs that considered events as basic individuals. Later process philosophers have defined things as "processual complexes possessing a functional unity instead of substances individuated by a qualitative nature of some sort" (Rescher 1996). Things, in this view, are construed as "manifolds of processes". This project, however, has never been accomplished in a rigorous way and in accordance to modern science.

There have been attempts to use the calculus of individuals of Leonard (1940) to provide an outline of a formal ontology of events, e.g. Martin (1978), but the topological structure based on the relation of precedence, attributed to the set of all events, is far too poor to account for some very general features of the world. More structure, in particular a metric structure, is required to deal with the totality of events. This fact was already noticed by Russell (1927), but a full theory was never developed.

Later ontological discussions about events have focused in the characterisation of events and their criteria of identity. The well-known views of Kim (1973), Brand (1977), and Davidson (1980) invoke spatiotemporal categories and cannot serve as a basis for a constructive theory of spacetime upon basic events. Quine's doctrine of the collapse of the categories of physical objects and events into spatiotemporal particulars (Quine 1960) is not a constitutive, but an eliminative theory. Lombard (1986) and Bunge (1977) understand events as changes in things, and hence they consider events as derivative of an ontology of physical objects. Such thing-based ontology, although attractive at some level of description for the physical sciences, presents problems related to the violation of Lorentz symmetry: basic things should have an absolute minimum length. The existence of such a length is incompatible with Lorentz invariance and requires an absolute system of reference, which blocks the path to a relativistic theory of spacetime.

I want to offer in this paper a more elaborate event ontology and briefly discuss its relevancy for the foundations of spacetime theories. In particular, I want to propose a theory about basic timeless events and their possible place as constitutive elements in the world. The theory might be useful for the foundations of some promising approaches to quantum gravity, such as the causal set program. In any case, the theory I present is a sketch that can be expanded in different ways to provide an ontological framework for different areas in science and philosophy.

A Theory of Basic Events and Processes

I assume that there are events. My writing and your reading of this line are series of events or processes. Of course, there are people that have denied the existence of events. Parmenides denied events or happenings because for something to happen, something should go out of existence, and something that previously did not exist should appear. But nothing can come from nothing, because what is not does not exist, and what does not exist has no causal power. I have sustained that this is a powerful argument (Romero 2012), but its correct interpretation requires a 4-dimensional approach. Allow me now to accept events. I consider here events as basic entities, primitive concepts of an ontological basis. The full meaning of what I understand by an 'event' will be given by the role played by the term in the proposed axiomatic system. The generating basis of the system is

$$\mathcal{B} = < E, \mathcal{E}, e^0, \star >, \tag{1}$$

where E is a set, $\mathcal{E}$ is the collection of all events, e^0 is a fiction called the null event, and $\star$ is a binary operation on E. The meaning of all these symbols will become clear through a set of axioms. In what follows I assume as background knowledge the predicate calculus, set theory, semantics, and real analysis. I adopt standard logical notation. The symbol $\triangleq$ denotes the semantic relation of representation (Bunge 1974a, b). The symbol $\vdash$ is used to mean 'is a theorem'.

In the following theory events are the only individuals that can be values of bound variables. The first axioms are:

- P_1. $(\forall e)_E\ (e \star e = e)$.
- P_2. $(\forall e_1)_E (\forall e_2)_E\ (e_1 \star e_2 \in E)$.
- P_3. $(\forall x)_{\mathcal{E}} (\exists e)_E\ (e \triangleq x)$.
- P_4. $(\forall x)_{\mathcal{E}}\ (e_1 \triangleq x \wedge e_2 \triangleq x) \Rightarrow (e_1 = e_2)$.
- P_5. $(\exists e^0)(\forall e)_E\ (e^0 \star e = e \star e^0 \equiv e)$.
- P_6. $\neg(\exists x)_{\mathcal{E}}\ (e^0 \triangleq x)$.

A few comments are in order. The first two axioms characterise the operation $\star$ as a binary operation (closed on E), that is idempotent on the same individual. Axiom P_2 states that the set E contains both basic and composed events (see definitions D_1-D_6 below). The axiom P_3 is of semantic nature: it states that for each event occurring in the world there is an element in the set E such that it represents the event. Notice that $\mathcal{E}$ is not a set, as E, but a collection of individuals. P_4 establishes that the representation of events is unique. P_5 introduces e^0, which is a neutral element under operation $\star$ in E. The next axiom states that this individual, e^0, is syncategorematic, i.e. it is a fiction that does not represent any real event; see Bunge (1966) for details. The element e^0 is introduced for formal purposes, in order to give E some basic mathematical structure. I emphasise: there are not null events in the world.

After these axioms, I introduce some useful definitions:

- D_1. An event $e_1 \in E$ is composed $\Leftrightarrow (\exists e_2, e_3)_E\ (e_1 = e_2 \star e_3)$.
- D_2. An event $e_1 \in E$ is basic $\Leftrightarrow\ \neg\, (\exists e_2, e_3)_E\ (e_1 = e_2 \star e_3)$.
- D_3. $e_1 \subset e_2\ \Leftrightarrow e_1 \star e_2 = e_2\ \ (e_1$ is part of $e_2\ \Leftrightarrow e_1 \star e_2 = e_2)$.
- D_4. $\mathrm{Comp}(e) \equiv \{e_i \in E \mid e_i \subset e\}$ is the composition of e.
- D_5. $E^0 = E \cup \{e^0\}$.
- D_6. If $e \in E$ is composed by basic events, it is called a *process* and denoted by p.

These definitions give the concept of composition and the relation of 'being part of', which depends entirely on the basic operation of composition. A process is any composed event. In what remains of this paper I shall use, for simplicity, the word 'event' as meaning 'basic event', and 'process' for 'composed event'. Note that both events and processes belong to the collection $\mathcal{E}$ and any e and p are elements of the set E. Also notice that the symbol '$\subset$' is not being used in its standard sense of 'subset' but in the ontological (actually mereological) sense of "is part of".

The following theorems are immediate:

$$\vdash (\forall e)_{E^0}\ (e^0 \subset e).$$

$$\vdash < E^0,\ \star,\ e^0 > \text{ is a commutative monoid of idempotents.}$$

The structure of a monoid is essentially that of a semi-group with neutral element.

Processes, considered as individuals, have descriptions, such as duration and complexity, and then admit predicates. I use capital letters to denote unitary predicates and relations. There is no need, however, to admit properties as values of the bound variables in the formulation of the event ontology. I shall have some nominalistic scruples on this point. I introduce the operation of abstraction from a collection of individuals. Let us consider a formula with a single variable x that runs *only* over processes: '$(- - x - -)$'. This formula can be atomic or complex (i.e. formed by atomic formulae connected by standard logic functors). The formula predicates of each individual x such and such a property. We can abstract a virtual (i.e. fictitious) class from such a formula forming the collection (Martin 1969), p.125:

$$P = \{y : - - y - -\}.$$

Hence, properties are introduced as classes of individuals sharing descriptions. The identity criterion for properties is immediate.

- D_7. $F = G \Leftrightarrow (\forall p)_E(Fp \wedge Gp \Rightarrow Fp = Gp)$.
- D_8. $R = S \Leftrightarrow (\forall p_1)_E(\forall p_2)_E \ldots (\forall p_n)_E(Rp_1, \ldots, p_n \wedge Sp_1, \ldots, p_n \Rightarrow Rp_1, \ldots, p_n = Sp_1, \ldots, p_n)$.

The first definition means that two properties are identical if and only if they have the same value for any process that satisfies both. The second definition is just the extension from singular properties to relations among several processes. The identity criterion for events is given by

– P$_7$. $(\forall e_1)_E(\forall e_2)_E(e_1 = e_2 \Leftrightarrow \forall F : Fe_1 = Fe_2)$.

This is Leibniz's identity of the indiscernibles. It is valid for events of any kind: basic ones and processes.

Given the previous definition of F in terms of collections of individuals (events and processes), the universal quantification does not require second order logic. It follows immediately that

$$\vdash (\forall e)_E(e = e),$$

$$\vdash (\forall p)_E(p = p),$$

i.e., every event is identical to itself. Trivially, the same is valid for processes. It is convenient now to define two important relations between processes: *overlapping* and *separateness*. Two processes overlap if and only if they have common events. Two processes are separate if and only if they do not overlap. Formally,

– D$_9$. $p_1 O p_2 \Leftrightarrow (\exists p_i)_E(p_i \subset p_1 \wedge p_i \subset p_2)$.
– D$_{10}$. $p_1 \backslash p_2 \equiv \neg(p_1 O p_2)$.

The composition of all actual events and processes is the World (W):

$$\neg(\exists e)_E \neg(e \subset W).$$

There is nothing that is not part of the World. The World, W, should not be confused with the Universe, $\mathcal{U}$, the composition of all things in a thing-based ontology as the one given by Bunge (1977) and Romero (2013). The Universe can change, i.e. events and processes take place in the Universe. The World, the composition of all events, cannot change itself because it is not a thing. In an ontology of events, the totality of events is changeless, otherwise there would be an event not included in the totality, which is absurd. Events do not change, they simply *are*. In the sense used here, the Universe can evolve, but not the World, which is fixed. The World is the maximal processes; it is the process of the Universe (the maximal thing admitted by a thing ontology).

Composition is not an adequate ordering relation. So far, the set E is a mess of elements representing basic events and processes. Some events are part of some processes, but there is no order. I introduce some order now. I want to equip E with a relation that would allow to impose an ordering among basic events of any given list. I cannot adopt a simple relation of "before than", as Reichenbach (1980), Carnap (1958), Grünbaum (1973), and Martin (1978) did, because not all events can be ordered by such a relation without further specification: we know from relativity theory that such an order can be inverted by choosing an appropriate reference system in the case of space-like events. The World simply is not that way. Not all events can be related to each other by a precedence relation. I need to introduce a stronger structure on the set of all events E, if I want to represent with this a set the actual World. To achieve this goal, I stipulate that E is a *metric space*[2].

– D_{11}. E is a metric space if for any two elements e_1 and e_2 of E, there is number $d(e_1,\ e_2)$, called the *interval* between e_1 and e_2 in accordance with the postulates:

M1. $d(e_1,\ e_2) = 0$ iff $e_1 = e_2$.
M2. $d(e_1,\ e_2) + d(e_2,\ e_3) \geq d(e_1,\ e_3)$ with $e_3 \in E$.
Lindenbaum (1926) has demonstrated that from these two axioms it follows that[3]:

$$\vdash \quad d(e_1,\ e_2) = d(e_2,\ e_1).$$

Only in case that $d^2(e_1,\ e_3) > 0$, there is a precedence relation between e_1 and e_3. I postulate:

– P_8. E is a metric space.

Then,

– D_{12}. The event represented by e_1 *precedes* (or is *earlier than*) the event represented by e_3 iff $d^2(e_1, e_3) > 0$.

In short, $e_1 \prec e_3$. Events such that $d^2 > 0$, $d^2 = 0$, and $d^2 < 0$ are called *time-like*, *null*, and *space-like* events, respectively. Notice that d^2 can be negative since non-Euclidean metrics are possible. For instance, for a Minkowskian metric $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ the interval is an imaginary number if the spatial separation of the events (i.e. the part of the metric with signature $(-, -, -)$) is greater than the temporal one (signature $+$). For events and processes with non-real intervals the precedence relation can be reversed just choosing an adequate coordinate representation of the manifold. Hence, precedence is a partial ordering relation and not an absolute one in the context of a general geometry.

Given any event represented by $e \in E$, the *future* of e is the set Fut $= \{e' : d^2(e, e') > 0 \ \wedge \ e \prec e'\}$. Similarly, the *past* of e is the set Past $= \{e' : d^2(e, e') > 0 \wedge \neg(e \prec e')\}$. Every event has its own past and future, that depend on the metric d of the space E.

Some relevant theorems:

$$\vdash \ < E,\ \prec > \text{ is a partially ordered set.}$$
$$\vdash \quad (\forall e_1,\ e_2)_E\ [e_1 \prec e_2 \Rightarrow \neg(e_2 \prec e_1)].$$
$$\vdash \quad \neg(\exists e)_E(e \prec e^0 \vee e^0 \prec e).$$

All this can be easily generalised to processes.

[2]If a weaker structure such as a causal ordering is imposed, then the ordering will be only partial, and we would be unable to accomodate space-like events in the theory. So I adopt a strong structure for the whole World, and then I shall show how local time can emerge from a partial ordering of time-like events.
[3]For Euclidean spaces it is also the case that $d(e_1,\ e_2)\ \geq 0$.

Perhaps it is convenient at this point to remind that a set is partially ordered if the following conditions are fulfilled:

- Reflexive: For all $x \in E$, $x \preceq x$.
- Antisymmetric: For all $x,\ y\ \in E$, $x \preceq y \preceq x$ implies $x = y$.
- Transitive: For all $x,\ y,\ z \in E$, $x \preceq y \preceq z$ implies $x \preceq z$.

Here, $\preceq\ \equiv\ (\prec \vee =)$.

Once the set E has been equipped with a metric structure, I can make the fundamental semantic assumption of the event ontology: The World is represented by a metric space. In symbols:

- P_9. $E \triangleq W$.

Here, E is a mathematical construct and W is the composition of all events, i.e. the maximal existent in an event ontology. It follows that

$$\vdash \quad \neg(\exists e)_E(e \prec W \vee W \prec e).$$

There is no previous or subsequent event to the World, since simply there is not any event outside W. This implies, in turn, that:

$$\vdash \quad E \text{ is closed.}$$

There is no preceding event to the World. Creation, if is understood as a causal relation among events, e.g. Bunge (1979), is not even an option for the World in this ontology.

A final step in the formulation of the event ontology is the formal construction of things out of events and processes. This can be done defining things as classes of processes sharing some properties, P, Q, etc:

$$X = \langle P,\ Q, ...\rangle\, p.$$

This formula is true of all processes that satisfy P, Q, ... In this way things are bundles of events defined by shared properties, which are abstracted from conditions imposed on the events. The thing "Socrates", for instance, is a cluster of events sharing their occurrence in Greece, previous to such and such other events, including processes like "talking with Plato", and so on. Note the similarity with the qualitative insight proposed by Russell (1914).

I close this section with the remark that there are two relations in the event ontology I am presenting: a relation of composition, that is basic and allows events to form processes, and a partial ordering relation among the elements of E that is a consequence of the metric structure we attribute to these set. As far as the metric structure is postulated these relations must be considered independent. There is a third relation, causation, that can be introduced at the current level, and is derivative of the way composition acts upon events to produce some processes. I turn to it now.

Causation

Causation is a mode of process generation based on composition (Bunge 1979). It is not the only way of generating processes. Particle decays, such as those of the muon, and other quantum processes generate series of events without causal origination: the existence of no previous event is necessary for the occurrence of the decay. The event of decay is legal (it occurs in conformity to the probabilistic laws of quantum electroweak theory), but not causal. I adopt the following definition of causal interaction between events: two events represented by e_1 and e_2 are causally related iff there is at least a process p such that both events are components of p, and there never occurs an instance of p in which $e_2 \subset p$ and $\neg(e_1 \subset p)$. Then, I say that e_1 is a cause of e_2.

$$e_1 \rhd e_2.$$

In other words, the process p involving e_2 can never occur without the existence of e_1. The World is legal and determinate, but not strictly causal. There are events that are not causally related and processes that are not causally originated. They result from spontaneous (although lawful) events; for additional details see Bunge (1977, 1979) and Romero and Pérez (2012).

– D_{13}. e_1 is called "cause" and e_2, "effect".

– P_{10}. $(\exists e_1)_E (\exists e_2)_E \ (e_1 \rhd e_2)$.
– P_{11}. $\neg(\exists e)_E \ (e \rhd e)$.
– P_{12}. There are events that belong to the same process but are not causally related.

P_{10} states that some events , but not all, are causally related. P_{11} postulates that no event is cause of itself (Ulfbeck and Bohr 2001). Also notice that P_{12} allows spontaneous events, like quantum occurrences, to be part of a processes and belong to what is called in the physical literature the "causal past" of a given event.

Spacetime

I call spacetime to the ontological system formed by all events and processes. It is the World, with all its events and the restrictions on the way events are. Spacetime, then, being an emergent entity from a system of structured events, is substantival. I do not endorse a pure metric or manifold substantivalism as characterised by Hoefer (1996), but a constructive substantivalism that can be reduced to pure event relationalism. This intermediate position will be discussed in detail elsewhere (Romero 2016).

The mathematical representation of the World *on large scales* can be improved imposing some additional constraints on the set E. To the metric postulates $M1$ and $M2$ I add now the following postulates:

P_{13}. The set E is a C^∞ differentiable, 4-dimensional, real pseudo-Riemannian manifold.

P_{14}. The metric structure of E is given by a tensor field of rank 2, g_{ab}, in such a way that the differential interval ds between two events is given by: $ds^2 = g_{ab} dx^a dx^b$.

A real 4-D manifold is a set that can be covered completely by subsets whose elements are in a one-to-one correspondence with subsets of $\Re^4$. The manifold is pseudo-Riemannian if the tangent space in each element is flat but not Euclidean. Each element of the manifold represents one (and only one) event. Notice that it is incorrect to say that spacetime *is* the manifold (a position know as manifold substantivalism); spacetime is *represented* by the manifold and its metric structure (see below). We adopt 4 dimensions because it seems enough to give 4 real numbers to provide the minimal characterisation of an event. We can always provide a set of 4 real numbers for every event, and this can be done independently of the intrinsic geometry of the manifold. If there is more than a single characterisation of an event, we can always find a transformation law between the different coordinate systems. This is a basic property of manifolds.

I have introduced the continuum through the adoption of a manifold structure. This is a major step and I shall come back to the implications of adopting the continuum hypothesis in the next section.

I am ready now to introduce the Equivalence Principle and the specification of the metric through two additional postulates:

P_{15}. The tangent space of E at any point is Minkowskian, i.e. its metric is given by a symmetric tensor η_{ab} of rank 2 and signature -2.

P_{16}. The metric of E is determined by a rank 2 tensor field T_{ab} through the Einstein field equations:

$$G_{ab} - g_{ab}\Lambda = \kappa T_{ab}. \tag{2}$$

In these equations G_{ab} is the so-called Einstein tensor, formed by second order derivatives of the metric. In the second term on the left, Λ is called the cosmological constant, whose value–according to observations–is thought to be small but not null. The constant κ on the right side is -8π in units of $c = G = 1$. Finally, T_{ab} represents the source of the metric field, and satisfies conservation conditions ($\nabla_b T^{ab} = T^{ab}_{;b} = 0$) from which the equations of motion of physical things (i.e. bundles of events) can be derived. The solutions of such equations are the histories of things: 4D subsets of E. The solutions can be seen as continuous series of events (processes) represented on the manifold E. The Einstein's field equations are a set of ten non-linear partial differential equations for the metric coefficients.

Postulates P_{15} and P_{16} given above, with an adequate formal background (Bunge 1967; Covarrubias 1993; Perez Bergliaffa et al. 1998; Romero 2016) imply the theory of general relativity. The conceptual representation of spacetime $\mathcal{ST}$ is given by a 4-dimensional manifold equipped with a metric. In standard relativistic notation:

$$W = \mathcal{ST} \hat{=} \langle E, g_{ab} \rangle .$$

General Relativity, then, can be obtained from our ontology just with some simple additional constraints upon the set E that represents the totality of events. It is a natural extension of the proposed ontology that applies to processes with large number of events, in such a way that they can be represented by continuous functions. I insist with an important point: spacetime *is not* a manifold (i.e. a mathematical construct) but the "totality" (the composition in our characterisation) of all events and processes plus some metric structure. A specific model of the World requires the specification of the source of the metric field. As we have seen, this is done through another field, called the "energy-momentum" tensor field T_{ab} (Hawking and Ellis 1973). Hence, a model of the World is:

$$M_W = \langle E, g_{ab}, T_{ab} \rangle \, .$$

Since the ontic basis of the model is the *totality* of events, the World is ontologically determined. This does not imply that the World is necessarily *predictable* from the model. In fact, Cauchy horizons can appear in the manifold E for many prescriptions of T_{ab} e.g. Joshi (1993). One thing is the World, and another our representations of the World. Not all models of the World admit full predictability since in many of them the Cauchy problem cannot be well posed.

In the World, objects are 4-dimensional bundles of events (Heller 1990). Beginning and end, are just boundaries of objects, in the same way that the surfaces and boundary layers are limits of 3-dimensional slices of such objects. The child I was, long time ago, is just a temporal part of me. The fact that these parts are not identical is not mysterious or particularly puzzling, since spacetime, although changeless itself, is composed of events. We can understand the intrinsic changes of the World as asymmetries in the geometry of spacetime (Romero 2013).

Although so far I have presented spacetime as a structured system of events and processes, I have not shown that its structure naturally emerges from basic relations among basic events. To exhibit the mechanism that enforces such an emergence, i.e. to construct the metric structure upon an operation such as composition of basic events, is a major problem for any ontology of spacetime, and arguably, the main challenge of most approaches to quantum gravity. Nevertheless, I think that the theory of events I have outlined might help to formalise some promising proposals to constructive spacetime theories such as the so-called causal set approach. In what follows I shall present some preliminary steps towards providing an ontological foundation for such theory, and some hints about how to proceed towards the transition from discrete to continuum representations.

Discrete Spacetime

As far as we can decompose a given process into more basic events, in such a way that E can be approximated by a compact non-denumerable metric space, the continuum representation for the totality of events will work. But if there are atomic[4] events, there will be a sub-space of E that is countable (or denumerable if it is infinite) and ontologically basic. There is, in such a case, a discrete substratum underlying the continuum manifold, which is, ultimately, a large number approximation. Since the

[4] I use the word "atomic" in the original Greek sense of $\acute{\alpha}\tau o\mu o\varsigma$, "uncut", "individual", "not decomposable". It should be considered as synonimous of "basic", introduced in D_2.

quantum of action is given by the Planck constant, it seems a reasonable hypothesis to assume that atomic events occur at the Planck scale, $l_P = \sqrt{\hbar G/c^3}$. If there are atomic events, the continuum hypothesis breaks down and a new postulate should be introduced:

$$P_{discrete}.\mathrm{Card}(E) < \aleph_1.$$

The continuum representation would be only an approximation that is adequate for complex processes and large numbers of basic events. The continuum spacetime is then a large-scale emergent property, absent at the more basic ontological level. This is similar to, for instance, considering the mind as a collection of complex processes of the brain, emerging from arrays of 'mindless' neurons. The word 'emergence', in the present context, means apparition of qualitative novelty (Bunge 2003). The postulate $P_{discrete}$ says that the cardinality of the set of basic events is not in correspondence with that of the real numbers. As shown by Cantor in his famous proof, there should be possible to establish a one-to-one correspondence between natural numbers and basic events.

If this view is correct, discrete spacetime should be represented by a theory about the relations among basic events, yielding the ontological emergence of spacetime and classical gravitation for large numbers of events. The basic substratum of the World would be purely ontological instead of physical; the physical realm emerges at scales where dynamics makes sense.

Atomic events and their relations can be represented by a partially ordered set: a *poset*, see Bombelli et al. (1987) . It can be proved, under some assumptions, that the dimension, topology, differential structure, and metric of the manifold where a poset is embedded is determined by the poset structure (Malament 1977). If the order relation is interpreted as a causal relation, the posets are called *causal sets* (or *causets*). We have already seen that this relation obtains in terms of the basic relation of composition in our ontology.

A given poset can be embedded into a Lorentzian manifold. An embedding is a map taking elements of the poset into points in the manifold such that the order relation of the poset matches the causal ordering of the manifold. A further criterion is needed, however, before the embedding is suitable. If, on average, the number of poset elements mapped into a region of the manifold is proportional to the volume of the region, the embedding is said to be faithful (Sorkin 1990; Walden 2010). The poset is then called *manifold-like*.

A conjecture (called *hauptvermutung*) is usually made to ensure that the same poset cannot be faithfully embedded into two different spacetimes that are dissimilar on large scales. Alternatively, a poset can be generated by *sprinkling* points (events) from a Lorentzian manifold. By sprinkling points in proportion to the volume of the spacetime regions and using the causal order relations in the manifold to induce order relations between the sprinkled points, a poset can be produced that (by construction) can be faithfully embedded into the manifold.

To maintain Lorentz invariance[5] this sprinkling of points must be selected randomly using a Poisson process. Thus, the probability of sprinkling n points (events) into a region of volume V is:

$$P(n) = \frac{(\rho V)^n e^{-\rho V}}{n!}, \quad (3)$$

where ρ is the density of the sprinkling.

A *link* in a poset is a pair of elements $e_1,\ e_2 \in E$ such that $e_1 \prec e_2$ but with no $e_3 \in E$ such that $e_1 \prec e_3 \prec e_2$. In other words, e_1 and e_2 represent directly linked events.

A *chain* is a sequence of elements $e_0,\ e_1, \ldots, e_n$ such that $e_i \prec e_{i+1}$ for $i = 0, \ldots, n-1$. The length of a chain is n, the number of links used. A chain represents a specific type of process.

A geodesic between two poset elements can then be introduced as follows: a geodesic between two elements $e_i,\ e_f \in E$ is a chain consisting only of links such that $e_0 = e_i$ and $e_n = e_f$. The length of the chain, n, is maximal over all chains from e_i to e_f. In general there will be more than one geodesic between two elements. The length of a geodesic should be directly proportional to the proper time along a time-like geodesic joining the two spacetime points if the embedding is faithful.

A major challenge is to recover a realistic spacetime structure starting from a numerable poset. A step in the direction of solving the problem is a classical model in which elements are added according to probabilities. This model is known as classical sequential growth (CSG) dynamics (Rideout and Sorkin 2000). The classical sequential growth model is a way to generate posets by adding new elements one after another. Rules of how new elements are added are specified and, depending on the parameters in the model, different posets result. The direction of growing gives rise to a global time, which does not exist at the fundamental poset event level. In the large number limit, the poset becomes manifold-like. The local time we ‘feel’ is given by the local causal ordering of the events and not by the global ‘cosmic’ time.

Another challenge is to account for the remaining referents of general relativity, namely, gravitating objects. I have proposed above that physical objects can be understood as clusters of processes, and hence they can emerge as inhomogeneities in the growing pattern of events. This conjecture is supported by the observation that whatever exists seems to have energy, and energy is just the capability to change.[6] The most populous the bundle of events is, the larger the associated energy results. In this view, spacetime curvature emerges as well, just as a measure of the number of basic events. Objects, physical things, would be nothing else than clusters of events.

Any object has energy and any object can be defined as the result of a myriad of events. Objects, then, appear as a large number approximation to clusters of events. They inherit energy from the events that form them. In such a context, I can define energy as an additive quantity associated with composition. I postulate:

[5] Lorentz invariance is incompatible with most approaches to quantum gravity and with ontologies based on things, since in a Lorentzian world it is impossible to have an *absolute* minimum length.

[6] I notice that a thing-based ontology, such as Bunge’s, is an emergent ontology of the system here presented, valid for any level well above the Planck scale.

P_{17}. $(\forall e)_E(\exists \mathcal{W})(\mathcal{W}$ is a real function $\mathcal{W}: E \rightarrow \Re)$.

P_{18}. If $\text{Comp}(e) = \{e_1, e_2, ..., e_n\}$ then $\mathcal{W}(e) = \mathcal{W}(e_1) + \mathcal{W}(e_2) + ... + \mathcal{W}(e_n)$, where all e_i are basic events.

Let us define:

D_{14}. $\text{Eff}(e) = \{e_i$, such that e_i is an effect of $e\}$

D_{15}. $\mathcal{W}(e) \equiv \text{Card [Eff}(e)\text{]}$: "energy" of e,

Then,

$$\vdash \mathcal{W}(e_1) > \mathcal{W}(e_2) \rightarrow \text{Card [Eff }(e_1)\text{]} > \text{Card [Eff }(e_2)\text{]}.$$

In words, if the energy of an event is greater than the energy of another, then the former event produces more events than the latter (its effect is stronger). It is conceivable, but not necessarily true, that all events might have originated in a single, very energetic event. Notice that the effects of an event can be infinite in number, but this does not imply an infinite energy for the chain, since conservation of energy requires that if there are more than a single effect, the energy is divided among the succesive events, in a similar way as it occurs in a particle cascade. Insofar as there are more than one basic event, they can be differentiated by their sole intrinsic property (energy) and by their relational properties. Composed events (processes), on the other hand, have emergent properties.

I illustrate the above considerations in Fig. 1, which shows a Hasse-like diagram; see Dowker (2013). This diagram is a graph-theoretic representation of a finite partially ordered set. The dots represent events and the arrows indicate the asymmetric link between events. Events connected by successive arrows are proceses. I have added circles centred at each event. The area of these circles represents the energy of the event. Since energy is conserved, at each level[7] of generation, L_i, the total area in linked circles is constant. I do admit spontaneous basic events: these appear in the graphic without being generated by previous events. Global time emerges in the graph as the direction of growth. For the emergence of spatial dimensions see Perez Bergliaffa et al. (1998). After a large number of levels a continuum manifold is a good representation. The clustering of events giving rise to curvature is pictorially indicated in the upper part of the figure.

The transition from clustering to curvature is mediated by energy. If $E' \subset E$ has n elements, then

$$\mathcal{W}(E') = \Sigma_{i=1}^{n} \mathcal{W}(e_i), \quad e_i \in E', \tag{4}$$

and we can introduce an energy density $\rho = \mathcal{W}(E')/V$, where V is the volume of E' in the metric space E. This energy density forms a component of a tensor field on E that is related to the curvature of E by Einstein's field equations. The implementation of this proposal will be presented elsewhere.

[7] Levels are define by space-like classes of events.

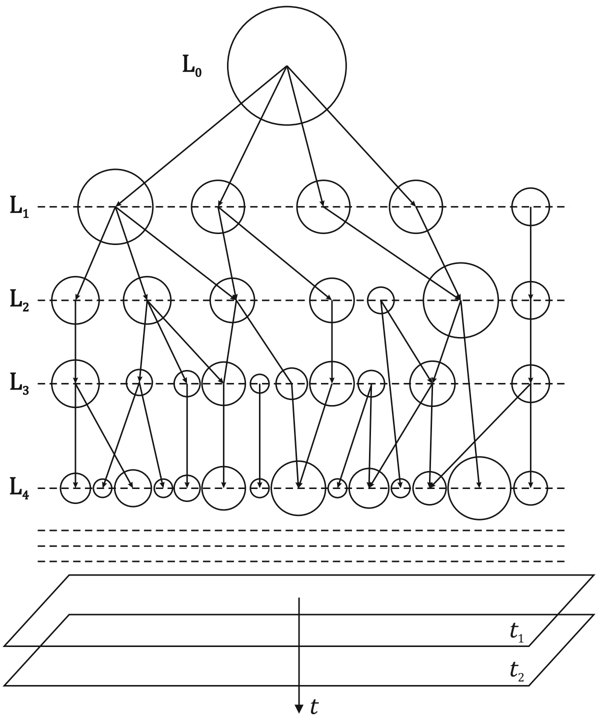

Fig. 1 Graphic representation of discrete event generation and transition to spacetime. The circles around each event represent energy, defined as the capacity to generate new events. See main text for details

The World, under the perspective presented in this paper, would be a maelstrom of events; the things, people, the galaxies of the universe, would arise as a patterned poset in that storm.

Conclusions

In this paper I have outlined a formal ontological system that takes events as primitive individuals. I have shown how both things and continuum spacetime might emerge from a discrete system of basic events. The theory provides foundations for constructive approaches to an ontological theory of spacetime, such as causal set theory. The most basic intrinsic property of events is energy, defined here as the capacity to generate new events. The mode of event generation is something to be investigated by physics and does not pertain to ontology. Higher level properties, as entropy, manifest in the growing process of the net of discrete events. I shall discuss them in a forthcoming communication.

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