Algebr Represent Theor (2013) 16:1001–1015
DOI 10.1007/s10468-012-9343-5

Representation Dimension of Cluster-Concealed Algebras

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Received: 14 May 2011 / Accepted: 16 March 2012 / Published online: 27 April 2012

Abstract We prove that the representation dimension of a cluster-concealed algebra B is three. We compute its representation dimension by showing an explicit Auslander generator for the cluster-tilted algebra.

Keywords Auslander generator · Cluster-concealed · Cluster-tilted algebra · Representation dimension

1 Introduction

The concept of representation dimension for artin algebras was introduced by Auslander [3], motivated by the connection of arbitrary artin algebras with representation finite artin algebras. He expected this notion to give a reasonable way of measuring how far an artin algebra is from being of representation finite type. The representation dimension is a Morita-invariant of artin algebras and characterizes artin algebras of finite representation type. It was shown by Auslander in [3] that an

Presented by Raymundo Bautista.

This work is part of the Phd Thesis of Alfredo Gonzalez Chaio, under the supervision of Sonia Trepode, in course at Universidad Nacional de Mar del Plata. The first author was supported by a grant from CONICET. The second author is a researcher of CONICET, Argentina. We would like to thank the referee for the very useful suggestions that let us simplify some proofs and improve the form of this article.

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algebra A is representation-finite if and only if rep.dim$A \leq 2$. Later, Iyama proved in [14] that the representation dimension of an artin algebra is always finite, using a relationship with quasihereditary algebras. The interest in representation dimension revived when Igusa and Todorov showed that the representation dimension is related to the finitistic dimension conjecture. They proved that if an artin algebra has representation dimension at most three, then its finitistic dimension is finite [13]. Recently, Rouquier showed in [17] an exterior algebra with representation dimension 4. In fact, he has constructed examples of algebras with arbitrarily large representation dimensions.

On the other hand, Cluster algebras were introduced by Fomin and Zelevinsky [18]. Later, Marsh-Reineke-Zelevinsky found that there is a deep connection between cluster algebras and quiver representations. Buan et al. [6] defined the cluster category and developed a tilting theory using a special class of objects, namely the cluster tilting objects. In [7], Buan-Marsh-Reiten introduced the cluster-tilted algebras as endomorphism algebras $\mathrm{End}_{\mathcal{C}}(T)$ of a cluster-tilting object T in a cluster category $\mathcal{C}$. These algebras are connected to tilted algebras, which are the algebras of the form $\mathrm{End}_H(T)$ for a tilting module T over a hereditary algebra H. This last fact motivates us to investigate the relationship between the module theory of cluster-tilted algebras and the module theory of hereditary algebras.

A cluster concealed algebra is given by $B = \mathrm{End}_{\mathcal{C}}(\tilde{T})$ where $\tilde{T}$ is a cluster tilting object induced by a preprojective tilting H-module. The aim of this paper is to compute the representation dimension of cluster-concealed algebras by showing an explicit Auslander generator. In order to do this, tilting and torsion theory of hereditary algebras became very useful tools. Also the concept of slice and local slice, the last one defined in [1], became a key tool to find Auslander generators for cluster-concealed algebras. The notions of covariantly finite categories [5] are very useful for the proof, together with their relationship with torsion pairs in tilted algebras [4].

In Section 2, we give some notations and preliminary concepts needed for proving our main result. In Section 3, we present our main theorem and some previous results required for proving it.

2 Preliminaries

Through this paper we are going to use the following notation. A will denote a finite dimensional algebra over an algebraically closed field and mod A the category of all finitely generated right A-modules. By ind$A$ we will denote the full subcategory of indecomposable modules in mod A and we represent the subcategory generated by a module M by add$M$.

We will denote by H a finite dimensional hereditary algebra and by $\mathcal{D}^b(H)$ the bounded derived category of mod H. We will denote by [ ] the shift functor. We will identify the objects concentrated in degree zero with the corresponding H-modules.

2.1 Tilting Theory

We start this section by giving the definition of tilting module. For more details on tilting modules see [12]. Let A be an algebra and T an A-module. T is said to be a tilting module in mod A if it satisfies the following conditions

(a) $pdT \leq 1$.
(b) $\mathrm{Ext}^1_A(T, T) = 0$.
(c) A_A admits a copresentation $0 \to A \to A T_0 \to T_1 \to 0$ with T_0 and T_1 in add$T$.

If a module satisfies condition (b), we say that the module is exceptional. Equivalently, we say that T is a tilting module if $pdT \leq 1$, is exceptional and has n non-isomorphic indecomposable direct summands, where n is the number of non-isomorphic simple modules.

Remark 2.1 We recall that for a hereditary algebra H, an H-module T is said to be a tilting module if T is exceptional and at least n of its indecomposable direct summands are non-isomorphic.

A tilting module is said to be basic if all its direct summands are non-isomorphic. The endomorphism ring of a tilting module over a hereditary algebra is said to be a **tilted algebra**. In particular, hereditary algebras are tilted algebras.

We recall that a **path** from X to Y is a sequence $X = X_0 \to X_1 \to \ldots \to X_t = Y$ with $t > 0$ of non-zero non-isomorphisms between indecomposable modules. Given $X, Y \in \mathrm{ind}A$, we say that X is a **predecessor** of Y or that Y is a **successor** of X, provided that there exists a path from X to Y. A tilting module T is **convex** if, for a given pair of indecomposable summands of T, X Y in add T, any path from X to Y contains only indecomposable modules in add T. Following [2], we say that a set Σ_T in mod A is a **complete slice** if $T = \bigoplus_{M \in \Sigma_T} M$ is a convex tilting module with $\mathrm{End}_A T$ hereditary. For the original definition of complete slice, we refer the reader to [15, 16]. Tilted algebras are characterized by the existence of a complete slice in its module category.

For a given tilting module T in mod H there exists two full disjoint subcategories of mod H, namely

$$\mathcal{F}(T) = \{X \in \mathrm{mod}H \text{ such that } \mathrm{Hom}_H(T, X) = 0\}$$

$$\mathcal{T}(T) = \{X \in \mathrm{mod}H \text{ such that } \mathrm{Ext}^1_H(T, X) = 0\}$$

the free torsion class and the torsion class, respectively.

Furthermore, if T is a convex tilting module, then $\mathrm{mod}H = \mathcal{F}(T) \bigcup \mathcal{T}(T)$. We have that, in this case, $\mathcal{F}(T)$ is closed under predecessors and $\mathcal{T}(T)$ is closed under successors.

2.2 Cluster Categories and Cluster Tilted Algebras

For the convenience of the reader, we start this section recalling some definitions and results of cluster categories from [6]. Let $\mathcal{C}$ be the **cluster category** associated to H given by $\mathcal{D}^b(H)/F$, where F is the composition functor $\tau_{\mathcal{D}}^{-1}[1]$. We represent by $\tilde{X}$ the class of an object X of $\mathcal{D}^b(H)$ in the cluster category. We recall that $\mathrm{Hom}_{\mathcal{C}}(\tilde{X}, \tilde{Y}) = \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\mathcal{D}^b(H)}(X, F^iY)$. We also recall that $S = \mathrm{ind} H \bigcup H[1]$ is a fundamental domain of $\mathcal{C}$. If X and Y are objects in the fundamental domain, then we have that $\mathrm{Hom}_{\mathcal{D}^b(H)}(X, F^iY) = 0$ for all $i \neq 0, 1$. Moreover, any indecomposable object in $\mathcal{C}$ is of the form $\tilde{X}$ with $X \in S$.

We say that $\tilde{T}$ in $\mathcal{C}$ is a **tilting object** if $\mathrm{Ext}^1_{\mathcal{C}}(\tilde{T}, \tilde{T}) = 0$ and $\tilde{T}$ has a maximal number of non-isomorphic direct summands. A tilting object in $\mathcal{C}$ has finite summands.

There exists the following nice correspondence between tilting modules and basic tilting objects.

Theorem [6, Theorem 3.3.]

(a) *Let T be a basic tilting object in $\mathcal{C} = \mathcal{D}^b(H)/F$, where H is a hereditary algebra with n simple modules.*

   (i) *T is induced by a basic tilting module over a hereditary algebra H', derived equivalent to H.*
   (ii) *T has n indecomposable direct summands.*

(b) *Any basic tilting module over a hereditary algebra H induces a basic tilting object for $\mathcal{C} = \mathcal{D}^b(H)/F$.*

In [7], Buan, Marsh and Reiten introduced the cluster-tilted algebras as follows. Let $\tilde{T}$ be a tilting object over the cluster category $\mathcal{C}$. We recall that B is a **cluster tilted algebra** if $B = \mathrm{End}_{\mathcal{C}}(\tilde{T})$. It is also known that, if $\tilde{T}$ is a tilting object in $\mathcal{C}$, the functor $\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \ )$ induces an equivalence of categories between $\mathcal{C}/\mathrm{add}(\tau\tilde{T})$ and mod B. We will call this equivalence the [BMR]-equivalence.

Thus using the above equivalence, we can compute $\mathrm{Hom}_B(X', Y')$ in terms of the cluster category $\mathcal{C}$ as follows:

$$Hom_B(\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}), \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{Y})) \simeq \mathrm{Hom}_{\mathcal{C}}(\tilde{X}, \tilde{Y})/\mathrm{add}(\tau\tilde{T}),$$

where $X' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X})$ and $Y' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{Y})$ are B-modules.

Remark 2.2 Suppose $X' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X})$ and $Y' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{Y})$. If for every $f : \tilde{X} \to \tilde{Y}$, f factors through $\mathrm{add}(\tau\tilde{T})$ in $\mathcal{C}$ then we have that $Hom_B(X', Y') = 0$.

2.3 Covariantly and Contravariantly Finite Subcategories

In this section, we recall some facts on approximation morphisms, and covariantly and contravariantly finite categories from [5] (also see [4]).

First we begin by recalling the notion of approximations. Let $\mathcal{X}$ be a full subcategory of mod A closed under direct summands and isomorphisms. We say that Y is covariantly finite over $\mathcal{X}$ if there exists a morphism $g : Y \to X$, with $X \in \mathcal{X}$, such that $\mathrm{Hom}_A(g, X') : \mathrm{Hom}_A(X, X') \to \mathrm{Hom}_A(Y, X')$ is surjective for all $X' \in \mathcal{X}$.

Dually we say that an A-module Y is contravariantly finite over $\mathcal{X}$ if there exists a morphism $f : X \to Y$, with $X \in \mathcal{X}$, such that $\mathrm{Hom}_A(X', f) : \mathrm{Hom}_A(X', X) \to \mathrm{Hom}_A(X', Y)$ is surjective for all $X' \in \mathcal{X}$. Furthermore, we say that the morphism $f : X \to Y$ is a right $\mathcal{X}$-approximation of Y and that the morphism $g : Y \to X$ is a left $\mathcal{X}$-approximation of Y. A morphism $f : B \to C$ is right minimal if $g : B \to B$ is an endomorphism, and if $f = fg$ then g is an automorphism. A right $\mathcal{X}$-approximation $f : X \to Y$ of Y is minimal if f is right minimal. Two right minimal $\mathcal{X}$-approximations are isomorphic. Moreover, by [5, Proposition 3.9], an A-module Y is covariantly finite over $\mathcal{X}$ if and only if there exists a left $\mathcal{X}$-approximation of Y.

The category $\mathcal{X}$ is called covariantly finite in mod A if every A-module Y has a left minimal $\mathcal{X}$-approximation. Contravariantly finite subcategories are defined dually but we will not use them.

Recall from [4] that $\mathrm{Gen}(T) = \mathcal{T}(T)$ is covariantly finite.

2.4 Representation Dimension

We recall that an A-module M is a generator for mod A if for each $X \in$ mod A there exists an epimorphism $M' \to X$ with $M' \in \mathrm{add}(M)$. Observe that A is a generator for mod A. Dually, we say that an A-module M is a cogenerator if for each $Y \in$ mod A there exists a monomorphism $Y \to M'$ with $M' \in \mathrm{add}(M)$. Note that DA is a cogenerator for mod A. In particular, any module M containing every indecomposable projective and every indecomposable injective module as a summand is a generator-cogenerator module for mod A.

The original definition of representation dimension (we will denote it by rep.dim) of an artin algebra A is due to Auslander. For more details on this topic, we refer the reader to [3]. The following is a useful characterization of representation dimension, in the case that A is a non semisimple algebra, also due to Auslander. The representation dimension of an artin algebra is given by

$$\mathrm{rep.dim} A = \inf\{\mathrm{gl.dim}\, \mathrm{End}_A(M) \mid M \text{ is a generator-cogenerator for mod} A\}.$$

A module M that reaches the minimum in the above definition is called an **Auslander generator** and $\mathrm{gl.dim}\, \mathrm{End}_A(M) = \mathrm{rep.dim} A$ if M is an Auslander generator.

The representation dimension can also be defined in a functorial way, which will be more convenient for us. The next definition (see [2, 10, 11, 17]) will be very useful for the rest of this work.

Definition 2.3 The representation dimension rep.dim$A$ is the smallest integer $i \geq 2$ such that there is a module $M \in \mathrm{mod} A$ with the property that, given any A-module X,

(a) there is an exact sequence

$$0 \to M^{i-1} \to M^{i-2} \to \ldots \to M^1 \xrightarrow{f} X \to 0$$

with $M^j \in \mathrm{add}(M)$ such that the sequence

$$0 \to \mathrm{Hom}_A(M, M^{i-1}) \to \ldots \to \mathrm{Hom}_A(M, M^1) \to \mathrm{Hom}_A(M, X) \to 0$$

is exact.

(b) there is a exact sequence

$$0 \to X \xrightarrow{g} M'_1 \to M'_2 \to \ldots \to M'_{i-1} \to 0$$

with $M'_j \in \mathrm{add}(M)$ such that the sequence

$$0 \to \mathrm{Hom}_A(M'_{i-1}, M) \to \ldots \to \mathrm{Hom}_A(M'_1, M) \to \mathrm{Hom}_A(X, M) \to 0$$

is exact.

Following [10], we say that the module M has the $i-1$-resolution property and that the sequence $0 \to M^{i-1} \to M^{i-2} \to \ldots \to M^1 \to X \to 0$ is an add$M$-approximation of X of length $i-1$. Observe, that $f : M^1 \to X$ is a right add(M)-approximation of X and $g : X \to M'_1$ is a left add(M)-approximation of X.

Remark 2.4 Either condition (a) or (b) imply that gl.dim $\mathrm{End}_A(M) \leq i$ ([11, Lemma 2.2]). Then, if $M \in \mathrm{mod}\,A$ and $i \geq 2$, the following statements are equivalent.

- M satisfies (a) and (b) of the definition.
- M satisfies (a) and M contains an injective cogenerator as a direct summand.
- M satisfies (b) and M contains a projective generator as a direct summand.

3 Representation Dimension for Cluster Concealed Algebras

In this section we will present our main result. Let $\tilde{T}$ be a tilting object in a cluster category $\mathcal{C}$ and let $B = \mathrm{End}_{\mathcal{C}}(\tilde{T})$ be the associated cluster-tilted algebra. As we are interested in computing the representation dimension of cluster concealed algebras, we will assume through this section that $\tilde{T}$ is induced by a preprojective tilting module T on mod H. Then $\tilde{T}$ will be a tilting object in the transjective component of $\mathcal{C}$. We will consider H, and thus B, of infinite representation type. Then, without loss of generality, we may choose T and τT without projective summands.

For a given complete slice Σ on a component of the hereditary algebra H, we can always construct a convex tilting module of the form $U = \bigoplus_{E \in \Sigma} E$. We will not distinguish between the complete slice Σ and the convex tilting module U. We are going to identify both by Σ. Let Σ be a complete slice on a preprojective component of the hereditary algebra H, such that $\Sigma \subset \mathcal{T}(T)$. Thus, $\mathcal{T}(\Sigma) \subset \mathcal{T}(T)$. Note that this condition implies that Σ does not have projective summands.

Let $\Sigma' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{\Sigma})$, be the image of $\tilde{\Sigma}$ by the [BMR]-equivalence [7], which is a local slice in mod B [1].

Using the [BMR]-equivalence and the fact that $(\mathcal{F}(\Sigma), \mathcal{T}(\Sigma))$ is a split torsion theory and ind $H \cup H[1]$ is a fundamental domain for $\mathcal{C}$, we get a partition of the indecomposable modules in mod B as follows. Let Y be an indecomposable module in mod B, then Y belongs to one of the following classes.

(1) $Y \simeq \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{Q})$ with $Q = P[1]$ where P is a projective indecomposable in mod H.
(2) $Y \simeq \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{G})$ with $G \in \mathcal{F}(\Sigma) \subset \mathrm{mod}\,H$.
(3) $Y \simeq \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X})$ with $X \in \mathcal{T}(\Sigma) \subset \mathrm{mod}\,H$.

Observe that if Y is indecomposable also Q, X or G are indecomposable.

In the class (1), we have a finite number of non-isomorphic indecomposable B-modules Y, since the number of non-isomorphic indecomposable projective modules in mod H is finite. In the class (2), we also have a finite number of indecomposables B-modules. In fact, since we choose Σ in the preprojective component of mod H, we have that $\mathcal{F}(\Sigma)$ has only a finite number of non isomorphic indecomposable H-modules.

Consider the B-modules, $Q' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau\tilde{H})$ and $\mathcal{G} = \bigoplus_G \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{G})$ with G non-isomorphic indecomposable modules in $\mathcal{F}(\Sigma)$.

We will prove that the B-module $M' = \Sigma' \oplus \mathcal{G} \oplus Q'$ is an Auslander generator for mod B. Through this section we will establish that M' satisfy condition (a) of Definition 2.3 for $i = 3$ and is a generator-cogenerator in mod B. Then by Remark 2.4 M' will be an Auslander generator.

First we begin by proving that M' satisfies the first part of (a), that is, we are going to show that there exists a short exact sequence

$$0 \to M'_2 \to M'_1 \to X' \to 0$$

for every $X' \in \mathrm{mod}\, B$, with M'_2 and $M'_1 \in \mathrm{add}(M')$.

Clearly, M' trivially approximates the modules in classes (1) and (2).

Hence we only need to prove the existence of such a sequence for the modules in class (3). We will show that there exists a short exact sequence $0 \to K \to E \to X \to 0$ in mod H with $K, E \in add(\Sigma)$ that induces an exact sequence in mod B

$$0 \to K' \to E' \to X' \to 0$$

with $E', K' \in add(\Sigma')$ for every X' in class (3). Furthermore, we will prove that this induced sequence is an $add(M')$-approximation sequence for any X' in class (3). We start by showing the existence of the exact sequence $0 \to K' \to E' \to X' \to 0$.

Let X' be in class (3). Then we have that $X' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X})$ with $X \in \mathcal{T}(\Sigma) \subset \mathrm{mod} H$.

Now, since $X \in \mathcal{T}(\Sigma) = \mathrm{Gen}(\Sigma)$ there exists an epimorphism

$$f : E \to X \to 0$$

which is a right add(Σ)-approximation of X. Since H is hereditary and $X \in \mathrm{Gen}(\Sigma)$ by [2, Proposition p.432], we have that $K = \mathrm{Ker}\, f \in \mathrm{add}(\Sigma)$ and hence that the short exact sequence

$$0 \to K \to E \to X \to 0$$

is an add(Σ)-resolution of length two for X in mod H. This short exact sequence induces a triangle in $\mathcal{D}^b(H)$

$$K \to E \to X \to K[1]$$

which induces a triangle in the cluster category $\mathcal{C}$

$$\tilde{K} \to \tilde{E} \to \tilde{X} \to \tau\tilde{K} \quad (*_1)$$

We will prove that the image of this triangle by the functor $\mathrm{Hom}_{\mathcal{C}}(\tilde{T},\ )$ induces the desired sequence in mod B. Before proving that, we establish the following technical result.

Lemma 3.1 *Let Σ be a convex tilting module without projective summands and let E be a module in $add(\Sigma)$ and $X \in \mathcal{T}(\Sigma)$. Then $Hom_{\mathcal{C}}(\tilde{X}, \tau\tilde{E}) = Hom_{\mathcal{D}^b(H)}(X, E[1])$.*

Proof Recall that Σ does not have projective summands. Since $E \in \mathrm{add}(\Sigma)$, E is not a projective module and thus $\tau E \in \mathrm{mod}\, H$.

Since X and τE are H-modules, then

$$\mathrm{Hom}_{\mathcal{C}}(\tilde{X}, \tau\tilde{E}) = \mathrm{Hom}_{\mathcal{D}^b(H)}(X, \tau E) \oplus \mathrm{Hom}_{\mathcal{D}^b(H)}(X, E[1])$$

and all the other summands are zero. We shall prove that $\mathrm{Hom}_{\mathcal{D}^b(H)}(X, \tau E)$ is zero.

Since X and τE are H-modules, we have that

$$\mathrm{Hom}_{\mathcal{D}^b(H)}(X, \tau E) = \mathrm{Hom}_H(X, \tau E).$$

On the other hand, since $E \in \mathrm{add}(\Sigma)$, we have that $\tau E \in \mathcal{F}(\Sigma)$. By hypothesis, we have that $X \in \mathcal{T}(\Sigma)$. Hence $\mathrm{Hom}_{\mathcal{D}^b(H)}(X, \tau E) = \mathrm{Hom}_H(X, \tau E) = 0$ since $(\mathcal{F}, \mathcal{T})$ is a torsion pair. Then the result follows. □

We are now in a position to prove that the modules in class (3) are generated by modules in $\mathrm{add}(\Sigma')$. The following proposition shows that M' satisfies the first part of Definition 2.3(a).

Proposition 3.2 *Let T be a preprojective tilting module over mod H. Let Σ be a complete slice on the preprojective component of $\Gamma(mod H)$ such that Σ is contained on $\mathcal{T}(T)$. Then*

- (i) *The class* (3) *is generated by Σ' in mod B.*
- (ii) *There exists a short exact sequence of the form $0 \to K' \to E' \to X' \to 0$ with $E', K' \in add(\Sigma')$ for every $X' \in$* (3).

Proof Let $X' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}) \in (3)$. We may assume that $X \notin \mathrm{add}(\Sigma)$, otherwise there is nothing to prove. We have a triangle $\tilde{K} \to \tilde{E} \to \tilde{X} \to \tau\tilde{K}$ in $\mathcal{C}$, with K, E in $\mathrm{add}(\Sigma)$, and the functor $\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \ )$ induces an equivalence between $\mathcal{C}/\mathrm{add}(\tau\tilde{T})$ and $\mathrm{mod} B$.

Applying this functor to the triangle, we obtain a long exact sequence in mod B as follows

$$... \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau^{-1}\tilde{X}) \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{K}) \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{E}) \to$$

$$\to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}) \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau\tilde{K}) \to ... \quad (*)$$

We want to construct an epimorphism from a module in $\mathrm{add}(\Sigma')$ to $X' \in (3)$. We have that $\Sigma' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{\Sigma})$ and that E is an $\mathrm{add}(\Sigma)$-approximation of X. Consider $E' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{E})$ and $f' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{f})$. We will prove that $f' : E' \to X'$ is an epimorphism. In fact, we shall see that $\mathrm{Hom}_B(\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}), \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau\tilde{K})) = 0$.

To do this it is enough, by Remark 2.2, to show that any morphism $h : \tilde{X} \to \tau\tilde{K}$ in $\mathcal{C}$ factors through $add(\tau\tilde{T})$.

Since $K \in \mathrm{add}(\Sigma)$, by Lemma 3.1, it follows that $h \in \mathrm{Hom}_{\mathcal{D}^b(H)}(X, K[1])$. Since $F\tau T = T[1]$ in $\mathcal{C}$, it is enough to show that h factors through $T[1]$ in $\mathcal{D}^b(H)$.

Without loss of generality we may assume that there exists H' hereditary, derived equivalent to H, such that X, $T[1]$ and $K[1]$ are H'-modules. In fact, take $H' = \mathrm{End}_H \tau^{-1}\Sigma$. It follows that $T[1]$ can be identified with a tilting H'-module and $\mathcal{T}(T[1])$ with a full subcategory of $\mathrm{mod} H'$.

Moreover, since K is in $\mathrm{add}(\Sigma)$, $K[1]$ will be a injective H'-module. Since $X \in \mathrm{mod}\ H$, we have that

$$0 = \mathrm{Hom}_{\mathcal{D}^b(H)}(\tau^{-1}T[1], X) \simeq \mathrm{Hom}_{\mathcal{D}^b(H')}(\tau^{-1}T[1], X) = \mathrm{Hom}_{H'}(\tau^{-1}T[1], X)$$

since $\tau^{-1}T[1]$ and X are H'-modules. Then we get that $X \in \mathcal{F}(\tau^{-1}T[1]) \subseteq \mathrm{mod}\ H'$. Now, since $X \in \mathcal{F}(\tau^{-1}T[1]) = \mathrm{Cogen}(T[1]) \subset \mathrm{mod}\ H'$, there exists a monomorphism $g : X \to T'$ with $T' \in \mathrm{add}(T[1])$. Since $K[1]$ is an injective H' module $\mathrm{Hom}_{H'}(g, K[1]) : \mathrm{Hom}_{H'}(T', K[1]) \to \mathrm{Hom}_{H'}(X, K[1])$ is surjective. Then there exists $k : T' \to K[1]$ such that $kg = h$, it is, we have the following commutative diagram

$$\begin{array}{ccc} X & \xrightarrow{h} & K[1] \\ & \searrow & \uparrow \\ & & T' \end{array}$$

that is, h factors through $\mathrm{add}(T[1])$. It follows that $\tilde{h} : \tilde{X} \to \tau\tilde{K}$ factors through $\mathrm{add}(\tau\tilde{T})$.

Therefore we have a long exact sequence

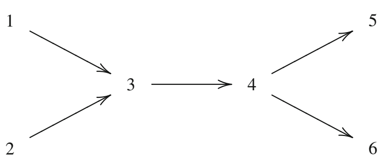
$$\begin{array}{ccccccc} \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{E}) & \xrightarrow{f'} & \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}) & \longrightarrow & \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau\tilde{K}) & \longrightarrow & \ldots \\ & & & \searrow \ \nearrow & & & \\ & & & 0 & & & \end{array}$$

and it follows that $f' : \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \widetilde{E}) \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \widetilde{X})$ is an epimorphism with $E' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \widetilde{E}) \in \mathrm{add}(\Sigma')$. This finishes the proof of (i).

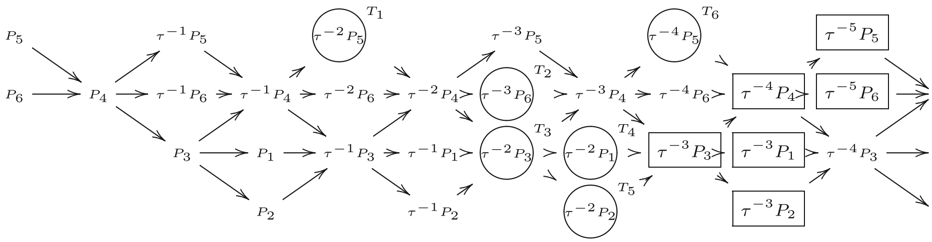
It only remains to show that the morphism $\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tau^{-1}\tilde{X}) \to \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{K})$ vanishes. To prove that, we note that $\tau^{-1}\Sigma$ is a complete slice contained in $\mathcal{T}(T)$ and $\tau^{-1}X \in \mathcal{T}(\tau^{-1}\Sigma)$. By (i) it follows that any $h \in \mathrm{Hom}_{\mathcal{C}}(\tau^{-1}\tilde{X}, \tilde{K})$ factors through $\mathrm{add}(\tau\tilde{T})$ in $\mathcal{C}$. Then we get that $0 \to K' \to E' \to X' \to 0$ is an exact sequence. □

Now we show an example which illustrates the situation in the above proposition, we consider a hereditary algebra H, a preprojective tilting module T, a complete slice Σ, and the algebra $H' = End_H \tau^{-1}\Sigma$.

Example 3.3 Let H be the hereditary algebra $\tilde{D}_5$ given by the quiver



The preprojective component of $\Gamma(\mathrm{mod} H)$ is the following



Consider $T = \tau^{-2}P_5 \oplus \tau^{-3}P_6 \oplus \tau^{-2}P_3 \oplus \tau^{-2}P_1 \oplus \tau^{-2}P_2 \oplus \tau^{-4}P_5$. We have that T is a preprojective tilting H-module. Let $\Sigma = \tau^{-3}P_3 \oplus \tau^{-4}P_4 \oplus \tau^{-3}P_1 \oplus \tau^{-3}P_2 \oplus \tau^{-5}P_5 \oplus \tau^{-5}P_6$. We observe that Σ is a complete slice contained in $\mathcal{T}(T)$. In this example we can observe that the indecomposable modules on class (2) are obtained from the indecomposable modules which are predecessors of the slice Σ. The modules in class (3) are those obtained from the successors of Σ. Finally, we have the modules of class (3) which are obtained of the projective's shifts.

For example, if we choose $X = \tau^{-4}P_3$ we have the following add(Σ)-resolution of X

$$0 \to \begin{smallmatrix} 2 & & 2 \\ & 5 & \\ & 5 & \\ 3 & & 3 \end{smallmatrix} \to \begin{smallmatrix} 2 & & 2 \\ & 5 & \\ & 6 & \\ 3 & & 3 \end{smallmatrix} \bigoplus \begin{smallmatrix} 1 & & 2 \\ & 3 & \\ & 3 & \\ 2 & & 2 \end{smallmatrix} \bigoplus \begin{smallmatrix} 2 & & 1 \\ & 3 & \\ & 3 & \\ 2 & & 2 \end{smallmatrix} \to \begin{smallmatrix} 3 & & 3 \\ & 6 & \\ & 7 & \\ 4 & & 4 \end{smallmatrix} \to 0$$

where $K = \tau^{-3}P_3$ and $E = \tau^{-4}P_4 \oplus \tau^{-3}P_1 \oplus \tau^{-3}P_2$.

Another example, this time we choose a preinjective module $\tilde{X} = \tilde{I}_1$ we have the following add(Σ)-resolution of X

$$0 \to (\Sigma_1)^3 \to \Sigma_5 \oplus \Sigma_6 \oplus (\Sigma_4)^2 \oplus \Sigma_3 \to I_1 \to 0.$$

Consider H' the hereditary algebra $\mathrm{End}_H(\tau^{-1}\Sigma)$, then H' is derived equivalent to H, and the objects in the derived category of H, $T[1]$ and $\Sigma[1]$ can be identified with H'-modules. In fact, let $X \in \mathcal{T}(\tau^{-1}\Sigma)$ then X can be identified with an H'-module.

We will prove now that M' satisfies the second part of (a). We know that M' trivially approximates the modules in the classes (1) and (2), so it is enough to prove that

$$0 \to \mathrm{Hom}_B(N', K') \to \mathrm{Hom}_B(N', E') \to \mathrm{Hom}_B(N', X') \to 0$$

is exact for any $N' \in$add(M'), $X' \in (3)$.

Since $\mathrm{Hom}_B(M', \ \ )$ is left exact, we only need to prove that any morphism $h' : M'_1 \to X'$ with $M'_1 \in \mathrm{add}(M')$ factors trough E', that is there exists $k' : M'_1 \to E'$ such that $h' = f'k'$ or equivalently that the morphism

$$\mathrm{Hom}_B(M', E') \stackrel{\mathrm{Hom}_{B(M',f')}}{\longrightarrow} \mathrm{Hom}_B(M', X')$$

is an epimorphism.

In order to do this, we need the next technical result.

Lemma 3.4 *Let H be a hereditary algebra and Σ be a complete slice. Then. for any indecomposable H-module G in $\mathcal{F}(\Sigma)$ and any indecomposable H-module X in $\mathcal{T}(\Sigma)$, we have that*

$$Hom_{\mathcal{C}}(\tilde{G}, \tilde{X}) = Hom_H(G, X).$$

Proof We can assume that $\tilde{G}$ and $\tilde{X}$ are in the fundamental domain of $\mathcal{C}$. Therefore, by [6], $\mathrm{Hom}_{\mathcal{C}}(\tilde{G}, \tilde{X}) = \mathrm{Hom}_{\mathcal{D}^b(H)}(G, X) \oplus \mathrm{Hom}_{\mathcal{D}^b(H)}(G, F(X))$. We shall prove that $\mathrm{Hom}_{\mathcal{D}^b(H)}(G, F(X)) = 0$.

If X is an injective H-module, then $\tau^{-1}X = P[1]$ with P a indecomposable projective module in $\mathrm{mod}H$, then $F(X) = \tau^{-1}X[1] = P[2]$ and

$$\mathrm{Hom}_{\mathcal{D}^b(H)}(G, F(X)) = \mathrm{Hom}_{\mathcal{D}^b(H)}(G, P[2]) = \mathrm{Ext}^2_H(G, P) = 0,$$

since H is hereditary. Otherwise if X is not an injective H-module , $\tau^{-1}X \in \mathrm{mod}H$. Then

$$\mathrm{Hom}_{\mathcal{D}^b(H)}(G, \tau^{-1}X[1]) = \mathrm{Ext}^1_H(G, \tau^{-1}X) \simeq D\mathrm{Hom}_H(\tau^{-1}X, \tau G),$$

where the last isomorphism is given by the Auslander-Reiten formula.

We may assume that G is not a projective H-module, since otherwise $\mathrm{Ext}^1_H(G, \tau^{-1}X) = 0$. Then we have that $\tau G \in \mathcal{F}(\Sigma)$ and $\tau^{-1}X \in \mathcal{T}(\Sigma)$ since $\mathcal{T}(\Sigma)$ is closed under successors and $\mathcal{F}(\Sigma)$ is closed under predecessors. Therefore, $\mathrm{Hom}_H(\tau^{-1}X, \tau G) = 0$. Thus, $\mathrm{Hom}_{\mathcal{D}^b(H)}(G, \tau^{-1}X[1]) = 0$ and $\mathrm{Hom}_{\mathcal{C}}(\tilde{G}, \tilde{X}) = \mathrm{Hom}_{\mathcal{D}^b(H)}(G, X) = \mathrm{Hom}_H(G, X)$ since G and X are H-modules.

□

Now, we are a in position to prove that $f' : E' \to X'$ is an $\mathrm{add}(M')$-approximation of X'.

Proposition 3.5 *Let M' be the B-module, $M' = \Sigma' \oplus Q' \oplus \mathcal{G}$, where $\mathcal{G}$, Q' and Σ' are considered as before. Then the map $Hom_B(N', f') : Hom_B(N', E') \to Hom_B(N', X')$ is an epimorphism for every module $X' \in (3)$, every $N' \in add(M')$ and where E' is the $add(\Sigma')$-approximation constructed before.*

Proof Let $X' \in (3)$. Again as in Proposition 3.2, we may assume that $X' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, X) \notin \mathrm{add}(\Sigma')$ since otherwise there is nothing to prove. Then $X \in \mathcal{T}(\tau^{-1}\Sigma)$.

Let N' be an indecomposable direct summand of M'.

We will prove that the result holds for N' in the following two cases:

(i) $N' \in \mathrm{add}(\Sigma' \oplus \mathcal{G})$
(ii) $N' \in \mathrm{add}(Q')$

Then the Proposition follows for M' by the additivity of the functor $\mathrm{Hom}_B(N',\ )$.

Assume that $N' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{N}) \in \mathrm{add}(\Sigma' \oplus \mathcal{G})$. Using the [BMR]-equivalence, we have that $\tilde{N}$ is induced by an indecomposable H-module $N \in \mathcal{F}(\tau^{-1}\Sigma)$. We are going to show that there exists a commutative diagram

$$\begin{array}{ccc} & & N' \\ & \swarrow & \downarrow \\ E' & \to & X' \end{array}$$

for every non-zero map $h' : N' \to X'$. That is, there exists a $k' : N' \to E'$ in mod$B$ such that $f'k' = h'$.

Consider $h' : N' \to X'$. Hence,

$$h' \in \mathrm{Hom}_B(N', X') = \mathrm{Hom}_B(\mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{N}), \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X})) \simeq \mathrm{Hom}_{\mathcal{C}}(\tilde{N}, \tilde{X})/\mathrm{add}(\tau\tilde{T}).$$

Then there exists $\tilde{h} \in \mathrm{Hom}_{\mathcal{C}}(\tilde{N}, \tilde{X})$ such that $h' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{h})$. Since $h' \neq 0$, we may assume that $\tilde{h}$ does not factor through add$(\tau\tilde{T})$ in $\mathcal{C}$. Let us compute $\mathrm{Hom}_{\mathcal{C}}(\tilde{N}, \tilde{X})$

$$\mathrm{Hom}_{\mathcal{C}}(\tilde{N}, \tilde{X}) = \mathrm{Hom}_{\mathcal{D}^b(H)}(N, X) \oplus \mathrm{Hom}_{\mathcal{D}^b(H)}(N, FX)$$

Since $N \in \mathcal{F}(\tau^{-1}\Sigma)$ and $X \in \mathcal{T}(\tau^{-1}\Sigma)$, by Lemma 3.4, we infer that $\mathrm{Hom}_{\mathcal{C}}(\tilde{N}, \tilde{X}) = \mathrm{Hom}_H(N, X)$ and $\tilde{h} : N \to X$ can be identified to a morphism in mod H.

Hence, $h' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{h})$ with $\tilde{h} \in \mathrm{Hom}_H(N, X)$. Since $f : E \to X$ is an add(Σ)-approximation of $X \in$ mod$H$ and Σ is a complete slice in mod H, any morphism from N to X, factorize through E since $\mathcal{T}(\Sigma)$ is covariantly finite, see [4]. Then, there exists $k : N \to E$ such that the following diagram commutes

$$\begin{array}{ccc} & & N \\ & \swarrow & \downarrow \\ E & \to & X \end{array}$$

that is, $fk = h$. This diagram induces the following commutative diagram in $\mathcal{C}$

$$\begin{array}{ccc} & & \tilde{N} \\ & \swarrow & \downarrow \\ \tilde{E} & \to & \tilde{X} \end{array}$$

Applying to the above diagram $\mathrm{Hom}_{\mathcal{C}}(\tilde{T},\ )$ we obtain a diagram as follows

$$\begin{array}{ccc} & & \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{N}) \\ & \swarrow & \downarrow \\ \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{E}) & \to & \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{X}) \end{array}$$

Taking $k' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{k})$, we have that $f'k' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{f}\tilde{k}) = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{h}) = h'$. Then, $h' : N' \to X'$ factors through E'. Observe that $k' \neq 0$, otherwise $\tilde{h}$ would factorize by add$(\tau\tilde{T})$. This finishes the proof of the first case.

It only remains to prove that, if we have a map from a B-module $Q' = Hom_{\mathcal{C}}(\tilde{T}, \tilde{Q})$ to any module $X' \in (3)$, this map factors through E'. It follows from the previous case by replacing H with $H' = \mathrm{End}_H \tau H$. Then $Q = P[1] \cong \tau P$ in $\mathcal{C}$ is identified with the projective H'-module τP. It follows that $\tau P \in \mathcal{F}_{H'}(\Sigma)$ and, if X in not an injective H-module, then $X \in$modH' and $X \in \mathcal{T}_{H'}(\Sigma)$. By (i), it follows that $\mathrm{Hom}_B(\tau P, f')$ is an epimorphism as desired.

In the case that X is an injective H-module, then $FX \in$add $H[2]$. So

$\mathrm{Hom}_{\mathcal{C}}(\tilde{Q}, \tilde{X}) = \mathrm{Hom}_{\mathcal{D}^b(H)}(P([1], X) \oplus \mathrm{Hom}_{\mathcal{D}^b(H)}(P[1], FX) = 0$ Then, there is nothing to prove in this case. This finishes the proof of the proposition. □

By Proposition 3.5 there exists an add(M')-resolution of length at most two, for every module in mod B. Note that we have proved that M' satisfies the condition (a) of Definition 2.3 for $i = 3$, which implies that gl.dim $\text{End}_B(M') \leq 3$. We now proceed to state and prove our main result.

Theorem 3.6 *Let B be a cluster-concealed algebra of infinite representation type. Then rep.dim$B = 3$.*

Proof Let M' be the module of the above proposition. Observe that if I is an injective module in mod B, then $I = \text{Hom}_{\mathcal{C}}(\tilde{T}, \tau^2\tilde{T})$. Since τT does not have projective summands and $\mathcal{F}(T) \subset \mathcal{F}(\Sigma)$, then $\tau^2 T \in \mathcal{F}(\Sigma)$. Therefore any indecomposable injective module in mod B is a direct summand of $\mathcal{G}$. The same occurs for any indecomposable projective $P \in$mod B. In fact, $P = \text{Hom}_B(\tilde{T}, \tilde{T})$ and again T lies in $\mathcal{F}(\Sigma)$. It follows that add($\mathcal{G}$) contains every indecomposable projective and every indecomposable injective module. Then $\mathcal{G}$ is a generator-cogenerator for mod B, and hence M is a generator-cogenerator for mod B. Finally, applying Proposition 3.5, we have that gl.dim $\text{End}_B(M') \leq 3$ getting that rep.dim$B \leq 3$. □

We give now an application of our main theorem. Let A be a basic connected finite dimensional algebra. A is said to be of minimal infinite type if it is of infinite representation type and for each vertex e in the quiver of A, we have that A/AeA is of finite type. Recall that, for any cluster-tilted algebra B with a vertex e in the quiver of B, we have that B/BeB is again a cluster-tilted algebra (see [8] Section 2).

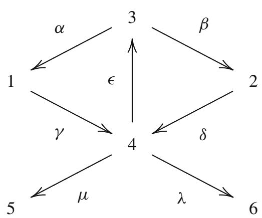
Recall that all minimal cluster-tilted algebras of infinite representation type are endomorphisms algebras of preprojective tilting modules over tame hereditary algebras (see [9]). So, as an immediate consequence of Theorem 3.6, we have the following corollary.

Corollary 3.7 *Let B be a minimal cluster-tilted algebra of infinite representation type. Then rep.dim$B = 3$.*

Finally, we give an example showing that the given proof can not be extended to a tilting module T having regular indecomposable summands.

Example 3.8 Let H be the hereditary algebra of the Example 3.3. Consider the tilting module $T = \tau^{-2}P_5 \oplus \tau^{-3}P_6 \oplus \tau^{-2}P_4 \oplus \tau^{-2}P_1 \oplus \tau^{-2}P_2 \oplus S_3$. Observe that the simple module associated to the vertex 3 is a regular module over mod H.

Let B =$\text{End}_{\mathcal{C}}(\tilde{T})$. The algebra B is given by the quiver



with the relations $\epsilon\alpha = \epsilon\beta = \gamma\epsilon = \delta\epsilon = 0$ and $\alpha\gamma = \beta\delta$.

Now, we are going to show that the existence of the exact sequence $0 \to K' \to E' \to X' \to 0$ in mod B depends on the module X'. We note that, since one of the summands of T is regular, no preprojective complete slice can be entirely contained in $\mathcal{T}(T)$. Therefore, any choice of Σ will not satisfy the hypothesis of Proposition 3.2. So, the conclusion of the proposition does not follow. In fact, the conclusion is false in some cases. For example, if we choose $\Sigma = \tau^{-3} H$, then $\Sigma' = \mathrm{Hom}_{\mathcal{C}}(\tilde{T}, \tilde{\Sigma})$ is given by the following direct sum of indecomposable B-modules indicated by their vector dimension

$$\Sigma' = \Sigma'_1 \oplus \Sigma'_2 \oplus \Sigma'_3 \oplus \Sigma'_4 \oplus \Sigma'_5 \oplus \Sigma'_6 =$$

$$= \begin{array}{ccc} & 1 & \\ 0 & & 1 \\ & 2 & \\ 1 & & 1 \end{array} \bigoplus \begin{array}{ccc} & 1 & \\ 1 & & 0 \\ & 2 & \\ 1 & & 1 \end{array} \bigoplus \begin{array}{ccc} & 1 & \\ 1 & & 1 \\ & 3 & \\ 2 & & 2 \end{array} \bigoplus \begin{array}{ccc} & 1 & \\ 0 & & 0 \\ & 2 & \\ 1 & & 1 \end{array} \bigoplus \begin{array}{ccc} & 1 & \\ 0 & & 0 \\ & 1 & \\ 0 & & 1 \end{array} \bigoplus \begin{array}{ccc} & 1 & \\ 0 & & 0 \\ & 1 & \\ 1 & & 0 \end{array}.$$

Consider $X = I_3$ the indecomposable injective H-module associated to the vertex 3. Then there is an exact sequence

$$0 \to {\Sigma_5}^2 \oplus {\Sigma_6}^2 \oplus \Sigma_4 \oplus \Sigma_3 \to {\Sigma_1}^3 \oplus {\Sigma_2}^3 \to I_3 \to 0$$

in mod H which is an add(Σ)-resolution of I_3.

The induced sequence

$$0 \to {\Sigma'_5}^2 \oplus {\Sigma'_6}^2 \oplus \Sigma'_4 \oplus \Sigma'_3 \to {\Sigma'_1}^3 \oplus {\Sigma'_2}^3 \to I'_3 \to 0$$

is not exact. In fact this sequence is given by

$$0 \longrightarrow \begin{array}{ccc} & 6 & \\ 1 & & 1 \\ & 9 & \\ 5 & & 5 \end{array} \longrightarrow \begin{array}{ccc} & 6 & \\ 3 & & 3 \\ & 12 & \\ 6 & & 6 \end{array} \longrightarrow \begin{array}{ccc} & 1 & \\ 2 & & 2 \\ & 3 & \\ 1 & & 1 \end{array} \longrightarrow 0$$

which is not exact.

Finally, we observe that, for this particular choice of $\Sigma = \tau^{-3} H$, the induced sequence is not exact for every B-module X' in (3). This example shows that the hypothesis of Proposition 3.2 cannot be omitted.

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