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# ON THE BLACK HOLE INNER MECHANICS

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**Abstract**

With the aim of investigating to what extent the statement of the recently proposed laws of black hole inner mechanics hold, here I study the case of higher-curvature theories of gravity. I focus my attention on certain gravity theories that emerge in the low energy limit of string theory. I prove that, in such a scenario, one of the statements of the black hole inner mechanics -the statement about the product of entropies associated to black hole horizons resulting independent of the mass- does not hold. I make use of this observation to speculate about the origin of the quantization of mass.

*Key words:* black holes; holography; general relativity.

**Resumen**

Sobre la mecánica interna de los agujeros negros. Con la motivación de investigar hasta qué punto son válidas las afirmaciones de las leyes de la mecánica interna de los agujeros negros recientemente propuestas, estudio aquí el caso de teorías de gravitación de alta curvatura. Enfoco mi atención en ciertas teorías de gravitación que emergen en el límite de bajas energías de la teoría de cuerdas. Pruebo que, en un escenario tal, una de las afirmaciones de la mecánica interior de los agujeros negros –la afirmación de que el producto de las entropías asociadas a los horizontes de los agujeros negros resulta independiente de la masa– no es válida. Hago uso de esta observación para presentar algunas digresiones sobre el origen de la cuantización de la masa.

*Palabras clave:* agujeros negros; holografía; relatividad general.

## Introduction

Recently, the possibility of formulating the thermodynamics associated to the black hole inner Cauchy horizons was investigated. By exploring a considerably large class of explicit black hole and black (st)ring solutions, the authors of [1] concluded that there exists sufficient evidence suggesting that one can actually formulate a first law of black hole inner mechanics, somehow mimicking what happens with the standard black hole thermodynamics associated to the outer event horizon.

One can begin with the observation that, for all the black hole solutions examined, the following relation holds

$$-dM = T_- dS_- - \Sigma_{a=1} \Omega_-^a dJ_a - \Sigma_{i=1} \Phi_-^i dQ_i, \quad (1)$$

where $M$, $J_a$, and $Q_i$ are the Arnowitt-Deser-Misner quantities associated to the mass, the angular momenta, and the electric charges of the solution, respectively. $\Omega_-^a$ and $\Phi_-^i$ represent the chemical potentials associated to the extensive variables $J_a$ and $Q_i$, i.e. to the $a$-th momenta and $i$-th charge. The potentials $\Omega_-^a$ and $\Phi_-^i$ are defined on the inner horizon. $S_\pm$ in (1) represents the entropy associated to the inner horizon, which in a theory whose gravitational part is given by the Einstein-Hilbert action happens to take the Bekenstein-Hawking form $S_- \sim A_-$, with $A_-$ being the area of the inner Cauchy horizon.

Equation (1) is totally analogous to the standard first law of black hole thermodynamics associated to the external horizon, and a similar relation had been previously observed for the particular case of Kerr black hole. Whether or not the whole idea of thermodynamics associated to the internal Cauchy horizon has a sensible physical interpretation is an open question; however, it could possibly provide an interesting mathematical artifice to work out the microscopic description of black holes.

A second observation of crucial importance to formulate a black hole inner mechanics concerns the product of the areas A- and A+, the latter being the one associated to the outer horizon. It turns out that, for the large class of black holes examined, the product A+A- is always independent of the mass; that is, the quantity A+A- happens to depend only on the quantized charges $J_a$ and $Q_i$, (see [2,3,4,5]). This simple observation led to the statement that the product of the entropy associated to the external and the internal horizons obeys a quantization condition of the form

$$S_+S_- = 4\neq^2 n, \tag{2}$$

with $n$ being an integer number.

This statement, on the other hand, is in consonance with the fact that, in the conformal field theory (CFT) description of black hole microstates, the product S+ S- is given by the difference between the number of right-moving ($N_R$) and left-moving $N_L$) degrees of freedom [6]. Therefore, within the holographic framework, a quantization condition like (2) would simply follow from the level matching condition of the CFT. In fact, there are well known examples of CFT descriptions of black holes thermodynamics for which Cardy-like formulas [7] hold both for the inner and for the outer areas, namely

$$A\pm = 8\sqrt{N_R} \pm 8\sqrt{N_L}, \tag{3}$$

in such a way that the relation S+ S- = $4\pi 2(N_R$-$N_L)$ is obtained by multiplying the areas.

With the aim of investigating to what extent the assertions of the black hole inner mechanics can be taken as general statements, in this paper we analyze how the inclusion of higher-curvature terms in the gravity action can affect the results of [1]. In the last few years we have learned that including higher-curvature terms in gravity actions introduces new features in the context of holography [8-10]; in turn, it would be interesting to investigate whether the statements about the inner black hole mechanics remain valid or, on the contrary, have to be refined in some way when one augments Einstein-Hilbert action with, for instance, $R^2$-terms.

We will observe that, if higher-curvature terms are included, the product of the entropies associated to the internal and external horizons can actually depend on the black hole mass, regardless of whether the theory exhibits diffeomorphism anomaly cf. [11]. At first glance it may seem that this observation is in contradiction with the analysis of [1], however it is not the case: Without a deeper understanding of the holographic description of black hole thermodynamics one could hardly expect the general arguments of [1] to apply without modification to theories that include higher-curvature terms, specially in the case of asymptotically flat solutions for which a description in terms of a $CFT_2$ is uncertain. Nevertheless, it is still an important question to know whether assertions like (1) and/or (2) still hold in more general theories.

**Higher-curvature black holes**

We consider the example of Einstein-Gauss-Bonnet gravity action coupled to electrodynamics in d=5 dimensions. That is, consider the Lanczos-Lovelock gravity action coupled to the Maxwell term

$$S = 1/(16\pi G) \int d^5x\sqrt{g}\, R\, \Big(-2\Lambda - (1/4)\, F_{ij}F^{ij} + \alpha\big(R_{ijkl}R^{ijkl} - 4\, R_{ij}R^{ij} + R^2\big)\Big). \tag{4}$$

Where the first two terms are the Einstein-Hilbert action of General Relativity, $R^i{}_{jkl}$ is the Riemann curvature tensor associated to the space-time metric $g_{ij}$, $\Lambda$ is the cosmological constant, and $F_{ij}$ is the field strength associated to the Maxwell field $A_i$. The three terms of order $R^2$ in the action above represent higher-curvature corrections to five-dimensional General relativity, whose strength is governed by the coupling constant $\alpha$. Constant $\alpha$ has dimensions of length$^2$, so that the $R^2$-terms introduce short-distance corrections to the theory. Constant G is the Newton constant; hereafter we will adopt the notational convention $G=1$.

The equations of motion of this theory are of second order [12] and can be explicitly solved in several scenarios. In particular, the theory admits maximally symmetric solutions with effective cosmological constant

$$|\Lambda_\pm| = l_\pm^{-2} = -3/(2\alpha)\,(1 \pm \sqrt{(1+4\alpha\Lambda/3)}) \tag{5}$$

Here, we will consider $\Lambda_- = 0$; nevertheless, it is worthwhile mentioning that the same considerations hold for the case $\Lambda_- < 0$. About this background the theory is free of ghosts [13].

Einstein-Gauss-Bonnet theory also admits static spherically symmetric solutions of the form [13-18]

$$ds^2 = -F(r)\,dt^2 + dr^2/F(r) + r^2 d\Theta_3^{\,2},$$
$$A = A_0(r)\,dt \qquad (6)$$

where $d\Theta_3^{\,2}$ is the metric of unit 3-sphere, and the function *F(r)* is

$$\frac{F(r) = 1 + r^2/(4\alpha) - r^2/(4a)}{\sqrt{(1 + 16M\alpha r^{-4} - 8Q^2\alpha r^{-6} + 4\alpha\Lambda/3)}} \qquad (7)$$

Maxwell field has the Coulomb from in five dimensions; namely

$$A_0(r) = Q/r^2. \qquad (8)$$

For certain range of parameters, solution (6)-(7) describes asymptotically flat charged black holes of mass $M$ (more precisely, the mass is proportional to the parameter $M$ as in our conventions we have absorbed a numerical factor in the definition of the parameter) and electric charge $Q$. As it happens with Reissner-Nördstrom solution of general relativity, the metric above exhibits two horizons; these are located at

$$r_\pm = M - \alpha \pm \sqrt{((M-\alpha)^2 - Q^2)} \qquad (9)$$

That is, $r+2+ r-2=2(M-\alpha)$, $r+ r-=Q$.

It is interesting to notice that horizons exist provided $|Q|+\alpha<M$. This means in particular that, if $Q=0$, solutions with mass parameter in the range $0<M\leq\alpha$ exhibit a naked curvature singularity. This is reminiscent of the black hole of $d=3$ General Relativity. For $M=0$ metric (6) coincides with that of AdS5 space. That is, in this theory we have a natural mass gap given by $M_0=\alpha$.

## Black hole thermodynamics and inner mechanics

Thermodynamics of black holes in Einstein-Gauss-Bonnet theory was first studied in Ref. [19]. In particular, it can be shown that charged black hole solutions (6)-(7) satisfy the first law of black hole thermodynamics

$$dM = T_+ dS_+ + \Omega_+ dQ \qquad (10)$$

where $\Omega+=\mathbf{A}_0(r_+)$. The black hole entropy is given by

$$S_+ = (\pi^2/2)\,(r^3_+ - 12\alpha r_+) \qquad (11)$$

Notice that this expression corresponds to the Bekenstein-Hawking area law plus a correction of order $\alpha$. This means that, when trying to explore the laws of black hole inner mechanics in this setup one actually has to deal with quantity (11) and its inner analogue (13), and not simply with the areas.

Remarkably, solutions (6)-(7) also satisfy the inner black hole analogue of the first law. In fact, one verifies

$$dM = T_- dS_- - \Omega_- dQ, \qquad (12)$$

with $\Omega\text{-}=A_0(r\text{-})$ and with the inner entropy $S$-being

$$S_- = -(\pi^2/2)\,(r^3_- - 12\alpha r_-) \qquad (13)$$

Then, from expressions (12) and (13) we find

$$S_+S_- = -\pi^4/4\,Q^3 + 6\pi^4\alpha QM - 42\pi^4\alpha^2 Q \qquad (14)$$

We observe from this expression that only the first and the third terms are independent of the mass. The first term, as expected, is of the form $A+A\text{-}\sim Q3$ and independent of $M$, while the next-to-leading term in $\alpha$ does depend on $M$. Besides, the way the mass parameter $M$ enters in expression (14) suggests that, if one assumes that a quantization condition like (2) does hold, then the mass of these $R^2$-corrected black hole solutions should be quantized in units of the fundamental mass-gap $M0=\alpha$.

It is worthwhile mentioning that the same feature is observed in lower- and higher-dimensional black holes. Probably, the simpler example to verify this is consider odd-dimensional Lovelock theory in $d=2k+1$ dimensions with $R^k$-terms. The same feature is found in some black holes in $d=3$ massive gravity, for which a dual description in terms of a CFT2 is well established. Such $d=3$ black holes also exhibit a gap in the mass spectrum.

## Conclusions

The conclusion of this analysis may be presented as a refined version of the black hole inner mechanics statement: When higher-curvature corrections to the Einstein-Hilbert action are considered, the product of the entropy associated to the inner and outer horizons may depend on the mass parameter of the black hole. This is the case regardless the higher-curvature model presents diffeomorphism anomaly or if it does not. However, in the examples examined, when S+S- depends on the mass $M$, the black hole solutions happen to exhibit a gap in the mass spectrum. Then, the arguments in [1] may be reversed and used to suggest that the mass of such higher-curvature black holes has to be quantized in units of a fundamental scale that depends on each theory.

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