Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **420,** 3298–3302 (2012) doi:10.1111/j.1365-2966.2011.20252.x

Dark matter and dark energy accretion on to intermediate-mass black holes

C. Pepe,[1,2]★ L. J. Pellizza[1,2] and G. E. Romero[2,3]

[1]*Instituto de Astronomía y Física del Espacio, Casilla de Correo 67, Suc. 28, 1428 Buenos Aires, Argentina*
[2]*Consejo Nacional de Investigaciones Científicas y Técnicas, Rivadavia 1917, C1033 AAJ, Buenos Aires, Argentina*
[3]*Instituto Argentino de Radioastronomía, C.C.S., 1894 Villa Elisa, Pcia. de Buenos Aires, Argentina*

Accepted 2011 November 23. Received 2011 November 18; in original form 2011 October 12

ABSTRACT

In this work we investigate the accretion of cosmological fluids on to an intermediate-mass black hole at the centre of a globular cluster, focusing on the influence of the parent stellar system on the accretion flow. We show that the accretion of cosmic background radiation and the so-called dark energy on to an intermediate-mass black hole is negligible. On the other hand, if cold dark matter has a non-vanishing pressure, the accretion of dark matter is large enough to increase the black hole mass well beyond the present observed upper limits. We conclude that either intermediate-mass black holes do not exist, or dark matter does not exist, or it is not strictly collisionless. In the latter case, we set a lower limit for the parameter of the cold dark matter equation of state.

Key words: accretion, accretion discs – black hole physics – globular clusters: general.

1 INTRODUCTION

Intermediate-mass black holes (IMBHs) are hypothetical compact objects with masses in the range 10^2–$10^4\,M_\odot$, proposed to explain ultraluminous X-ray sources (ULXs), which exceed by 1–2 orders of magnitude the Eddington limit for the accretion luminosity of stellar mass black holes (Angelini, Loewenstein & Mushotzky 2001; Fabbiano 2006). These objects are thought to form in the centres of globular clusters, where the stellar density is high enough to trigger the runaway merging of stellar mass compact remnants (Gürkan, Freitag & Rasio 2004; Portegies Zwart et al. 2004; Freitag, Gürkan & Rasio 2006). Careful measurements of stellar kinematics and density profiles of globular clusters suggest indeed that some of these clusters may harbour IMBHs, although the evidence is still not conclusive (e.g. Baumgardt, Makino & Hut 2005; Noyola, Gebhardt & Bergmann 2008). The properties of an extended central X-ray source in NGC 6388 also suggest an IMBH as a possible explanation for its central engine (Nucita et al. 2008). The fact that many ULXs in nearby galaxies are associated with globular clusters provides another argument for their existence. However, alternative explanations can be found for these observations, hence the existence of IMBHs is still a matter of discussion.

The detectability of IMBHs depends on their masses, since the effects on their surroundings increase with mass. The mass of an IMBH is determined by its initial mass and the accretion of matter and energy from their surroundings. This accretion can occur in discrete events, such as the infall of the stripped envelope of a star passing near the IMBH (e.g. Miocchi 2007), or by the continuous infall of any media in which the IMBH is immersed, such as cosmological fluids (dark energy, dark matter, the cosmic microwave background) or the intracluster medium of the globular cluster.

The solution of the steady spherical accretion flow of a classical fluid on to an isolated compact object has been obtained by Bondi & Hoyle (1944). Michel (1971) worked out the corresponding solution of the general relativistic fluid equations in the Schwarzschild metric, considering a fluid composed of massive particles, while Babichev, Dokuchaev & Eroshenko (2004) developed the same model for massless particle fluids. In all cases, an important prediction of the models is that the accretion rate scales as the square of the mass of the compact object. Applying Babichev et al. (2004) model to an IMBH, one can readily see that the total mass change due to the accretion of dark energy during the entire life of the IMBH is negligible. However, this might not be the case for dark matter. Several authors have investigated the accretion of dark matter by supermassive black holes (MacMillan & Henriksen 2002; Munyaneza & Biermann 2005; Zelnikov & Vasilev 2005a,b; Peirani & de Freitas Pacheco 2008). Guzmán & Lora-Clavijo (2011) computed time-dependent accretion models, finding that if dark matter is pressureless, it must contribute with a major fraction of the present black hole mass. The same reasoning might be applied to IMBHs, suggesting that they might have accreted hundreds to thousands of solar masses in their lifetime.

However, IMBHs are not isolated objects; they reside in the centre of massive stellar systems. The gravitational field of these systems might influence the accretion flow, enhancing the accretion rate, an effect not taken into account in the models of Babichev et al. (2004) and Guzmán & Lora-Clavijo (2011). Hence, an investigation

★E-mail: carolinap@iafe.uba.ar

of the effects of the host stellar systems of IMBHs on the accretion flow seems relevant, as it would allow us to make more precise predictions about the accretion rates on to IMBHs and their final masses, which can be compared with the available observations.

In this paper we explore simple models for the steady spherical accretion flow on to an IMBH located at the centre of a globular cluster, taking into account the influence of the cluster. In Section 2 we develop our models, based on relativistic fluid dynamics in the gravitational field of both the black hole and the cluster. In Section 3 we apply them to compute the accretion rates for different kinds of fluids and we estimate the final masses of IMBHs. Finally, in Section 4 we discuss the impact of our results on the current knowledge of IMBHs and summarize the results of our work.

2 THE MODEL

Our model is based on the following hypotheses. The accretor is an IMBH of mass M located at the centre of a globular cluster. The mass distribution of the stellar system (cluster plus black hole) is assumed to be spherical, hence the space–time interval can be written as $ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$, where r, θ, ϕ are the usual spherical coordinates, t is the time and ν and λ are functions of r only. These functions are determined by the mass distribution of the stellar system, which in turn can be described by a single function $m(r)$ that gives the mass m enclosed by a sphere of radius r. The Schwarzschild metric is the particular case in which $m(r) = M$, and hence $\nu = -\lambda = \ln(1 - 2M/r)$. Throughout this work, we use the natural system of units in which the gravitational constant G and the speed of light c are equal to unity. The accreted medium can be described as a relativistic fluid with an equation of state $p = \omega\rho$, where p is its pressure and ρ its energy density. Its energy–momentum tensor is $T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor and $u^{\mu} = dx^{\mu}/ds$ is the fluid four-velocity with $u^{\mu}u_{\mu} = 1$.

We aim at computing the accretion rate on to the IMBH, $\dot{M} = -4\pi \lim_{r\to 2M} r^2 T_0^r$, for which we integrate the equations of motion of the fluid, subject to the boundary conditions that far from the globular cluster the density ρ_∞ and pressure p_∞ are constant, and the fluid is at rest ($u_\infty = 0$). Following Babichev et al. (2004), the integration of the time component of the energy–momentum conservation law $T^{\mu\nu}_{;\nu} = 0$ gives the energy conservation equation:

$$(p + \rho)\left(e^{-\nu} + e^{\lambda-\nu}u^2\right)^{1/2} ur^2 e^{\frac{1}{2}(\lambda+3\nu)} = C_1, \tag{1}$$

where C_1 is a constant of integration. Another integral of motion is obtained by projecting the energy–momentum conservation law on the four-velocity, $u_{\mu}T^{\mu\nu}_{;\nu} = 0$. Integrating this equation gives the energy flux equation:

$$e^{\int_\infty^\rho \frac{d\rho'}{\rho'+p}} e^{\frac{1}{2}(\nu+\lambda)} ur^2 = C_2, \tag{2}$$

where C_2 is another constant. Combining equations (1) and (2) we obtain

$$(p + \rho)e^{-\int_\infty^\rho \frac{d\rho'}{p+\rho'}}\left(1 + e^{\lambda}u^2\right)^{1/2} = \rho_\infty + p_\infty. \tag{3}$$

Considering the expression shown above, the accretion rate can be evaluated as

$$\dot{M} = -4\pi(\rho_\infty + p_\infty)C_2, \tag{4}$$

where we have used that in the limit $r \to 2M$ the influence of the globular cluster vanishes and the metric approaches to that of Schwarzschild, for which $\lambda + \nu = 0$.

The constant C_2 determines the accretion flux on to the black hole and can be calculated by fixing the parameters at any point. Michel (1971) has shown that the relativistic hydrodynamical equations for accretion on to an isolated black hole have a critical point, which can be used to determine C_2. To obtain the critical point in our case we follow Michel (1971) and write equations (1) and (2) in their differential form, combining them into

$$\left[(e^{-\nu} + e^{\lambda-\nu}u^2)^{-1}e^{\lambda-\nu}u - \frac{\omega}{u}\right] du + \left[\frac{1}{2}(e^{-\nu} + e^{\lambda-\nu}u^2)^{-1}\left(-e^{-\nu}\nu' + (\lambda' - \nu')u^2e^{\lambda-\nu}\right) + \frac{\lambda'}{2} + \frac{3\nu'}{2} + \frac{2}{r} - (1+\omega)\left(\frac{\nu'}{2} + \frac{\lambda'}{2} + \frac{2}{r}\right)\right] dr = 0, \tag{5}$$

where we used the equation of state. We can see from this expression that if one of the bracketed factors vanishes, there is a turnaround point and the solutions are double valued. Only solutions that pass through the critical point where both brackets vanish simultaneously are single valued, leading to

$$\frac{1}{2}\nu'(r_c)(1 - \omega) - \frac{2\omega}{r_c} = 0, \tag{6}$$

and

$$u_c^2 = \frac{\omega}{1 - \omega}. \tag{7}$$

Here, the subscript 'c' stands for the corresponding variables evaluated at the critical values. It is apparent from equation (7) that there is no critical point for $\omega < 0$ (dark energy). We will treat this case separately later, and focus now on the case $\omega > 0$. Misner, Thorne & Wheeler (1973) have shown that for a spherical stellar system,

$$\nu'(r) = \frac{2m(r)}{r\left[r - 2m(r)\right]}. \tag{8}$$

Hence r_c is defined by the implicit equation:

$$\frac{m(r_c)}{r_c} = \frac{2\omega}{1 + 3\omega}. \tag{9}$$

Evaluating equations (2) and (3) at the critical point, we replace C_2 in equation (4) to obtain the final expression in terms of known quantities:

$$\dot{M} = -\pi(1+\omega)\rho_\infty\left[e^{\nu_c}\left(1 + \frac{\omega}{1-\omega}\right)\right]^{-1/2\omega} \times \left(\frac{\omega}{1-\omega}\right)^{1/2} m^2(r_c)\left(\frac{1+3\omega}{\omega}\right)^2 e^{(1/2)\nu_c}. \tag{10}$$

Note that $\dot{M}$ is proportional to $m^2(r_c)$ which, depending on the location of the critical point, can range from M^2 to $(M_{gc} + M)^2$, where M_{gc} is the mass of the globular cluster. Hence, as $M_{gc} \gg M$, the accretion can be greatly enhanced if the critical point is located in the outer regions of the cluster. Equation (9) shows that for a given cluster, the location of the critical point depends solely on ω.

In the next sections we apply this model to several fluids with different values for the equation of state parameter ω, and analyse the resulting accretion rates.

3 ACCRETION RATES

3.1 Perfect relativistic fluid

As an example we consider the perfect relativistic fluid, for which ω is of the order of unity and equal to the square of the sound speed

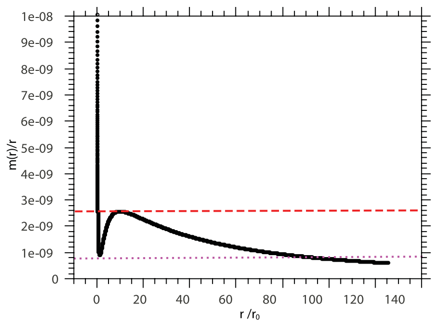

Figure 1. Ratio $m(r)/r$ for a typical globular cluster hosting an IMBH. The dashed and dotted lines define, respectively, the maximum and minimum values for $2\omega/(1 + 3\omega)$ for which equation (9) has three solutions.

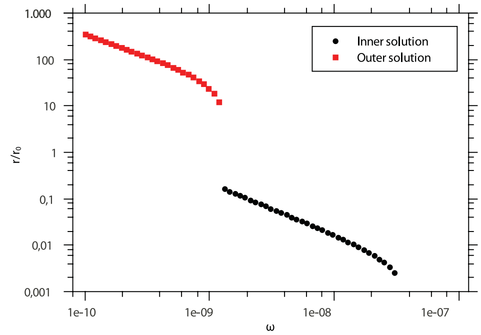

Figure 2. Critical radius for a typical globular cluster/IMBH system, as a function of the equation of state parameter ω. Circles and rectangles correspond to the inner and outer solutions of equation (9), respectively.

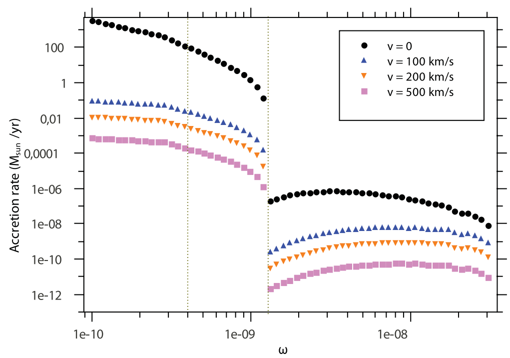

Figure 3. Accretion rate for a typical globular cluster/IMBH system, as a function of the equation of state parameter ω, for different velocities of the cluster relative to the fluid. Dotted vertical lines correspond to the upper and lower limits of the transition region.

in the medium in units of c. This model describes, for example, the cosmic microwave background ($p = \rho/3$). The description of the globular cluster plus black hole mass distribution $m(r)$ was taken from the model of Miocchi (2007), with a scale radius $r_0 =$ 0.35 pc, a concentration $c_{gc} = 1.8$ and a central black hole mass of $M = 3000\,M_\odot$. These parameters, taken from Harris (1996), were chosen to represent the mass distribution of NGC 6388, one of the globular clusters with the strongest evidence in favour of the existence of an IMBH.

Equation (9) shows the individual contribution of the black hole and the globular cluster in a very practical way. A plot of the left-hand side of this equation for our model is shown in Fig. 1. The globular cluster contribution is the small peak at $r/r_0 \sim 10$, while that of the IMBH is the asymptotic growth at $r \to 0$. We can see that for relativistic fluids ($\omega \sim 1$) the right-hand side of equation (9) takes values of the order of unity, several orders of magnitude higher than the maximum globular cluster contribution. The critical radius then occurs at $r \ll r_0$, where the black hole dominates the scene, and we can approximate the metric by that of Schwarzschild and solve for r_c, u_c and $\dot{M}$. The problem in this case reduces to that of an isolated IMBH, and the accretion rate depends only on M^2, as predicted by Babichev et al. (2004). As a toy example, for our model, the accretion rate of the cosmic microwave background is $\dot{M} = 1.7 \times 10^{-29}\,M_\odot\,yr^{-1}$, a completely negligible value resulting from both the M-dependence of $\dot{M}$ and the extremely low photon density of the cosmic background during the matter-dominated era in which stellar systems formed. This value is in excellent agreement with the results of de Freitas Pacheco (2011).

3.2 Cold dark matter

Cold dark matter is a non-relativistic fluid with negligible thermal energy and pressure, hence ρ is essentially the rest energy density. For this fluid, the equation of state $p = \omega\rho$ with $\omega \ll 1$ can be used if we assume an isothermal flow. The same description applies to any perfect baryonic fluid at low temperature. Fig. 1 shows that for a fluid with $\omega \lesssim 4 \times 10^{-8}$, the accretion flow is indeed modified by the presence of the globular cluster.

For very low values of ω ($\omega \lesssim 10^{-9}$, below the dotted line in Fig. 1), the critical point lies far from the cluster centre (Fig. 2), and $m(r_c) \approx M + M_{gc}$. Hence, we expect the accretion rate to be greatly enhanced. There is also a transition region in which equation (9) has three solutions (between the dotted and dashed lines in Fig. 1). However, only two of them are physically admissible. If the middle one were the critical radius, the other two must be turn around points, violating the boundary conditions on the flow velocity. Hence, only the inner and outer solutions are possible, but only one of them corresponds to the critical radius. This radius can be found by integrating the flow equation (5) for both solutions. Only in one case the integration converges, indicating that this is the correct value of r_c. In the case of the cluster considered here, the full integration of the flow shows that the critical point is the inner solution for $\omega > \omega_t = 1.5 \times 10^{-9}$, and the outer one otherwise. At this transition value, the position of the critical point changes abruptly (Fig. 2). This change, probably an artefact due to the simplicity of our model, does not affect our conclusions as we will show later.

In Fig. 3 we show the results for the accretion rates derived from equation (10), and using $\rho_\infty = \rho^{DM}c^2$ (see Table 1), where ρ^{DM} is the dark matter density at the Galactocentric distance of the globular cluster, taken from the work of Klypin, Zhao & Somerville (2002). We can see the abrupt change in the accretion rate, which follows the corresponding change in r_c. At $\omega_u \approx 3 \times 10^{-10}$, r_c reaches the globular cluster tidal radius. For $r \gg r_t$ the globular cluster cannot

Table 1. Model parameters.

Cluster scale radius (r_0)	0.35 pc
Cluster tidal radius (r_t)	44 pc
Cluster concentration (c_{gc})	1.8
Cluster Galactocentric distance (R_{gc})	3.1 kpc
Black hole mass (M)	3000 $M_\odot$
Dark matter density at R_{gc} (ρ^{DM})	4.0×10^{-21} kg m^{-3}
Dark energy density (ρ^{DE})	7.7×10^{-27} kg m^{-3}

be considered an isolated object any more, as the tidal field of the Galaxy becomes dominant. The predictions of our model are not reliable in such a case.

It is important to discuss the effect of the cluster (and thus the IMBH) motion relative to the dark matter fluid on the accretion rate previously calculated. Hoyle & Lyttleton (1939) have derived a formula for the accretion rate of a classical fluid by an object moving through it, which is similar to that of Bondi & Hoyle (1944). According to the formula of Hoyle & Lyttleton (1939), the relation between the accretion rate $\dot{M}(v)$ on to an object moving with velocity v with respect to the fluid and the accretion rate $\dot{M}_0$ on to an object at rest is

$$\dot{M}(v) = \dot{M}_0 \frac{c_s^3}{\left(v^2 + c_s^2\right)^{3/2}}, \tag{11}$$

where $c_s^2 = c^2\omega$ is the sound speed in the medium. Computing the Newtonian limit for equation (10), it can be easily seen that in the limit $\omega \to 0$ the accretion rate approaches that of Bondi & Hoyle (1944). Hence, we assume that the correction factor given by equation (11) can be used to derive $\dot{M}(v)$ from $\dot{M}_0$ in our models. In Fig. 3 we show the accretion rate values corresponding to cluster velocities v equal to 0, 100, 200 and 500 km s^{-1}, which are typical for the motion of Galactic halo objects such as globular clusters.

If we neglect the cosmological evolution of the dark matter density, we can compute a rough estimate of the amount of matter ΔM accreted by the black hole along the globular cluster lifetime $\Delta t \sim 10$ Gyr. For $\omega \lesssim 10^{-9}$, $\dot{M}$ depends on $m(r_c$, with $r_c \gg r_0$, hence the change in the accretion rate due to the IMBH growth is negligible. Hence, we can compute $\Delta M \approx \dot{M}\Delta t \gtrsim 10^4\,\mathrm{M}_\odot$ for any reasonable value of v. This is not consistent with present upper limits of Lanzoni et al. (2007), Lutzgendorf et al. (2011) and Cseh et al. (2010) for the mass of the IMBH in NGC 6388. On the other hand, for $\omega \gtrsim 10^{-9}$ the accretion rate is low enough to avoid this discrepancy. This result is remarkable, as it implies that either the globular cluster does not host an IMBH or there is a stringent lower limit on the cold dark matter equation of state parameter, which cannot be null but at least of the order of 10^{-9}.

3.3 Dark energy

In previous sections, we based our calculations on the existence of a critical radius and, hence, a critical velocity for the flow. However, we can see from equation (9) that for unstable fluids ($\omega < 0$; see Fabris & Martin 1997; Carroll, Hoffman & Trodden 2003, for reviews on this subject) there is no critical point outside the black hole horizon ($r_c < 2M$). Following Michel (1971) and Babichev et al. (2004), we assume that in this case the instabilities in the flow cause the growth of the fluid velocity up to the speed of light at the black hole horizon, and use this as a boundary condition instead of the critical values. With this assumption, the accretion rate of dark energy becomes identical to that of Babichev et al. (2004):

$$\dot{M} = 16\pi(1+\omega)\rho_\infty M^2. \tag{12}$$

This has a simple interpretation: in this regime, the pressure forces of a relativistic fluid are of the order of the gravitational forces produced by the black hole near its horizon, hence much greater than those of the gravitational field of the cluster elsewhere, and the system behaves as if the black hole were isolated, as in the model of Babichev et al. (2004). Another interesting property of equation (12) is that $\dot{M}$ is negative when $\omega < -1$ (i.e. for phantom energy), leading to a decrease of the black hole mass. It is important to point out that this would be the case only if the generalized second law is violated, as de Freitas Pacheco & Horvarth (2007) have shown. For our cluster plus black hole model, and using the present dark energy density $\rho^{\mathrm{DE}}c^2$ as ρ_∞ (see Table 1), we obtain

$$\dot{M} = 9.5 \times 10^{-34}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}(1+\omega)\left(\frac{M}{\mathrm{M}_\odot}\right)^2. \tag{13}$$

As the coefficient of the right-hand side of equation (13) shows, the accretion rate and the mass gained or lost by an IMBH of 10^2–$10^4\,\mathrm{M}_\odot$ during the past 10 Gyr is negligible. Hence, we can conclude that the accretion of dark energy by an IMBH does not affect the evolution of the accretor mass.

4 DISCUSSION AND CONCLUSIONS

In this work we investigated the spherical, steady-state accretion of cosmological fluids on to an IMBH at the centre of a globular cluster, taking into account the influence of the parent stellar system on the accretion flow. We also include in our models a correction for the effects of the motion of the black hole through the fluid. For relativistic perfect fluids ($p \sim \rho$) and dark energy, our results show that the presence of the cluster does not affect significantly the flow, hence our results coincide with those derived from models of accretion on to isolated black holes, such as those of Babichev et al. (2004). The conclusions of these authors hold, particularly that the accretion rate is proportional to $(p_\infty + \rho_\infty)M^2$. As pointed out by Babichev et al. (2004), this implies that, depending on the nature of the dark energy, the IMBH mass can either increase (for $\omega > -1$) or decrease (for $\omega < -1$). For $\omega = -1$ the mass remains unchanged. For the particular case of IMBHs with masses in the range 10^2–$10^4\,\mathrm{M}_\odot$ and a typical globular cluster, our calculations show that the black hole mass is not affected by the accretion of either dark energy or radiation from the cosmic microwave background.

However, for the accretion of cold dark matter the situation changes. We found that if dark matter is collisionless or has a very low speed of sound, the accretion rate no longer scales as the square of the black hole mass, but as the square of the mass inside the critical radius. This is due to the effect of the globular cluster gravitational field on the flow. As the critical radius can be well outside the cluster core, this mass can be much greater than the IMBH mass, and hence the accretion rate can be enhanced by a factor of the order of 10^4–10^6. We also found that the accretion rate scales as c_s^{-3}. These results lead to final IMBH masses greater than $10^4\,\mathrm{M}_\odot$, well above the upper limits given by present observations. We point out that this enhancement is independent of the details of our simple model, particularly of the abrupt change in the critical radius observed as the speed of sound decreases. Even if this change were replaced by a smooth behaviour, the critical radius must approach the cluster outer regions as ω decreases, greatly enhancing the accretion rate. Hence, there will still be a lower limit for ω at which the mass accreted by the IMBH during its lifetime exceeds the observational upper bounds. Our models estimate this limit at $\omega \sim 10^{-9}$, which corresponds to a sound speed of $c_s \sim 10$ km s^{-1}.

The last result has an important impact on our knowledge of IMBHs and dark matter. If IMBHs exist at the centres of globular clusters and dark matter has a low sound speed, IMBH masses should have grown to values beyond the observed upper limits. To restore the agreement with observations we are forced to assume that dark matter, if it exists, must have a sound speed at least of the order of $c_s \sim 10$ km s^{-1}. Indeed, Guzmán & Lora-Clavijo (2011) arrive at the same conclusion investigating the accretion on to supermassive black holes using more sophisticated, time-dependent accretion models. Another possibility to check our conclusions is to investigate the accretion on to stellar mass black holes. Some of these objects have known masses and their velocities are well established. For example, Mirabel et al. (2001) calculated the orbit of the black hole X-ray nova XTE J1118+480. Computing the accretion rate for this black hole (with a mass of $M_{\rm BH} = 6.9\,{\rm M}_{\odot}$ and a velocity of $v_{\rm BH} \sim 140\,{\rm km\,s^{-1}}$, taken from these authors) we obtain negligible accretion rates. Hence, stellar mass black holes do not seem to be useful for testing the dark matter equation of state. Thereby, if IMBHs were finally detected at the centres of globular clusters and detailed models of their accretion developed, these objects would turn into extraordinary tools to investigate this issue.

ACKNOWLEDGMENTS

We would like to thank the referee for his/her useful comments. This work was partially supported by grant PICT 2007-00848 from Argentine ANPCyT.

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