OVERLAPPING IMPROVED ELEMENT-FREE GALERKIN AND FINITE ELEMENT METHODS FOR THE SOLUTION OF NON-LINEAR TRANSIENT HEAT CONDUCTION PROBLEMS WITH CONCENTRATED MOVING HEAT SOURCES

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Abstract. A novel overlapping approach, termed the Overset IEFG-FE method (Ov-IEFG-FEM), is proposed for solving transient heat conduction problems with concentrated moving heat sources. This mesh-less/mesh-based chimera-type method combines improved element-free Galerkin (IEFG) and finite element (FE) methods. The Ov-IEFG-FEM uses a coarse FE mesh to discretise the problem geometry, while a set of overlapping patch nodes moves with the heat source, enhancing accuracy via the IEFG technique to capture marked thermal gradients. In regions outside the heat source area where accuracy requirements are lower, the thermal problem is solved using the FE method (FEM). The method involves solving the problem over these two overlapping domains and transferring numerical information between the approximations performed on both. The Ov-IEFG-FEM aims to provide an enriched solution by coupling temperature fields computed on the patch nodes and the coarse background mesh using IEFG and FE methods, respectively. Numerical experiments demonstrate the method potential in accurately and efficiently solving transient heat conduction problems with concentrated moving heat sources, including marked non-linear aspects related to temperature-dependent properties and phase change phenomena.

Key words: *Heat conduction, Moving heat sources, Overset, Element-free Galerkin, Finite element*

1 Introduction

Most numerical solutions reported in the literature for transient heat conduction problems with moving heat sources are based on mesh-based discretisation techniques such as the finite element method or the finite volume method, often requiring significant refinements along the scanning path to achieve an appropriate capture of high temperatures and marked thermal gradients. It is also well-known that performing mesh refinements along the heat source path or adaptive re-meshing techniques can be very cumbersome, and sometimes even unfeasible in problems involving heat sources following curved scanning paths in complex 3-D geometries[1–3]. Mesh-less or mesh-free methods have a emerged as an interesting alternative to the most commonly used mesh-based techniques due to two main reasons[4]: (i) the capability of easily attain higher-order approximations with continuous derivatives, and (ii) the enhanced flexibility of adding or removing nodes during adaptive local refinements. Such versatility of mesh-less methods has also enabled the implementation of such numerical techniques in the solution of both linear and non-linear applied problems involving concentrated moving heat sources[1,2]. Although the potential of mesh-less methods, these numerical techniques still pose noteworthy challenges in terms of computational efficiency, which primarily arise from the need for identifying neighbouring nodes that define the support domain for numerical approximations and the construction of shape functions via more computationally expensive unconventional procedures[1,2,4]. The emergence of hybrid mesh-less/mesh-based approaches has introduced very interesting alternatives that combine the strengths of mesh-less methods with the less computationally demanding approximations usually involved in standard mesh-based techniques[5]. The mesh-less element-free Galerkin (EFG) method shares similarities with FEM, notwithstanding differences in the construction of shape functions and assembly of the algebraic system of equations. These analogies primarily arise from both numerical techniques being developed in a weak formulation of the governing equations, promoting the development of hybrid EFG-FEM-based approaches[5]. These techniques improve computational efficiency by using EFG methods only in regions demanding high numerical accuracy[5], and the less computationally expensive FEM is used in the rest of the problem domain. Although hybrid EFG-FEM approaches have provided excellent results in scenarios demanding high accuracy[5], these techniques commonly require well defined coupling boundaries where EFG and FEM regions share common nodes. Implementing these approaches in transient heat conduction with concentrated moving heat sources might pose challenges due to the need of redefining the EFG-FEM coupling boundaries at each time step, such EFG computations are performed only near the moving heat source location. This communication aims to highlight the potential of a recently developed chimera-type scheme based on the Improved EFG (IEFG) and the FEM to overcome these challenges. The proposed Overset IEFG-FEM (Ov-IEFG-FEM) offers an enriched accurate solution, smoothly transitioning from EFG to FEM regions dispensing with predefined topological relationship.

2 Governing equations and problem description

The Ov-IEFG-FEM will be used to solve a problem emulating the thermal conditions of the direct metal laser sintering (DMLS) of AlSi10Mg alloys, with the geometric features depicted in Fig. 1. The enriched solution is obtained solving the governing equations of transient heat conduction in the domain Ω_{FEM} with boundaries $\Gamma_{\mathrm{FEM}} = \Gamma_D \cup \Gamma_N$ and the domain Ω_{IEFG} with boundaries Γ_{IEFG}, and performing an appropriate coupling via a reciprocal transfer of information. The thermal problem in Ω_{FEM} is:

$$\begin{gathered}\rho c_p \frac{\partial T}{\partial t} = \nabla.(k\nabla T) + \dot{Q} \qquad \text{in} \quad \Omega_{\mathrm{FEM}} \times [0,t_f],\\ T = T_D \qquad \text{on} \quad \Gamma_D \times [0,t_f], \quad \text{and} \qquad k\nabla T \cdot \hat{n} = q_N \qquad \text{on} \quad \Gamma_N \times [0,t_f],\end{gathered} \tag{1}$$

whereas the thermal problem in Ω_{IEFG} is:

$$\begin{gathered}\rho c_p \left(\frac{\partial T}{\partial t} - \vec{v}\cdot\nabla T\right) = \nabla.(k\nabla T) + \dot{Q} + \rho H_f \left(\frac{\partial f_s}{\partial t} - \vec{v}\cdot\nabla f_s\right) \qquad \text{in} \quad \Omega_{\mathrm{IEFG}} \times [0,t_f],\\ T = \tilde{T}_{\mathrm{FEM}} \qquad \text{on} \quad \Gamma_{\mathrm{IEFG}} \times [0,t_f].\end{gathered} \tag{2}$$

It is important to note that the effect of the moving heat source can be incorporated either through the volumetric term $\dot{Q}$ or as a concentrated surface heat flux via a Neumann condition. The choice depends on the specific model used for the moving heat source, as discussed in [1,2]. Phase change terms, dependent on f_s and H_f, are exclusively incorporated within Ω_{IEFG} since melting/solidification only occurs in proximity to the moving heat source represented by the surface Gaussian distribution $q_N = 2\eta \dot{Q}_T/(\pi r_0^2)e^{-2\left[(x-||v||\times t)^2+y^2\right]/r_0^2}$. The temperature fields in the weak formulation of Eqs. (1) and (2) are approximated using standard linear interpolating finite element basis functions $\varphi^{(I)}_{\mathrm{FEM}}(\vec{x})$ and improved moving least squares (IMLS) approximations $\varphi^{(I)}_{\mathrm{IEFG}}(\vec{x})$, respectively:

$$T_{\mathrm{FEM}}(\vec{x}) = \sum_{I=1}^{n_{\mathrm{FEM}}} \varphi^{(I)}_{\mathrm{FEM}}(\vec{x})\hat{T}^{(I)}_{\mathrm{FEM}} \quad \text{for all } \vec{x} \in \Omega_{\mathrm{FEM}}, \qquad T_{\mathrm{IEFG}}(\vec{x}) = \sum_{I=1}^{n_{\mathrm{IEFG}}} \varphi^{(I)}_{\mathrm{IEFG}}(\vec{x})\hat{T}^{(I)}_{\mathrm{IEFG}} \quad \text{for all } \vec{x} \in \Omega_{\mathrm{IEFG}}. \tag{3}$$

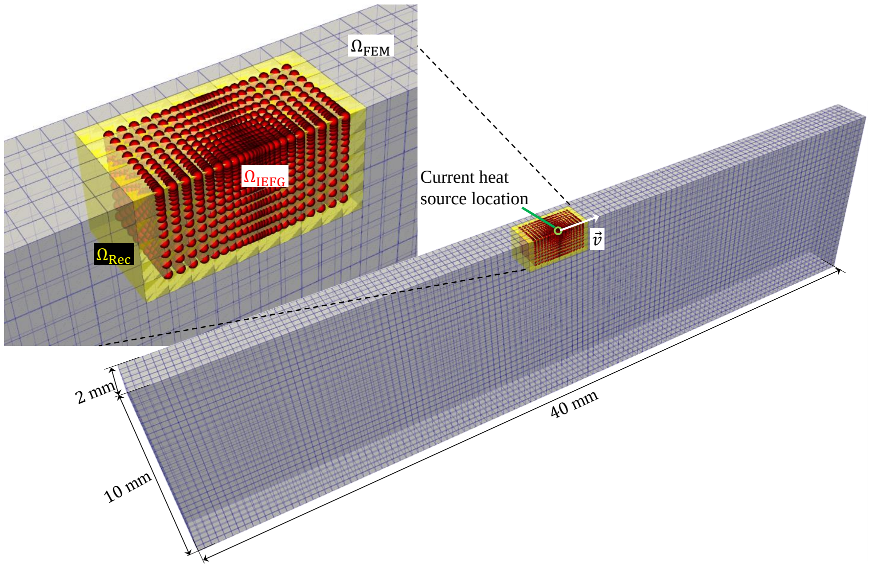


Figure 1: Detail on the discretisation of Ω_{FEM} and representation of Ω_{IEFG}. The nodes representing Ω_{IEFG} constantly moves tracking the scanning path, according to the the heat source velocity $\vec{v}$. This movement ensures that, at every time step, the heat source remains precisely centered within the arrangement of nodes.

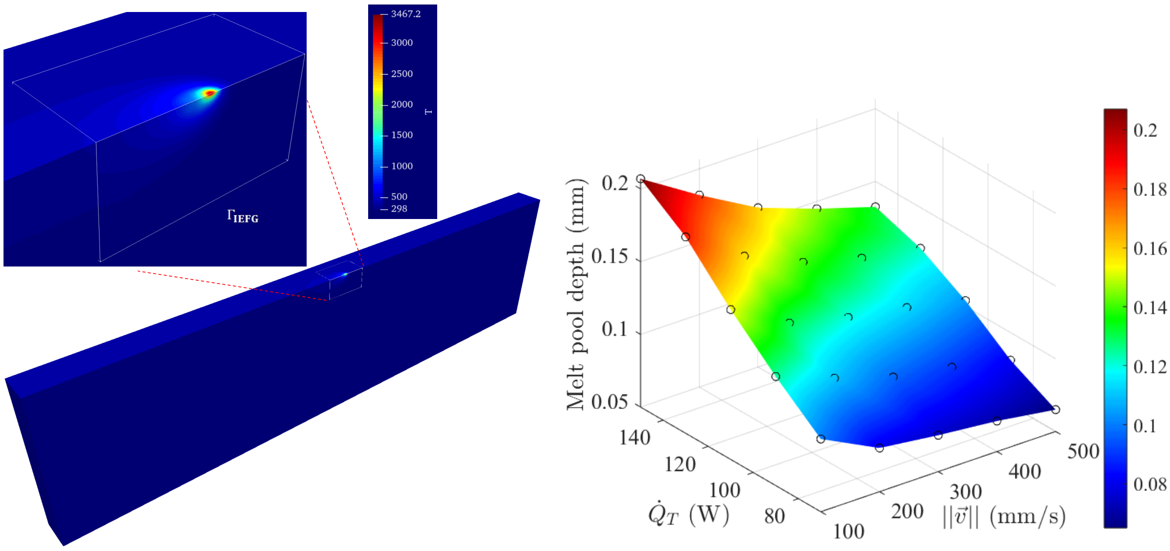
Substituting the approximations given in (3) into the weak formulation of Eqs. (1) and (2) gives rise to the following systems of equations:

$$\begin{aligned} \mathbf{C}_{\mathrm{FEM}}\dot{\hat{\mathbf{T}}}_{\mathrm{FEM}} + \mathbf{K}_{\mathrm{FEM}}\hat{\mathbf{T}}_{\mathrm{FEM}} &= \mathbf{Q}^{(T)}_{\mathrm{FEM}} \qquad \text{in} \quad \Omega_{\mathrm{FEM}} \\ \mathbf{C}_{\mathrm{IEFG}}\dot{\hat{\mathbf{T}}}_{\mathrm{IEFG}} + \left(\mathbf{K}_{\mathrm{IEFG}} - \mathbf{A}_{\mathrm{IEFG}} + \mathbf{K}^{(p)}_{\mathrm{IEFG}}\right)\hat{\mathbf{T}}_{\mathrm{IEFG}} &= \mathbf{Q}^{(T)}_{\mathrm{IEFG}} + \mathbf{Q}^{(p)}_{\mathrm{IEFG}} \qquad \text{in} \quad \Omega_{\mathrm{IEFG}} \end{aligned} \tag{4}$$

The Dirichlet condition $\tilde{T}_{\mathrm{FEM}}$ on the immersed boundaries Γ_{IEFG} is obtained through local reconstruction of FEM-based results via IMLS approximations over the sub-domain $\Omega_{\mathrm{Rec}} \in \Omega_{\mathrm{FEM}}$ (yellow region in Fig. 1). This allows the transfer of information from Ω_{FEM} to Ω_{IEFG} using the penalty matrix $\mathbf{K}^{(p)}_{\mathrm{IEFG}}$ and vector $\mathbf{Q}^{(p)}_{\mathrm{IEFG}}$ in the system of equations for Ω_{IEFG}. Numerical information from IEFG-based results is reciprocally transferred to the FE mesh using IMLS approximations $T_{\mathrm{IEFG}}(\vec{x})$ to compute temperatures at FE mesh nodes within Ω_{IEFG}. These nodal values are then prescribed in the system of equations for Ω_{FEM}. The iterative procedure continues until convergence is achieved in the coupling along Γ_{IEFG} and with respect to non-linearities related to phase change and temperature-dependent thermal properties.

3 Numerical results

The Ov-IEFG-FEM simulations for the thermal problem have been performed with laser power absorptivity $\eta = 0.95$ and heat source effective radius $r_0 = 100,\mu$m. Results for total heat source power $\dot{Q}_T = 150$ W with scanning speed $||\vec{v}|| = 500$ mm/s are shown in Fig. 2 (a), with a sensitivity analysis on melt pool depth to variations in these parameters presented in Fig. 2 (b). The outcomes demonstrate seamless coupling along Γ_{IEFG} and accurate capture of thermal gradients near the heat source. This indicates an appropriate information transfer between overlapping domains and solution of non-linearities. The sensitivity analysis on melt pool depth to variations in $\dot{Q}_T$ and $||\vec{v}||$ suggests a consistent thermal model as it behaves as expected in a wide range process parameters. Solving the phase change effects as a heat source term improves the convergence during the non-linear thermal problem numerical solution.



(a) Temperature field (b) Dependence of melt pool depth with $\dot{Q}_T$ and $||\vec{v}||$.

Figure 2: Numerical solution for the DMLS thermal problem, via the Ov-IEFG-FEM.

4 Conclusions

The Ov-IEFG-FEM introduces a novel approach to solving non-linear transient heat conduction problems with concentrated moving heat sources. By using a coarse FE mesh and overlapping patch nodes for IEFG computations, the method achieves enhanced accuracy, seamlessly coupling temperature fields and capturing thermal gradients. The sensitivity analysis confirms stability across a wide range of process parameters, and incorporating phase change effects as a heat source term has improved convergence in the numerical solution of material non-linearities.

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