Physica A 392 (2013) 278–286

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Interplay between social debate and propaganda in an opinion formation model

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ARTICLE INFO

Article history:
Received 16 February 2012
Received in revised form 23 July 2012
Available online 9 August 2012

Keywords:
Sznajd model
Stochastic resonance

ABSTRACT

We introduce a simple model of opinion dynamics in which a two-state agent modified Sznajd model evolves due to the simultaneous action of stochastic driving and a periodic signal. The stochastic effect mimics a social temperature, so the agents may adopt decisions in support for or against some opinion or position, according to a modified Sznajd rule with a varying probability. The external force represents a simplified picture by which society feels the influence of the external effects of propaganda. By means of Monte Carlo simulations we have shown the dynamical interplay between the social condition or mood and the external influence, finding a stochastic resonance-like phenomenon when we depict the noise-to-signal ratio as a function of the social temperature. In addition, we have also studied the effects of the system size and the external signal strength on the opinion formation dynamics.

1. Introduction

Interacting agent-based models are becoming increasingly important in behavioral, social, and political science [1–4]. There is compelling evidence that collective phenomena emerging in social contexts can be produced by basic agent-agent interactions [3,5]. Statistical physics represents the natural tool to study how global complex properties can emerge from purely local rules. The models and techniques of this discipline have been widely applied to the characterization of the collective social behavior of individuals, such as culture dissemination [6], spreading of linguistic conventions [7], and dynamics of opinion formation [8].

Basically, a statistical physics approach tries to grasp the essential features of the emerging social behaviors, and therefore takes into account simple rules of opinion formation in which agents update their internal states or opinion, through interactions with their neighbors, the rest of the community or through external factors.

In opinion dynamics, recent studies focus on the emergence of cooperative phenomena including spatial organization, the formation of coherent structures (political parties), and the transition from unity to discord [8–13]. In the simplest variant, models assume that the individuals can adopt two different opinions. The opinion of a given individual may be influenced by that of the neighbors, making it to change with a certain probability. Also, some recent papers have considered the possibility of a third, intermediate or undecided, group [14,15].

Some studies have considered the interaction among members of the community and external perturbations that society may receive and those related to the influence of internal social dynamics as well. Such considerations are particularly

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doi:10.1016/j.physa.2012.07.076

relevant for societies which continuously evolve. Moreover, the evolution of the topology and the dynamical processes can drive each other with complex feedback effects.

On the one hand, in Ref. [16] an Ising-like model of opinion formation was analyzed. When subject to the action of an external modulation and noise showing the existence of a stochastic resonance phenomenon [17], implicating that there is an optimal noise level for a population to respond to an external fashion modulation. On the other hand an important aspect of social systems recently studied has been the presence of some agents called contrarians, namely people who are in a nonconformist opposition. That is, people who always adopt the opposite opinion to the majority [8,18,19]. In Ref. [20] it was shown that a contrarian-like effect can spontaneously emerge when stochastic driving is included in the model. This randomness in the update of an agent opinion is meant to be a highly simplified description of the interplay between fashion/propaganda and a collective climate parameter, which is usually referred to as social temperature of the system [21–24].

The possibility of some external stochastic and/or deterministic influence on the agents in an opinion formation model, particularly regarding the possibility of some form of stochastic resonance [17], was recently analyzed by several authors [16,23,24]. It would be of great interest to study such a resonance effect (particularly its dependence on the size of the system [24]) in our model. This can actually be done by including a fashion external field (for instance a periodic signal) combined with the noise effect coming from the *social temperature.*

In this paper, we therefore investigate how the interplay of an internal movement due to a certain society turmoil and an external signal, modeling a propaganda action in a simple way, can exert an influence over the opinion of the society. In the next section we introduce the model and define the two effects acting over the system (social temperature and propaganda), and describe the simulation method. In Section 3 we show and discuss the results and, in the last section we draw some conclusions.

2. Model and simulation method

2.1. The model

The Sznajd model is an Ising-like model describing a simple mechanism of taking up decisions in a closed community. The model allows to each member of the community to have two attitudes, to vote for option *A* or to vote for option *B*. These two attitudes are identified with the state of spins variables up or down respectively. Dynamics is introduced in the model by means of convincing rules applied in a sequential manner in which a selected pair of adjacent spins influence their nearest neighbors through a given criterion. The Sznajd model with *social temperature* was previously considered in Ref. [20].

Each trial consists of choosing one agent at random, for example the agent i. Let s_i be the value of the opinion of agent i at time t. The Sznajd rules are:

- If $s_i \times s_{i+1} = 1$, then s_{i-1} and s_{i+2} agents adopt the opinion of the pair $[i, i+1]$.
- If $s_i \times s_{i+1} = -1$, then the agent s_i takes the value of the s_{i-1} one and the s_{i+1} agent adopts the value of the s_{i+2} one.

We define agents i and $i+1$ as the *discussion* agents since in the present context they are the first agents to argue about a given topic. The $i-1$ and $i+2$ agents are the ones who make further discussion with the previous agents and somehow their ideas modify or are modified by the discussion agents. So the latter are called the *modification* agents. We call R_1 to the mentioned rule, which is a variant of the Sznajd model, that was previously presented in Ref. [21].

2.2. Propaganda and social temperature

Propaganda will be introduced as an external periodic effect which has the following form: an agent i is chosen and R_1 is applied to the four agents ($i, i+1, i-1$ and $i+2$). On each application of rule R_1, two of the four agents have the possibility of changing their opinion value. For each recently modified agent, the rule is applied with probability p and the opposite way to the established rule is performed with probability $1-p$, where p is calculated in the following form

$$p = \Lambda \ \exp\left(\frac{\alpha + q \times H}{T}\right) \tag{1}$$

where q is the new value of the opinion of the considered agent recently modified according to rule R_1 (the opinion of every agent is considered separately), α is a constant value related to the strength of nearest neighbor interactions which just defines the units by which temperature is measured (here we considered $\alpha = 1$). The parameter T provides the mechanism for some individuals to have the chance to react in the opposite form of that established by the Sznajd rule. In other words, it plays a role analogous to the temperature in thermodynamic systems; for this reason it is called *social temperature*. The model is completed by defining H, the propaganda parameter, which is a periodic function of the form

$$H = H_0 \times \sin(\omega t), \tag{2}$$

where H_0 is the amplitude of the applied field (we take $0 \leq H_0 \leq 1$) and $\omega = 2\pi/P$ ($P =$ period) is the angular frequency of the external signal.

The normalization constant takes the form

$$\Lambda^{-1} = \exp\left(\frac{\alpha + q \times H}{T}\right) + \exp\left(-\frac{\alpha + q \times H}{T}\right). \tag{3}$$

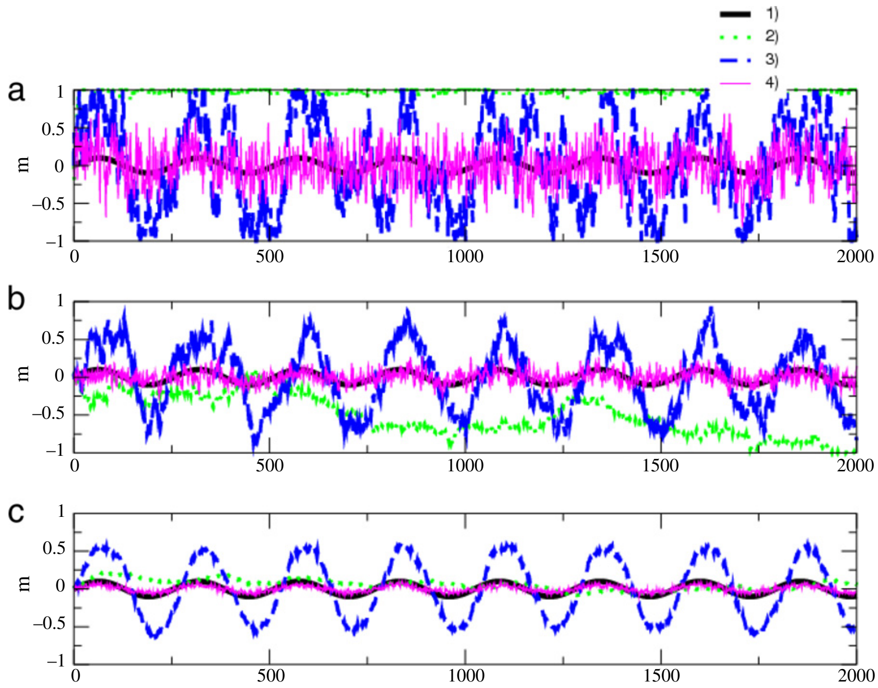

Fig. 1. External field (line 1) and mean opinion, m, as a function of time, t, for three different sizes of the system (a) $L = 32$, (b) $L = 256$ and (c) $L = 4096$ and for three different social temperatures (2) $T = 0.4$, (3) $T = 1.0$ and (4) $T = 5.0$. In all cases the amplitude of the external field is $H_0 = 0.1$ and the period is $P = 256$.

Note that, when q and H have the same direction ($q \times H > 0$), the probability p of retaining the new value (obtained according to R_1) is increased. On the other hand, when $q \times H < 0$, the probability p decreases and the selected agents are more likely to change opinion to the opposite sense. So, in both cases, the factor $q \times H$ contributes to align the agents in the same direction of the field H.

Note that $p \to 1$ when $T \to 0$ and $p \to 0.5$ when $T \to \infty$. If the temperature tends to 0, the effect of the fashion disappears.

Each time step consists on L trials, where L is the total number of agents considered.

3. Results and discussion

We define the opinion of the population, m, at each time t of the simulation as the mean value of the individual opinion

$$m = \frac{\sum_i s_i}{L}, \tag{4}$$

hence, we have that $-1 \leq m \leq 1$.

Fig. 1 shows the external field (line 1) and the mean opinion as a function of time, for different social temperatures, T, and three different sizes of system L in a single realization. In all these cases, the amplitude of the field was set as $H_0 = 0.1$, and the period of the field was $P = 256$. Besides, in all cases agents were initially distributed in a random way, with probability $p = 0.5$ of having $+1$ (-1) opinion.

It can be observed that, for low temperature ($T = 0.4$) and for the H_0 simulated, m does not follow the external field. In the case of small size of the system ($L = 32$) and in this particular simulation, it tends to 1 (but it also could be to -1 in other realizations) and remains stable. The system exhibits a kind of consensus for low temperatures and small system sizes. When increasing the size of the system ($L = 256$) the opinion still tends to an extreme (-1 in this particular case), but more slowly. And for even larger sizes of the system ($L = 4096$) it oscillates around $m = 0$. As the system size increases, the consensus tends to disappear. As temperature is increased ($T = 1.0$), the value of m follows the external field, however a phase lag between the propaganda and the opinion signal is observed, and the amplitude of m is considerably large (and seems to decrease monotonously with the system size). For higher temperatures ($T = 5.0$ in this case), we can observe that the value of m also follows the field, but the amplitude is lower than for intermediate temperature and the phase lag tends to disappear.

Fig. 2 shows the distribution of m (probability of obtaining each particular value of m) for four different temperatures and $L = 512$ in the cases of (a) absence of the external field, and (b) for an external field of amplitude $H_0 = 0.1$ and period

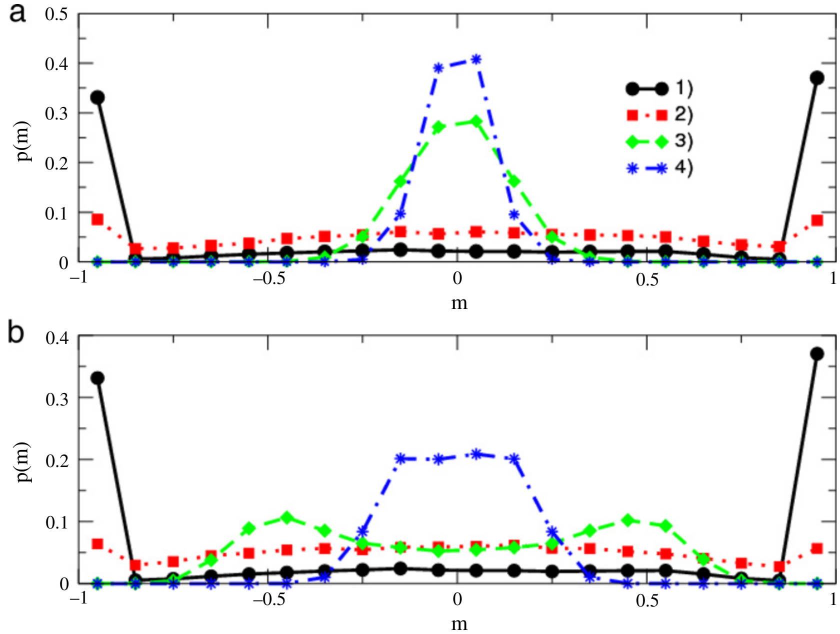

Fig. 2. Histograms of distribution probabilities of m for (a) $H_0 = 0.0$ and (b) $H_0 = 0.1$ and $P = 256$, $L = 512$ and four different temperatures (1) $T = 0.0$, (2) $T = 0.4$, (3) $T = 1.0$, and (4) $T = 2.0$. Average over 1024 simulations.

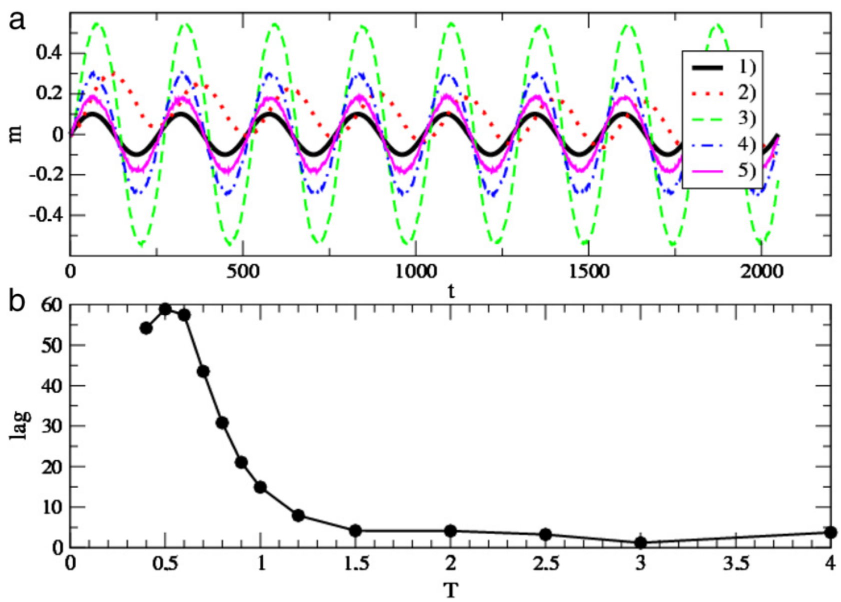

Fig. 3. Upper panel: mean opinion, m, as a function of time for a system of $L = 256$, $H_0 = 0.1$, $P = 256$ and an average over 500 simulation runs. (1) Periodic external signal, (2) $T = 0.5$, (3) $T = 1.0$, (4) $T = 1.5$, and (5) $T = 2.0$. Lower panel: lag as a function of temperature for a system of $L = 256$, $H_0 = 0.1$, $P = 256$ and an average over 500 simulation runs.

$P = 256$, averaged over 1024 simulations. For each simulation, the average was taken over the last 2048 temporal steps, that is in the stationary regime. It can be seen that when there is no social temperature ($T = 0.0$), the mean opinion adopts the values 1 or -1 in both cases. That means that consensus is reached. In the case (a) of absence of external field, as temperature increases, the mean opinion adopts values around zero. In the presence of propaganda this tendency is also observed, but not for intermediate values (around $T = 1.0$) for which two maxima at $m = \pm 0.5$ can be clearly distinguished.

Fig. 3 shows, in the upper part, the mean opinion, m, averaged over 500 realizations, as a function of time for a system of $L = 256$, $H_0 = 0.1$, $P = 256$, and different temperatures. From the figure it is apparent that there exists a delay between the external field and the mean opinion m, specially for low temperatures. The lower part of Fig. 3 shows the lag as a function

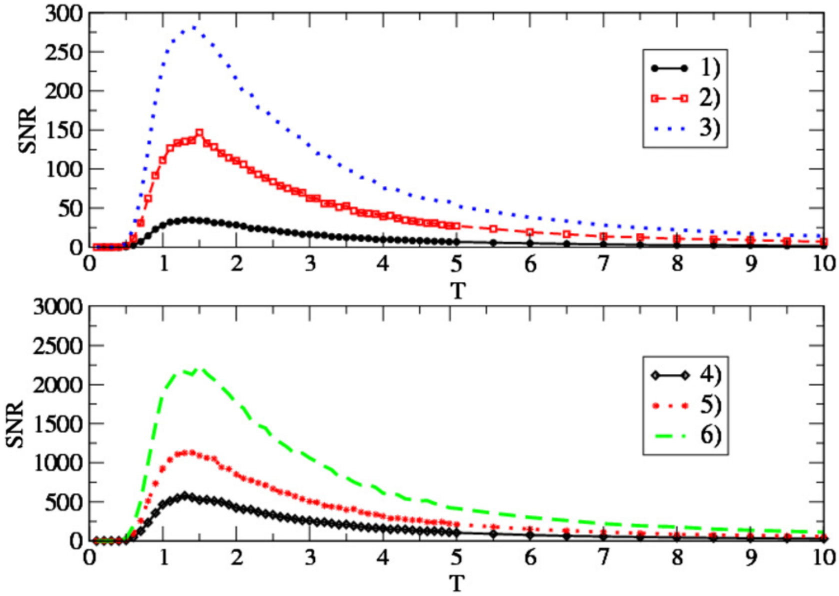

Fig. 4. SNR as a function of T for different sizes of the system, for the case of $H_0 = 0.1$ and period $P = 256$. (1) $L = 32$, (2) $L = 128$, (3) $L = 256$, (4) $L = 512$, (5) $L = 1024$, and (6) $L = 2048$.

of temperature for the same systems. Here, the lag is defined as

$$lag = \frac{(m_{max} - H_{max}) + (m_{min} - H_{min})}{2} \tag{5}$$

averaged over eight periods. The lag presents an abrupt drop and remains more or less stable for temperatures higher than $T = 1.5$. This graph allows to distinguish three regions: low temperature, where the system does not follow the external signal; intermediate temperature, where the system follows the external signal with a delay; and high temperature, where the system response is in phase with the external signal.

In order to study resonant effects, we have calculated the Fourier transform of m as a function of time in the presence of the external field, hence, the signal-to-noise ratio (SNR) for the frequency corresponding to that of the field, was defined as

$$SNR = \frac{\int_{w_0-\delta}^{w_0+\delta} S(w)\mathrm{d}w}{\int_{w_0-\delta}^{w_0+\delta} S_{back}(w)\mathrm{d}w} \tag{6}$$

where

$$S(w) = \lim_{\tau\to\infty} \int_{-\infty}^{\infty} \langle m(t)m(t+\tau)\rangle \exp(-\mathrm{i}w\tau)\mathrm{d}\tau \tag{7}$$

where S_{back} means the value of S in the background of that region.

So, in the graph of the Fourier transform, we have a peak at the frequency corresponding to that of the external field, H. The value of SNR is the area of that peak, normalized with respect to the basis, $S(w)$, i.e., divided by the height of the basis of the peak, in order to take into account the noise. So, SNR is a measure of the intensity of the response of m to the oscillation of the external field.

For each temperature, we have calculated the value of SNR averaging over 1000 realizations.

Fig. 4 shows SNR as a function of T for different sizes of the system, for the case of $H_0 = 0.1$ and period $P = 256$, averaged over 1000 simulation runs. It can be noticed that there is a resonant behavior and the resonant temperature is approximately 1.3–1.5. The exact position of the peak, as well as its height, depends on the system size.

Fig. 5 shows the value of the temperature (top) and the height of the SNR divided by L (bottom) for the maximum of the peaks shown in Fig. 4 as a function of L. The value of SNR/L is approximately constant, as SNR_{max} grows almost linearly with the size of the system.

In order to study the field frequency's influence on the SNR behavior, in Fig. 6 we show SNR vs T for different frequencies, for the case of $L = 64$ and $H_0 = 0.1$ (upper panel). It can be noticed that the curves are almost coincident, so that the influence of the frequency in this model, under the parameters considered, is not important. It also shows the SNR as a function of the frequency for different temperatures (lower panel). For $T = 1.0$ there is an initial increment of SNR with P and from $P = 256$ it remains almost constant. For higher values of T, SNR is approximately independent of P.

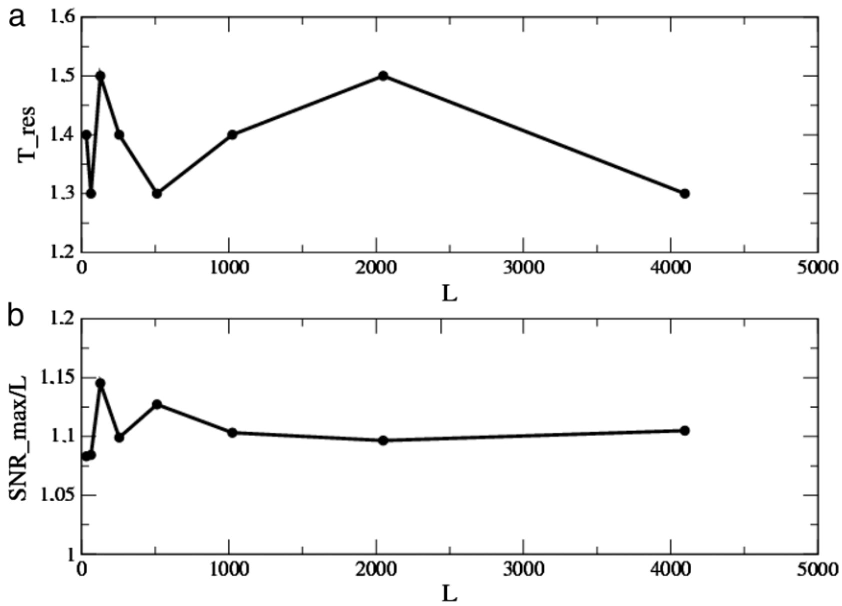

Fig. 5. Resonant temperature (upper panel) and SNR_{max}/L (lower panel), for the curves shown in Fig. 4 as a function of L. $P = 256$ and $H_0 = 0.1$.

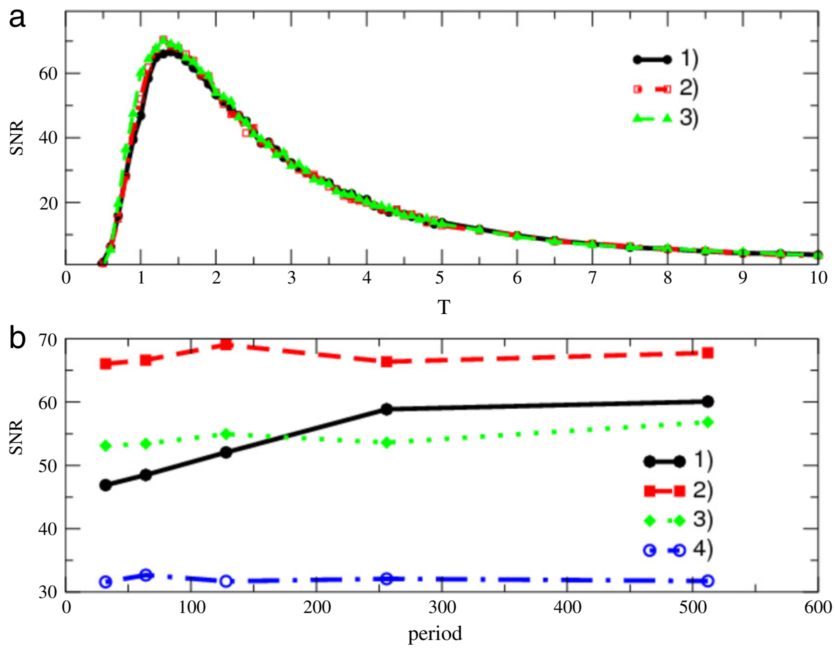

Fig. 6. (a) SNR as a function of T for three different frequencies: (1) $P = 32$, (2) $P = 128$, and (3) $P = 512$. (b) SNR as a function of the frequency for different temperatures: (1) $T = 1.0$, (2) $T = 1.5$, (3) $T = 2.0$, and (4) $T = 3.0$. In all these cases, $L = 64$ and $H_0 = 0.1$.

Another important question is related with the effect on the system's response of the strength of the external signal. We have also studied the effect of H_0 (the amplitude of the external field) on the system's response.

Fig. 7 shows the evolution of m for different values of H_0 and different temperatures. In these cases, the size of the system was set at $L = 256$ and the period of the signal was $P = 256$. It can be seen that for low values of H_0, the mean opinion follows the signal only slightly, specially for low temperatures. As H_0 increases, the mean opinion follows tightly the signal, even for low temperatures. At low and intermediate values of H_0, intermediate temperatures ($T = 1.0$) are optimal.

Fig. 8 shows the evolution of SNR with T, for different values of H_0. For relatively low values of H_0 the shape of the curves are similar, and the values of SNR are higher for higher values of H_0. As H_0 approaches 1.0, something strange occurs with the curves. The value of SNR becomes very high for low temperatures. For intense fields, there exists a local minimum and a local maximum and it can be induced that, at low temperatures, the value of m follows very well the external signal.

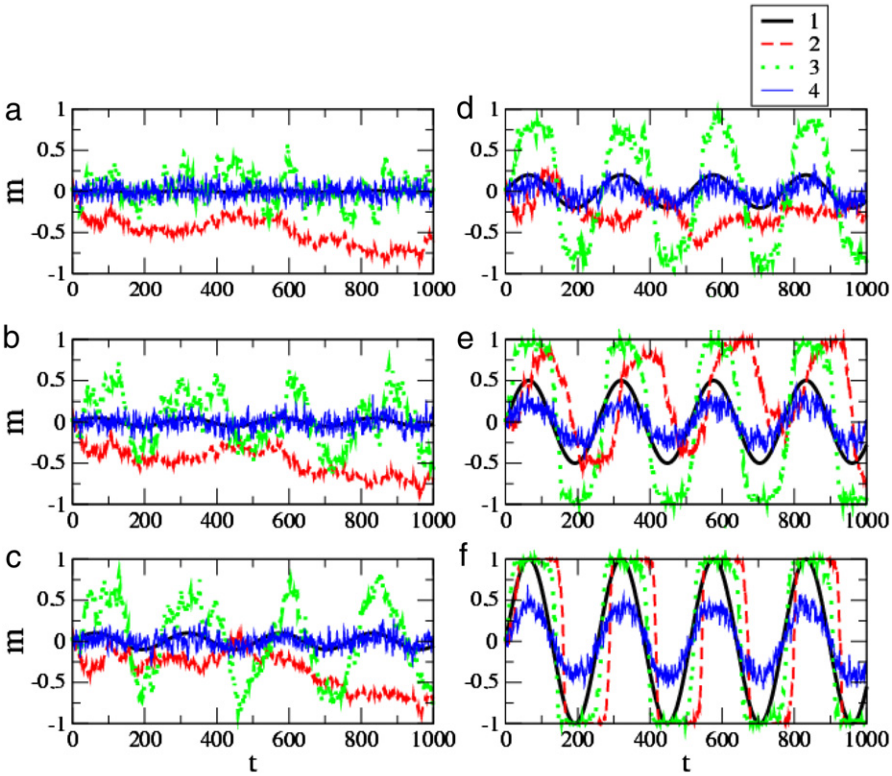

Fig. 7. External field (line 1) and mean opinion, m, as a function of time, t, for different values of H_0 (amplitude of the external field): (a) $H_0 = 0.01$, (b) $H_0 = 0.05$, (c) $H_0 = 0.10$, (d) $H_0 = 0.20$, (e) $H_0 = 0.50$ and (f) $H_0 = 1.00$ and different social temperatures: (2) $T = 0.4$, (3) $T = 1.0$ and (4) $T = 5.0$. In all cases the size of the system is $L = 256$ and the period is $P = 256$.

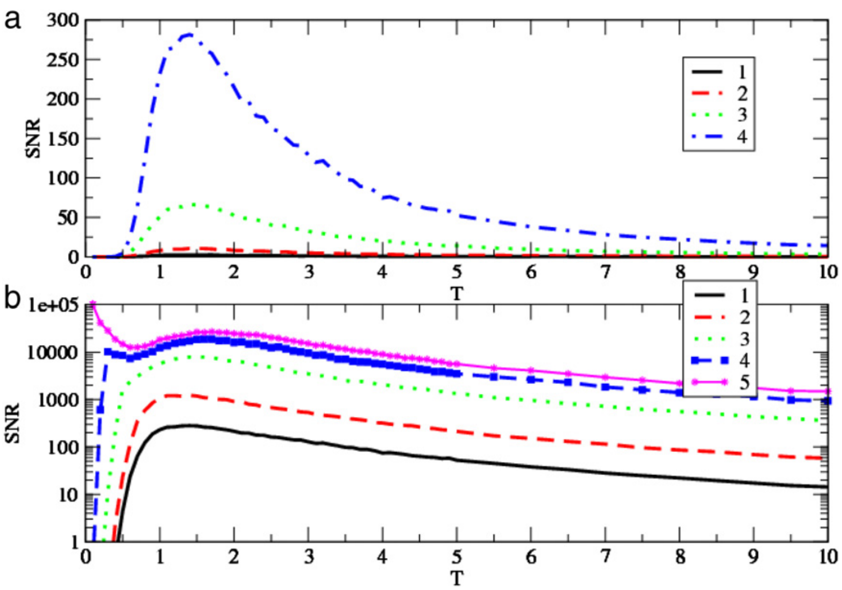

Fig. 8. (a) SNR as a function of T for four different values of H_0: (1) $H_0 = 0.01$, (2) $H_0 = 0.02$, (3) $H_0 = 0.05$, and (4) $H_0 = 0.1$. (b) SNR as a function of T for different values of H_0: (1) $H_0 = 0.1$, (2) $H_0 = 0.20$, (3) $H_0 = 0.50$, (4) $H_0 = 0.80$, and (5) $H_0 = 1.0$. Note the logarithmic scale in this case. In all these cases, $L = 256$ and $P = 256$.

Fig. 9 shows SNR as a function of H_0 for different temperatures. It can be seen that, for low temperatures ($T = 0.1$, for instance), SNR is very small for low H_0, but approximately from $H_0 = 0.8$ it grows abruptly, taking very high values around $H_0 = 1.0$. For high temperatures, SNR grows approximately linearly with H_0 (note the logarithmic scale). From the figure it

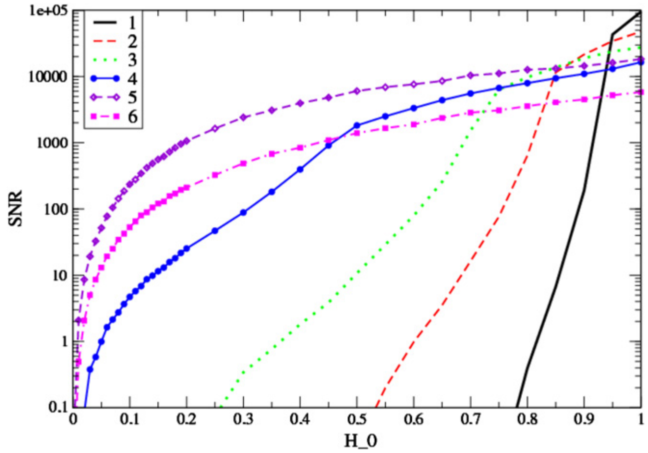

Fig. 9. SNR as a function of H_0 for different values of T: (1) $T = 0.1$, (2) $T = 0.2$, (3) $T = 0.3$, (4) $T = 0.5$, (5) $T = 1.0$, and (6) $T = 5.0$. Note the logarithmic scale. In all these cases, $L = 256$ and $P = 256$.

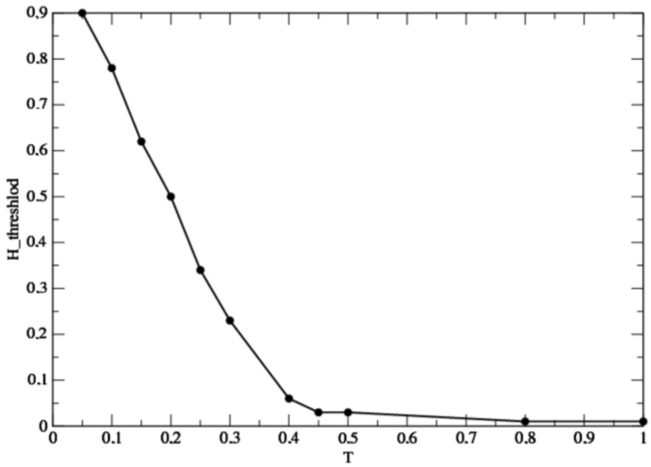

Fig. 10. Value of H_0 for which SNR is at least 0.1 for each temperature ($H_{threshold}$). In all these cases, $L = 256$ and $P = 256$.

may be apparent that there exists a critical H_0 for low temperatures, where below it, there is no system's response signal. Moreover, for low H_0 SNR is more important for high temperatures than for low ones. However, this tendency seems to change for high or intense H_0.

It is worth remarking here that, as it can be seen from Figs. 8 and 9, even for high H_0, where an anomalous behavior is observed for low temperatures (because of a perfect mapping to the external signal), a stochastic resonance phenomenon keeps going for intermediate temperatures.

Finally Fig. 10 shows the critical value of H_0 below which there is no response signal for each temperature, for the case of $L = 256$ and $P = 256$. For $T = 0.05$, the value of H_0 needed to have a response is about 0.9. As temperature increases, the $H_{threshold}$ decreases until $T = 0.8$, where it reaches a constant value of 0.01.

4. Conclusions

We have studied an opinion model with *social temperature* and *fashion*, that is a variant of the Sznajd model with two possible states for each agent (+1 or −1), meaning two possible opinions about some specified topic or the preference for one of two possible candidates in an election. The *social temperature* induces an effect analogous to that of *contrarians*, that is people who react in the opposite form of that established by the model. The *fashion* is introduced as an external field that induces people to follow one or the other tendency periodically.

We have shown that in a simple formation opinion model, the interaction of the internal social movement and external propaganda can produce an enhancement on the adoption of an opinion. In other words, when social temperature is adequate there exists a resonant phenomenon when a given external perturbation excites the system. It is worth remarking here that, at least for the present model, the external frequency is not relevant in the system response.

For intermediate H_0, when we observe the evolution of the mean opinion with time, we show the existence of three regions depending on the considered social temperature. These regions show how the internal and external influences interact with each other through the system's response.

For small temperatures the system evolves in a random way, that is the agents do not follow the propaganda signal. Somehow agents do not pay attention to the propaganda when they mainly follow the Sznajd rule. For intermediate temperatures there exists an optimal interplay between internal noise and external signal. Besides, it is worth mentioning here that there exists a delay or lag between the propaganda signal and the system response. For large social temperature, internal noise makes people to follow the external signal.

When we study the Fourier transform of the mean opinion, and the SNR, we find a temperature for which the response of the system is maximum, that is *stochastic resonance*. The height of the peak depends on the size of the system (increasing more or less linearly with L), but apparently, the dependence of SNR with the frequency is weak, at least for not too high frequencies. This indicates that for an optimal "propaganda bombardment" it is convenient to keep a low frequency "mode" in its submission.

We have also studied the effect of the amplitude of the external field and we have found that, in general, the signal-to-noise relationship grows with H_0, but the changes are very abrupt for low temperatures and high fields.

Acknowledgments

MCG and JAR are grateful to CONICET, Argentina, for continued support. HSW acknowledges financial support from MICINN, Spain, through Project FIS2010-18023.

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