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Efficient Tool for the Scheduling of Multiproduct Pipelines and Terminal Operations

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This paper addresses the problem of scheduling a transmission pipeline carrying several petroleum products from a single oil refinery to a unique distribution center over a monthly horizon. The proposed approach is based on a very efficient mixed-integer linear programming (MILP) continuous-time formulation that is capable of determining the optimal pipeline batch sequence and lot sizes as well as the schedule of lot injections in the line and the timing of product deliveries to the distribution terminal. Moreover, the MILP model rigorously accounts for customer product demands on a daily basis, key terminal operations like lot settling periods for quality control tasks, and a predefined set of alternative lot sizes to get better control of tank availability. The approach also does not require the division of pipeline segments into a number of single-product packs of known capacities since the volume scale is also handled in a continuous manner. Results found for several examples involving the schedule of a real-world single-source single-destination multiproduct pipeline under different operational scenarios show that the proposed method leads to better pipeline schedules than previous approaches in a more rigorous way and at much lower computational cost.

1. Introduction

Pipelines are by far the most important mode of transportation of refined petroleum products from refineries to distribution centers closer to large consuming areas. Their main purpose is to supply the required products to pipeline terminals at the right time so that the product quality downgrading due to interface and tank mixing and the transportation cost are both minimized. Pipeline systems are normally composed by gathering, transmission, and delivering lines. Gathering lines collect refined products from different production facilities and convey them to pipeline terminal stations. Large volumes of commodities are then shipped through long, high-pressure transmission (or trunk) pipelines with relatively large diameters to several distribution centers. The batch schedule in transmission lines attempts to minimize the amount of transmix generated at batch interfaces by making batches as large as possible. Products from different shippers meeting the same specifications can be unified and sent through the pipeline together as a single batch to several, distant delivering points. It is the so-called fungible operational mode. Some transmission pipelines are rather simple, connecting a single source to a single destination, while others are very complex with many sources, destinations, and connections to other pipelines. In contrast, lower-pressure delivering lines connect distribution terminals to closer, final markets and feature shorter lengths, smaller diameters, and many branches. They carry smaller lots to many more delivery points, with each lot destined to a single customer, i.e. the segregated operational mode. In other words, large batches in trunk lines are split up into smaller lots moving down through delivering pipelines to large customers and local markets directly. However, delivery of products from distribution centers to final consumers or gasoline stations is mostly made by truck and rail.[1]

The batching operation is a central feature in the efficient distribution of refined products by pipeline. A batch is a quantity of a given product or grade that will be injected in the line before pumping another one. Batching makes possible to meet daily product demands at large consuming markets by shipping lots of different products or grades of the same product in sequence through the same pipeline, instead of using a separate pipe for each one. To optimize operations, schedulers carefully establish the batch sequence and lot sizes that minimize product degradation and maximize customer satisfaction. More stringent regulations and the proliferation of product qualities lead to more batching, thus increasing the number of interfaces and the extent of batch cross-contamination. If the products are similar, the resulting mixture at the interface is added to the lower value product. If they are dissimilar, the hybrid product (the transmix) must be sent to a separate storage and reprocessed.

There are two nonpipe components that are critical parts of the delivery infrastructure and play an important role in the smooth and efficient operation of pipeline networks: distribution terminals and storage tanks. Pipeline terminals can be regarded as distribution hubs where product supplies from different sources are consolidated before sending them on to other directions and destinations. Such terminals have few dedicated tanks for each product to mostly facilitate lot discharging operations from the line and quality control procedures rather than using them for long-term storage. Batches discharged from the pipeline fully fill the assigned tanks and are kept inside them until quality control and approving tasks have been completed. Since the terminal has limited storage, the key to efficient terminal operations is the optimal coordination among incoming and outgoing flows. A lack of coordination can easily shut down the pipeline until the problem is solved.[2] This paper addresses the problem of scheduling a single transmission pipeline operating on fungible mode and carrying refined products from an oil refinery to a unique distribution center in order to meet customer demands on a daily basis. Key terminal operations like lot settling periods for quality control tasks and a discrete set of product-dependent lot-sizes for a better use of the storage capacity will additionally be considered.

Current Pipeline Scheduling Approaches. As stated by Rejowski and Pinto,[3] the scheduling of pipeline operations is a very complex problem that usually presents a small number of feasible solutions. The major scheduling decisions are concerned

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with the sequence of products to be shipped through the line, the lot sizes, the timing of lot injections and the allocation of batches to one or several distribution terminals. Efficient planning and scheduling tools are needed to lowering pumping and inventory costs, and simultaneously increasing customer satisfaction by reducing backorders and product contamination. A pipeline scheduler typically develops a monthly plan involving an hourly program of pipeline and terminal operations. The problem of scheduling multiproduct pipelines has received an increasing attention in the last ten years. Two different kinds of scheduling methodologies have been proposed: knowledge-based search techniques[4] and mixed-integer mathematical programming formulations. Depending on whether or not the pipeline volume and/or the time horizon are discretized, model-based scheduling methods can be classified into two types: discrete and continuous mixed-integer linear programming (MILP) approaches. Pure discrete formulations not only use a discrete time representation but also divide every pipeline segment into a significant number of single-product packs of equal or different sizes.[5−7] As a result, relatively short time horizons comprising just a few days are to be considered to limit the model size. The MILP discrete formulation of Rejowski and Pinto[5] based on disjunctive programming addresses the problem of scheduling a real-world single pipeline that conveys refined products to multiple destinations in series while allowing intermittent operations due to high peak electricity periods. Inventory, pumping and interface costs are minimized by optimizing the sequence of product lot injections, the batch sizes and the allocation of batches to pipeline terminals. All product demands are due at the end of the time horizon. The approach was later modified by adding special and nonintuitive integer cuts in order to minimize product contamination inside the pipeline segments and improve the computational performance of the MILP.[8]

The first MILP continuous formulation for the scheduling of a single multiproduct pipeline connecting a refinery to multiple depots was developed by Cafaro and Cerdá.[9,10] The proposed problem representation assumed that the pipeline is operated on a fungible mode and product demands are due at the end of the scheduling horizon. Neither time discretization nor division of pipeline segments into single-product packs were required. The MILP model permits to optimally establish the sequence of batch injections, the lot sizes, the pump rates, the start/end times of the pumping runs, the interface volumes to be reprocessed, and the amounts and types of products delivered to every depot tankage during lot injections. In addition, the model is able to track the location and size of product lots traveling along the line as well as product inventory levels in refinery and depot tanks at the start/end of every lot injection. Another interesting feature is the handling of pipeline shutdown periods. In this regard, the optimal schedules reported by Cafaro and Cerdá[9,10] for a pair of real-world examples showed that the method favors to stopping pipeline operations during high-pumping cost intervals, i.e. an intermittent pipeline operation. Likewise prior static scheduling methodologies, the approach assumes that product delivery requirements at pipeline terminals remain unchanged throughout the planning horizon. Moreover, it considers a single-period horizon and a unique due-date for all product deliveries to terminals at the horizon end. Reddy et al.[11] presented a mixed-integer nonlinear programming (MINLP) formulation and a novel MILP-based solution approach for optimizing crude oil unloading, storage, and processing operations in a multi-CDU (crude distillation unit) refinery receiving crude from very large multiparcel crude carriers through a single-buoy mooring (SBM) pipeline.

Relvas et al.[12] studied the scheduling of a real-world pipeline transporting a variety of oil derivatives from a single refinery to a unique distribution center. The final terminal contains a tank farm where each tank is devoted to a specific product. The authors have intended to close a gap in the multiproduct pipeline scheduling literature by focusing the study on the end-of-the-pipe. In previous papers, the internal dynamics in distribution terminals were partially considered. However, several strict quality control procedures may significantly affect the tank inventory management at the pipeline terminal. Such procedures are related to the so-called lot settling period during which every lot should stay in the assigned tank until quality control and lot-approving tasks are completed. The lot settling period starts just at the time the lot has been fully unloaded from the pipe. Daily client information is another important issue being addressed. In previous work, demands were considered at the end of the time horizon or at a few intermediate due dates.[13] In contrast, Relvas et al.[12] assumed that monthly product demands given on a daily basis are known by the pipeline operator before the start of the next month. Then, a close tracking of product inventories becomes necessary. Furthermore, pipeline scheduling methodologies usually account for the available storage capacity at any time on some aggregate level. Generally, any lot of a particular product fully fills up one or several dedicated tanks. Then, the lot size exactly matches the total capacity of the tanks assigned to it rather than being a free positive value. Considering that a limited number of tanks is available for each product and some of them contain lots on settling periods, it may occur that the tank farm cannot accommodate an arriving lot at a particular moment, and the pipeline operation should be stopped. Relvas et al.[12] developed a MILP problem formulation that combines pipeline operation with a rigorous inventory management in the distribution center. Time and volumes scales were modeled through a continuous representation. The approach can be regarded as an extension of the MILP formulation of Cafaro and Cerdá[10] for a single terminal with three additional features: handling of client demands on a daily basis, operational issues like the settling period for each new lot discharged to the terminal, and a predefined set of alternative lot sizes for each product. Another contribution of Relvas et al.[12] was the use of operational objectives such as the maximization of both the pipeline usage level and the total product inventories at the end of the time horizon. Real-world examples involving fixed product sequences or mixed sequences with a few “holes” to be filled up by the model were solved in reasonable CPU times. However, the mathematical formulation becomes extremely large when the choice of the complete product sequence and the settling period constraints are simultaneously considered. In such cases, the optimal schedule cannot be found after a CPU time limit of 2 h. In addition, the lots last injected and, consequently, the final product inventories hardly match future product requirements. As a result, a feasible schedule for the next monthly horizon may not exist.

More recently, Rejowski and Pinto[3] introduced an MINLP continuous-time formulation for the static scheduling of a single pipeline supplying a number of oil derivatives to multiple consumer markets. However, each pipeline segment is still composed by several packs with equal or different capacities to account for possible reductions in the pipeline diameter. Therefore, a discrete-type approach is still used to handle the lot size process, the movement of product lots along the pipeline and the lot discharging operations at the terminal. In addition, the horizon length is divided into an arbitrary number of time intervals of adjustable duration to allow variations in the pumping flowrate. Compared with discrete-time formulations, fewer time intervals are required to find the optimal schedule. The approach also takes into account the influence of the pumping flow rate on the transportation costs. Booster stations providing the energy for the product movement inside the pipeline are typically composed by several pumps connected in series or parallel configurations. By considering the dependency of the booster station yield on the products flow rate, the approach can establish the optimal pump rate conditions. The MINLP model was applied to a real-world example first introduced by Rejowski and Pinto[5] involving the transportation of six products to five terminals over a scheduling horizon of 100−130 h. Though the computational cost was drastically reduced with regards to pure discrete approaches, the solution time is still significant accounting for the length of the time horizon.

Cafaro and Cerdá[13,14] developed an efficient MILP continuous-time framework for the dynamic scheduling of pipelines over a multiperiod moving horizon. At the completion time of the current period, the planning horizon moves forward and the rescheduling process based on updated problem data is triggered again over the new instance of the planning horizon. Pumping runs can be extended over two or more periods and a different sequence of batches may be pumped each week. Moreover, multiple due-dates just occurring at period ends are considered. The approach has successfully solved a real-world pipeline scheduling problem over a rolling horizon always comprising four weekly periods. Results show that the sequence of pumping runs finally executed by the pipeline dispatcher along the time horizon looks quite different from the one found through static pipeline scheduling techniques. Later lots injected in the line during the current horizon should be planned to meet next-month delivery requirements, especially at long transmission pipelines. By updating the schedule at the start of every week, the pumping runs become shorter and its number rises. Besides, the pipeline utilization level shows a sizable increase.

This paper is concerned with the scheduling a single transmission pipeline carrying several petroleum products from an oil refinery to a unique distribution center over a monthly horizon. The proposed approach accounts for client demands on a daily basis, lot settling periods constraints and a discrete set of candidate lot sizes for each product all at once. It consists of a very efficient MILP continuous formulation that is capable of determining the complete product sequence, the lot sizes, the timing of pipeline lot injections and product deliveries at the distribution terminal, the schedule of lot settling periods, and simultaneously monitoring customer demand satisfaction and product inventory levels on a daily basis. Since the pipeline has no intermediate terminals, the size of any product lot flowing along the line does not change at all during the journey to the final destination. As a result, batch-size tracking is not needed and consequently a much simpler problem representation than the one proposed by Cafaro and Cerdá[10] has been developed. Results found for several examples involving the scheduling of a real-world single-source single-destination multiproduct pipeline under different operational scenarios show that the proposed approach leads to better pipeline schedules in a more rigorous way and at much lower computational cost.

2. Problem Definition

Given (a) a multiproduct pipeline connecting an oil refinery to a unique distribution center; (b) the number and type of products to be transported through the pipeline; (c) the forbidden two-product sequences; (d) the daily product demands at the distribution center; (e) the alternative lot sizes and the maximum storage capacity for each product; (f) the initial pipeline conditions (sequence of batches inside the line at $t = 0$ and their sizes); (g) the initial inventory of every product in terminal tanks; (h) the maximum number of lot injections throughout the time horizon; (i) the constant product pumping rate; (j) the minimum settling time for each product; and (k) the multiperiod time horizon composed by a specified number of daily periods, it should be established:

(1) the sequence of product lots to be pumped in the pipeline;

(2) the selected lot sizes and the starting/end times of lot injections;

(3) the amounts and types of products delivered to storage tanks from batches arriving to the terminal during every lot injection;

(4) the starting time for the lot settling period, i.e. the time at which it has been completely loaded in the terminal tank;

(5) the end time of the settling period at which the lot is released to meet client demands;

(6) the product inventory management at the unique terminal by simultaenously considering product batches on settling period, released product lots, and client demands on a daily basis.

3. Model Assumptions

To develop the problem mathematical formulation, the following assumptions have been made:

(1) A single multiproduct pipeline with unidirectional flow is considered.

(2) The pipeline remains completely full of incompressible liquid products at any time. The only way to get a volume of product out of the line at the terminal is by injecting an equal volume at the origin.

(3) Product batches are sequentially injected in the pipeline one after another, with no physical barrier between them. They move along the pipeline at turbulent flow to retard mixing.

(4) The "transmix" or contamination volume between a particular pair of refined products is a known constant.

(5) Every batch is pumped at a product-dependent fixed-flow rate. The way of relaxing this assumption is explained in sections 4.2 and 4.6.

(6) The distribution center contains a tank farm with dedicated storage units of known capacity for each product.

(7) At most one terminal tank at a time is connected to the pipeline during discharge operations and the setup time for switching from one tank to another is negligible. Nonetheless, idle time intervals between consecutive pumping runs and the related discharge operations can arise.

(8) After delivering a full batch to the terminal, it should stay in the assigned tanks for quality control and lot-approving tasks during at least the lot settling time. The length of the settling period is a product-dependent datum.

(9) During the settling period, the product lot is not available to meet customer demands. At any time, there will be a certain inventory of product p ready for clients and an additional amount on settling period.

(10) Any tank in the distribution center can be in one of the following states: (i) receiving a lot and gradually filling to full capacity; (ii) fully filled and waiting for the completion of the settling period; (iii) ready to unload the product batch to meet customer demands; (iv) empty and staying idle until the next product lot arrives.

(11) The volume of a batch injected in the pipeline exactly matches the total capacity of the assigned storage tanks at the terminal.

(12) Daily product demands for the next month are known at the time of planning the lot product sequence and the internal operations at the distribution center.

(13) The refinery production schedule has been developed taking into account the daily product demands at the distribution terminal. Therefore, there is no need to monitor product inventory levels in refinery tanks.

(14) Daily terminal demands are due at the end of every day.

4. Problem Mathematical Formulation

In the proposed problem formulation, the mathematical constraints have been grouped into eight major categories: (1) *Batch-defining constraints* deal with the allocation of oil refined products to batches, the sequence of batch injections, the pumping run durations, and the interface volume between consecutive product lots. Since their expressions do not depend on the number of pipeline terminals, batch-defining constraints are similar to those first introduced by Cafaro and Cerdá.[10,14] (2) *Lot-sizing constraints* select the batch volumes from a discrete product-dependent set of lot sizes. This problem feature has already been considered by Relvas et al.[12] However, the new lot-sizing constraint formulation produces a substantial saving in binary variables. (3) *Pipeline shutdown periods* deal with prespecified shutdown intervals for maintenance work. (4) *Batch tracking constraints* monitor the location of batches moving along the line at the end of every pumping run. Since a pipeline system with a unique distribution terminal is considered, batch-tracking constraints are much simpler than those first proposed by Cafaro and Cerdá[10,14] and later used by Relvas et al.[12] (5) *Terminal delivery constraints* control the occurrence of discharge operations from the pipeline to terminal tanks while inserting a new lot. As product batches are delivered to a single terminal, the modeling of these constraints is greatly simplified with regards to previous approaches. (6) *Lot settling constraints* ensure the completion of quality-control tasks on discharged product lots before releasing them to the market. Compared with Relvas et al.,[12] more rigorous expressions have been developed. (7) *Customer demand constraints* monitor the daily period at which a product lot loaded in a terminal tank is ready to meet customer demands. Relvas et al.[12] have already considered daily product requirements at the unique pipeline terminal, but the new approach presents more rigorous and less complex demand satisfaction constraints. (8) *Product inventory tracking constraints* aim to keeping product inventory levels within the specified limits and avoiding, if possible, product shortages. In order to account for the lot settling period, new inventory balance equations properly handling product batches on settling period and product inventories ready to meet customer demands have been developed.

4.1. Batch-Defining Constraints. Product Allocation. A batch $i \in I^{\text{new}}$ to be pumped in the pipeline can at most contain a single refined petroleum product $p \in P$. Since each pumping run can be extended over multiple time periods, a single set of batch injections I^{new} for the whole time horizon is just considered. Let the binary variable $y_{i,p}$ denote the allocation of product p to lot $i \in I^{\text{new}}$. Then,

$$\sum_{p \in P} y_{i,p} \le 1 \qquad \forall\, i \in I^{\text{new}} \tag{1}$$

For a fictitious batch $i \in I^{\text{new}}$ never pumped in the pipeline, the assignment variable $y_{i,p}$ is equal to zero for every $p \in P$. The cardinality of I^{new} should at least match the number of pumping runs performed at the optimal schedule. As the alternative lot sizes for each product are problem data, a simple expression for computing a good estimation of $|I^{\text{new}}|$ is given by:

$$|I^{\text{new}}| = \sum_{p \in P} \frac{1}{\langle b \rangle_p} \left(\sum_{t \in T} \text{dem}_{p,t} \right) \tag{2}$$

where $\langle b \rangle_p$ is the mean lot size for product p and $\text{dem}_{p,t}$ is the pth-product demand at the daily period t.

When no fictitious batches at all arise at the best problem solution, there is some chance that $|I^{\text{new}}|$ is not large enough and the true optimal pipeline schedule has not yet been discovered. If so, the value of $|I^{\text{new}}|$ should be increased by one and the resulting problem formulation is to be solved again. This iterative procedure must be stopped when some elements of I^{new} are never performed at the optimum. Even if the value of $|I^{\text{new}}|$ is rather restrictive and lower than required, it will be later shown that a nonoptimal solution is still found because product shortages are allowed by the proposed MILP formulation. In other words, a feasible schedule may include late product deliveries and/or nonsatisfied product demands at the horizon end.

Batch Sequencing. The injection of a new batch $i \in I^{\text{new}}$ in the pipeline origin should start after dispatching the previous one ($i - 1$) and performing the required changeover operation.

$$C_i - L_i \ge C_{i-1} + \tau_{p',p}(y_{i-1,p} + y_{i,p'} - 1) \qquad \forall\, i \in I^{\text{new}};\ p, p' \in P \tag{3}$$

$$L_i \le C_i \le h_{\max} \qquad \forall\, i \in I^{\text{new}} \tag{4}$$

The continuous variable C_i represents the completion time for the pumping run of batch $i \in I^{\text{new}}$, the variable L_i is the related duration, and the parameter $h_{\max}$ is the overall length of the scheduling horizon. In turn, $\tau_{p',p}$ stands for the changeover time between consecutive injections of products p and p'. For a pair of nonfictitious batches ($i - 1$, i), only one of the constraints (3) will become binding at the optimum. The active constraint will be related to products $p, p' \in P$ whenever the new batches ($i - 1$) and i contain products p and p', respectively.

Pumping Run Length. A fictitious batch $i \in I^{\text{new}}$ never injected in the pipeline ($\sum_p y_{i,p} = 0$) will feature a length L_i equal to zero. In turn, the shortest/largest batch sizes for product p will set the bounds on the pumping run length of a nonfictitious batch containing p.

$$\sum_{p \in P} l_{\min,p} y_{i,p} \le L_i \le \sum_{p \in P} l_{\max,p} y_{i,p} \qquad \forall\, i \in I^{\text{new}} \tag{5}$$

where

$$l_{\min,p} = \frac{1}{\text{vb}_p} (\min_{s \in S_p} b_s);\ l_{\max,p} = \frac{1}{\text{vb}_p} (\max_{s \in S_p} b_s) \qquad \forall\, p \in P \tag{6}$$

and the parameter vb_p stands for the selected pumping rate of any new batch of product p. Furthermore, the set S_p includes all possible batch sizes for product p.

Fictitious Batches. In order to accelerate the search for the optimal schedule, fictitious batches $i \in I^{\text{new}}$ featuring $\sum_p y_{i,p} = 0$ and obviously $L_i = 0$ at the optimum should be left at the end of the batch sequence. If N_R is the number of pumping runs being executed, the last elements $\{|I^{\text{new}}| - N_R\}$ of the set I^{new} are reserved for fictitious batches never injected in the pipeline. Therefore, the following constraints should be added to the problem formulation:

$$\sum_{p\in P} y_{i,p} \le \sum_{p\in P} y_{i-1,p} \quad \forall\, i \in I^{new} \tag{7}$$

Interface Volume between Consecutive Batches. By convention, batch $(i - 1) \in I$ has been pumped in the line just before batch $i \in I$. Then the volume of the interface between those consecutive batches will never be lower than the parameter $IF_{p',p}$. Such parameter denotes the volume of the transmix between products p and p', just in case batches $(i - 1)$ and i contain products p and p', respectively (see Figure 1). Otherwise, the constraint will become redundant. Likewise previous approaches, the value of $IF_{p',p}$ for any ordered pair of products (p', p) is assumed to be known and independent of the pump rate. In contrast to discrete representations, the proposed continuous model is able to account and trace the location of transmix volumes from the origin to the last distribution terminal.

$$WIF_{i,p',p} \ge IF_{p',p}(y_{i-1,p} + y_{i,p'} - 1) \quad \forall\, i \in I, i > 1\; p, p' \in P \tag{8}$$

Forbidden Product Sequences. Because of product contamination, some sequences of products in the pipeline are forbidden. If (p',p) represents a forbidden sequence, a pair of batches containing products p and p' must not be consecutively pumped in the pipeline. Then, the following constraint is added to the problem formulation,

$$y_{i-1,p} + y_{i,p'} \le 1 \quad \forall\, i \in I^{new} \tag{9}$$

4.2. Lot-Sizing Constraints. As stated in the problem definition, the volume of a new batch to be pumped in the line should be selected from a discrete product-dependent set of lot-sizes. To handle this problem feature, also considered by Relvas et al.,[12] additional 0−1 variables are to be defined. In the opposite case (i.e., the continuous lot-size case), previously studied by Cafaro and Cerdá,[10,14] the lot size (Q_i) is just a continuous problem variable whose value is freely selected by the model. Consequently, the constraints presented in this section are not needed for the continuous lot-size case.

One of the major shortcomings of the discrete lot-size problem formulation proposed by Relvas et al.[12] is the large number of binary variables required to choose the size of the new batches to be pumped in the pipeline. Since the set of alternative lot sizes varies with the product, Relvas et al.[12] defined a three-index binary variable $B_{i,p,s}$ in order to select the lot size s for a new batch i containing product p. However, the product assigned to batch i is not known beforehand. Therefore, the domain of the index s should include the whole set of candidate sizes for all products instead of just the ones for product p. In this way, many binaries $B_{i,p,s}$ are to be defined. If $|I| = 35$ batches, $|P| = 6$ products and $|S| = 16$ candidate sizes, then 3360 binaries $B_{i,p,s}$ are required in the model of Relvas et al.,[12] despite at most three candidate sizes for each product are considered. A more compact model can be developed by handling the allocation of products to batches through the two-index binary variables $y_{i,p}$ and the selection of lot sizes by means of the two-index binaries $v_{i,s}$. By doing so, the number of binaries is cut down by a factor of 6, i.e. it drops to 560. The new formulation of the lot-size constraints is given by eqs 10−12,

$$\sum_{s\in S_p} v_{i,s} = y_{i,p} \quad \forall\, i \in I, p \in P \tag{10}$$

where S_p is the set of candidate batch sizes for product p. If product p has been assigned to batch i ($y_{i,p} = 1$), one of the candidate sizes $s \in S_p$ for product p should be selected. In other words, only one of the binaries $v_{i,s}$ with $s \in S_p$ should be equal to one while the others remain all equal to zero. Let us assume that b_s is the sth-lot size candidate for product p. Then, the amount of product p contained in batch i ($QP_{i,p}$) is given by

$$QP_{i,p} = \sum_{s\in S_p} b_s v_{i,s} \quad \forall\, i \in I, p \in P \tag{11}$$

and the size of a new batch i to be inserted in the line is computed through eq 12,

$$Q_i = \sum_{p\in P} QP_{i,p} = \sum_{p\in P}\sum_{s\in S_p} b_s v_{i,s} \quad \forall\, i \in I \tag{12}$$

For a nonfictitious batch $i \in I^{new}$, just a single variable $QP_{i,p}$ for any $p \in P$ will take a positive value. Moreover,

$$\begin{aligned} LP_{i,p} &= \left(\frac{1}{vb_p}\right) QP_{i,p} \quad && \forall\, i \in I^{new}, p \in P \\ L_i &= \sum_{p\in P} LP_{i,p} && \forall\, i \in I^{new} \end{aligned} \tag{13}$$

where $LP_{i,p}$ is driven to zero if $QP_{i,p} = 0$.

When the assumption (5) is relaxed and the pumping rate for any product p can be adjusted within the range (vb_{min}, vb_{max}), eqs 5 and 6 should be replaced by the following eq 13′:

$$(b_s/vb_{min})v_{i,s} \le L_i \le (b_s/vb_{max})v_{i,s} \quad \forall\, i \in I^{new}, p \in P, s \in S_p \tag{13′}$$

4.3. Pipeline Shutdown Periods. Nonprespecified Pipeline Shutdowns. Since a discrete number of lot sizes for each product is available and a fixed pumping rate has been adopted, it is quite likely that the sum of the pumping run durations cannot exactly match the length of the time horizon h_{max}. As a result, some idle time will usually arise along the time horizon. Such a pipeline shutdown period can be minimized through an optimal choice of the product lot sizes. However, the alternative lot sizes depend on the product. Then, one can expect a very short pipeline shutdown when the sequence of products to be pumped in the line is not fixed beforehand but chosen through the proposed formulation. The time interval during which such nonprespecified pipeline shutdowns occur will be optimally selected by the model so that all customer demands are timely satisfied. A shutdown interval is inserted by the model between consecutive runs $(i - 1)$ and i whenever $C_i - L_i > C_{i-1}$. Nonprespecified pipeline shutdown operations can also occur because of temporary shortages of storage capacity for a certain product at the distribution center. In this case, the pipeline

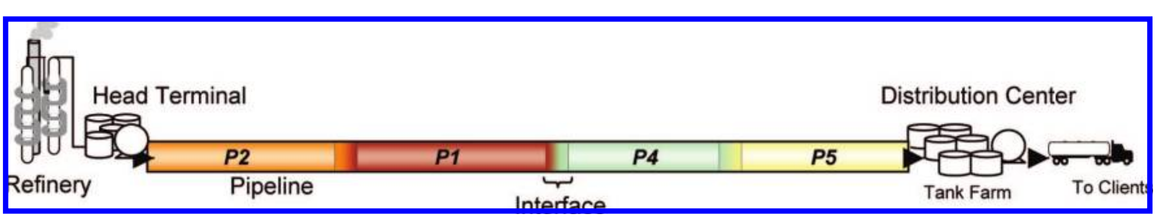

Figure 1. Single unidirectional multiproduct pipeline system.

activity will be restarted when enough tank capacity becomes again available by shipping refined products to the market.

Prespecified Pipeline Shutdowns. Let us consider a prespecified ordered set of pipeline shutdowns $k \in K$ for maintenance work to be done during the time period (s_k, e_k). Pipeline shutdowns have been arranged so that $e_k < s_{k+1}$. To enforce prespecified shutdown periods, new binary variables $w_{i,k}$ should be defined. Pumping run i will end before the start of the shutdown period k whenever $w_{i,k} = 0$. Otherwise, run i begins after time e_k and $w_{i,k} = 1$. Then,

$$C_i \le s_k + (h_{\max} - s_k)w_{i,k} \quad \forall\, i \in I^{\text{new}}, k \in K \tag{14}$$

$$C_i - L_i \ge e_k w_{i,k} \quad \forall\, i \in I^{\text{new}}, k \in K \tag{15}$$

$$w_{i,k+1} \le w_{i,k};\ w_{i-1,k} \le w_{i,k} \quad \forall\, i \in I^{\text{new}}, k \in K \tag{16}$$

According to the last constraints (16), pumping run i will end before period $k+1$ ($w_{i,k+1} = 0$) if completed before time s_k ($w_{i,k} = 0$). Moreover, run i will begin after period k if started after time e_{k+1}. Similarly, run ($i - 1$) will end before period k if run i finishes before s_k. The presence of shutdown periods increases the chance of inventory shortages at the pipeline terminal.

4.4. Batch Tracking Constraints. When a single unidirectional pipeline transporting multiple oil derivatives from a unique refinery to a single distribution terminal is considered, much simpler batch-tracking constraints than those introduced by Cafaro and Cerdá[10] can be developed. Since there is no product extraction from the line between the origin and the unique distribution terminal, the content of any lot i will not change along the line until it arrives at the distribution center. It will remain steadily equal to its initial size Q_i. Therefore, it makes no sense to track the size of batch i at the completion of a new pumping run i' ($W_i^{(i')}$) and to keep the variable $W_i^{(i')}$ in the problem formulation. Moreover, the upper volumetric coordinate of batch i after completing the pumping run of batch $i' \ge i$ (i.e., lot i' follows lot i) is equal to the total volume put in the line by injecting batches i, $i+1$, $i+2$, ..., i', from time ($C_i - L_i$) to time $C_{i'}$.

$$F_i^{(i')} = \sum_{l=i}^{i'} Q_l \quad \forall\, i, i' \in I^{\text{new}}\ (i \le i') \tag{17}$$

Let σ be the volumetric coordinate of the unique distribution terminal, i.e. the pipeline total volume. If $F_i^{(i')} \ge \sigma$, then a portion of or the whole batch i has been loaded in the assigned terminal tanks after completing the injection of batch i' at time $C_{i'}$. For an old batch $i \in I^{\text{old}}$ already in the line at the initial time of the current horizon with a starting upper coordinate F_i^{o}, its movement along the pipeline as new batches are injected can be tracked through the following eq 18,

$$F_i^{(i')} = F_i^{\text{o}} + \sum_{\substack{l \in I^{\text{new}} \\ l \le i'}} Q_l \quad \forall\, i \in I^{\text{old}}, i' \in I^{\text{new}} \tag{18}$$

By excluding the variable $W_i^{(i')}$ from the problem formulation, a significant saving in constraints is obtained.

4.5. Product Deliveries to the Distribution Terminal. Let us define the binary variable $x_i^{(i')}$ to indicate whether or not some amount of product contained in batch $i \in I$ moving along the pipeline can be diverted to the unique destination while injecting a new batch $i' \ge i$. The condition $x_i^{(i')} = 1$ denotes that a portion of batch i arrives at the terminal while injecting a later batch i' in the pipeline. Otherwise, $x_i^{(i')} = 0$ and no material from batch i can be loaded in the assigned terminal tank. Such feasibility condition for the partial/total transfer of batch i from the pipeline to the terminal is given by the following set of equations,

$$D_{\min} x_i^{(i')} \le D_i^{(i')} \le D_{\max} x_i^{(i')} \quad \forall\, i \in I, i' \in I^{\text{new}}(i \le i') \tag{19}$$

$$D_i^{(i')} \le F_i^{(i')} - \sigma x_i^{(i')} \quad \forall\, i \in I, i' \in I^{\text{new}}(i \le i') \tag{20}$$

If $x_i^{(i')} = 0$, then eq 19 drives $D_i^{(i')}$ to zero and eq 20 reduces itself to $F_i^{(i')} \ge 0$ (redundant condition). Otherwise, $D_{\min} \le D_i^{(i')} \le D_{\max}$ and $D_i^{(i')} \le F_i^{(i')} - \sigma$, where $[F_i^{(i')} - \sigma]$ is the volume of batch i "located" beyond the terminal coordinate σ, i.e. the volume of lot i already loaded in the depot storage tank. There are two additional conditions to be satisfied. First, the total amount of product transferred from lot i to the assigned terminal tanks can never be greater than the original size of batch i given by either Q_i for a new lot i or Q_i^{o} (a problem datum) for an old lot i in transit at time zero.

$$\sum_{\substack{i' \in I^{\text{new}} \\ i \le i'}} D_i^{(i')} \le Q_i \quad \forall\, i \in I^{\text{new}} \tag{21}$$

$$\sum_{i' \in I^{\text{new}}} D_i^{(i')} \le Q_i^{\text{o}} \quad \forall\, i \in I^{\text{old}} \tag{22}$$

Moreover, the total volume of products delivered to the terminal from one or several lots $i \in I$ while injecting a later batch i' must be exactly equal to the size of batch i' (the liquid incompressibility condition).

$$\sum_{\substack{i \in I \\ i \le i'}} D_i^{(i')} = Q_{i'} \quad \forall\, i' \in I^{\text{new}} \tag{23}$$

Whenever $F_i^{(i'-1)} - \sigma > Q_i$, the whole lot i has been entirely loaded into the terminal tank at time $C_{i'-1}$. Then, constraint 21 drives both variables $D_i^{(i')}$ and $x_i^{(i')}$ to 0, to meet the condition $D_i^{(i')} \ge D_{\min} x_i^{(i')}$. The parameter $D_{\min}$ is adopted equal to a sufficiently small quantity. Consequently, the condition $x_i^{(i')} = 0$ holds if either lot i is still in transit to the terminal at time $C_{i'}$ or the lot i has been entirely delivered to the terminal during a previous pumping run $k < i'$. Though much simpler than the constraints proposed by Relvas et al.,[12] the set of eqs 19−23 still permits the rigorously control the transfer of products from the pipeline to the distribution terminal.

To analyze the effectiveness of eqs 17−23 let us consider a simple example (see Figure 2). At the completion time $C_{i'-1}$ of the last pumping run ($i' - 1$) = B4, there are four batches B4, B3, B2, and B1 inside the pipeline containing products P3, P1, P2, and P1, respectively. Suppose that the farthest batch B1 has been partially delivered to the terminal during pumping run ($i' - 1$), and $D_{\text{B1}}^{(i'-1)} = F_{\text{B1}}^{(i'-1)} - \sigma = 21\,000 - 18\,000 = 3000$ volumetric units. The next pumping run i' injects batch B5 containing product P4 with a size $Q_{\text{B5}} = 10\,000$. By eq 17, $F_{\text{B1}}^{(i')} = F_{\text{B1}}^{(i'-1)} + Q_{i'} = 21\,000 + 10\,000 = 31\,000$. Similarly, $F_{\text{B2}}^{(i')} = 24\,000$; $F_{\text{B3}}^{(i')} = 19\,000$; $F_{\text{B4}}^{(i')} = 13\,000$; and $F_{\text{B5}}^{(i')} = 10\,000$. As prescribed by eq 20, just batches B1, B2, and B3 featuring $F_i^{(i')} > \sigma$ can be loaded in terminal tanks. Product delivery from batch B2 to terminal tanks, $D_{\text{B2}}^{(i')}$, is limited by the original size of B2 (Q_{B2}) through eq 21. Then, $D_{\text{B2}}^{(i')} \le Q_{\text{B2}} = 5000$, while the transfer of batch B3 is bounded by eq 20 to the beyond-the-terminal portion of lot B3; i.e. $D_{\text{B3}}^{(i')} \le F_{\text{B3}}^{(i')} - \sigma = 19\,000 - 18\,000 = 1000$. Furthermore, eq 23 establishes the product delivery from batch B1 to the terminal during the injection of lot $i' = \text{B5}$ featuring a size $Q_{i'} = 10\,000$:

$$D_{\text{B1}}^{(i')} = Q_{i'} - D_{\text{B2}}^{(i')} - D_{\text{B3}}^{(i')} \ge 10\,000 - 5\,000 - 1000 = 4000$$

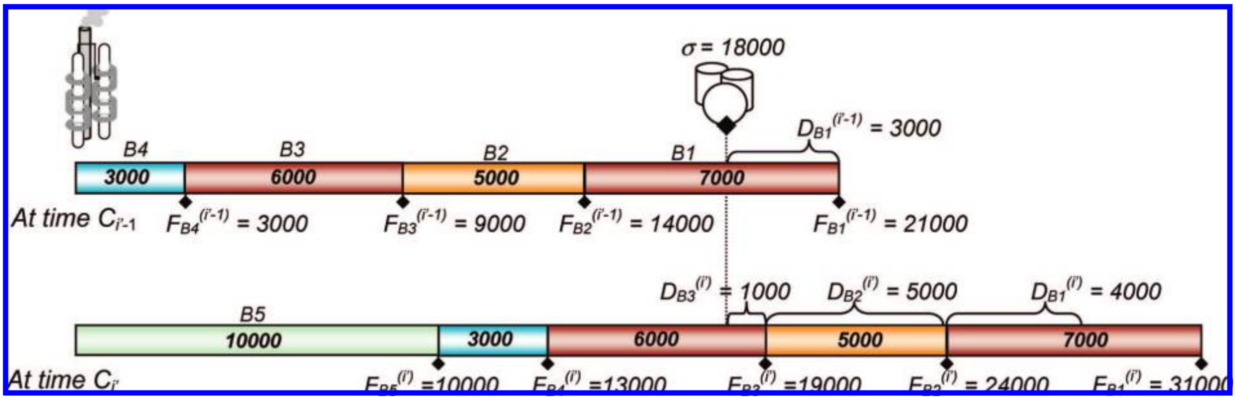

Figure 2. Simple example illustrating batch tracking constraints 19−23.

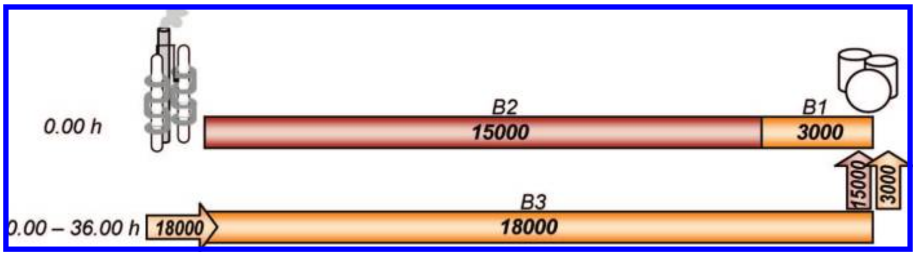

Figure 3. Simple example illustrating a bad estimation of the lot arrival time to the terminal.

Similarly, eq 21 limits the value of $D_{B1}^{(i')}$ to the current size of B1: $D_{B1}^{(i')} \le Q_{B1} - D_{B1}^{(i'-1)} = 7000 - 3000 = 4000$. Consequently, the only way to satisfy all the equations is: $D_{B1}^{(i')} = 4000$; $D_{B2}^{(i')} = 5000$; and $D_{B3}^{(i')} = 1000$ as shown in Figure 2.

4.6. Lot Settling Period Condition. Additional constraints were included in the formulation of Relvas et al.[12] to account for real-world internal operations in the distribution center. Before releasing discharged product lots to the market, they should remain in terminal tanks during at least a certain settling time st_p for quality control and lot-approving tasks. At any time, there will be a certain inventory of product p at the terminal ready to meet customer demands coexisting with some lots of product p on their settling period waiting for quality approval in pipeline terminal tanks.

To deal with the setting time period condition, Relvas et al.[12] have made two limiting assumptions:

(a) The length of the settling period is the same for all products though in practice nonstandard refined products may require longer quality control tasks.

(b) The settling period for a lot i starts when it ends the pumping run pushing either the entire lot i or the last portion of lot i to the assigned tank at the pipeline terminal.

However, several batches can be loaded in terminal tanks while injecting a new batch in the pipeline and lot i may be the first one being pushed to the terminal. Indeed, the settling period should really start just at the time the whole lot i has been loaded into the terminal tank and this event may occur much earlier than the end of the pushing pumping run. Let us consider a simple example described in Figure 3 to understand why the selected initial time for the settling period in Relvas et al.[12] may be a poor approximation. At time zero, two batches B2 and B1 are inside the pipeline when the injection of a new batch B3 begins. As shown in Figure 3, the sizes of lots B3, B2, and B1 are 18 000, 15 000, and 3000 vu, respectively. Therefore, lots B2 and B1 are entirely transferred to terminal tanks when the pumping of B3 finishes. Assuming a pumping rate of 500 vu/h, the delivery of lot B1 to the distribution center ends at time (3000/500) = 6 h, while the pumping of B3 is completed at time (18 000/500) = 36 h, i.e. 30 h after the start of the settling period for lot B1. If the settling period lasts 24 h, the lot B1 would be already available to meet customer demands six hours before ending the injection of lot B3 in the line.

To avoid both assumptions, the binary variable $x_i^{(i')}$ denoting that lot i is either still in transit or has already arrived at the unique terminal during the injection of batch i' has been reused. Whenever $x_i^{(k)} = 0$ and $F_i^{(k)} < \sigma$ for all $k = i, i+1, ..., i'$, the lot i remains entirely in the pipeline during the pumping of any lot $k \ge i$. If $x_i^{(i')} = 1$, the lot i has arrived at the terminal and the unloading operation from the line has already started. In case the discharge of lot i is completed during run $(i' - 1)$, then the value of $x_i^{(i')}$ drops again to 0. Let the parameter st_p denote the settling time for product p and the variable RT_i stand for the time at which lot $i \in I$ is released to meet customer demands. Then, the lot setting period constraint is given by

$$RT_i \ge (C_{i'} - L_{i'}) + G_i^{(i')} + \sum_{p\in P} st_p y_{i,p} - H(1 - x_i^{(i')}) \quad \forall i \in I, i < |I|, i' \in I^{new}, i' \ge i \quad (24)$$

where $G_i^{(i')}$ is the amount of time required to push batch i out of the pipeline to the terminal during the injection of the new batch $i' \in I^{new}$ starting at time $(C_{i'} - L_{i'})$. Therefore, every product lot i loaded in terminal tanks during a pumping run has a characteristic time RT_i called the release-time of lot i, that represents the earliest time at which it can be delivered to clients. To reduce the release time of batch i (RT_i) the model tends, if necessary, to diminish the value of $G_i^{(i')}$ as much as possible. A lower bound of $G_i^{(i')}$ is given by the set of constraints 25.

$$G_i^{(i')} \ge \left(\frac{1}{vb_p}\right)\left[\sigma y_{i',p} - F_{i+1}^{(i'-1)} - \left(Q_i - \sum_{\substack{l=i \\ l\in I^{new}}}^{i'} D_i^{(l)}\right)\right] \quad \forall i \in I, i' \in I^{new}(i < i'), p \in P \quad (25)$$

For every pair $i < i'$, at most a single constraint 25 will hold, i.e. the one with $y_{i',p} = 1$. However, such an instance of constraint 25 will also be redundant if batch i has already left the pipeline at the start of run i' because: $\sigma - F_{i+1}^{(i'-1)} < 0$. Conversely, eq 25 will not be redundant for any run $i' > i$ such that $\sigma < F_i^{(i')} \le \sigma + Q_i$. It should be observed that eq 24 allows, if necessary, to extend the settling period of lot i and by so doing delaying its release time RT_i to the market. If a pumping rate interval (vb_{min}, vb_{max}) rather than a constant value vb_p is given, then vb_p should be replaced by vb_{min} in eq 25 to determine a conservative lower bound for $G_i^{(i')}$.

Let us further analyze eq 24. If $x_i^{(i')} = 0$ and $F_i^{(i')} \le \sigma$, then

$$\sum_{\substack{l=i \\ l\in I_{new}}} D_i^{(l)} = 0$$

and the lot setting period constraint becomes redundant.

$$RT_i \ge C_{i'} - L_{i'} + G_i^{(i')} + \sum_{p\in P} st_p y_{i,p} - H \qquad \forall\, i \in I, i < |I|, i' \in I^{new}, i' \ge i$$

H is an estimation of the latest time at which the last lot injected in the pipeline during the current horizon may arrive to the final terminal, i.e. $H = h_{max} + [\sigma/\min_p(vb_p)]$. Let us now assume that $x_i^{(i')} = 1$ and some portion of batch $i \in I$ still remains in the line after completing the injection of lot i'. Then, the following condition holds: $\sum_{l=i}^{i'} D_i^{(l)} < Q_i$ and the corresponding constraint 25 for batch i will yield a lower bound for $G_i^{(i')}$ equal to L_i. Consequently,

$$RT_i \ge C_{i'} + \sum_{p\in P} st_p y_{i,p} \qquad \forall\, i \in I, i < |I|, i' \in I^{new}, i' \ge i$$

The highest lower bound for RT_i will be set by run j during which the transfer of lot i to the terminal is completed and $\sum_{l=i}^{j} D_i^{(l)} = Q_i$. Assuming that lot j contains product p, such a bound will be equal to the time at which lot i has been fully loaded in the terminal tank.

$$G_i^{(j)} \ge \left(\frac{1}{vb_p}\right)(\sigma - F_{i+1}^{(j-1)}), \qquad RT_i \ge C_j - L_j + \left(\frac{1}{vb_p}\right)(\sigma - F_{i+1}^{(j-1)}) \tag{25'}$$

$F_{i+1}^{(j-1)}$ is the lower coordinate of lot i at the start of run j and $[\sigma - F_{i+1}^{(j-1)}]$ is the volume of lot i still in the line at the beginning of run j. In case lot i is needed as soon as possible to meet customer demands, the constraints 25′ associated to both run j and batch i will be active at the optimum. In other words, the settling period will begin immediately after the entire lot i is inside the terminal tank.

Tightening Cut. The batch i can arrive at the unique terminal just after the preceding one ($i - 1$). In case both lots i and ($i - 1$) are transferred to the terminal before the end of the scheduling horizon, the corresponding constraints 24 will provide their release times. If instead a part of or the entire lot i is still in the pipeline at the end of the current horizon, constraint 26 will provide a lower bound on its release time RT_i to occur during the next scheduling horizon.

$$RT_i - \sum_{p\in P} st_p y_{i,p} \ge RT_{i-1} - \sum_{p\in P} st_p y_{i-1,p} + \left(\frac{Q_i}{\max_{p\in P}(vb_p)}\right) \qquad \forall\, i \in I(i > 1) \tag{26}$$

The last term on the RHS accounts for the estimated time required to push the entire lot i out of the pipeline to the distribution center. It assumes that the batch i will be pushed into the terminal tank at the highest pumping rate and that the pipeline is continuously operated. For the other lots reaching the end of the pipeline before time h_{max}, eq 24 gives a fair, nonstrict lower bound of RT_i. In the Appendix, simpler expressions for eqs 13 and 24−26 resulting from assuming a common fixed pumping rate for every product are presented.

4.7. Customer Demand Constraints. So far, the problem time events are those at which the pumping runs $i \in I^{new}$ are completed, i.e. the completion times C_i. From such time events, it can be established the times RT_i, $\forall\, i \in I^{new}$ at which new lots of products are released and added to the available inventories at the distribution center (the supply side). However, product demands are associated to daily periods. To precisely monitor product inventory levels available to meet customer demands (the demand side), the start/end daily times {0, 24, 48, 72, 96, 120, etc.} are also important fixed time points to be considered. Let us define the binary variable $r_{i,t}$ to denote that the released lot i is available to meet customer demands at period t or subsequent periods whenever $r_{i,t} = 1$. In other words, lot i will be released within the time interval [dd_{t-1}, dd_t] only if $r_{i,t} = 1$, where dd_t is the end time of the daily period t. Consequently,

$$\sum_{t\in T} dd_{t-1} r_{i,t} \le RT_i \le \sum_{t\in T} dd_t r_{i,t} \qquad \forall\, i \in I \tag{27}$$

Since the release time for a nonfictitious lot i should belong to only one time period t, then the following condition is to be satisfied:

$$\sum_{t\in T} r_{i,t} = \sum_{p\in P} y_{i,p} \qquad \forall\, i \in I \tag{28}$$

However, some batches will be either on settling period or still in the pipeline when the current horizon is completed, i.e. at $t = h_{max}$. Since RT_i for any of them will be larger than h_{max}, a few daily periods {tf1, tf2, etc.} with upper extreme limits $h_{max} < dd_{tf1} < dd_{tf2} < dd_{tf3} <$ etc. should be added at the end of the time horizon. In this way, the proposed model will be able to assign the release time of every batch to a single period t of the extended time horizon T featuring a length $H > h_{max}$.

In the formulation of Relvas et al.,[12] the start/end daily times never arise as explicit time points and the tracking of product inventories from the demand side is also made at the completion time of a new pumping run. Assuming that the injection of a new lot starts at the beginning of day 3 and finishes at the end of day 6, then the aggregate demand from day 3 to day 6 of any product is tardily removed from the available inventory at the end of day 6. Such an aggregate treatment of product demands can allow nondetected shortages of products on days 3 to 5, i.e. it may lead to infeasible schedules. Despite that, the number of additional binaries and constraints required by the model of Relvas et al.[12] to monitor available inventory levels is still higher than the one needed in the proposed formulation, though it provides a poorer tracking of product inventories. In addition to the variables $x_i^{(i')}$, two new sets of binary variables are required by the approach of Relvas et al.[12] to check the feasibility of the operations at the pipeline terminal. The first one is needed to estimate the time RT_i at which a new lot i is released, and the remaining one permits to determine the times at which client demands are to be removed from the product inventories. In addition, the set of related constraints is clearly much larger than the one included in our formulation as shown in section 5.

4.8. Monitoring Product Inventories at Daily Periods. Let $QR_{i,p,t}$ be a problem variable that is equal to the saleable content of lot i only if the following two conditions hold: (i) lot i

contains product p ($QP_{i,p} > 0$) and (ii) lot i is released to the market at period t ($r_{i,t} = 1$). Since a nonfictitious lot i always conveys a single product ($y_{i,p} = 1$ for some $p \in P$) and its release-time RT_i must belong to a particular time period t ($r_{i,t} = 1$ for some period $t \in T$), then just a single variable $QR_{i,p,t}$ related to lot i will be positive and the other ones must be zero. Such a pair of conditions is given through constraints 29 and 30.

$$\sum_{p\in P} QR_{i,p,t} \le Q_{max} r_{i,t} \quad \forall\, i \in I, t \in T \tag{29}$$

$$\sum_{t\in T} QR_{i,p,t} = QP_{i,p} - \sum_{p'\neq p} WIF_{i,p,p'} \quad \forall\, i \in I, p \in P \tag{30}$$

If lot i is not released at period t, constraint 29 will drive all variables $QR_{i,p,t}$ to zero. Let us now assume that lot i does not contain product p, then $QP_{i,p} = 0$ because of eqs 10 and 11, respectively. Consequently, eq 30 forces all variables $WIF_{i,p,p'}$ and $QR_{i,p,t}$ to be zero. If terminal demands for the daily period t are due at the start of the day, then $r_{i,t}$ should be replaced by $r_{i,t-1}$ in eq 29.

Let $ID_{p,t}$ denote the inventory of product p available to meet customer demands at period t. Its value is given by,

$$ID_{p,t} = ID_{p,t-1} + \sum_{i\in I} QR_{i,p,t} - dem_{p,t} - B_{p,t-1} + B_{p,t} \quad \forall\, p \in P, t \in T \tag{31}$$

where the variable $B_{p,t}$ stands for backorders of product p promised for period t to be satisfied at the next period ($t + 1$) because of product shortages. Moreover, the level of inventory for any product p must be kept within the allowable range at every period.

$$(ID_{min})_p \le ID_{p,t} \le (ID_{max})_p \quad \forall\, p \in P, t \in T \tag{32}$$

Let $IA_{p,t}$ be the total inventory of product p at period t, including the lots of product p on settling period. In turn, $QA_{i,p,t}$ stands for the amount of product p in lot i that has been completely discharged from the pipeline at period t. Assuming that the settling period st_p lasts n_p days, then: $st_p = 24\, n_p$. Since a lot fully discharged from the pipeline in period t will be released on period $t + n_p$,

$$QA_{i,p,t-n_p} = QR_{i,p,t} \quad \forall\, i \in I, p \in P, t \in T \tag{33}$$

Then, the total inventory of product p at period t is given by eq 34,

$$IA_{p,t} = ID_{p,t} + \sum_{i\in I}\sum_{\theta=1}^{n_p} QR_{i,p,t+\theta} \quad \forall\, p \in P, t \in T \tag{34}$$

Moreover, the maximum and minimum inventory constraints are enforced during the true scheduling horizon $T - \{tf1, tf2, etc.\}$ by eq 35.

$$(IA_{min})_p \le IA_{p,t} \le (IA_{max})_p \quad \forall\, p \in P, t \in T - \{tf1, tf2, etc.\} \tag{35}$$

In this way, the usage of the storage capacity over the monthly horizon has been planned at an aggregate level. After discovering the best pipeline planning, a detailed pipeline schedule at the level of individual tanks must be developed. To guarantee the discovery of a feasible detailed schedule, one may assume at the planning stage that some small percentage of the available storage capacity will remain permanently idle.

4.9. Objective Function. The problem goal is to minimize the total pipeline operating cost including (i) the cost of underutilizing pipeline transportation capacity; (ii) the cost of reprocessing the interface material between consecutive batches ($WIF_{i,p',p}$); (iii) the cost of holding product inventory in terminal tanks ($IA_{p,t}$); and (iv) the cost of product backorders ($B_{p,t}$) being tardily delivered to their destination.

$$\min z = \rho\left(h_{max} - \sum_{i\in I^{new}} L_i\right) + \sum_{p\in P}\sum_{\substack{p'\in P\\ p'\neq p}}\sum_{\substack{i\in I\\ i>1}} cf_{p,p'} WIF_{i,p',p} + \sum_{p\in P}\sum_{t\in T} ci_{p,t}(dd_t - dd_{t-1}) IA_{p,t} + \sum_{p\in P}\sum_{t\in T} cb_{p,t} B_{p,t} \tag{36}$$

When the available pth-product inventory is not enough to cover the demand $dem_{p,t}$, a nonzero backorder of product p at time period t will occur ($B_{p,t} > 0$). In order to buildup product inventories at the end of the scheduling horizon with a similar profile of future product demands, potential backorders $B_{p,t}$ on the additional periods {tf1, tf2, etc.} have also been considered. During those periods going from time h_{max} to time H, batches that have been injected during the current scheduling horizon but are on settling period or still remain in the pipeline at the horizon end will be loaded in terminal tanks and subsequently released to meet customer requirements. To account for potential backorders at the early stage of the next month, projected product inventories are compared with projected product demands at periods {tf1, tf2, etc.}. In this way, backorders for future periods tf1, tf2, etc. (located at the beginning of the coming scheduling horizon) can also be minimized by pumping lots of products at the later stage of the current month that properly match projected product requirements. This is another model improvement with regards to the formulation of Relvas et al.[12]

5. Results and Discussion

The real-world case study introduced by Relvas et al.[12] involving the transportation of six different oil derivatives (P1−P6) by pipeline from a unique oil refinery to a single distribution center has been tackled. This unidirectional multiproduct pipeline has a length of 147 km and a capacity of 18 000 volumetric units, and it carries four liquid products and two liquefied gases. The oil refinery and the pipeline are both owned and operated by the same company. The tank farm at the distribution center comprises dedicated storage tanks for liquid products and spheres for gases. In order to fill up exactly either one or multiple tanks, the batch size for each product can be selected from a limited number of size options, i.e. at most three per product (see Table 1). The maximum storage capacity for every product is a problem datum also given in Table 1.

Table 1. Maximum Storage Capacities and Alternative Lot Sizes for Each Product

product (p)	$ID_{max,p}$ (vu)	alternative lot sizes (in vu)		
		option 1	option 2	option 3
P1	81500	21800	18000	17300
P2	32000	16000	8000	
P3	24000	16000	8000	
P4	27800	16000	8000	3800
P5	10320	3440	1720	860
P6	13120	6560	4920	8200

Similarly to Relvas et al.,[12] the pumping rate of any product is assumed to be fixed at 519.4 vu/h to compare model computational performances. Moreover, maintenance shutdown periods are not expected over the planning horizon. Client demands are known by the refiner two weeks before the

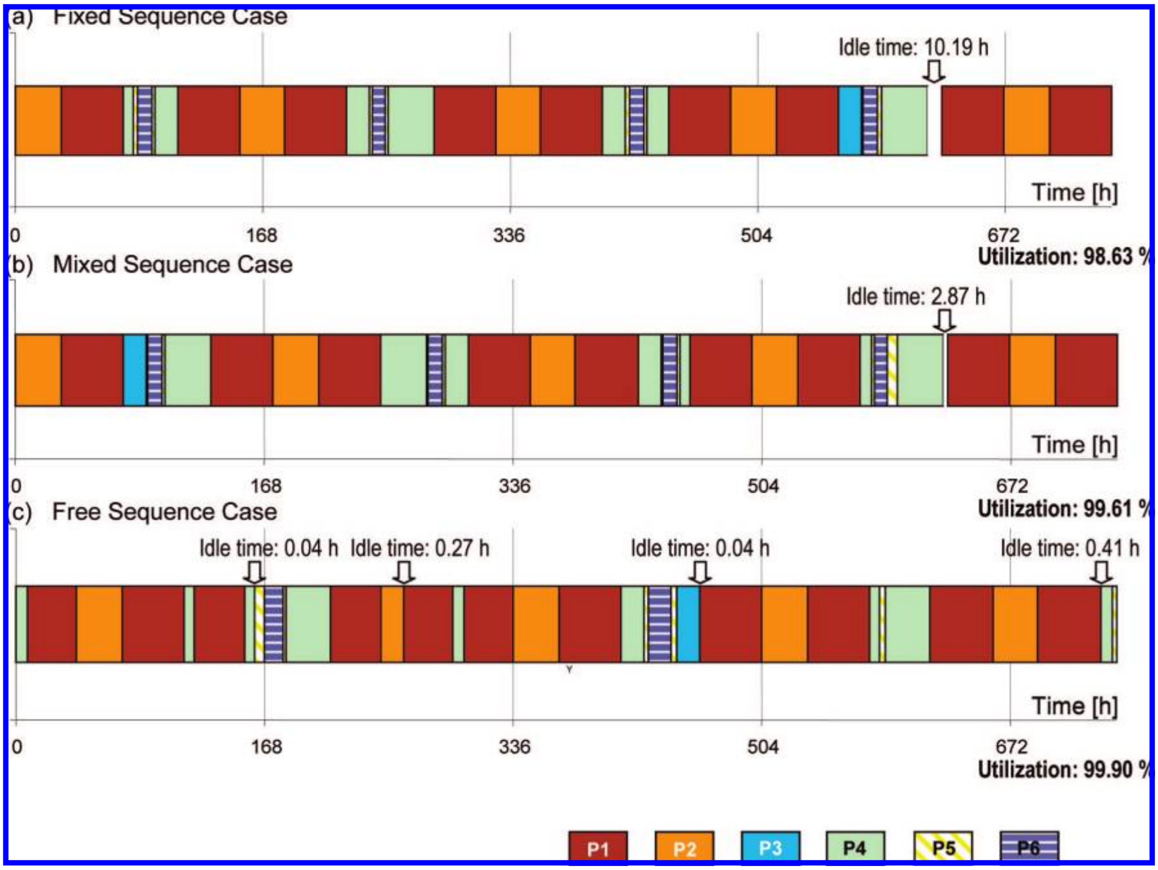

Figure 4. Product sequences and pipeline utilization level for examples 1−3.

Table 2. Monthly Demand and Initial Inventory for Each Product (vu)

product	P1	P2	P3	P4	P5	P6
monthly demand (vu)	198043	64800	14642	68244	10934	16955
initial inventory (vu)	52397	17565	18569	19888	10027	7309

Table 3. Allowed Preceding Products for Each Oil Derivative

	allowed preceding products					
	P1	P2	P3	P4	P5	P6
P1		×	×	×		
P2	×					
P3	×			×	×	
P4	×		×		×	
P5			×	×		×
P6					×	

beginning of each month. Since the distribution center is one of the major refinery clients, its monthly product demands are considered at the time of developing the refinery production plan. In other words, enough product inventories at refinery tanks are assumed to be available to fulfill the pipeline pumping run schedule. Therefore, no control of product inventory levels in refinery tanks will be needed. The monthly delivery requirements and the initial inventory for every product at the distribution center is shown in Table 2, while allowed product sequences are included in Table 3. For instance, P5 is the only product that can be injected immediately before P6 while P1 can merely be preceded by either P2, P3, or P4. As in the work of Relvas et al.,[12] a minimum settling time of 24 h for product certification has been adopted. Since 35 batches are usually pumped in the line during a 31-day period, as reported by the refinery in charge of the pipeline operation, the cardinality of the set I^{new} is fixed at 35. It is also assumed that daily product demands are to be satisfied and, therefore, removed from the available inventories at the start of every day. The monthly product demand reported in Table 2 is supposed to be uniformly distributed along the month with some few exceptions. In fact, there is no demand for P3 and P5 at some particular day of every week. Moreover, the pipeline is initially full of product P1.

In real-life operations, the product sequence generally follows some particular batch cycle. Such a sequencing pattern often results from systematic practical approaches developed by schedulers to avoid undesired product contamination. However, the product sequence is a key scheduling decision to be established by solving the proposed formulation. In order to compare the performance of our pipeline scheduling approach with regards to previous work, three different instances of the case study previously defined by Relvas et al.[12] have been considered. They will be called examples 1−3. In example 1, the sequence of products to be shipped through the pipeline is arbitrarily adopted by the scheduler (i.e., a prefixed product sequence) before solving the problem formulation to just optimize the lot sizing process. Example 2 assumes that the pipeline scheduler has adopted an incomplete predefined product sequence with a limited number of open positions to be filled with predefined allowable products. In this case, the problem model is aimed at optimally assigning products to open positions and determining the size of every batch. In example 3, the proposed formulation permits to establish both the complete sequence of products to be injected in the pipeline and the lot sizing. Relvas et al.[12] adopted as stopping criterion a maximum resource time of 7200 CPU s or a final solution within a maximum relative tolerance of 5%. In this work, a relative tolerance of 2% was selected.

Table 4. Fixed, Mixed, and Free Product Sequences Adopted or Discovered for Examples 1−3

Example	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	I13	I14	I15	I16	I17	I18
1	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2
2	P1	P2	P1	P3	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2
3	P1	P4	P1	P2	P1	P4	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P1	P2

Example	I19	I20	I21	I22	I23	I24	I25	I26	I27	I28	I29	I30	I31	I32	I33	I34	I35
1	P1	P4	P5	P6	P5	P4	P1	P2	P1	P3	P5	P6	P5	P4	P1	P2	P1
2	P1	P4	P5	P6	P5	P4	P1	P2	P1	P4	P5	P6	P5	P4	P1	P2	P1
3	P1	P4	P5	P6	P5	P3	P1	P2	P1	P4	P5	P4	P1	P2	P1	P4	P5

Table 5. Comparison of Model Performance for Examples 1−3, Including the Settling Time Period

	Relvas et al. (2006)		proposed MILP formulation		
	fixed sequence	mixed sequence	fixed sequence	mixed sequence	free sequence (for each subproblem)
constraints	30703	30703	4659	4667	894; 1001; 950; 1159
continuous variables	16259	16259	9945	9945	995; 1158; 1078; 1476
binary variables	4687	4695	1841	1857	292; 327; 310; 397
CPU time (s)	355.083	4120.795	21.921	48.890	2.3 + 40.2 + 44.9 + 9.26
number of iterations	59975	1183378	62306	150200	14270 + 291434 + 397663 + 38965
nodes explored	150	3200	2248	5335	460 + 17713 + 18981 + 1000
pipeline usage (%)	96.6	96.5	98.6	99.6	99.9
relative gap (%)	4.23	4.63	1.84	0.91	1.55; 0.28; 0.01; 1.88

5.1. Example 1. In the first instance of the case study, the sequence of product lots to be pumped in the pipeline is adopted by the scheduler before solving the problem, i.e. the fixed sequence case. Only the size of every lot to be pumped in the line is a model decision. Moreover, a suitable base product string that accounts for the product sequence feasibility matrix is assumed to be used by the pipeline scheduler to heuristically build up the fixed product sequence. In other words, the product sequence is composed by a number of base product strings. However, the base product string has two open positions and a pair of alternative products for each "hole". In this way, the base string may be slightly modified accounting for the monthly demands of such products. The base eight-product string is given by the following: P1−P2−P1−P4/P3−P5−P6−P5−P4/P3, where P4 and P3 are the alternative options for the two open string positions. In this way, four variants of the base product string can be generated by the scheduler. Similar to Relvas et al.,[12] a fixed product sequence involving four complete and one incomplete base product strings has been chosen (see Table 4). Such a number of base strings comes from the fact that 35 lots will be pumped throughout the scheduling horizon and each string only involves 8 lots. From Table 4 it follows that P4 was the scheduler choice for the two open positions in the base product string (P1−P2−P1−P4−P5−P6−P5−P4) except for the first hole of the fourth base product string where P3 replaces P4.

The proposed mathematical model that takes into account the lot settling period constraints has been solved all at once without applying any decomposition strategy on an Intel 2.80 GHz processor with GAMS/CPLEX 10.0.[15] Computational results shown in Table 5 include the number of binary/continuous variables and constraints in the problem formulation, the required CPU time, the number of nodes explored, the pipeline usage level, and the relative gap at the end of the branch-and-cut search. For the sake of comparison, Table 5 also presents the results reported by Relvas et al.[12] By not tracking the lot sizes in the line at the end of every lot injection and using a better modeling of the lot settling periods, the number of problem constraints drops by a factor of 6.6 and the number of binary variables falls 2.5 times. Moreover, the problem has been solved to optimality and, surprisingly, the solution time compared with Relvas et al.[12] diminishes 16.2 times from 355 to 22 s. Another major improvement is the reduction of the pipeline idle time in 15 h and the resulting increase of the utilization level to 98.63% from 96.6% reported by Relvas et al.[12] The higher pipeline utilization comes from a better choice of the lot sizes and the possibility to reach the true problem optimum by adopting a lower relative gap as stopping criterion.

Figure 4a depicts the prefixed sequence of pumping runs and their durations while Figure 5 describes the variations of product inventory levels ($IA_{p,t}$) with time. One major difference between Figure 5 and the results reported by Relvas et al.[12] is that the curves in Figure 5 clearly present the classical "saw-tooth" pattern. There is a positive slope when a new lot of product *p* is released from the settling period and a negative slope if just demand of product *p* is to be satisfied. Another problem target being considered is that the final product inventory profile almost matches future product load profile. By taking into account product delivery requirements for the next horizon while injecting the later elements of the lot sequence, the proposed approach is also able to get a good match between final available inventories and future load profiles (see Table 6 and Figure 6a). It has been assumed that product demands for the next horizon are similar to those given in Table 2 for the current horizon. Figure 6b shows the results reported by Relvas et al.[12] When our approach is applied, the mismatch between final product inventories and projected load profiles goes from −7.69% for P1 to +4.32% for P2, i.e. an average absolute mismatch of 4.37%. If the model of Relvas et al.[12] is used, the mismatch interval is given by {−18.72; +11.99} with an average of 7.26%.

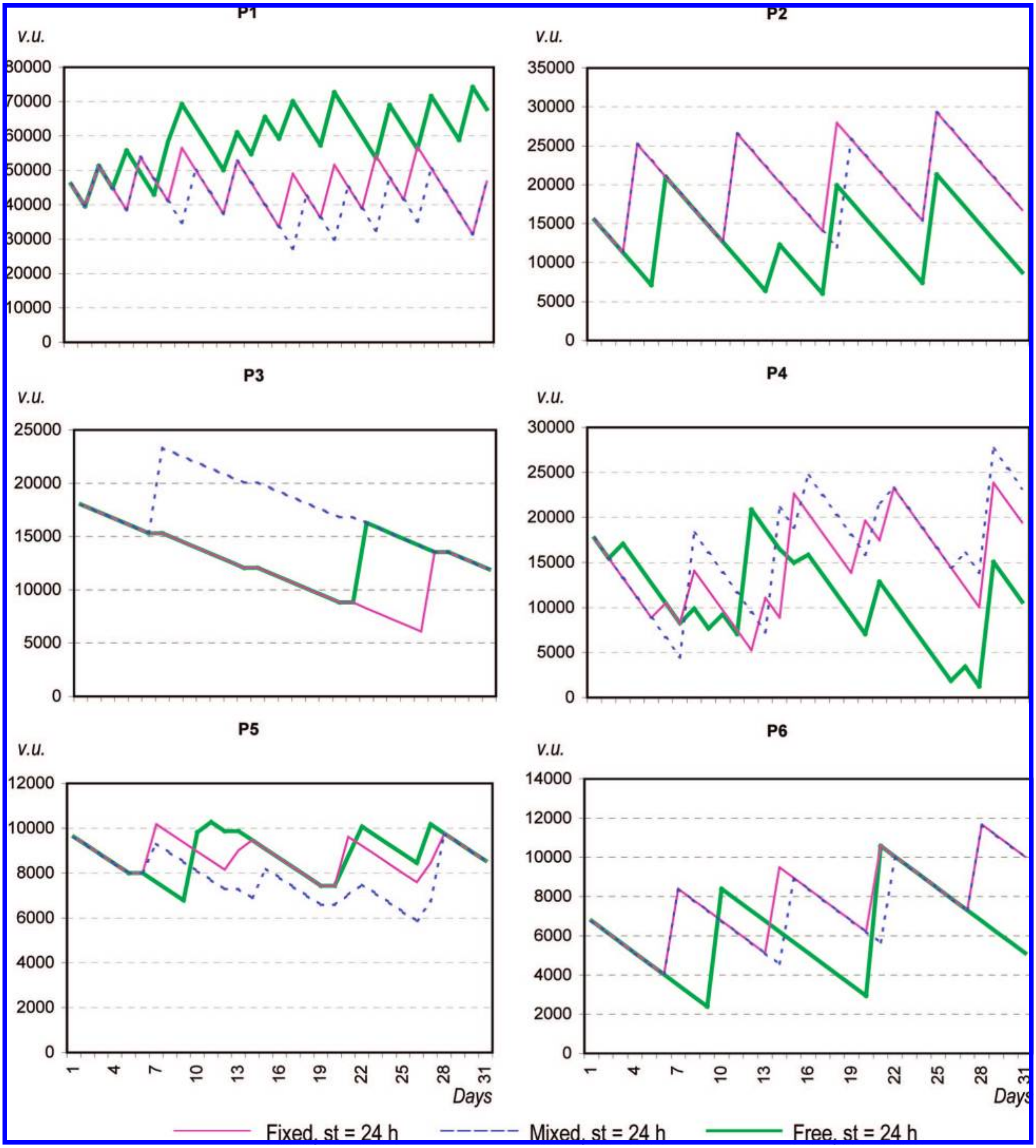

Figure 5. Variation of product inventories throughout the scheduling horizon.

5.2. Example 2. In the second instance of the case study, the assignment of product P3 or P4 to open positions on the base product strings is left to the model. This is the so-called mixed product sequence case since part of the product sequence has been prefixed and the remaining one is selected by the model. Despite the fact that the problem size for the mixed-sequence case slightly increases with regards to example 1, Relvas et al.[12] reported a CPU time 11.6 times larger and a number of explored nodes at least 20 times higher (see Table 5). In contrast, the required CPU time and the number of nodes to be explored when the proposed formulation is applied to the mixed-sequence case both show a much reasonable increase with respect to example 1, i.e. they only rise by a factor of 2. The comparison of both formulations based on their computational performances for the mixed-sequence case leads to the conclusion that our approach is by far more efficient and robust. In fact, the CPU time decreases 84 times from 4120 to 49 s, though the relative gap at the optimum is reduced by a half (see Table 5).

At the same time, the pipeline usage for the mixed-sequence instance rises to 99.6% from the previous value 98.6% found for example 1, and the pipeline idle time drops from 10.2 to 2.9 h. These improvements come from the additional degrees of freedom (the assignment of P3/P4 to open positions) through which a better choice of the lot sizes can be made. The sequence of pumping runs and their durations (sizes) provided by the proposed formulation are shown in Figure 4b. A couple of changes consisting of an earlier insertion of product P3 and the injection of larger amounts of P4 in the line can be observed. The latter change comes from the higher pipeline utilization

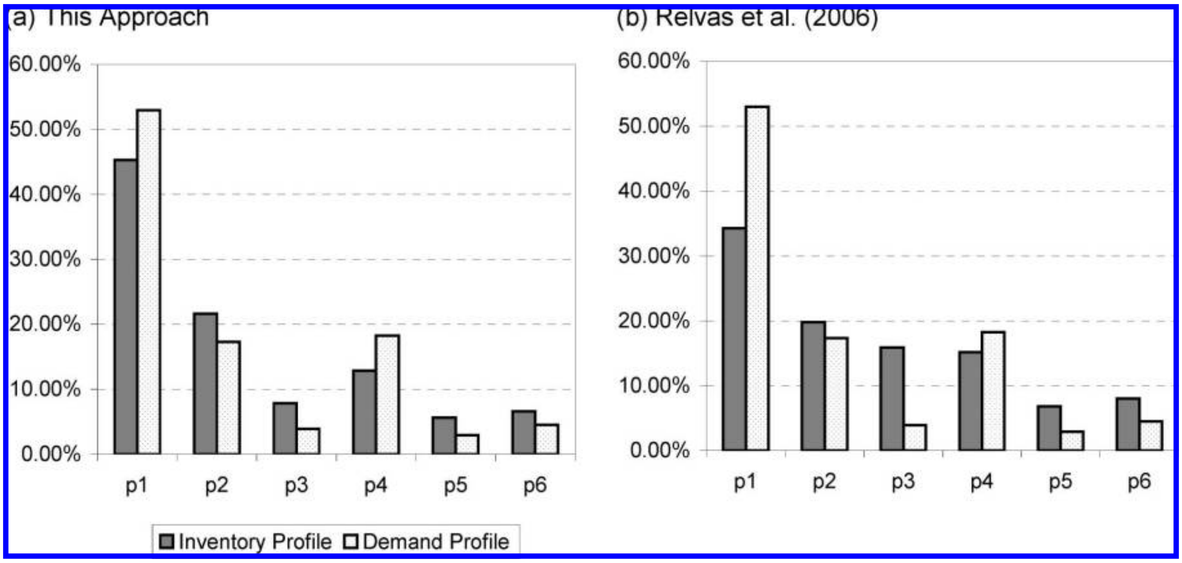

Figure 6. Matching between projected inventories and future load profiles (fixed sequence case).

Table 6. Mismatch between Future Demand and Projected Inventory Profiles in Example 1

			this approach			Relvas et al. (2006)		
	projected monthly demands	demand profile (%)	final inventories	inventory profile (%)	profile deviation (%)	final inventories	inventory profile (%)	profile deviation (%)
P1	198043	53.01	68554	45.32	−7.69	42954	34.29	−18.72
P2	64800	17.34	32765	21.66	4.32	24765	19.77	2.42
P3	14642	3.92	11927	7.88	3.92	19927	15.91	11.99
P4	68244	18.27	19444	12.85	−5.41	19044	15.20	−3.06
P5	10934	2.93	8553	5.65	2.73	8552	6.83	3.90
P6	16955	4.54	10034	6.63	2.09	10034	8.01	3.47

Table 7. Mismatch between Future Demand and Projected Inventory Profiles in Example 2

		this approach		Relvas et al. (2006)	
	demand profile (%)	inventory profile (%)	profile deviation (%)	inventory profile (%)	profile deviation (%)
P1	53.01	44.21	−8.80	33.81	−19.20
P2	17.34	21.13	3.78	19.82	2.47
P3	3.92	7.69	3.77	15.94	12.03
P4	18.27	14.99	−3.28	15.56	−2.71
P5	2.93	5.52	2.59	6.84	3.92
P6	4.54	6.47	1.93	8.03	3.49

level. Variations of product inventory levels ($IA_{p,t}$) with time for the mixed-sequence case present almost the same pattern observed in example 1, except for P3 and P4 due to the changes just described (see Figure 5). Mismatch between final product inventories and projected load profiles goes from −8.80% for P1 to +3.78% for P2; i.e. an average absolute mismatch of 4.03%. For the model of Relvas et al.,[12] the mismatch interval is given by {−19.2; +12.03} with an average of 7.30% (see Table 7).

5.3. Example 3. In the free-sequence case, the complete product sequence and the lot sizes are all optimized through the proposed MILP mathematical model. Similarly to Relvas et al.,[12] it becomes necessary to apply a decomposition technique because a much larger problem is to be tackled. The 31-day scheduling horizon has been divided into four periods {0−168}, {168−336}, {336−504}, and {504−744}, with the times given in hours, and the pipeline schedules for them were sequentially determined. The initial conditions (pipeline linefill and product inventory levels) for every period are given by the optimal solution to the preceding one. In this way, a noncyclic schedule significantly different from the ones found in examples 1 and 2 has been obtained. However, some product subsequences adopted in the previous examples still remain in the optimal solution like (P4−P1−P2−P1−P4) and (P1−P4−P5−P6−P5−P4).

In contrast to Relvas et al.,[12] the settling period was still considered. Despite that, the overall CPU time required to sequentially solve the four subproblems with a relative gap lower than 0.02 amounts to only 97 s (see Table 5). From Figure 4c, the number of lots of P1 and P4, i.e. the products with the largest monthly load, increases from 10 to 12 and from 7 to 9, respectively, at the expense of diminishing the batches of P5 and P6 by 2. Consequently, the average inventory of product P1 clearly rises with a simultaneous decline in the final inventories of P2, P4, and P6 (see Figure 5). Moreover, the pipeline usage climbs to 99.90% from 99.61% for the mixed-sequence case. Figure 7 depicts the product lots in pipeline transit at the end of every pumping run. The scheduling of one-day settling periods for the lots of product P1 arriving at the terminal is shown in Figure 8. No overlapping of lot settling periods is observed. A total of 11 lots of P1 are planned to be loaded in terminal tanks before time h_{max} including five of 21 800 volumetric units, five of 17 300, and one of 18 000.

Relvas et al.[12] could not find the optimal solution for the free-sequence case within the imposed total time limit of 7200 s though they ignored the lot settling period. Indeed, they applied a decomposition strategy that divides the monthly horizon into two equal-length periods and the resulting pair of scheduling problems were sequentially solved. However, neither of them could be solved up to optimality within a time limit of 3600 s. The solutions for the two scheduling periods feature optimality gaps of 0.061 and 0.0934, respectively, and the pipeline usage for the combined schedule is worse than the ones also reported by Relvas et al.[12] for the fixed and mixed product sequences.

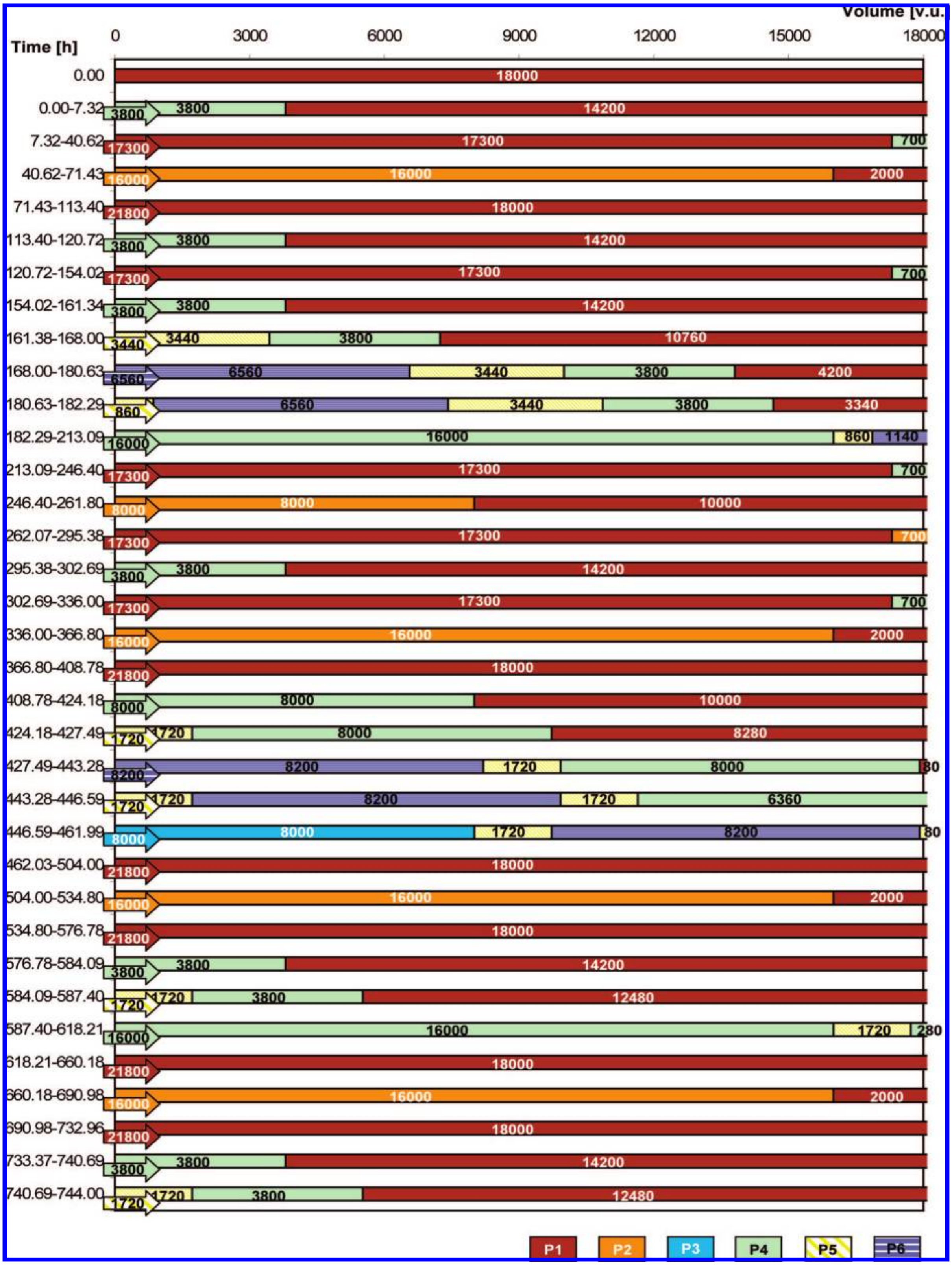

Figure 7. Optimal pipeline schedule for the free sequence solution.

6. Conclusions

An efficient MILP continuous-time tool for the scheduling of a multiproduct transmission pipeline connecting a unique oil refinery to a single distribution terminal over a monthly horizon has been developed. The approach neither requires to divide pipeline segments into a number of single-product packs of known capacities since the volume scale is also handled in a continuous manner. Problem decisions to be optimized by the formulation include the selection of the lot volumes shipped through the line from a limited set of alternative sizes. For a better use of storage resources, the model assumes that every lot volume exactly matches the total capacity of the assigned tanks, and therefore the possible product lot sizes can be known beforehand. The approach provides the optimal sequence and

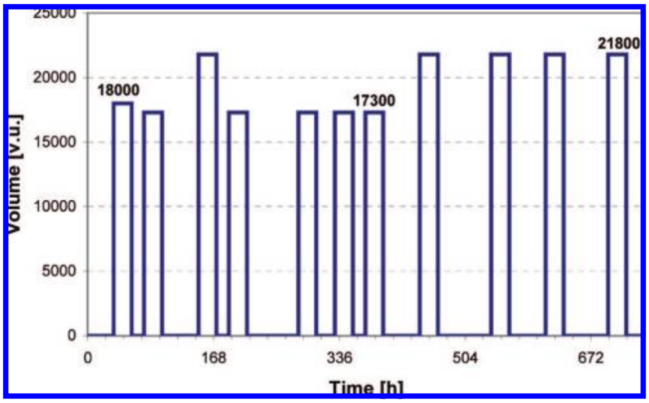

Figure 8. Timing of the settling periods for all lots of product P1 in example 3.

timing of product lot injections in the pipeline and lot discharging operations at the terminal. Furthermore, it is able to manage a distinct pump rate for each product.

Another important model feature is the efficient handling of key terminal operations like settling time periods for batch quality approval. The settling period rigorously starts at the exact time the delivery of a product batch from the line into terminal tanks is completed. A product-dependent settling time is considered. By introducing 0−1 variables to identify the daily period at which a lot is released to meet customer demands, the approach can make a detailed tracking of available quality-certificated product inventories and customer demand satisfaction on a daily basis. The new problem representation is much simpler than the one previously introduced by Relvas et al.[12] for the same pipeline distribution problem. Such a lower model complexity comes from simply avoiding the batch-size tracking along the line since the lot size remains unchanged until it arrives at the unique terminal. However, the tracking of the batch location is still performed to retain control on the timing of lot discharging operations at the depot. In this way, the model size is drastically reduced by significantly decreasing the number of binary/continuous variables and constraints. As a result, the approach presents a much better computational performance than previous ones and, consequently, the CPU solution time drops by a factor of nearly 100.

A real-world example involving the transportation of six different oil derivatives from a unique oil refinery to a single distribution center has been tackled. Three different problem instances were considered: (a) a prefixed product lot sequence, (b) an incomplete prefixed lot sequence with a limited number of open positions to be filled with predefined allowable products, and (c) a free lot sequence with all positions to be optimally filled up by the model. For all of them, the proposed formulation finds the optimal solution at much lower computational cost. When the free sequence scenario is tackled, the number of constraints decreases almost 17 times, the number of binary variables is reduced by a factor greater than 10 and the required CPU time decreases 84 times with regards to previous approaches despite these ones ignored the settling period. Moreover, the pipeline usage level rises up to 99.9% for the free sequence case and final product inventories better match next-month load profiles thus preventing future product backorders. Since the problem involves as many as 35 lot injections over a 31-day horizon, a decomposition strategy was required for the free sequence case.

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Appendix

Simpler Model for a Common Pumping Rate for Every Product

When every oil product is injected and conveyed through the pipeline at the same pumping rate, the problem formulation becomes much simpler. Let the parameter vb (without the p subscript) denote the pipeline pumping rate in volume units per hour. Then eq 13 reduces to

$$L_i = \left(\frac{1}{\mathrm{vb}}\right) Q_i \quad \forall\, i \in I^{\mathrm{new}} \tag{A1}$$

where Q_i stands for the volume of the new lot $i \in I^{\mathrm{new}}$, and L_i is the related pumping run duration. Furthermore, eq 24 can be simplified as follows:

$$\mathrm{RT}_i \geq (C_{i'} - L_{i'}) + \frac{\sigma - F_{i+1}^{(i'-1)}}{\mathrm{vb}} + \sum_{p \in P} \mathrm{st}_p y_{i,p} - H(1 - x_i^{(i')}) \quad \forall\, i \in I, i < |I|, i' \in I^{\mathrm{new}}, i' \geq i \tag{A2}$$

Meanwhile, the lower bound on the remaining transportation time $G_i^{(i')}$ given by eq 25 can be expressed in a much simpler form:

$$G_i^{(i')} \geq \left(\frac{1}{\mathrm{vb}}\right)[\sigma - F_{i+1}^{(i'-1)}] \quad \forall\, i \in I, i' \in I^{\mathrm{new}} (i < i') \tag{A3}$$

Moreover, the tightening cut 26 can be replaced by

$$\mathrm{RT}_i - \sum_{p \in P} \mathrm{st}_p y_{i,p} \geq \mathrm{RT}_{i-1} - \sum_{p \in P} \mathrm{st}_p y_{i-1,p} + L_i \quad \forall\, i \in I (i > 1) \tag{A4}$$

In this case, the last term on the RHS stands for the precise time required to push the entire lot i out of the pipeline to the distribution center, assuming that batch $(i - 1)$ and i are delivered in the same run or in subsequent runs with no interruptions.

Nomenclature

(a) Sets

I = chronologically arranged lots ($I^{\mathrm{old}} \cup I^{\mathrm{new}}$)
I^{new} = new batches to be injected during the planning horizon
I^{old} = old batches inside the pipeline at the start of the planning horizon
K = prespecified pipeline shutdown periods for maintenance work
P = refined petroleum products
S_p = candidate batch sizes for product p
T = daily periods of the extended scheduling horizon

(b) Parameters

b_s = size or volume of lot candidate s
$\mathrm{cb}_{p,t}$ = unit backorder penalty cost for tardily satisfying a requirement due at period t
$\mathrm{cf}_{p,p'}$ = unit reprocessing cost of interface material involving products p and p'
$\mathrm{ci}_{p,j}$ = unit inventory holding cost for product p at depot tanks

dd_t = upper extreme of time period t
$\mathrm{dem}_{p,t}$ = overall demand of product p to be satisfied at the start of time period t
$D_{\min}$, $D_{\max}$ = minimum/maximum delivery size from a batch to the distribution terminal
F_i^{o} = initial upper pipeline coordinate of old batch i
H = estimation of the time at which the last injected batch will be released in the terminal
$h_{\max}$ = true scheduling horizon length
$(\mathrm{IA}_{\max})_p$ = maximum allowed inventory level for product p
$(\mathrm{IA}_{\min})_p$ = minimum allowed inventory level for product p
$(\mathrm{ID}_{\max})_p$ = maximum allowed inventory level for product p ready to meet client demands
$(\mathrm{ID}_{\min})_p$ = minimum allowed inventory level for product p ready to meet client demands
$\mathrm{IF}_{p',p}$ = volume of interface between batches containing products p and p'
$l_{\min,p}$, $l_{\max,p}$ = minimum/maximum length of a new batch injection of product p
Q_i^{o} = initial pipeline volume of old batch i
$Q_{\max}$ = maximum lot injection size
s_k, e_k = starting/ending time of prespecified shutdown period k
st_p = settling time for every lot of product p
vb_p = pumping rate of product p
ρ = unit-time penalty cost for underutilizing pipeline capacity
σ = volumetric coordinate of the distribution terminal from the refinery
$\tau_{p',p}$ = changeover time between injections of products p and p'

(c) Variables

Continuous Variables

$B_{p,t}$ = backorder of product p due at period t to meet at period $t+1$
C_i, L_i = completion time/length of pumping run $i \in I^{\mathrm{new}}$
$D_i^{(i')}$ = volume of batch i loaded in depot tanks while injecting batch i'
$F_i^{(i')}$ = upper coordinate of batch i from the origin at time $C_{i'}$
$F_{i+1}^{(i')}$ = lower coordinate of batch i from the origin at time $C_{i'}$
$G_i^{(i')}$ = amount of time required to push batch i out of the pipeline during injection i'
$\mathrm{IA}_{p,t}$ = total inventory of product p at period t, including lots on settling periods
$\mathrm{ID}_{p,t}$ = inventory of product p in depot tanks ready for clients at time period t
$\mathrm{LP}_{i,p}$ = length of injection i containing product p
Q_i = initial size of the new batch i
$\mathrm{QA}_{i,p,t}$ = volume of product p in lot i completely discharged from the pipeline at period t
$\mathrm{QP}_{i,p}$ = volume of product p injected in the pipeline while pumping batch i
$\mathrm{QR}_{i,p,t}$ = volume of product p contained in lot i released in the terminal at time period t
RT_i = time at which batch i is released after the settling period
$\mathrm{WIF}_{i,p',p}$ = interface volume between batches i and $(i-1)$ containing products p and p'

Binary Variables

$r_{i,t}$ = variable denoting that batch i is released in the terminal within time period t
$x_i^{(i')}$ = variable denoting that a portion of batch i is transferred to depot tanks while injecting i'
$y_{i,p}$ = variable denoting that batch i contains product p
$v_{i,s}$ = variable denoting that lot volume candidate s is selected for the new batch i
$w_{i,k}$ = variable denoting that the injection of lot i begins after prespecified shutdown period k

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