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Analysis of 56-plet positive parity baryon decays in the $1/N_c$ expansion

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The partial decay widths of positive parity baryons belonging to **56**-plets of $SU(6)$ are analyzed in the framework of the $1/N_c$ expansion. The channels considered are those with emission of a single π, K, or $\bar{K}$ meson, and the analysis is carried out to subleading order in $1/N_c$ and to first order in $SU(3)$ symmetry breaking. The results for the multiplet $[\mathbf{56}, 0^+]$, to which the Roper resonance belongs, indicate a poor description of the widths at leading order, requiring important next to leading order corrections. For the multiplet $[\mathbf{56}, 2^+]$, the P wave decays in the nonstrange sector are well described at leading order, while the F wave decays require the next to leading order corrections, which turn out to be of natural magnitude. $SU(3)$ breaking effects are poorly determined, because only few decays with a K meson in the final state are established, and their widths are not known with sufficient accuracy.

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I. INTRODUCTION

One of the most important objectives in hadronic physics is the increasingly accurate determination of the properties of baryon resonances, the search for predicted and yet unobserved resonances, and the theoretical description and understanding of the resonances' observables. The study of baryons, which is complementary to that of mesons, plays, indeed, an important role in exposing nonperturbative aspects of QCD, such as the ordering of states into approximately linear Regge trajectories, the various strong and electromagnetic transition observables, and the remarkable identification of spin-flavor multiplets in the known spectrum of baryons. This latter property was identified back in the 1960s [1], and in QCD it can be explained in terms of the $1/N_c$ expansion: in the large N_c limit, baryons must fill multiplets of the contracted spin-flavor symmetry group $SU^c(6)$ [2,3]. In the real world with $N_c = 3$, we do have evidence of an $O(3) \times SU(6)$ multiplet structure, in particular, thanks to the well-established ground state octet and decouplet states which are accommodated in a $[\mathbf{56}, \ell^P = 0^+]$, and the convincingly established $[\mathbf{56}, 2^+]$ and $[\mathbf{70}, 1^-]$ multiplets in the second resonance region, which, although incomplete, have enough known states for a good identification of them. For other multiplets, the identification needs further scrutiny, based on mass formulas in particular. One such a multiplet is the one containing the Roper resonance $N(1440)$, which is assumed to be a $[\mathbf{56}, 0^+]$. Only a few of the resonances which would be assigned to that multiplet are established.

While in the large N_c limit there must be a contracted $SU^c(6)$ symmetry in the baryon sector, its breaking happening at $\mathcal{O}(1/N_c)$, there is no justification of principle for the observed approximate $O(3)$ symmetry. Within a given $O(3) \times SU(6)$ multiplet, the $O(3)$ symmetry is recovered in the large N_c limit if the multiplet corresponds to the symmetric spin-flavor representation, but it is broken at $\mathcal{O}(N_c^0)$ for a mixed-symmetry multiplet. Thus for the states we discuss in this paper, and provided we neglect configuration mixings [4], i.e. mixings with, for instance, a mixed-symmetry multiplet, the breaking of the $O(3)$ symmetry is a subleading effect in $1/N_c$.

Based on the $O(3) \times SU(6)$ symmetry scheme, it is possible to implement a $1/N_c$ expansion in baryons as an effective theory built in terms of effective operators associated with the observable to be analyzed. This framework has been utilized in the analysis of baryon masses [5,6], strong decays [7–10], magnetic moments [11], electromagnetic helicity amplitudes [12], etc. From these works, it transpires that the ordering of effects according to the $1/N_c$ power counting is remarkably well manifested for most observables that have been studied, with some exception in the case of the $[\mathbf{56}, 0^+]$ Roper multiplet, which is part of the present analysis.

This work extends the analysis of the strong decays of positive parity baryons studied within $SU(4)$ [10] to the **56**-plet of $SU(6)$. The analysis is carried out to subleading $\mathcal{O}(1/N_c)$ and to first order in $SU(3)$ symmetry breaking, and the framework follows a similar implementation as in the case of decays in $SU(4)$ [10]. An earlier analysis of the decays of the $[\mathbf{56}, 0^+]$ baryons in the $1/N_c$ expansion was made in Ref. [8]. The present work presents a full fledged analysis to the orders mentioned for that multiplet as well as for the $[\mathbf{56}, 2^+]$.

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The inputs utilized for the analysis are the partial decay widths as given in terms of fractions of the Breit-Wigner widths by the Particle Data Group [13]. Most of these inputs have changed only slightly over the last few editions of the particle listings, and we refer to the latest edition for references to the most recent partial wave analyses.

The work is organized as follows: in Sec. II, we outline the implementation of the $1/N_c$ expansion for the decays; in Sec. III, we give the results for the analyses of the $[\mathbf{56}, 0^+]$ and the $[\mathbf{56}, 2^+]$ baryon decays, followed by the conclusions in Sec. IV. For completeness, an appendix provides details on the $SU(3)$ isoscalar factors needed in the calculations.

II. THEORETICAL BACKGROUND

A. Baryon states in $O(3) \times SU(6)$

The excited baryon states in the large N_c limit must fill towers of states, which correspond to irreducible representations of a contracted spin-flavor symmetry group $SU^c(2N_f)$. The states of angular momentum of order unity in these towers have mass splittings $\mathcal{O}(1/N_c)$. As already mentioned, in the real world with $N_c = 3$, it is observed that there is in addition an approximate $O(3)$ symmetry, which does not follow from the $1/N_c$ expansion but is, rather, from a phenomenological observation. This symmetry is most clearly displayed in the spectrum of the known excited baryons. Thus, the classification of the excited baryons in terms of the symmetry group $O(3) \times SU(2N_f)$ is the proper approach. In large N_c, the contracted symmetry group will emerge as a subgroup of that larger group [14].

For three light flavors, we need the group $SU(6)$, which has 35 generators, namely $\{S_i, T_a, G_{ia}\}$, with $i = 1, 2, 3$, and $a = 1, \ldots, 8$, where the first three are the generators of the spin $SU(2)$, the second eight are the generators of flavor $SU(3)$, and the last 24 can be identified as an octet of axial-vector currents in the limit of zero momentum transfer. The algebra of $SU(6)$ has the following commutation relations that fix the normalizations of the generators:

$$[S_i, S_j] = i\epsilon_{ijk}S_k, \qquad [T_a, T_b] = if_{abc}T_c,$$
$$[S_i, G_{ja}] = i\epsilon_{ijk}G_{ka}, \qquad [T_a, G_{ib}] = if_{abc}G_{ic},$$
$$[G_ia, G_jb] = \frac{i}{4}\delta_{ij}f_{abc}T_c + \frac{i}{2}\epsilon_{ijk}\left(\frac{1}{3}\delta_{ab}S_k + d_{abc}G_{ck}\right), \tag{1}$$

where d_{abc} and f_{abc} are the $SU(3)$ symmetric and anti-symmetric invariant tensors, respectively.

The states of interest in this work belong to the totally symmetric irreducible representation $\mathbf{S}$, which is given by a Young tableau consisting of a single row of N_c boxes. These states correspond at $N_c = 3$ to the $\mathbf{56}$-plet of $SU(6)$. Another important representation is the mixed symmetric, which consists of a row with $N_c - 1$ boxes and a second row with a single box, that for $N_c = 3$ corresponds to the $\mathbf{70}$-plet of $SU(6)$. We denote, by ℓ, the $O(3)$ quantum number of the states. The states belonging to the $[\mathbf{56}, \ell]$ multiplet are then given by

$$|(\ell, S)JJ_3; R = (p, q), Y, II_3>_{\mathbf{S}} = \sum_{m,S_3}\langle \ell m, SS_3 \mid JJ_3\rangle \times |SS_3; R = (p, q), Y, II_3>_{\mathbf{S}} \mid \ell m\rangle, \tag{2}$$

where the label $\mathbf{S}$ indicates that the state belongs to the symmetric spin-flavor representation, S is the spin quantum number associated with the spin subgroup of $SU(6)$, J is the total angular momentum of the baryon, $R = (p, q)$ indicates the $SU(3)$ irreducible representation given in terms of the usual labeling of a Young tableu with $p + 2q = N_c$, and Y and I are, respectively, the hypercharge and the isospin.

From the decomposition of the $\mathbf{S}$ representation of $SU(6)$ as a sum of direct products of irreducible representations of $SU(2) \otimes SU(3)$, it results that $p = 2S$. This latter relation is a consequence of the fact that, for the $\mathbf{S}$ representations, the two factors in the direct products involved in the decomposition have the same Young tableau (p, q). The relation then follows from the relation $p = 2S$ in $SU(2)$. The $p = 2S$ relation is a generalization of the $I = S$ relation for the $\mathbf{S}$ representations of $SU(4)$.

The correspondence of multiplets between generic $N_c > 3$ and $N_c = 3$ is as follows: (i) $(p = 1, q = \frac{N_c - 1}{2}) \to (\mathbf{8}, S = \frac{1}{2})$ and (ii) $(p = 3, q = \frac{N_c - 3}{2}) \to (\mathbf{10}, S = \frac{3}{2})$. Clearly, all this holds for odd N_c.

For $\ell = 0$, which also includes the ground state baryons, we have the $J = S = 1/2$ octet and the $J = S = 3/2$ decuplet. For $\ell = 2$, we have octets with $J = 3/2$, $5/2$ and decuplets with $J = 1/2$ through $7/2$.

B. $O(3) \times SU(6)$ framework for decays

In the following, the framework for implementing the calculation of decays via emission of a single pseudoscalar meson is described. We first give the partial decay widths in terms of reduced matrix elements of the effective baryonic operators, which describe the transition between the initial excited baryon and the final ground state baryon. If we altogether neglect $SU(3)$ symmetry breaking, the partial widths could be expressed solely in terms of $SU(3)$ reduced matrix elements of the effective operators. However, $SU(3)$ symmetry breaking is noticeable in the decays, and must be included. This is done with effective operators, generated by the octet component of the quark masses [i.e., by the term $\frac{1}{2\sqrt{3}}(m_u + m_d - 2m_s)\lambda_8$ in the quark mass matrix], and thus the partial widths are, rather, given in terms of reduced matrix elements of the isospin $SU(2)$. We start from the general expression for a partial decay width of an initial excited baryon state $|(\ell, S^*)J^*J_3^*; R^*, Y^*, I^*I_3^*\rangle$ to a final state $|SS_3; R, Y, II_3\rangle$

emitting a pseudoscalar meson in the state $|\tilde{\ell}\tilde{\ell}_3;\tilde{R},\tilde{Y},\tilde{I}\tilde{I}_3\rangle$. The effective operator must carry the same quantum numbers of the meson. The partial decay width is then given by

$$\Gamma = \frac{k}{8\pi^2}\frac{M_B}{M_{B^*}}\frac{1}{(\hat{I}^*\hat{J}^*)^2}\sum_{J_3^*,I_3^*,S_3,I_3,\tilde{\ell}_3,\tilde{I}_3}|\langle SS_3;R,Y,II_3 \mid B^{[\tilde{\ell},\tilde{R}]}_{[\tilde{\ell}_3,\tilde{Y},\tilde{I},\tilde{I}_3]} \mid (\ell,S^*)J^*J_3^*;R^*,Y^*,I^*I_3^*\rangle|^2, \tag{3}$$

where M_{B^*} and M_B are the masses of the excited and final baryon, respectively, k is the meson momentum, and the notation $\hat{j}\equiv\sqrt{2j+1}$ is used. The baryonic transition operator is denoted by $B^{[\tilde{\ell},\tilde{R}]}_{[\tilde{\ell}_3,\tilde{Y},\tilde{I},\tilde{I}_3]}$, where the upper labels display the angular momentum of the operator and its $SU(3)$ representation, and the lower labels display the corresponding projections. The operator is built as a linear combination in a basis of effective operators, which is ordered in powers of $1/N_c$:

$$B^{[\tilde{\ell},\tilde{R}]} = \left(\frac{k}{\Lambda}\right)^{\tilde{\ell}}\sum_n C_n^{[\tilde{\ell},\tilde{R}]}(k)B_n^{[\tilde{\ell},\tilde{R}]}, \tag{4}$$

where the B_n are the operators in the basis, and the coefficients C_n encode the dynamics of the decay amplitudes. In this work, they are determined by fitting to the empirical partial decay widths. A centrifugal factor is included, which is expected to carry the chief momentum dependence of the transition amplitude, with the arbitrary scale Λ to be chosen to be 200 MeV in what follows.

The effective operators in the basis can be expressed in terms of spin-flavor operators in the following general form:

$$B^{[\tilde{\ell},\tilde{R}]}_{n[\tilde{\ell}_3,\alpha]} = \sum_{m,j_3}\langle \ell m, j_n j_3 \mid \tilde{\ell}\tilde{\ell}_3\rangle \xi^\ell_m \mathcal{G}^{[j_n,\tilde{R}]}_{n[j_3,\alpha]}, \tag{5}$$

where ξ^ℓ is the tensor operator, which gives the transition between the $O(3)$ state of the excited baryon and the ground state baryon, and $\mathcal{G}_n^{[j,\tilde{R}]}$ is a spin-flavor operator. Without any loss of generality, ξ^ℓ is normalized such that $\langle 0 \mid \xi^\ell_{m'} \mid \ell m\rangle = (-1)^{\ell-m}\delta_{m-m'}$. The spin-flavor operator, which is a tensor with angular momentum j_n, can be built as tensor products of the $SU(6)$ generators.

We can express the partial decay widths in terms of the reduced matrix elements (RMEs) of the basis operators as follows:

$$\Gamma = \frac{k}{8\pi^2}\left(\frac{k}{\Lambda}\right)^{2\tilde{\ell}}\frac{M_B}{M_B^*}\frac{\hat{I}^2}{(\hat{I}^*\hat{J}^*)^2}|\sum_n C_n^{[\tilde{\ell},\tilde{R}]}(k) \times \mathbf{B}_n(\{S,R,Y,I\},\{(\ell,S^*)J^*,R^*,Y^*,I^*\},\{\tilde{\ell},\tilde{R},\tilde{Y},\tilde{I}\})|^2, \tag{6}$$

where the $\mathbf{B}_n$ are the RMEs. These can also be expressed in terms of RMEs of spin-flavor operators. For operators B_n, which do not involve $SU(3)$ symmetry breaking, one obtains

$$\mathbf{B}_n(\{S,R,Y,I\},\{(\ell,S^*)J^*,R^*,Y^*,I^*\},\{\tilde{\ell},\tilde{R},\tilde{Y},\tilde{I}\}) = (-1)^{j_n+J^*+\ell+S}\frac{\hat{J}^*\hat{\tilde{\ell}}}{\sqrt{\dim R}}\begin{Bmatrix} J^* & S^* & \ell \\ j_n & \tilde{\ell} & S\end{Bmatrix} \times \sum_\gamma \begin{pmatrix} R^* & \tilde{R} & R \\ Y^*I^* & \tilde{Y}\tilde{I} & YI\end{pmatrix}_\gamma \langle S,R \parallel \mathcal{G}_n^{[j_n,\tilde{R}]} \parallel S^*,R^*\rangle_\gamma, \tag{7}$$

where, with obvious notation, there appear a $SU(2)$ 6-j symbol, $SU(3)$ isoscalar factors, and the reduced matrix element of the corresponding spin-flavor operator. γ labels the possible multiplicities for coupling the product of representations $R^*\otimes\tilde{R}$ to R in $SU(3)$. Throughout, the $SU(2)$ conventions are those of Edmonds [15], and the $SU(3)$ conventions are those established in the article by Hecht [16].

In the case of $SU(3)$ symmetry breaking operators, we proceed as follows. The symmetry breaking is due to the mass difference between the s quark and the u and d quarks. To first order in the quark masses, this symmetry breaking is implemented at the level of the spin-flavor operators according to

$$\mathcal{G}^{[j_n,\tilde{R}]}_{n,\gamma_n} = \frac{1}{\Lambda}[\mathcal{M}_q^8\mathcal{G}_n^{[j_n,R_n]}]^{\tilde{R}}_{\gamma_n}, \tag{8}$$

where $\mathcal{M}_q^8$ is the octet component of the quark mass matrix, Λ an arbitrary scale to render operators dimensionless, and γ_n indicates the particular coupling $\mathbf{8}\otimes R_n$ to $\tilde{R}$. For the case of interest here, $\tilde{R}=\mathbf{8}$, and therefore R_n can be $\mathbf{1}$, $\mathbf{8}$, $\mathbf{10}$, $\overline{\mathbf{10}}$, or $\mathbf{27}$. These possibilities give rise to a large proliferation of $SU(3)$ breaking operators at 2-body level. Fortunately, for the decays of **56**-plet baryons, those 2-body operators are of higher order in $1/N_c$, and thus contribute corrections to the leading order decay amplitudes of order $(m_s-m_{u,d})/N_c$, which are beyond the accuracy of the present analysis. This is quite different in the case of the **70**-plet, as it will be discussed elsewhere. The RME of an $SU(3)$ breaking operator will then be given by

$$\mathbf{B}_n(\{S,R,Y,I\},\{(\ell,S^*)J^*,R^*,Y^*,I^*\},\{\tilde{\ell},\tilde{R},\tilde{Y},\tilde{I}\}) = (-1)^{j_n+J^*+\ell+S}\frac{\hat{J}^*\hat{\tilde{\ell}}}{\sqrt{\dim R}}\begin{Bmatrix} J^* & S^* & \ell \\ j_n & \tilde{\ell} & S\end{Bmatrix} \times \begin{pmatrix} 8 & R_n & \tilde{R} \\ 00 & \tilde{Y}\tilde{I} & \tilde{Y}\tilde{I}\end{pmatrix}_{\gamma_n}\sum_\gamma \begin{pmatrix} R^* & R_n & R \\ Y^*I^* & \tilde{Y}\tilde{I} & YI\end{pmatrix}_\gamma \times \langle S,R \parallel \mathcal{G}_n^{[j_n,R_n]} \parallel S^*,R^*\rangle_\gamma, \tag{9}$$

where γ_n indicates the $SU(3)$ recoupling corresponding to the operator $\mathbf{B}_n$. With the definition of effective operators used in this work, all coefficients $C_n^{[\ell,\tilde{R}]}(k)$ in Eq. (4) are of zeroth order in N_c. In addition, we will normalize the operators in such a way that the *natural* size of all coefficients would be the same. The leading order of the decay amplitude is in fact N_c^0 [4]. At this point, it is important to comment on the momentum dependence of the coefficients. The spin-flavor breakings in the masses, of both excited and ground state baryons, give rise to different values of the momenta k. In the **56**-plets the mass splittings are, however, $\mathcal{O}(1/N_c)$ or order $m_s - m_{u,d}$, therefore, those effects on k are taken into account automatically in the expansion we are performing. Thus, we can ignore any momentum dependence of the coefficients $C_n^{[\ell,\tilde{R}]}$ as such effects are absorbed into the operators.

C. Operator basis

The construction of a basis of spin-flavor operators follows similar lines as in previous works on baryon decays [10]. The main difference between the $SU(4)$ and $SU(6)$ cases is that, in the first case the matrix elements of a given spin-flavor operator are all of the same order in $1/N_c$, while in the latter case they are not. For instance, the emission of K mesons is suppressed by a factor $1/\sqrt{N_c}$ with respect to the emission of pions. This is because, in the $1/N_c$ counting, baryons are considered to have strangeness $\mathcal{O}(N_c^0)$. Note that this does not represent any $SU(3)$ symmetry breaking. Because of this issue, the $SU(3)$ preserving operators are classified according to the order in $1/N_c$ at which they contribute in pion emission. The order of an n-body operator is given by $\nu = n - \kappa$, where κ is the coherence factor determined by the number of coherent generators that appear in the product building the operator. Details on the derivation of this power counting can be found in [4].

The decays of the positive parity baryons involve odd partial waves, and we analyze here the P and F wave decays. The $SU(3)$ preserving operators for the emission of the pseudoscalar octet are as follows. There is only one 1-body operator, namely G_{ia}/N_c. This operator is of leading order, and gives contributions to the decay widths into pions at $\mathcal{O}(N_c^0)$. Next, we have 2-body operators, which are built by multiplying a pair of generators such that one can couple them to $j = 1$ or 2 and to $R = \mathbf{8}$. One can construct six such products, which upon using the $SU(6)$ reduction formulas [17] and keeping only operators up to $\mathcal{O}(1/N_c)$, only two operators are left, namely $1/N_c^2\{S, G\}^{[j=1,\mathbf{8}]}$ and $1/N_c^2\{S, G\}^{[j=2,\mathbf{8}]}$. One can show that 3-body operators contribute to amplitudes at $\mathcal{O}(1/N_c^2)$, which is beyond the order of this work.

To the order we are working, $SU(3)$ breaking is described by two 1-body operators, namely $f_{8ab}G_{ib}/N_c$ and $d_{8ab}G_{ib}/N_c$. The first operator does not contribute to decays involving a π meson, and the second one can be redefined by adding to it a nonbreaking piece in such a way that it does not contribute to matrix elements involving a π meson. Because of the small number of empirically known decay channels into K mesons, we will not be able to fit the effects of both operators. For this reason, only the operator $d_{8ab}G_{ib}/N_c$ will be utilized, which puts some limitations to the predictivity of the present analysis.

The $1/N_c$ counting for the reduced matrix elements of operators is finally given as follows: (i) For $SU(3)$ preserving LO operators, the amplitudes $B^* \to \pi B$ are $\mathcal{O}(N_c^0)$ and $B^* \to KB$ or $B^* \to \bar{K}B$ are $\mathcal{O}(N_c^{-1/2})$; next to leading order (NLO) amplitudes are simply an extra factor $1/N_c$ in all cases. (ii) For the $SU(3)$ breaking operator, once redefined to have vanishing contributions to amplitudes with pion emission, it contributes at $\mathcal{O}(m_s N_c^{-1/2})$.

Table I summarizes the set of spin-flavor operators relevant to the decays of the **56**-plet baryons as needed in this work.

The RMEs of the spin-flavor operators between the **56**-plets can be expressed in terms of the $SU(3)$ reduced matrix elements of G and of $\{S, G\}$. The RMEs of G are given in Table II, and the ones of $\{S, G\}^{[j]}$ are related to those of G by the following formula:

$$\begin{aligned}\langle R' \parallel \{S, G\}^{[j,\mathbf{8}]} \parallel R\rangle_\gamma &= (-1)^{S+S'}\hat{j}\langle R' \parallel G \parallel R\rangle_\gamma \Bigg((-1)^j \hat{S}'\sqrt{S'(S'+1)} \\ &\quad\times \begin{Bmatrix} S' & S' & 1 \\ 1 & j & S \end{Bmatrix} + \hat{S}\sqrt{S(S+1)}\begin{Bmatrix} S & S & 1 \\ 1 & j & S' \end{Bmatrix}\Bigg), \quad (10)\end{aligned}$$

where, as mentioned earlier, S and S' in the **56**-plet are determined by the respective $SU(3)$ representation R and R'. In this work we only need the cases $j = 1, 2$. Note that for $j = 1$ the right-hand side of (10) vanishes if $S = S'$, which implies, in particular, that $\langle \mathbf{8} \parallel \{S, G\}^{[1,\mathbf{8}]} \parallel \mathbf{8}\rangle_\gamma = \langle \mathbf{10} \parallel \{S, G\}^{[1,\mathbf{10}]} \parallel \mathbf{10}\rangle_\gamma = 0$. This, in particular, means

TABLE I. Spin-flavor operators

Operator	Order
$\mathcal{G}_1 \equiv \frac{1}{N_c} G$	$\mathcal{O}(N_c^0)$
$\mathcal{G}_2 \equiv \frac{1}{N_c^2}\{S, G\}^{[j=1]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_3 \equiv \frac{1}{N_c^2}\{S, G\}^{[j=2]}$	$\mathcal{O}(1/N_c)$
$\mathcal{G}_{SB} \equiv \frac{1}{N_c}(d_{8ab} - \delta_{ab}/\sqrt{3})G_{ib}$	$\mathcal{O}((m_s - m_{u,d})/\sqrt{N_c})$

TABLE II. $SU(3)$ RMEs of the spin-flavor operator G between $SU(3)$ multiplets in the $SU(6)$ **56**-plet. $\langle \mathbf{10} \parallel G \parallel \mathbf{8}\rangle = \langle \mathbf{8} \parallel G \parallel \mathbf{10}\rangle$, and $\langle \mathbf{10} \parallel G \parallel \mathbf{10}\rangle_{\gamma=2} = 0$ when $N_c = 3$.

RME
$\langle \mathbf{8} \parallel G \parallel \mathbf{8}\rangle_{\gamma=1} = \sqrt{\frac{(N_c+1)(N_c+5)}{2}}$
$\langle \mathbf{8} \parallel G \parallel \mathbf{8}\rangle_{\gamma=2} = -\sqrt{\frac{(N_c^2-1)(N_c+5)(N_c+7)}{32}}$
$\langle \mathbf{8} \parallel G \parallel \mathbf{10}\rangle = -\frac{1}{4}\sqrt{(N_c^2-1)(N_c+5)(N_c+7)}$
$\langle \mathbf{10} \parallel G \parallel \mathbf{10}\rangle_{\gamma=1} = (N_c+3)\sqrt{\frac{10(N_c-1)(N_c+7)}{(45+N_c(N_c+6))}}$
$\langle \mathbf{10} \parallel G \parallel \mathbf{10}\rangle_{\gamma=2} = -\frac{1}{\sqrt{8}}\sqrt{\frac{(N_c^2-1)(N_c-3)(N_c+5)(N_c+7)(N_c+9)}{(45+N_c(N_c+6))}}$

that the NLO operator does not contribute to the $[\mathbf{56}, 0^+]$ amplitudes $N(1440) \to \pi N$ and $\Delta(1600) \to \pi\Delta$.

Notice that the $1/N_c$ power counting is not given solely by the reduced matrix elements of the baryonic operator, but also involves the N_c dependence of the isoscalar factors. For this reason, we give in an appendix the expressions for general N_c of those isoscalar factors as needed in the present calculations.

III. RESULTS

In this section, we present and analyze the fits to partial widths. In general, the LO fits correspond to the results one would obtain from coupling the pseudoscalar meson to the excited quark, similar to the framework of the chiral quark model, leading to results similar to those of quark models [18]. Our analysis below shows the shortcomings of the LO approximation, and the need for the NLO contributions, which are given by 2-body effects in the decay amplitudes.

A. Results for the $[\mathbf{56}, 0^+]$ decays

The basis of operators in this case contains the LO $\mathcal{G}_1$ operator, the 2-body NLO $\mathcal{G}_2$ operator, and the $SU(3)$ breaking $\mathcal{G}_{SB}$ operator. We normalize these operators according to $\mathcal{G}_n \to \alpha_n \mathcal{G}_n$, with $\alpha_1 = 6/5$, $\alpha_2 = 1/\sqrt{2}$, $\alpha_{SB} = 2\sqrt{3}/5$, so that the corresponding matrix elements have natural magnitude. In this multiplet, the experimentally established states to be used in the analysis along with their respective star rating by the Particle Data Group are as follows: the Roper resonance $N(1440)$ $(****)$, the $\Lambda(1600)$ $(***)$ and the $\Sigma(1660)$ $(***)$ in the **8**, and only the $\Delta(1600)$ $(***)$ in the **10**.

The results of the fits are shown in Table III.

In all the leading order fits considered here denoted as LO, the decays with K or $\bar{K}$ mesons in a final state are not included, because their widths start at $\mathcal{O}(1/N_c)$. The LO fit involves only one operator, and has been carried out by assigning error bars to the input partial widths, obtained from the Particle Data Group, which are 30% or the experimental value if it is larger than 30%. This is to test whether or not the LO is consistent. In the case of the $[\mathbf{56}, 0^+]$ decays, where all experimental errors are larger than 30%, it turns out that the LO fit is not consistent, with a $\chi^2_{\rm dof} \simeq 2$. This was well known from previous work in $SU(4)$ [10], where the nonstrange channels were shown to be poorly described. The problem is best illustrated by the two ratios, which are parameter-free at LO: (i) $\Gamma(N(1440) \to \pi N)/\Gamma(N(1440) \to \pi\Delta) = 7.5$ (Theory) vs 2.6 ± 2.1 (Experimental), and (ii) $\Gamma(\Delta(1600) \to \pi N)/\Gamma(\Delta(1600) \to \pi\Delta) = 1.7$ (Theory) vs 0.3 ± 0.2 (Experimental). Thus, spin-flavor symmetry is badly broken in these decays; this is the most notable inconsistency of the $1/N_c$ expansion at LO, and points to the particular nature of the Roper multiplet. Another relevant ratio is $\Gamma(N(1440) \to \pi N)/\Gamma(\Delta(1600) \to \pi\Delta) = 1.64$ (Theory) vs 1.10 ± 0.63 (Experimental). This ratio is interesting because it is not corrected at NLO (the NLO operator, as mentioned earlier, does not contribute to these decay channels). At LO, the decay rate $\Lambda(1600) \to \pi\Sigma$ is remarkably close to the observed one, but not so well described at NLO. As in the other fits in this work, we include a fit denoted by LO*, in which we do not expand the matrix elements of the LO operator, and also include the $SU(3)$ breaking operator. The reason for doing this is that some of the reduced matrix elements and isoscalar factors are poorly represented by the first term in the expansion. This fit gives significant improvement over the LO one, shows that the $SU(3)$ breaking operator is nearly irrelevant, and it further exposes the problematic decay channels, which are the $\pi\Delta$ channels. Therefore, the NLO corrections must be very important, and they are provided by the single 2-body operator. The inclusion of these NLO cor-

TABLE III. Fit parameters and partial widths in MeV for P-wave decays of the $[56, 0^+]$ baryons. The theoretical errors of the results from the NLO fit are indicated explicitly; similar errors result for the NLO* fit. Please note that here and in subsequent tables "Exp" refers to "Experiment."

	$\chi^2_{\rm dof}$	dof	C_1	C_2	B_1
LO	2.2	6	8.8(1.3)	⋯	⋯
LO*	1.7	5	6.7(0.8)	⋯	4.2(3.2)
NLO	0.8	4	8.1(0.9)	−8.0(2.6)	2.7(4.6)
NLO*	0.9	4	8.7(1.0)	−9.3(4.3)	8.2(3.8)

	$N_{1/2}(1440)$		$\Lambda_{1/2}(1600)$			$\Sigma_{1/2}(1660)$				$\Delta_{3/2}(1600)$	
	πN	$\pi\Delta$	$\bar{K}N$	$\pi\Sigma$	$\pi\Sigma^*$	$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$	$\pi\Sigma^*$	πN	$\pi\Delta$
LO	90	12	0	73	24	0	41	58	6	93	55
LO*	148	13	36	57	19	1.6	32	61	6.5	97	90
NLO	214(49)	65(17)	38(28)	111(25)	95(27)	5(4)	63(14)	88(20)	25(7)	56(44)	131(30)
NLO*	250	45	36	97	66	1.6	55	103	23	51	153
Exp	211(88)	81(35)	34(25)	53(51)	⋯	24(20)	⋯	⋯	⋯	61(32)	193(76)

rections leads to a consistent fit. Since the NLO operator does not contribute to either $N(1440) \rightarrow \pi N$ or $\Delta(1600) \rightarrow \pi\Delta$ decays, the change in the C_1 coefficient from the LO to the NLO fit gives the correction to these decays. The large corrections to the widths $N(1440) \rightarrow \pi\Delta$ and $\Delta(1600) \rightarrow \pi N$ is where the importance of the NLO operator is shown. Following the criterion of normalizing the operators we indicated earlier, the coefficient C_2 is similar in magnitude to C_1, and thus one could regard it as of natural size. The dramatic NLO increase in the width $N(1440) \rightarrow \pi\Delta$ is due to the increase in C_1 to which the NLO contribution adds up. To be more quantitative, we compare the contributions by the NLO operator to those of the LO one, and they are $\pm 44\%$, which is slightly bigger than the expected $1/3$ for a natural size NLO correction. This seems to leave an open issue on how well the $1/N_c$ expansion converges for these decays. The NLO fit does not describe the decay $\Lambda(1600) \rightarrow \pi\Sigma$ as well as the LO one, but it is consistent within the errors. In the NLO* fit we do not expand the matrix elements. The main effect is to change the coefficients C_2 and B_1, and the reason is that the NLO operator does not contribute to the decays into K mesons in the NLO fit, while it does in the NLO* fit.

For the $SU(3)$ symmetry breaking we find, for all the fits in this work, two consistent solutions. In all cases, one of the solutions has a coefficient, which is unnaturally large, and we assume that such a solution is unacceptable. The only way to confirm this assumption is by determining empirically other widths sensitive to the symmetry breaking. In the case of the $[\mathbf{56}, 0^+]$, the role of the $SU(3)$ breaking operator cannot be established: it is irrelevant in the NLO fit and relevant in NLO* fit. The reason for this ambiguity is that only two decay widths are involved, and both have large error bars. Finally, the fits permit us to give rough predictions for the unknown channels.

B. Results for the $[\mathbf{56}, \mathbf{2}^+]$ decays

The basis of operators in this case involves the LO $\mathcal{G}_1$ operator, the NLO $\mathcal{G}_2$ and $\mathcal{G}_3$ 2-body operators, and the $SU(3)$ breaking $\mathcal{G}_{SB}$ operator. In this case, the normalization factors are taken to be as follows: (i) for the P-wave decays $\alpha_1 = 4/\sqrt{7}$, $\alpha_2 = 1$, $\alpha_3 = \sqrt{3}/2$, $\alpha_{SB} = 4/\sqrt{21}$, and (ii) for F-wave decays $\alpha_1 = 3/\sqrt{8}$, $\alpha_2 = 1/2$, $\alpha_3 = \sqrt{3/10}$, $\alpha_{SB} = \sqrt{3/8}$.

The experimentally established states in the $[\mathbf{56}, 2^+]$ to be used in the analysis are the following ones. In the octets: (i) $J = 3/2$: $N(1720)$ $(****)$ and $\Lambda(1890)$ $(****)$; (ii) $J = 5/2$: $N(1680)$ $(****)$, $\Lambda(1820)$ $(****)$, and $\Sigma(1915)$ $(****)$. In the decuplets: (i) $J = 1/2$: $\Delta(1910)$ $(****)$; (ii) $J = 3/2$: $\Delta(1920)$ $(***)$; (iii) $J = 5/2$: $\Delta(1905)$ $(****)$; (iv) $J = 7/2$: $\Delta(1950)$ $(****)$ and $\Sigma_{10}(2030)$ $(****)$.

Let us discuss the results for the P- and F-wave decays separately, which are respectively displayed in Tables IV and V.

(i) P-wave decays: All states with $J < 7/2$ have a P-wave decay channel. At LO, there is only one

TABLE IV. Fit parameters and partial widths in MeV for P-wave decays of the $[56, 2^+]$ baryons.

	$\chi^2_{\rm dof}$	dof	C_1	C_2	C_3	B_1
LO	1.6	6	2.8(0.4)	···	···	···
LO*	0.7	5	2.2(0.3)	···	···	−1.2(1.6)
NLO	0.9	3	1.9(0.2)	0.2(1.2)	0.4(2.8)	−4.2(2.2)
NLO*	1.2	3	2.2(0.3)	0.2(1.0)	0.5(2.3)	−1.3(1.6)

	$N_{3/2}(1720)$					$\Lambda_{3/2}(1890)$					$N_{5/2}(1680)$	$\Lambda_{5/2}(1820)$	$\Sigma_{5/2}(1915)$	
	πN	ηN	$K\Sigma$	$K\Lambda$	$\pi\Delta$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$	$\pi\Sigma^*$	$\pi\Delta$	$\pi\Sigma^*$	$\bar{K}\Delta$	$\pi\Sigma^*$
LO	21	0	0	0	1.7	0	19	0	0	1.8	8.1	7.7	0	2.2
LO*	36	1	0.1	3	2	38	16	4	0.3	1.6	9.2	6.6	9.6	2.5
NLO	27(6)	2.4(2)	0.2(0.1)	2.9(2)	2.5(3)	38(21)	16(4)	9.8(6)	1(0.6)	1.8(3)	10(5)	6.1(3)	7.2(4)	1.8(1)
NLO*	36	1	0.1	2.9	2.4	38	16	4	0.3	2	8.1	6	8.8	2.2
Exp	34(16)	···	···	18(17)	···	36(22)	8.5(6.4)	···	···	···	13(5.3)	···	···	···

	$\Delta_{1/2}(1910)$					$\Delta_{3/2}(1920)$					$\Delta_{5/2}(1905)$		
	πN	$K\Sigma$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	πN	$K\Sigma$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$
LO	31	0	3.5	0	0	17	0	13	0	0	14.5	0	0
LO*	35	13	6	0.4	0	19	8.6	23	2.2	0.8	26	1.7	0.1
NLO	45(25)	9.9(6)	5.7(8)	1(0.6)	0	22(13)	6.5(4)	19(15)	5.2(3)	0.6(0.3)	18(11)	4(3)	0.1(0.03)
NLO*	41	15	9	0.6	0.1	18	8.3	28	2.5	0.9	22	1.5	0.1
Exp	52(20)	···	···	···	···	28(19)	···	···	···	···	···	···	···

TABLE V. Fit parameters and partial widths in MeV for F-wave decays of the $[56, 2^+]$ baryons.

	$\chi^2_{\rm dof}$	dof	C_1	C_2	C_3	B_1
LO	3.7	11	0.44(0.03)	⋯	⋯	⋯
LO*	3.4	10	0.41(0.02)	⋯	⋯	−0.20(0.07)
NLO	0.7	6	0.52(0.02)	−0.76(0.10)	0.42(0.18)	0.26(0.08)
NLO*	0.83	7	0.51(0.02)	−0.32(0.06)	0.40(0.13)	−0.11(0.07)

	$N_{3/2}(1720)$	$\Lambda_{3/2}(1890)$	$\Delta_{3/2}(1920)$			$N_{5/2}(1680)$				$\Lambda_{5/2}(1820)$				
	$\pi\Delta$	$\pi\Sigma^*$	$\pi\Delta$	$\eta\Delta$	$K\Sigma^*$	πN	$\pi\Delta$	ηN	$K\Lambda$	$\bar{K}N$	$K\Xi$	$\pi\Sigma$	$\eta\Lambda$	$\pi\Sigma^*$
LO	4.5	5.6	8.7	0	0	22	1.1	0	0	0	0	12	0	1
LO*	8	7	22	0.3	0	55	1.8	0.2	0.1	42	0	14	0.6	1.3
NLO	27(3)	22(3)	59(13)	0.1(0.04)	0	84(6)	6(0.8)	0.1(0.03)	0	12(3)	0	29(2)	0.2(0.1)	4(0.6)
NLO*	13	12	77	0.8	0	83	2.8	0.3	0.1	52	0	21	0.7	2
Exp	⋯	⋯	⋯	⋯	⋯	88(8)	⋯	⋯	⋯	48(7)	⋯	9(3)	⋯	⋯

	$\Sigma_{5/2}(1915)$							$\Delta_{5/2}(1905)$				
	$\bar{K}N$	$\pi\Sigma$	$\eta\Sigma$	$K\Xi$	$\pi\Lambda$	$\bar{K}\Delta$	$\pi\Sigma^*$	πN	$\eta\Delta$	$\pi\Delta$	$K\Sigma$	$K\Sigma^*$
LO	0	18	0	0	15	0	0.6	13	0	12	0	0
LO*	3.6	29	1	0.5	18	2	1	20	0.2	30	0.7	0
NLO	3(0.7)	44(3)	0.4(0.1)	0.1(0.03)	36(3)	0.4(0.1)	2(0.4)	45(11)	0.1(0.02)	58(7)	0.1(0.03)	0
NLO*	4.4	43	1.1	0.7	27	2.6	1.6	44	0.3	65	1.2	0
Exp	12(7)	⋯	⋯	⋯	⋯	⋯	⋯	40(13)	⋯	⋯	⋯	⋯

	$\Delta_{7/2}(1950)$					$\Sigma_{7/2}(2030)$							
	πN	$\eta\Delta$	$\pi\Delta$	$K\Sigma$	$K\Sigma^*$	$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$	$\pi\Sigma^*$	$\bar{K}\Delta$	$\eta\Sigma$	$K\Xi^*$	$K\Xi$
LO	72	0	18	0	0	0	41	14	11	0	0	0	0
LO*	115	0.6	45	6	0	43	50	22	18	16	8	0	1.8
NLO	123(21)	0.2(0.1)	48(9)	0.9(0.2)	0	36(8)	35(8)	12(3)	17(4)	3(0.8)	3(1)	0	0.5(0.1)
NLO*	92	0.4	36	4.5	0	31	40	18	14	11	8	0	1.2
Exp	114(25)	⋯	⋯	⋯	⋯	35(7)	35(7)	13(5)	18(9)	⋯	⋯	⋯	⋯

operator and the fit is consistent as previously observed [10], giving $\chi^2_{\rm dof} \sim 1.6$. The main inconsistency appears in the decay $\Lambda(1890) \to \pi\Sigma$, and this problem gets improved but not solved at NLO. We also show a LO* fit as described earlier. This fit gives further improvement over the LO one, and shows the $SU(3)$ breaking operator to be insignificant. Since, at LO*, one already has a very good description, one expects the NLO corrections to be small, which is indeed the case. As it occurs for the $[\mathbf{56}, 0^+]$, the breaking of $SU(3)$ cannot be accurately determined because there are only two widths with large error bars which are affected. We note some differences between the NLO and NLO* fits: in the former case the $SU(3)$ breaking is rather large, while in the latter it is marginally relevant. Thus, the $1/N_c$ expansion works particularly well in P-wave decays, but the $SU(3)$ breaking cannot be clearly determined.

We have included predictions for the decay rates into η mesons. Although these widths are suppressed, starting at $\mathcal{O}(1/N_c^2)$, it is interesting to have an idea of their magnitude. The fact that no such decays for states assigned to **56**-plets are experimentally observed is a strong indication of the correctness of such assignments. For **70**-plet baryons, the η modes are not suppressed and start at $\mathcal{O}(N_c^0)$ [9].

(i) F-wave decays: All states with $J > 1/2$ have F-wave decays. Here, the LO results are problematic as shown by its $\chi^2_{\rm dof} = 3.7$. This is a one-parameter fit to 12 data. The main issue is that the widths for $N(1680) \to \pi N$ and $\Delta(1905) \to \pi N$ resulting from the fit are too small. The situation for these decays at LO is substantially worse than in the case of the analysis in $SU(4)$ [10]. This is due to the Λ and Σ decays that are included in the fits of the present work. The LO* fit, for which we use the experimental errors, is substantially better. This is because not expanding the matrix elements leads to an important enhancement of the rates $N(1680) \to \pi N$ and $\Delta(1905) \to \pi N$. We note that these channels are the main source of the large χ^2 at LO. Clearly, the NLO corrections must be important. The NLO fit has a minor problem of consistency with the bound

$\Gamma_{F\text{-wave}}(N(1680) \to \pi\Delta) < 2.6$ MeV, violating the bound by a relatively small amount. The main problem is that the decays $\Lambda(1820) \to \bar{K}N$ and $\pi\Sigma$ cannot be simultaneously fitted. The NLO fit gives $\Gamma(\Lambda(1820) \to \bar{K}N) < \Gamma(\Lambda(1820) \to \pi\Sigma)$, which is opposite to the empirical ordering. We find that by eliminating both channels as inputs to the NLO the χ^2 is acceptable. The discrepancy is in part resolved in the fit NLO*, for which the inconsistency remains in the decay $\Lambda(1820) \to \pi\Sigma$. We have, therefore, carried out the NLO* fit without that channel. Since there are no Λ^* baryons in the mass proximity of $\Lambda(1820)$ with which it could mix, the inconsistency should be resolved either by higher order terms in the $1/N_c$ expansion, which would be indication of poor convergence of the expansion for that channel, or by a better empirical value of the width, whose measurement dates back to the 1970s. The $SU(3)$ breaking effects are determined by three input widths, and turn out to be within the expectations. Note that the experimental errors for the F-wave widths are significantly smaller than for the P waves, and thus the predictions for the unknown widths should be better than for the P-wave decays.

IV. CONCLUSIONS

This work implemented the $1/N_c$ expansion for the decays of positive parity **56**-plet baryons, and analyzed the empirically known partial decay widths to $\mathcal{O}(1/N_c)$ and first order in $SU(3)$ symmetry breaking. As it had been well established in previous work within $SU(4)$, the decays of the $[\mathbf{56}, 0^+]$ baryons are poorly described at LO, requiring NLO corrections by a 2-body operator, which are almost 50% of the contributions from the LO operator at the level of the amplitude. These large corrections are signaling the special nature of this multiplet. We note that large NLO corrections are also necessary in the description of the helicity amplitudes of this multiplet [12]. These facts are indicative of a poorly converging $1/N_c$ expansion in the $[\mathbf{56}, 0^+]$ baryons. On the other hand, the decays of the $[\mathbf{56}, 2^+]$ baryons are well described, showing, for both P- and F-wave decays, natural size contributions by the NLO operators. We have noticed that for a considerable number of matrix elements better fits result if they are not expanded in $1/N_c$, in particular, in the case of the F waves. For this reason, we included fits LO* and NLO* to see that effect.

One point to note is that, in general, the empirical widths have rather large error bars, considerably larger than the 10%, which would allow for an accurate determination of the NLO effects. This impacts on the predictions one can deduce from the present analysis; in the case of the $[\mathbf{56}, 0^+]$ decays, the predictions should be taken with caution. In the case of the $[\mathbf{56}, 2^+]$ decays, the predictions for P-wave widths are within errors of the order of 50%, while for the F waves they are accurate at the 20% level, upon having eliminated the problematic channel $\Lambda(1820) \to \pi\Sigma$.

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APPENDIX: $SU(3)$ ISOSCALAR FACTORS

This appendix gives the isoscalar factors, which appear in the matrix elements needed in this work, which correspond to the emission of mesons belonging to an octet of $SU(3)$. They are given for the irreducible representations of $SU(3)$ as needed in this work for generic N_c. The representations are given in terms of the two labels defining the Young tableau, namely (p, q), where $p + 2q = N_c$. The isoscalar factors needed for the decays into the octet of mesons are denoted by

$$\begin{pmatrix} (p, q) & (1, 1) & (p', q') \\ Y_1 I_1 & Y_2 I_2 & YI \end{pmatrix}_\gamma. \tag{A1}$$

Here, (p', q') is the representation of the final ground state baryon and (p, q) of the initial excited baryon. For baryons, the correspondences between multiplets for generic odd N_c and $N_c = 3$ are as follows: $(p = 1, q = \frac{N_c-1}{2}) \to \mathbf{8}$, and $(p = 3, q = \frac{N_c-3}{2}) \to \mathbf{10}$. Table VI displays these correspondences explicitly.

TABLE VI. Representation correspondences for arbitrary odd N_c.

8 Baryons $(p,q)=(1,\frac{N_c-1}{2})$ State	(Y,I)	**10** Baryons $(p,q)=(3,\frac{N_c-3}{2})$ State	(Y,I)	Mesons $(p,q)=(1,1)$ State	(Y,I)
N	$(\frac{N_c}{3}, \frac{1}{2})$	Δ	$(\frac{N_c}{3}, \frac{3}{2})$	π	$(0, 1)$
Σ	$(\frac{N_c-3}{3}, 1)$	Σ^*	$(\frac{N_c-3}{3}, 1)$	η	$(0, 0)$
Λ	$(\frac{N_c-3}{3}, 0)$	Ξ^*	$(\frac{N_c-6}{3}, \frac{1}{2})$	K	$(1, \frac{1}{2})$
Ξ	$(\frac{N_c-6}{3}, \frac{1}{2})$	Ω	$(\frac{N_c-9}{3}, 0)$	$\bar{K}$	$(-1, \frac{1}{2})$

In the following, Tables VII, VIII, IX, and X display the isoscalar factors as needed for the calculations carried out in this work.

TABLE VII. Isoscalar factors for $\mathbf{8} \to \mathbf{8}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{(N_c+3)}$ and $f_2 = \frac{1}{(N_c+3)}\sqrt{\frac{(N_c-1)}{(N_c+7)}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively.

	N				Λ			
	ηN	πN	$K\Sigma$	$K\Lambda$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\gamma = 1$	N_c	3	$\sqrt{3(N_c-1)}$	$\sqrt{3(N_c+3)}$	$-\sqrt{\frac{3(N_c+3)}{2}}$	0	$N_c - 3$	$3\sqrt{\frac{N_c-1}{2}}$
$\gamma = 2$	3	$-(N_c+6)$	$\frac{N_c+15}{\sqrt{3(N_c-1)}}$	$-\sqrt{3(N_c+3)}$	$\sqrt{\frac{3(N_c+3)}{2}}$	$-\sqrt{\frac{(N_c+3)^3}{3(N_c-1)}}$	6	$\frac{9-N_c}{\sqrt{2(N_c-1)}}$

	Σ				
	$\bar{K}N$	$\eta\Sigma$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$
$\gamma = 1$	$3\sqrt{\frac{N_c-1}{2}}$	$N_c - 3$	$2\sqrt{6}$	0	$\sqrt{\frac{3(N_c+3)}{2}}$
$\gamma = 2$	$\frac{N_c+15}{\sqrt{2(N_c-1)}}$	$\frac{2(N_c-9)}{N_c-1}$	$-\sqrt{\frac{2}{3}}\frac{(N_c-3)(N_c+7)}{N_c-1}$	$\sqrt{\frac{(N_c+3)^3}{N_c-1}}$	$-\frac{5N_c+3}{N_c-1}\sqrt{\frac{N_c+3}{6}}$

	Ξ			
	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$\gamma = 1$	$-\sqrt{N_c+3}$	$3\sqrt{N_c-1}$	$N_c - 6$	3
$\gamma = 2$	$\frac{5N_c+3}{3(N_c-1)}\sqrt{N_c+3}$	$\frac{9-N_c}{\sqrt{N_c-1}}$	$\frac{7N_c-15}{N_c-1}$	$\frac{N_c^2+3N_c+36}{3(N_c-1)}$

TABLE VIII. Isoscalar factors for $\mathbf{10} \to \mathbf{10}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{\sqrt{45+N_c(N_c+6)}}$ and $f_2 = \sqrt{\frac{5(N_c-3)(N_c+5)}{(N_c+1)(N_c+9)(45+N_c(N_c+6))}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively.

	Δ			Σ^*			
	$\eta\Delta$	$\pi\Delta$	$K\Sigma^*$	$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$
$\gamma = 1$	N_c	$3\sqrt{5}$	$\sqrt{3(N_c+5)}$	$-\frac{3\sqrt{N_c+5}}{2}$	$N_c - 3$	$2\sqrt{6}$	$\sqrt{6(N_c+3)}$
$\gamma = 2$	3	$-\frac{N_c+6}{\sqrt{5}}$	$\frac{3-N_c}{\sqrt{3(N_c+5)}}$	$\frac{N_c-3}{2\sqrt{N_c+5}}$	$\frac{4(N_c+3)}{N_c+5}$	$-\frac{N_c^2+10N_c+33}{\sqrt{6}(N_c+5)}$	$\frac{3-N_c}{N_c+5}\sqrt{\frac{2(N_c+3)}{3}}$

	Ξ^*				Ω	
	$\bar{K}\Sigma^*$	$\eta\Xi^*$	$\pi\Xi^*$	$K\Omega$	$\bar{K}\Xi^*$	$\eta\Omega$
$\gamma = 1$	$-2\sqrt{N_c+3}$	$N_c - 6$	3	$3\sqrt{N_c+1}$	$-3\sqrt{\frac{N_c+1}{2}}$	$N_c - 9$
$\gamma = 2$	$\frac{2(N_c-3)\sqrt{N_c+3}}{3(N_c+5)}$	$\frac{5N_c+9}{N_c+5}$	$-\frac{N_c^2+9N_c+36}{3(N_c+5)}$	$\frac{(3-N_c)\sqrt{N_c+1}}{N_c+5}$	$\frac{N_c-3}{N_c+5}\sqrt{\frac{N_c+1}{2}}$	$\frac{6(N_c+1)}{N_c+5}$

TABLE IX. Isoscalar factors for $\mathbf{8} \to \mathbf{10}$ decays. The listed values should be multiplied by $f = \frac{\sqrt{2}}{\sqrt{(N_c+1)(N_c+5)}}$ to obtain the actual isoscalar factors.

N		Λ	
$\pi\Delta$	$K\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$
$-\sqrt{\frac{(N_c-1)(N_c+5)}{2}}$	$-2\sqrt{\frac{N_c-1}{3}}$	$-\sqrt{\frac{(N_c+3)(N_c-1)}{3}}$	$-\sqrt{2(N_c-1)}$

Σ				Ξ			
$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$	$\eta\Xi^*$	$\pi\Xi^*$	$\bar{K}\Sigma^*$	$K\Omega$
$\sqrt{N_c+5}$	2	$\frac{N_c+1}{\sqrt{6}}$	$\sqrt{\frac{2(N_c+3)}{3}}$	2	$\frac{2N_c}{3}$	$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c+1}$

TABLE X. Isoscalar factors for **10** → **8** decays. The listed values should be multiplied by $f = ((N_c + 7)(N_c - 1))^{-1/2}$ to obtain the actual isoscalar factors.

Δ		Ξ^*			
πN	$K\Sigma$	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$-\sqrt{(N_c-1)(N_c+5)}$	$2\sqrt{\frac{N_c+5}{3}}$	$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c-1}$	-2	$-\frac{2N_c}{3}$

Σ^*					Ω
$\eta\Sigma$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$	$\bar{K}\Xi$
-2	$\sqrt{2(N_c-1)}$	$-\frac{N_c+1}{\sqrt{6}}$	$-\sqrt{(N_c+3)(N_c-1)}$	$\sqrt{\frac{2(N_c+3)}{3}}$	$\sqrt{2(N_c+1)}$

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