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# Formal Operational Performance: Epochal and Sociocultural Differences in the First Level of Secondary School Students in Argentina*

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**Abstract:** The following research presents the outcomes of a cohort study investigating formal thinking skills among first and second-year secondary school students. A specially crafted instrument, the Logical Thought Performance Test for Adolescents (LTP-A), was employed to gauge the level of formal thought. The LTP-A assesses various aspects, including: combinatorial reasoning; proportional reasoning; permutation; inferences derived from exclusive and inclusive disjunction; biconditional and asymmetric implication; and, modus tollens. The study compares the achievements of four student groups from two educational institutions at two distinct time points, with a thirty-year gap. The independent variables include the sociocultural level and the epochal aspect. Methodologically, one-way analysis of variance and cluster analysis were performed, showing significant differences in relation to sociocultural level. Results suggest that the sociocultural factor outweighs epochal differences. Then, a content analysis of some answers was carried out to detect resolution strategies, some conceptual categories and types of errors. Conclusions explore the moderating role of students' sociocultural levels, and provide educational recommendations.

**Keywords:** *Epochal differences, formal thought, secondary school students, sociocultural level.*

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## Introduction

The study of formal thinking has gained particular relevance in research following the work of Jean Piaget and his team, specifically during the last four decades of the 20th century. Hence, it is necessary to cite works from this period, although the topic remains theoretically and practically relevant today.

Formal thinking structures play a fundamental role in the understanding of basic curriculum topics. Consequently, deficiencies or delays in the development of these structures emerge as a specific predictive factor of academic failure.

### *Formal Thinking*

In his examination of the stages of intelligence development, Piaget has identified the culminating stage as formal thought. He has contended that the most important functional character of formal operations is the inversion of meaning between the real and the possible, which is an obligatory correlate of necessity (Piaget, 1979; Piaget & Inhelder, 1985). In effect, hypothetical-deductive operations lead to necessary results because the starting point of the enunciator is no longer the factual data, but the totality of compatible data, the product of a combinatorial operation (Piaget, 1979). Indeed, what leads to propositional thought and underlies this is combinatorics based on the set of parts, which is the most general property of this stage. This stage succeeds in merging the two great forms of reversibility into a single system: inversion, or negation, and reciprocity, or symmetry.

* Dedicated to the memory of María Virginia Rapetti.

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In this stage, the individual operates as if they could match each propositional operation with an inverse and a reciprocal. For example, when faced with two factors that appear together (p.q), the individual supposes or questions whether p implies q ($p \rightarrow q$), searches for a counterexample (p.-q), and simultaneously hypothesizes $q \rightarrow p$, for which the individual tries to verify if p.-q does not occur (Piaget & Beth, 1961). In other words, the individual combines inversion, which nullifies the direct operation, with reciprocity, which compensates it when combined with the direct operation, leading to symmetry or equivalence. It is important to note that these are operational tools and not conscious operations.

The precondition for these propositional operations is combinatorics: a set system (n to n) that allows one to understand that a phenomenon can be the result of either a single factor or a combination of all or some of them. Underlying the operation of combining –implicitly at the beginning– is the system of 16 binary propositional operations, combinations of 1 to 1, 2 to 2, 3 to 3, 4, or none of the four basic possibilities: p.q v p.-q v -p.q v -p.-q.

By combining experimental factors, individuals generate combinatorics through statements corresponding to what they observe, thereby determining the relationships of conjunction, implication, and exclusion. Thus, in the Piagetian interpretation, as combinatorial operations progress, so does deductive reasoning. What leads to combinatorics is the attempt to dissociate factors by jointly using inversion and reciprocity. The combinatorial operation acquires propositional meaning when it is contrasted with other possible combinations. For example, when the individual does not immediately conclude, from the presence of p.q, the equivalence $p \equiv q$ or the implication $p \leftrightarrow q$ without first verifying that p.q is not accompanied by p.-q and/or -p.q.

In terms of proportion, in its logical aspect, it expresses the equivalence of relationships between two pairs of expressions involving compensation and reciprocity, constitutive of the INRC group. The individual must understand that reciprocal action compensates but does not nullify the identical, unlike the inverse. Meanwhile, the correlative is the product of the inverse (N) by the reciprocal (R), restoring the identical by nullifying the reciprocal. Mastery of these four relationships allows for the possession of a scheme of equilibrium.

According to Piaget, the group of these four transformations (INRC) is equivalent to a system of logical proportions, as understanding the equalities NR = IC; RC = IN; NC = IR involves making the following equivalences: $I/C \equiv R/N$; or, alternatively, $R/I \equiv C/N$.

Therefore, the scheme of proportion precedes propositional logic, and the explicitness, or awareness, of the operations it entails allows for the attainment of propositional operations.

As noted above, mastery of these structures is crucial for understanding basic curriculum topics and is, therefore, a specific predictor of academic failure. This is particularly so not only in disciplines related to exact sciences but also in the argumentative aspects of discourse in humanities and social sciences. It is important to acknowledge that this predictive role is reflected in school practices through grades, implying that the grading criteria should include proficiency in logical schemas. To the extent that this requirement is lacking or not sufficiently weighted in teachers' assessments, the correlation between formal operation ability and academic promotion will naturally be weaker.

According to Boujade and Giuliano (1994), the best predictors of secondary-level chemistry performance are prior knowledge and formal reasoning skills. Other authors (Molina Lara & Rada Arteaga, 2013; Rohaeti et al., 2019) have also identified significant differences in performance in Sciences and Mathematics, according to the formal operation level. Nevertheless, Franco de Camargo (1990) found no significant correlations between formal operational thinking and academic performance in students ranging from the 2nd to 8th grades, as well as in university students who were studying General Didactics.

In an exploration involving a sample of 700 first-year Engineering students at a university (Vázquez & Difabio de Anglat, 2009), significant correlations were observed between the level of formal thought and students' grades in entrance exams, partial exams in Algebra, Calculus, Physics, and Chemistry, as well as the final Chemistry exam. Ramírez Leal et al. (2018), in a sample of 103 undergraduate Mathematics students in their initial phase, found that only 16% were at the formal level, 27% in a transitional phase, and the rest at the concrete thinking level. The best performance was in tasks involving proportionality schemes, and the poorest one was in solving combinatorial problems and permutations, which affected their academic performance.

The Piagetian theory of formal thought has been questioned in two aspects: the age at which achievement occurs (Chapman, 1988; Lourenço, 2016; McLean & Riggs, 2022); and, the impact of sociocultural factors (Dasen, 1994; Feldman, 2012; Fischer & Granott, 1995; Nouri et al., 2022). Several studies have reported that formal thought is observed in only 50% of adults (e.g., Larivée et al., 2000). However, Lourenço and Machado (1996), and Lourenço (2016) argue that, in Piagetian theory, the emphasis is not primarily on age, but on the sequence of cognitive transformation achievements.

Piaget consistently emphasized that cognitive development involves social interaction, and is not merely the result of individual development (Piaget, 1950; Piaget & Inhelder, 1985). Other authors (e.g., Rohaeti et al., 2019) find results that support Piaget's thesis that the formal stage is achieved by the interaction between maturity, socialization, teaching, and culture transmission. According to Kuhn (2008), Piaget maintains that the achievement sequence is universal, but its temporal rhythm is affected by the quality and frequency of opportunities that the subjects have. Furthermore, formal structures might not emerge in all domains of knowledge and will be influenced by the subjects' aptitudes and interests, resulting in inter-individual variability that tends to increase at the end of the second decade of life.

Concerning the influence of sociocultural aspects, a challenge to the Piagetian system is the assumption of a universal epistemic subject, implying that the evolutionary stages of these acquisitions are relatively independent of cultural conditions and historical periods. While Piaget does not deny variations in achievements based on cultural differences, his theory focuses on the regularity of the achievement sequence, a point that others researchers have not called into question (cf. Lourenço, 2016, for a review).

The objective of this research is to assess the extent to which adolescents in the first and second years of secondary school employ operational schemes characteristic of formal thought to solve situations that are required of them. Moreover, the aim is to investigate sociocultural differences in the mastery of these schemes, along with variations attributed to epochal factors.

## Methodology

*Goals and Hypothesis*

The research aims to:

a) Evaluate changes in the level of formal operational performance over time in Argentine secondary school adolescents, for the epochal differences; and,

b) Explore differences in the achievements of formal operational performance related to sociocultural differences.

Two hypotheses are tested in this study:

a) The level of formal thinking in secondary school adolescents has decreased in recent decades; and,

b) Students belonging to a lower sociocultural level exhibit poorer performance in formal thinking.

*Research Design*

The study employs a cohort descriptive approach to meet its objectives. A mixed design is utilized to attain a comprehensive analysis of the data: quantitative analysis that involves the use of statistical methods to analyze numerical data collected from the sample, and content analysis of some item responses to detect resolution strategies, some conceptual categories and types of errors. The first author of this work conducted the content analysis of the responses by item; it was then reviewed by the second author, who agreed on the significance of the selected examples and their analysis to show variations in the type of response to each item.

*Sample*

The sample consists of 229 secondary school students from two distinct populations with diverse sociocultural backgrounds. Data were collected at two different time points, with a 30-year gap (1992 and 2022).

Samples I ($n$ = 50, $X$ age: 14 years, 6 months, *SD*: .398) and II ($n$ =54, $X$ age: 14 years, 5 months, *SD*: .401) were obtained from a high socioeconomic-cultural level school in the city of Buenos Aires (hereinafter "CEB") with an international baccalaureate program. Samples III ($n$ =72, $X$ age: 14 years and 7 months, *SD*: 0.393) and IV ($n$ =53, $X$ age: 14 years 6 months, *SD*: 0.387) were collected in the city of Paraná, Argentina, from a school (hereinafter "CP") with a technical-accounting program. All participants were in the 1st and 2nd year of secondary school. Sample III represented a low socioeconomic-cultural level, while Sample IV represented a medium-low level. Although the last two samples were collected in the same school, CP, the families of origin included a considerable number of professionals in Sample IV.

*Instrument*

The instrument, Logical Thought Performance Test for Adolescents (LTP-A; Vázquez et al., 2023), was elaborated *ad hoc,* using some items of the Longeot (1964) test and Roberge and Flexer test (FORT, 1982). It is made up of eleven items, which evaluate combinatorial reasoning, proportional reasoning, permutation, inferences derived from exclusive and inclusive disjunction, biconditional and asymmetric implication and *modus tollens*. The statement of each item is found in the section corresponding to the analysis of categories present in some of the responses.

In prior research (Vázquez et al., 2023), the psychometric properties of the instrument have been examined, and stability in the achievement sequences have been confirmed via a hierarchical analysis of items, using Guttman's (1974) scalogram. The IR index of 0.90 has been found to be acceptable. The consistency index of 0.75 has been considered very good, given the dichotomous nature of the items. Here, the data obtained from the sample meet the criteria of normality and reliability (see Table 1 and 2).

*Table 1. Normality*

| | Kolmogorov-Smirnov | | |
|---|---|---|---|
| | **Statistic** | **df** | **Sig.** |
| Total score | 0.119 | 229 | 0.200 |

a. Lilliefors significance correction

*Table 2. Reliability (Internal Consistency)*

| Reliability Statistics | |
|---|---|
| **Cronbach's Alpha** | **N of items** |
| 0.705 | 11 |

To verify the validity of the internal structure of the instrument, a factorial analysis was conducted (see Table 3).

*Table 3. Factorial Analysis*

| Component Matrix | |
|---|---|
| | **Component 1** |
| Combinatorial reasoning 2 | 0.733 |
| Combinatorial reasoning 1 | 0.717 |
| Permutation | 0.667 |
| Inclusive Disjunction | 0.642 |
| Proportional reasoning 2 | 0.617 |
| Proportional reasoning 1 | 0.599 |
| Reciprocal implication | 0.408 |
| Combinatorial reasoning 3 | 0.402 |
| Exclusive Disjunction | 0.400 |
| Simple Implication | 0.400 |
| Modus tollens | 0.390 |

Extraction method: principal component analysis. 1 extracted component

The KMO value of 0.813 confirms that the sampling is adequate. The difficulty index is 0.53, thus meeting the requirement of statistical theory, as this moderate level should range between 0.5 and 0.6. Therefore, the level of difficulty would be considered satisfactory. However, there is an excessive percentage of very difficult items (see Table 4), which, for the purposes of the proposed study, is appropriate, given that the aim is to assess the level of formal operational thought and the subjects in the sample are in an initial stage of this acquisition. Therefore, the level of difficulty found is expected.

*Table 4. Item Percentage According to the Difficulty Index*

| Very easy | Easy | Moderate | Difficult | Very difficult |
|---|---|---|---|---|
| 18% | 18% | 27% | 18% | 27% |

*Index Discrimination*

For this analysis, a biserial correlation is conducted between the total number of correctly answered dichotomous items and the correct responses to each individual item. According to this correlation, the acceptable minimum value typically varies among authors: 0.25 (considered a 'softer' cutoff) and 0.30 for other authors. Therefore, all items meet the requirement for discrimination (see Table 5).

*Table 5. Item Discrimination Level*

| | Pearson Correlation | Sig. (bilateral) |
|---|---|---|
| Combinatorial reasoning 2 | 0.703** | .000 |
| Combinatorial reasoning 1 | 0.684** | .000 |
| Permutation | 0.628** | .000 |
| Inclusive Disjunction | 0.601** | .000 |
| Proportional reasoning 2 | 0.596** | .000 |
| Proportional reasoning 1 | 0.593** | .000 |
| Reciprocal implication | 0.461** | .000 |
| Combinatorial reasoning 3 | 0.422** | .000 |
| Exclusive Disjunction | 0.410** | .000 |
| Simple Implication | 0.382** | .000 |
| Modus tollens | 0.321** | .000 |

*Instrument Consistency and Reproducibility*

To evaluate these two aspects of the instrument, a hierarchical item analysis is conducted using the Guttman scaling technique (Guttman, 1974; see also Escurra Mayaute & Salas Blas, 2014), which allows for establishing an order of skill acquisition. An IR index of 0.90 is obtained. This value meets the requirement of the technique (it should be equal to or greater than 0.90). The consistency index found –Cochran's Q 603.65, p > 0.000– is 0.75, which is considered a very good index.

Three groups are delineated based on item difficulty. Using k-means cluster analysis, three levels are identified: Preformal, Formal A, and Formal B (see Table 10). The most difficult group includes items assessing *modus tollens*, simple implication, and third-level combinatorics (items K, H, and J). Conversely, the least challenging group consists of first and second-level combinatorics, reciprocal implication, and exclusive disjunction (items A, B, E, and C). The remaining four items constitute an intermediate group.

Examining both correct and incorrect responses to the items arranged in this manner allows for testing a genetic hypothesis. In other words, analyzing the hierarchy of responses enables the assessment of whether the differential tests being constructed correspond to the underlying theory or not.

*Analyzing of Data*

The dichotomous scores of the instrument are converted into a continuous scale by assigning different point values to each item based on its relative difficulty. This approach ensures that the correct resolution of various items does not equate to the score of the more difficult ones. The point assignment is as follows:

1 point for items resolved by 100% to 81%;

3 points for items resolved by 80% to 59%;

4 points for items resolved by 58% to 28%; and,

8 points for items resolved by 27% to 0%.

Consequently, points are assigned as follows: 1 point for tasks involving the first level of combinatorial reasoning, reciprocal implication, and exclusive disjunction tests; 3 points for tasks related to the first level of proportion and the second level of combinatorial reasoning; 4 points for tasks encompassing permutation, the second level of proportion, inclusive disjunction, and *modus tollens*; and, 8 points for tasks requiring simple implication and the third level of combinatorial reasoning. A student who correctly solves all items obtains a total of 41 points. Using these differential scores, we established eight intervals: 0-5, 6-10, 11-15, 16-20, 21-25, 26-30, 31-35, and 36-41.

To examine the hypothesis of potential epochal differences in achievement levels, we compare the samples obtained in 1992 and 2022 in terms of formal operational performance, keeping the variable of the schools constant and taking the sampling year as the independent variable. One-way analysis of variance (ANOVA) is used to test for differences among the groups.

To examine the hypothesis of potential sociocultural differences in achievement levels, we compare the samples obtained in CEB and CP for formal operational performance in both 1992 and 2022. One-way ANOVA is employed to assess differences among the groups. To enrich the analysis, illustrative cases are taken. These are to categorize types of responses.

## Results

*Quantitative Results*

Table 6 shows the percentage of student's achievement in formal thinking performance by school and year. A comprehensive examination of Table 6 reveals that the item-difficulty sequence is maintained within each sub-sample, despite variations in percentages. Consequently, it becomes feasible to identify the three groups of clustered items through scaling analysis.

*Table 6. Secondary School Student's Achievement Percentages in Formal Thinking Performance by School and Cohort*

| **Variables** | **CEB1992** (*n* = 50) | **CEB2022** (*n* = 54) | **CP 1992** (*n* = 72) | **CP2022** (*n* = 53) |
|---|---|---|---|---|
| Combinatorial reasoning 1 | 96% | 87% | 63% | 60% |
| Combinatorial reasoning 2 | 80% | 80% | 35% | 49% |
| Proportional reasoning 1 | 74% | 69% | 44% | 32% |
| Reciprocal implication | 84% | 89% | 56% | 47% |
| Simple implication | 26% | 22% | 8% | 23% |
| Inclusive disjunction | 52% | 39% | 4% | 8% |
| Exclusive disjunction | 64% | 94% | 67% | 77% |

*Table 6. Continued*

| Variables | CEB1992 (*n* = 50) | CEB2022 (*n* = 54) | CP 1992 (*n* = 72) | CP2022 (*n* = 53) |
|---|---|---|---|---|
| Proportional reasoning 2 | 52% | 56% | 13% | 19% |
| Combinatorial reasoning 3 | 26% | 9% | 8% | 11% |
| Permutation | 60% | 48% | 10% | 23% |
| *Modus tollens* | 40% | 17% | 14% | 25% |

Statistically significant differences exist between cohorts. For students from CEB, the 1992 cohort demonstrated a higher performance in Combinatorial Reasoning 3 ($\eta p^2 = .10$); compared to the 2022 cohort. Conversely, the 2022 cohort achieved superior performance in Exclusive Disjunction ($\eta p^2 = .15$) compared to the 1992 cohort.

For CP students, the 2022 cohort demonstrated higher performance in Simple Implication ($\eta p^2 = .10$) compared to the 1992 cohort. Additionally, advancements were observed in two items belonging to the formal operational thinking Stage B. While there was also an improvement in *Modus tollens* reasoning, the difference did not reach statistical significance. Notably, twelve students from the 2022 cohort successfully solved Simple Implication, a purely formal achievement.

An interval analysis was conducted to examine potential differences attributable to epochal features. These intervals were calculated using the total score of the instrument, and differences between cohorts (1992 vs. 2022) were compared using one-way ANOVA (see Table 7).

*Table 7. Intervals of Secondary School Student's Achievement Percentages on Formal Thinking Performance by School and Cohort*

| Intervals | CEB1992 (*n* = 50) | CEB2022 (*n* = 54) | CP 1992 (*n* = 72) | CP2022 (*n* = 53) |
|---|---|---|---|---|
| 0 -5 | 2% | 9% | 46% | 26% |
| 6-10 | 18% | 22% | 33% | 42% |
| 11-15 | 12% | 15% | 10% | 19% |
| 16-20 | 20% | 22% | 7% | 9% |
| 21-25 | 30% | 24% | 3% | 2% |
| 26-30 | 4% | 8% | 1% | 0% |
| 31-35 | 8% | 0% | 0% | 2% |
| 36-41 | 6% | 0% | 0% | 0% |

Table 7 indicates that there were no significant differences in the intervals between CEB cohorts (1992 vs. 2022). Moreover, the 2022 CEB cohort did not achieve any results in the last two intervals.

For CP, there were fewer students in the 2022 cohort than in the 1992 cohort in the lowest interval ($\eta p^2 = .15$). Although no differences were observed in the remaining intervals between cohorts, there is a higher percentage of students from the 2022 cohort in three items indicative of formal operational thought at Stage B (see Table 6).

A one-way ANOVA was conducted. This is to compare the formal thinking performance of students, in total scores, across cohorts.

*Table 8. Mean Difference on Secondary School Student's Formal Thinking Performance by Cohort*

| | Cohort | | | |
|---|---|---|---|---|
| | 1992 | 2022 | | |
| School | *X* (*SD*) | *X*(*SD*) | F | p |
| CEB | 19.38 (9.062) | 15.25 (7.477) | 6.47 | 0.01 |
| CP | 6.43 (6.780) | 8.72 (6.089) | 3.79 | 0.54 |

In the CEB sub-sample, 1992 student's cohort showed a higher total score in formal thinking performance than 2022 cohort. However, the effect size is not important ($\eta p^2 = 0.06$). In the CP sub-sample, only a significant trend is verified.

A one-way ANOVA was conducted to compare students' formal thinking performance (total score) based on their sociocultural level (see Table 9). In both 1992 and 2022, CEB students demonstrated higher total scores in formal thinking performance than CP.

*Table 9. Mean Difference on Secondary School Student's Formal Thinking Performance by Sociocultural Level*

| | Cohort | | | |
|---|---|---|---|---|
| | CEB | CP | | |
| **Year** | ***X* (*SD*)** | ***X* (*SD*)** | **F** | **p** |
| 1992 | 19.38 (9.062) | 6.43 (6.780) | 81.480 | 0.00 |
| 2022 | 15.25 (7.477) | 8.72 (6.089) | 24.932 | 0.00 |

Size effect for the sociocultural factor is $\eta p^2$ = .33, obtained from one-way ANOVA. Regarding the epochal factor, the statistical analysis shows that there are no significant differences between cohorts ($\eta p^2$ = .045), nor is there an interaction.

Finally, a K-means cluster analysis was conducted using standardized scores to categorize secondary school students into one of the three aforementioned levels: pre-formal, formal level A, and formal level B. Table 10 presents the percentage of students in each cluster, organized by cohort and school.

*Table 10. Percentage of Students in Each Cluster of Formal Thinking Performance by Cohort and School*

| **Categories** | **CEB1992** (*n* = 50) | **CEB2022** (*n* = 54) | **CP 1992** (*n* = 72) | **CP2022** (*n* = 53) |
|---|---|---|---|---|
| Preformal | 22% | 33.3% | 87.5% | 85% |
| Formal A | 50% | 66.7% | 9.7% | 15% |
| Formal B | 28% | 0% | 2.8% | 0% |

Regarding the difference by cohort, in the case of CEB, significant differences occur only in the Formal B level, with a size effect equal to $\eta p^2$ = 0.411. On the other hand, in the case of CP, there is a moderate improvement since the percentage of subjects in the preformal level decreases, and that of the Formal A level increases, but the differences are not statistically significant.

It is worth noting that there is some improvement in the case of the PC and a slight decrease in the performance of CEB. In the latter case, the content analysis of some item responses showed errors due to deficient interpretation of the instructions and biases in attributional beliefs about failure. In the case of CP, differences in the characteristics of the subjects' families of origin were observed.

*Content Analysis of Some Item Responses*

In this section, variations in the types of responses to each item of the Logical Thought Performance Test for Adolescents are analyzed. The responses are presented in tables organized as follows: on the left, the item is displayed along with some examples, except for Table 14, which includes only cases where the exercise was not solved; on the right, the correct response and the reasons for the errors are provided.

*a) Combinatorial Reasoning*

With regard to the first form of combinatorial reasoning (simple arrangements), systematic achievements in creating simple arrangements have been observed in children as young as 6 years old in other studies (e.g., English, 2005; Scardamalia, 1977). For instance, when presented with four or five digits and the instruction: 'Form all the two-digit numbers that you can', children successfully completed the task. They also demonstrated an awareness of the method –that Piaget refers to as reflecting abstraction– that allowed them to control and correct their production: 'to be sure that they have not forgotten any number'.

Reflecting abstraction deals with the actions of the subject, not with the characteristics of the object. It transposes what it captures to a higher plane; for example, the subject conceptualizes an action, at the same time while reconstructing the action scheme, thus generalizing and integrating schemata (Piaget, 1977).

Thus, some examples suggest the possibility of exploring the scheme at an early age. Larivée and Normandeau (1985) have referred to similar studies, leading to the conclusion that children are capable of performing combinatorial operations using the systematic method from the age of seven. In addition, studies conducted by English (1993, 2007) with children aged 7 to 12 have demonstrated their application of systematic strategies to solve combinatorial problems.

In our samples from the 1992 cohort (CP), the level of achievement on item A (Combinatorial Reasoning 1) was 63%, marking the greatest difference (28%) compared to item B (Combinatorial Reasoning 2). This latter item requires generalization in a formula to calculate the number of possible combinations without writing them down (see Table 6).

*Table 11. Item A) on Combinatorial Reasoning: Examples of Responses and Analysis*

| | |
|---|---|
| A) "You buy a number in a lottery. Only two-digit numbers have been sold, and the numbers were formed with the digits 1, 2, 3, and 4. You have the number 11. To find out your chances of owning the winning number, find ALL the 2-digit numbers that could have been sold."<br>11, 12, 13, 14<br>21, 22, 23, 24<br>31, 32, 33, 34<br>41, 42, 43, 44<br>My chance is 1/16 (CEB 1992 student).<br>12, 13, 14<br>21, 22, 23, 24<br>31, 32, 33, 34<br>41, 42, 43, 44 (CP 2022 student answer) | The correct answer is: 1/16 (4x4), since the combinations are:<br>11, 12, 13, 14<br>21, 22, 23, 24<br>31, 32, 33, 34<br>41, 42, 43, 44. |
| B) "Could you tell how many two-digit numbers would be formed using the digits 1, 2, 3, 4, 5? Do it without writing the numbers, by mental calculation."<br>I can form 25 numbers (5x) (CEB 1992 student).<br>I don't know (CP 2022 student). | The correct answer is: 25 (5x5). |

On the other hand, in both cohorts at CEB, the percentage of successful resolution for the first level of combinatorial ability, simple arrangements, is 90%, with 80% of students achieving generalization. Given that the ages and educational level are the same in both schools, these differences can be attributed to sociocultural factors.

The greatest difficulty occurs in the third level of combinatorial reasoning (item I), where resolution involves constructing the 'set of parts' through a systematic and exhaustive method to identify all possible ways to combine elements, avoiding repetitions. This third level of combinatorial ability yielded the lowest percentages in both cohorts. Among the students' responses that manage to show a system, the following can be cited:

*Table 12. Item I) on Combinatorial Reasoning: Examples of Responses and Analysis*

| | |
|---|---|
| I) "A horticulturist planted tomatoes, to which he added four fertilizer products – which we call A, B, C and D– with which he managed to increase the size of the tomatoes. He wants to know whether this increase is due to the action of one or some of the fertilizers (perhaps none). How can he find out? Represent the solution using the letters."<br>A AB ABC ABCD AC AD<br>B BCD BC BD<br>C CD<br>D<br>(CEB 1992 student)<br><br>Tomato + A =<br>Tomato + B =<br>Tomato + C =<br>Tomato + D =<br>(CP 2022 student)<br><br>Try each of the fertilizers to see which one it is. Maybe it's not just one; it could be two, three, or all of them (CP 1992 student). | The right answer is:<br>A – B – C – D<br>AB – AC – AD – BC - BD – CD<br>ABC-ABD-ACD-BCD<br>ABCD<br>(all)<br>None<br>16 possibilities. |

These examples demonstrate different approaches to solving the problem, each outlining a method to arrive at a solution, although some combinations are omitted (CEB 1992 student). In our samples, CEB students from the 1992 cohort showed the highest percentage of correct solutions, 26%. In the remaining sub-samples, the percentage drops to a range between 8% and 11%. It's noteworthy that all students who achieved a complete solution belonged to Formal Level B (see Table 6).

*b) Permutation*

Permutation is an extension of the first level of combinatorial reasoning. It involves the arrangement of four elements taken in fours.

In our study, item (J) comprises four elements with a moderate difficulty index (0.48). The item consists of two permutation exercises, both featuring four elements. In the first exercise, students are required to complete the solution, while in the second one, they are tasked with anticipating the result without writing down all possible permutations; in essence, they need to generalize from the discovery of the law.

Longeot (1968) confirms that the learning of Cartesian products influences the development of the combinatorial scheme, especially in permutation problems. However, this acquisition is not generalized to operations in propositional logic.

In studies regarding the acquisition of permutation, researchers have identified three distinct stages (Larivée & Normandeau, 1985; Piaget & Inhelder, 1951). They are: 1) Intuitive resolution trials, characterized by attempts without a systematic procedure; 2) Beginning of generalization, involving partial systems with up to three elements; and, 3) Utilization of systematic strategies, where students leave one or two elements fixed while varying the rest, avoiding repetitions and ensuring the exhaustive use of the elements. Some students resort to the tree diagram. This stage, corresponding to the formal operational level, is typically achieved between 14 and 15 years of age.

In our sample, a notable discrepancy is observed in the success rates between CEB students, 50% - 60%, and CP students, between 10% and 20%. This outcome can be attributed to the influence of sociocultural factors. CP students only managed to generalize in 7% of cases (see Table 6). Some examples are transcribed below:

*Table 13. Item J) on Permutation: Examples of Responses and Analysis*

| | |
|---|---|
| J.<br>a) "You and three friends go to a restaurant; each one asks for a different dessert, so that everyone can taste the four (ice cream, cake, pancake, fruit). The waiter brings them all together. In what order do you taste them? Indicate all the possible orders using the initial letter to designate each dessert."<br><br>H T P F<br>H T F P<br>H P F T 6 * 4 = 24<br>H P T F<br>H F P T<br>H F T P<br>(CEB 1992 student).<br>(H = initial letter of ice cream in Spanish<br>T = initial letter of cake in Spanish) | The correct answer is:<br>HTPF – HTFP – HPTF – HPFT – HFTP - HFPT<br>THPF – THFP – TPHF – TPFH – TFHP – TFPH<br>PHTF – PHFT – PTHF – PTFH –PFHT – PFTH<br>FHTP – FHPT – FTHP – FTPH – FPHT – FPTH<br>24 possible combinations<br>b) 24: N! (factorial N: 4x3x2)<br><br>This answer demonstrates the use of a systematic strategy and the discovery of the underlying law. |
| b) "In a magical carousel with four figures (boat, horse, plane, and giraffe), they change places –sometimes all of them, sometimes one or two or more– every time they pass behind a screen. Could you say how many laps can it do without repeating the order in which the figures appear? Explain the method you used to solve it."<br>I solved the problem using the previous activity since there were also 4 objects; therefore, there are the same number of possible variations (CEB 1992 student). | Some students systematically and exhaustively solve exercise a). For exercise b), they anticipate the solution through mental calculation, providing a justification for the given solution that they anticipate. |

Other students correctly solve the first exercise. However, they fail to generalize. In the case of those that do not resolve the exercises:

*Table 14. Item J) on Permutation: Cases That Do Not Resolve the Exercise*

| | |
|---|---|
| It could be in order of personal preferences, agreeing among them; in alphabetical order F, H, P, T; anyone who wants can eat whatever she wants.' (CP 2022 student). | "Possible orders" are confused with subjective ordering criteria. |
| I think it would be 5 laps because if you pay for the carousel, it would not be many laps and not very few either (CEB 2022 student). | The order of the elements is confused with the sequence of actions, applying here an empirical criterion that shows a concrete operational level. |
| I don't know; carousels like that don't exist (CEB 2022 student). | This exemplifies concrete thought, where the student is unable to conceptualize the idea of "the possible" as his thinking is limited to factual reality. |

*c) Proportional Reasoning*

The proportional reasoning items (C and H) assess the ability to capture relationships of relationships, in one case, keeping constant one of the terms and, in the other case, varying both, which requires finding the common basis of comparison. The acquisition of this scheme serves as an indicator of the transition from the concrete to the formal level, signifying the ability to establish an equivalence relationship between two reasons (Obando et al., 2014).

The percentages of achievement in item C for CEB are around 70% without significant differences by cohort (1992 vs. 2022) and drop to 52% or 56% in item H. However, in CP, item C is solved correctly by 44% and 32% of the cases, according to the cohorts, and the H drops to 13% and 19%, respectively (see Table 6).

It is common to observe the use of percentages in the responses to item H, while, in item C, students compare the numbers of men and women to determine whether they are equal or not. Following are some sample responses.

*Table 15. Items C) and H) on Proportional Reasoning: Examples of Responses and Analysis*

| | |
|---|---|
| C) "The employees leave the factory at 6 pm. Thirty-one employees (22 men and 9 women) exit through the door on the right, while 27 (18 men and 9 women) exit through the door on the left. Is there a greater probability of seeing a woman leave first through either of the two exits? Yes or no? Why?" | |
| It doesn't matter because if I'm at the door on the right and she leaves through the one on the left, I'll have to run (CP 2022 student).<br>In the middle of the two doors because if I don't know through which door she is going to leave, the only place I can see her for sure is the middle (CEB 2022 student).<br>Fewer people leave through the door on the left, and a woman can be seen better that way (CP 2022 student). | These are examples of answers based on figurative criteria of an empirical-social nature. |
| If the man is a gentleman, the woman will go out first (CP 1992 student).<br>Women are going to exit faster through the door on the right, since, even though the same number of women leave, more men leave. Therefore, more people (CEB 1992 student). | In these cases, there is no proportionality scheme. |
| H) "Juan buys a number in a lottery that has 25 numbers, of which 5 have a prize (the other 20 are "losing numbers"). Silvia buys another number in a lottery of 10 numbers with 2 winners and 8 losers. Finally, Marcela buys a number in a lottery of 40 numbers, of which 8 have a prize and 32 are "losers". Which of them is more likely to have bought a winning number, and why?" | |
| Silvia has more chances because I like the numbers she has: 10 and 8 are winners (CP 2022 student). | Here, the student employs a "magical" criterion. |
| Silvia has more possibilities since she only has 10 numbers to choose from, and the others have many more, making it more difficult to find the exact winner (CP 1992 student).<br>All three of them have a chance of winning because it is a lottery (the numbers come out randomly). However, Silvia has a better chance since she plays in a 10-number lottery, even though there are only two winning numbers. It's a matter of luck (CEB 2022 student). | When a proportional scheme, which relates the parts to the whole or to each other, is not used, the global consideration of the quantities can be misleading. This is because it is assumed that where there are more elements, there are more possibilities (either against or in favor). |
| Everyone has the same chances since, in all cases, the numbers that have a prize represent 1/5 of the total that participates in the draw (CEB 1992 student). | Two distinct approaches emerge, one grounded in chance and the other in the calculation of probability and, consequently, in the use of the proportionality scheme. |

With respect to the student who employed a "magical" criterion, it is worth noting that he did not even comprehend each proposition in isolation. The text specifies not the purchase of numbers 10 and 8, but rather that within 10 numbers, two are winners and eight are losers. It is evident that any combination of two and eight numbers is possible; however, cultural convictions, linked to certain superstitions, are so strong that they exert a kind of "necessity" stronger than natural logic.

*d) Reciprocal or Biconditional Implication vs. Simple or Asymmetric Implication*

The reciprocal or biconditional implication (item D) is a compound proposition in which $p$ is a sufficient and necessary condition for $q$. It is true when both the antecedent and the consequent are true or both are false. Conversely, the conditional is false when one of the propositions is true and the other is false, or vice versa.

Simple or asymmetric implication (item E) represents a form of reasoning wherein, given a conditional statement, a valid inference occurs when the antecedent is affirmed or the consequent is denied. This item yields a difficulty index of 0.22 (very difficult), which aligns with the expected difficulty at the students' age.

The attainment of simple implication occurs as a late achievement. In all instances, the percentage of achievement in reciprocal implication, 84% and 89% in CEB, and 56% and 47% in CP, surpasses that of simple implication, 26% and 22% in CEB, and 8% and 23% in CP (see Table 6).

Among other responses to item D (reciprocal implication), the responses are interesting. Following are examples.

*Table 16. Item D) on Reciprocal or Biconditional Implication: Examples of Responses and Analysis*

| | |
|---|---|
| D) "Complete these arguments. If it is not possible in any case, put: it cannot be known.<br>I go to the cinema if and only if it rains:"<br>- It's raining'<br>Therefore... I go alone (CP 2022 student).<br>"- It does not rain"<br>Therefore... I'm going with someone (CP 2022 student) | This 1st student, who is at the preformal level, confuses the adverb "only" with the adjective "alone". |
| "- It's raining"<br>Therefore... I'm going to the movies (CP 1992 student).<br>"- It does not rain"<br>Therefore... it is not possible for me to know (CEB 2022 student). | Other students reduce reciprocal implication to simple implication. |
| "- It's raining"<br>That is why... "I stay at home watching TV" (CP 1992 student).<br>"- It does not rain"<br>That is why...I'm going out with a friend because the weather is nice (CP 2022 student). | Some students employ a socio-empirical criterion, so strong that it prevails over the most elementary logical evidence. |

In the case of item E (simple implication), a notable number of errors are observed. This stems from its reduction to reciprocal implication.

*Table 17. Item E) on Simple Implication: Examples of Responses and Analysis*

| | |
|---|---|
| E) "In class:<br>* If the Mathematics teacher gives a wrong indication, students are mistaken.<br>* If the students make a mistake, there will be no Mathematics prizes.<br>• We have observed that students make mistakes."<br><br>**Conclusions** –mark the correct(s) statement(s):<br>- There will be Mathematics prizes.<br>- *The teacher gave a wrong indication*.<br>- There will be no Mathematics prizes.<br>- The teacher did not give a wrong indication.<br>- It is not possible to know if the teacher gave a wrong indication.<br>(italics answer given by a CEB 2022 student) | The error in this student's answer (the statement in italics) could be interpreted as the interference of a preconception, stemming from a tendency to attribute errors to external causes. |

*e) Exclusive and Inclusive Disjunction*

The exclusive disjunction (item G) occurs in a proposition where there are two options, of which only one is true, and one excludes the other. The disjunction is false if both propositions are either true or false. On the other hand, the inclusive disjunction (item F) is a logical connection between two propositions, and is false only if both propositions are false; otherwise, the disjunction is true.

In our sample, the disjunction operation had a higher percentage of achievements in the case of the exclusive, and there were no differences between CEB and CP. However, there were differences by school in the inclusive disjunction, which, in the CP sample, had a relatively low proportion of resolution, between 4% and 8% (see Table 6).

According to the theory proposed by Johnson-Laird and Byrne (1991; Johnson-Laird, 2006), exclusive disjunction is easier because it requires fewer mental models. In our sample, the percentage of correct answers is 75% for the exclusive disjunction, but only 24% for the inclusive disjunction.

*Table 18. Items F) and G) on Inclusive and Exclusive Disjunction: Examples of Responses and Analysis*

| | |
|---|---|
| F) "Complete these arguments:<br>For dessert I will have fruit or ice cream (or both)<br>I eat fruit."<br><br>Therefore... I finished my lunch (CP 1992 student). | The words after "Therefore..." indicate the students' exact answers.<br>Some students do not attain the logical inference level required by the major premise. |
| G) "On Saturday, I will go to the cinema or to the theater (but not to both places). I went to the cinema."<br><br>Therefore... Today is Saturday (CP 2022 student). | |
| F) "I eat fruit."<br><br>Therefore... I liked it. (CEB 2022 student).<br><br>"I do not eat ice cream."<br><br>Therefore... I was looking forward to it (CEB 2022 student). | Some students rely on a socio-empirical or affective criterion. |
| G) "I went to the movies."<br><br>Therefore... I had fun. I didn't go to the theater. (CP 1992 student).<br><br>Therefore... I was tired (CP 1992 student). | |

*f) Modus Tollens*

The *modus tollens* is a rule of inference in propositional logic expressed as: "If P implies Q, and Q is not true, then P is not true". In our samples, item K evaluates this type of reasoning. Its difficulty index of 0.17 indicates that it is one of the most challenging in the test. In the 1992 CEB cohort, 40% of students successfully solved it, reflecting a commendable performance within the group. However, this success rate decreases to 17% in the 2022 cohort. In the 1992 CP cohort, the resolution rate was 14%, but it shows an improvement to 25% in the 2022 cohort (see Table 6).

## Discussion

In this research, the students' achievement in formal thinking skills during the initial stage of secondary school is compared over a span of thirty years. The study aims to investigate whether students' formal thinking skills vary based on epochal and sociocultural differences.

Concerning epochal differences, the results reveal divergent trends among CEB and CP students. In the case of CEB students, the 1992 cohort exhibited a superior performance in formal thinking compared to the 2022 students. Specifically, the 1992 cohort outperformed the 2022 cohort in the third level of Combinatorial Reasoning, *Modus tollens*, and Permutation. Conversely, the 2022 cohort demonstrated higher proficiency in Exclusive Disjunction compared to the 1992 cohort.

In the CP group, the 2022 students demonstrated a better performance in Simple Implication and Permutation compared to the 1992 cohort. Although the 2022 students ($X$ = 8.72, $SD$ = 6.09) exhibited an overall higher performance in formal thinking skills than the 1992 students ($X$ = 6.43, $SD$ = 6.78), these differences were not statistically significant.

It is noteworthy that there have been some improvements in formal thought within the CP group, contrasted with a slight decline in performance among the CEB participants. In the CEB case, the analysis of the protocols revealed errors stemming from a deficient interpretation of instructions and biases in attributional beliefs regarding failure. In the CP group, the observed improvement could be attributed to differences in family-of-origin characteristics between the two sub-samples. A higher percentage of parents of CP students in 2022 are professionals compared to the parents of students in 1992.

On the other hand, a cluster analysis was conducted to categorize secondary school students into one of three groups: preformal level, formal level A, and formal level B. For CEB students, it is observed that the majority reached formal thinking level A in both cohorts. However, in 1992, 28% of students reached formal level B, while none in the 2022 cohort achieved this level. These differences were found to be statistically significant ($\eta p^2 = .41$). In the case of CP students; the results indicate that the vast majority in both cohorts attained preformal thinking. Although CP 2022 students exhibited improvement in Formal A level and a decrease in the preformal level, these differences were not statistically significant.

Findings from previous research have been shown to align with the percentage distribution identified in both cohorts here. Bello (2014) has observed that only a small percentage, 15%, of 16-year-olds were capable of operating at the formal level. According to Bello, this underscores a misalignment between the curricular demands of secondary school and the cognitive abilities of students. In a study by Molina Lara and Rada Arteaga (2013) involving students aged 15 to 17, 22.4% exhibited concrete reasoning, 73.5% were in a transitional phase, and the remaining 4.1% demonstrated formal reasoning. Other authors (Hernández Suárez et al., 2013), in a sample of university students in their first year of the business administration career, have found that only 1.5% were at the formal level, 97% at the transitional level and 1.5% at the concrete thinking level. In a pedagogical intervention with secondary schools' students (Anaya García et al., 2019), 38% of the students were found to be at the transitional level, but only 3% reached the formal stage. Improvements there were verified in the resolution of proportionality problems, while the lowest achievements corresponded to combinatorial problems.

On the other hand, regarding the consideration of the formal stage, as defined by Piaget, some authors have questioned the importance of the relationship between the achievements of this stage and academic performance in curricular areas that do not belong to the fields of Mathematics and Physics. Kaya and Çıkış (2017), among others, have considered that, in the area of Drawing, student performance is more significantly related to the skills involved with creativity and have proposed to promote post-formal thinking, characterized by a dialectical type of thinking, particularly in discovering and solving problems. The approach they propose is characterized by the use of different perspectives of approach, with a holistic viewpoint.

In this present research, students' formal thinking performance was compared by sociocultural levels. In both 1992 and 2022, CEB students exhibited a higher total score in formal thinking performance compared to CP students ($\eta p^2 = .41$). For a comprehensive understanding of the data, a content analysis was carried out on the students' response protocols of the LTP-A, in order to detect resolution strategies, some conceptual categories and types of errors. The results obtained are discussed in the following subtests.

*a) Combinatorial Reasoning*

Combinatorial reasoning encompasses various levels, with the initial stage involving simple arrangements. Previous studies have documented systematic achievements in simple arrangements among children as young as 6 years old (e.g., English, 2005; Scardamalia, 1977). In both cohorts, 1992 and 2022, the CEB students achieved a higher percentage of success in the first and second levels of combinatorial ability compared to the CP samples. Since the ages and education levels are the same in both schools, these differences can be attributed to sociocultural factors.

The most difficult challenge arises in the third level of combinatorial reasoning (item I). In our samples, CEB students from the 1992 cohort exhibited the highest percentage of correct solutions, 26%, at this level. In the remaining sub-samples, the percentage drops to a range between 8% and 11%. All students who achieved a complete solution belonged to Formal Level B. These differences could be interpreted as influenced by epochal and sociocultural differences.

Combinatorial reasoning is a requisite for solving probability problems (Navarro-Pelayo et al., 1996), and functions as a crucial component of formal thought as it involves manipulating all possibilities. Consistent with our findings, it has been emphasized that acquiring this complex skill necessitates sustained instruction over an extended period (Lawson, 1995).

*b) Permutation*

Regarding Permutation, which extends the first level of combinatorial reasoning by involving the arrangement of four elements taken in fours, our findings reveal a notable discrepancy in success rates between CEB and CP students. CEB students demonstrated success rates between 50% and 60%, whereas CP students only generalized in 7% of cases. This discrepancy suggests that sociocultural factors may significantly influence permutation understanding and application among different student groups. Integrating these findings into educational practices could involve targeted interventions to enhance permutation skills among students, especially from lower sociocultural levels.

Among other studies, Arteaga Cezón et al. (2017) have analyzed the solution to permutation problems in a sample from the first three compulsory secondary education courses. They have found that the majority of students do not use formulas; instead, the strategies they apply are based on graphic schemes. Lamanna et al. (2022), in a sample of Italian middle school students, have found that more errors occur when students use a non-systematic enumeration strategy. In our research, we also find this type of strategy, which leads to failures in solving the task. Lamanna et al. presented to the subjects, among other problems, one related to a task of distributing four elements –similar to our item J– that is successfully solved when students apply the Cartesian product formula (4! = 24). In their sample, 44% of students without prior instruction solved the problem. Thus, permutation is simpler than second and third-level combinatorics, as verified in the results of our research. According to Lamanna et al.'s results, the most difficult permutation problem is the one with four elements chosen in groups of three.

Asiala et al. (1998) have approached the topic of the development of the permutation schema from another perspective. The authors have considered that the mental construction of the schema involves understanding permutation as an action, on a set of objects, as a process, or as both possibilities. This latter conception implies what Piaget (1977) calls the internalization of the action, with the subsequent reflecting abstraction, which would lead to the successful resolution of permutation tasks.

*c) Proportional Reasoning*

Proportional reasoning is assessed through two items (C and H), which evaluate the ability to comprehend relationships of relationships. In one case, the task involves maintaining one term constant, while, in the other, both terms vary, requiring the identification of a common basis for comparison. The acquisition of this reasoning scheme is an indicator of the transition from the concrete to the formal level. Notably, this type of reasoning is not only essential for mathematics learning but also plays a pivotal role in a diverse range of curricular subjects, spanning from Physics to Social Sciences.

Longeot (1968), in a study involving a sample from the first two years of middle school, has found that the acquisition of this skill occurs later than the levels of combinatorial reasoning but precedes the mastery of simple implication and permutation. This sequence is verified in our samples.

Both the CEB and CP samples exhibited a higher percentage of success in item C compared to item H in both 1992 and 2022, but CEB students demonstrated superior performance, achieving a higher percentage of correct responses in both items compared to their CP counterparts. These results indicate, on the one hand, a consistent difference in difficulty between the two items: in item C, there is an inequality (9/22 is less than 9/18), while in item H (related to John buying a lottery ticket), there is a proportion with a relation of 1/5. On the other hand, the performance difference between the two schools remains constant. This is once again attributed to the influence of sociocultural factors.

In instances of incorrect responses, a bias is evident, a phenomenon that has been corroborated by other authors (Batanero & Borovcnik, 2016; Cañizares & Batanero, 1997). Instead of employing reasoning grounded in relationships between variables, some students rely on subjective reasons. Hence, it is important to present students with reasoning tasks using everyday situations and guide their resolution through strategies that integrate formal reasoning rules. Bronkhorst et al. (2020) have defined this type of formal reasoning as: "...selecting and interpreting information from a given context, making connections, verifying and drawing conclusions based on interpreted information and the associated rules and processes." (p. 1,676).

*d) Implication*

Two items (D and E) assess implication. Item D evaluates reciprocal or biconditional implication, while item E assesses simple or asymmetric implication.

In each cohort, the percentage of achievement in reciprocal implication exceeds that in simple implication. Our findings align with Longeot (1968), who observed that this level of achievement is characteristic of the final stage of formal reasoning.

In a seminal paper addressing this question, Wason (1968) underscored the challenges observed in a sample of adolescents when grappling with the concept of implication and emphasized the impact of verbal formulation. The formulation *If p, then q*, implying a temporal sequence, is deemed less challenging than *q if p*. Simpler forms, such as *or not -p or q* and *it is not the case that p and not -q*, present alternative, more accessible expressions.

Longeot (1968) has pointed out that these schemes are not achieved at the ages of 14-15, but would be reached, though not entirely, between the ages of 16-17. A common error involves inferring the antecedent from "putting the consequent", whereas, in reality, no valid conclusion can be drawn from it except through its negation. Nevertheless, it is necessary to identify more transparent evaluation methods, as the analysis of responses in the simple, asymmetric implication items suggests that many formally correct answers might mask a reduction of all implications to the type of reciprocal implication.

*e) Disjunction*

Two items here evaluate disjunction. Item G assesses exclusive disjunction, while item F evaluates inclusive disjunction. In our four sub-samples, exclusive disjunction demonstrates a higher percentage of achievements, while differences among schools emerge in the case of inclusive disjunction. In this regard, CP cohorts exhibit a notably low-resolution rate.

Certainly, a distinction must be made between the weak "or" (the *sive* of classical logic) and the strong "or" (*aut*). Ennis (1976) has considered the former psychologically ambiguous, in opposition to the more general interpretation that, when nothing else is stated, the "or" should be interpreted as inclusive. For example, when one says: "I will go to the cinema or the theater", everyday "logic" interprets it as exclusive. If the mother says: "Either you study or you won't go out", the exclusive meaning is obvious.

Another reason justifying the difference in difficulty between exclusive and inclusive disjunction is that, while in item G there are two options, in item F it is necessary to consider three possibilities: either eat fruit, or eat ice cream, or both. According to Johnson Laird and Byrne (1991; see also Johnson-Laird, 2006), exclusive disjunction is easier because it requires fewer mental models. Roberge and Flexer (1979) reported that subjects of the same age as those in our sample easily made inferences involving exclusive disjunction but encountered significant difficulties with conditional rules.

Regarding the dimension of principles, we have identified parallels with Ennis (1976). Notably, one of the most prevalent fallacies involves interpreting the simple conditional as a biconditional. Ennis (1976) pointed out the serious practical implications of this error and argued that the biconditional interpretation is incorrect unless the context implies otherwise.

Nevertheless, we find this postulate less clear because, in everyday language, the interpretation of the simple conditional is often compelled. For example, if I say: "If it rains, I will take an umbrella," it is difficult to conceive that this implies: "Rain is one of the circumstances that cause me to go out with an umbrella."

According to Martín and Valiña (2002), research on disjunctive reasoning has suggested the polysemic nature of disjunctive expressions in natural language, where both content and context play significant roles. This phenomenon is evident in the examples from our samples, as well as in previous research (Martín & Valiña, 2003; Valiña & Martín, 2021).

For those reasons, we contend that instruction aimed at fostering logical competence should employ broad and meaningful contexts, as they make it easier to distinguish between the form and content of an argument. In this regard, Mayer (1983), García-Madruga (2019), and Bronkhorst et al. (2020) have suggested that logical performance increases when a real-world context is used. In the same sense, Widodo et al. (2019) have suggested that teaching interventions should identify the strategies used by students and promote the integration of prior knowledge with new knowledge.

*f) Modus Tollens*

*Modus tollens* proves to be the most challenging item on the test. The CEB cohort exhibits significant epochal differences, favoring the 1992 cohort. In contrast, the CP cohort shows a marginal increase in *Modus tollens* performance in 2022.

In summary, the content analysis confirms the findings of the quantitative analysis. Firstly, it documents the sequential and hierarchical development of formal thinking skills in adolescents, irrespective of the context of belonging. Secondly, it substantiates sociocultural disparities in formal reasoning performance.

Additionally, the content analysis uncovers specific categories influenced by individual or sociocultural biases. Among individual biases, empirical criteria surface, rooted in personal tastes or preferences; "magical criteria" linked to non-rational beliefs, such as luck or figurative elements, characteristic of concrete thinking. Attribution styles also interfere with reasoning. Successes in the test illustrate the application of logical schemes, aligning with Piagetian theory. Among sociocultural biases, cultural biases manifest in the application of socio-empirical criteria, grounded in group practices and customs, persisting even when contradictory to logical principles.

## Conclusion

Multiple studies affirm that formal thinking significantly influences academic performance. Therefore, it is pertinent to investigate the factors that contribute to its development. This study specifically delves into the potential influence stemming from epochal and sociocultural factors.

The results suggest that the sociocultural factor carries more weight than the one associated with epochal differences. In the case of the CP samples, this sociocultural factor would be mediated by the influence of the family context since, over the past decade, a significant percentage of professionals has been noted within the parent group.

In both CEB samples, the interpretation becomes more intricate. Through content analysis of the responses, it is observed that, within the 2022 cohort, many students made errors stemming from difficulties in understanding problem statements and lapses in attention during answer composition. Additionally, some errors were associated with false attributional beliefs of failure. In essence, it is not merely a matter of a lower level of formal operational thought. They solve problems that are more difficult while encountering errors in simpler ones. This phenomenon could be interpreted in connection with epochal factors, such as an emphasis on processing speed over profound reflection, thereby affecting attention span. However, these types of errors were not identified among the CP subjects, a discrepancy that may be linked to the mediating influence of their environment. CP students reside in a small city within a more traditional state, whereas the CEB group hails from the most populous metropolis in the country.

Regarding the role of human nature, it is noted that the sequence of items' difficulty remains consistent across various subsamples and over time. This observation suggests a discernible pattern of the evolution of formal thought associated with inherent regularities in the nature of intelligence. While the research also identifies the influence of the sociocultural context, this aspect is interpreted as the impact of the family on students' cognitive development.

The question of changes in logical performance due to cultural shifts across epochs is tied to the scrutiny of a universal epistemic subject, a critique that has been raised against Piagetian theory on the acquisition of formal thought (Chapman, 1988; Lourenço, 2016). In the results of our empirical research, there are not sufficient grounds to challenge this theory. This, however, does not imply that psycho-cognitive evolution is exempt from cultural changes. Rather, it allows us to discover that there is a potentiality of human nature that is maintained.

Nevertheless, nature, being the capacities that constitute the essence of each reality, does not operate in isolation from factors that condition it. In other words, psycho-cognitive development is not a purely sociocultural construct, nor is it detached from its context. While the context clearly influences, it does not deterministically govern this developmental process.

In conclusion, the results pertaining to the acquisition of formal thinking underscore the necessity to cultivate logical competence by engaging in the development of diverse operational schemes. This involves commencing with meaningful contexts that facilitate the differentiation between the form and the content in an argumentation process.

## Recommendations

In the development of logical thinking, the schooling process plays a fundamental role along with family factors. The family, indeed, is the natural environment both for the early development of a human being and for their continuous support throughout childhood and adolescence.

From a pedagogical perspective, this implies, on the one hand, the necessity to promote school-family interrelation, enabling students to cultivate intellectual habits, particularly in the realm of argumentation. On the other hand, it involves preventing potential biases in students' reasoning and offering opportunities for them to recognize proper argumentative methods. This can be achieved through proposing exercises contextualized within disciplinary content. Moreover, teachers may suggest the analysis of social discourses, especially those that, through references to values, might be more influenced by previously adopted positions, or by the ideological presentation of the problem, or by both.

Future research could undertake longitudinal studies on the acquisition of formal thinking during secondary school, particularly given the absence of identified studies of this nature for benchmarking our findings. Exploring the longitudinal effects of specific educational interventions and investigating the influence of sociocultural factors, such as family background and peer interactions, on the progression of formal thought processes could provide valuable insights. Additionally, further studies might delve into the role of technology in enhancing formal reasoning skills and explore its implications for contemporary education.

## Limitations

Regarding the limitations of this study, a small and intentional sample was employed. This makes it challenging to generalize our results to other similar samples. Additionally, a more rigorous assessment of the sociocultural variable may be necessary.

### Ethics Statements

The participants provided informed consent by voluntarily choosing to participate in this study. The confidentiality of the obtained information was respected.

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### Authorship Contribution Statement

Vázquez: Conceptualization, design, data acquisition, data analysis/interpretation, writing. Noriega-Biggio: further statistical analysis/interpretation. Difabio-de-Anglat: Editing/reviewing.

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