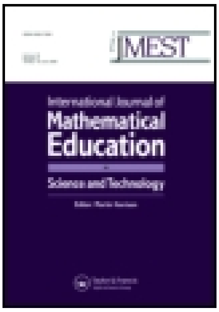

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This paper describes an action-research experience carried out with second year students at the School of Engineering of the National University of Entre Ríos, Argentina. Vector calculus students played an active role in their own learning process. They were required to present weekly reports, in both oral and written forms, on the topics studied, instead of merely sitting and watching as the teacher solved problems on the blackboard. The students were also asked to perform computer assignments, and their learning process was continuously monitored. Among many benefits, this methodology has allowed students and teachers to identify errors and misconceptions that might have gone unnoticed under a more passive approach.

Keywords: engineering mathematics education; error detection; action-research

1. Introduction

There exists a general consensus that pre-grad education in engineering and, in particular, bioengineering, must not only provide students with a solid training, but also help them develop skills and abilities that will allow them to be permanently up to date, engage in critical thinking, evaluate the information received, recognize when their training is obsolete and make decisions regarding their continuous education. To this end, it is necessary to organize learning so as to stimulate the development of such skills. For this reason, it is considered that the mathematics courses in the syllabus have the aim of not only providing the necessary skills to tackle specific subjects, but also of fostering the development of cognitive and metacognitive abilities that allow our students to continuously learn and deal with problem solving with ingenuity, creativity and social responsibility.

Thus, the need arises to change roles, and a challenge is presented: How can we turn into 'guiding' teachers and 'learning facilitators'? How can we give the student a space for leaving his role as a passive subject and becoming the centre of teaching–learning processes?

The challenge of finding answers to these questions was undertaken by teachers of the mathematics courses from the second year of the bioengineering course of study at the National University of Entre Ríos' School of Engineering. In these courses, concepts and methods from two branches of applied mathematics (vector calculus and differential equations) are dealt with. Acknowledging the inherent complexity of teaching–learning processes, this teaching team has deepened its analysis of the situation starting an

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action-research (A-R) process. An interdisciplinary space was born in which professionals from the mathematics, engineering, language and education sciences fields reflect and exchange knowledge with the ultimate goal of improving the practice of education.

2. Theoretical framework

The theoretical framework used includes inputs from the areas of general teaching, mathematical training, cognitive and cultural psychology and linguistic approaches that propose the teaching of reading and writing across the curriculum.

The process of defining a reference framework started by specifying the abilities that the students needed to strengthen. These are listed in Table 1.

The relationship between general skills to be developed and the different disciplines that offer theories and methods to achieve them is shown in Figure 1.

In the first place, problem resolution and design are two specific engineering activities so the development of the respective abilities constitutes a central point of attention when rethinking the teaching of engineering.[1] The problem-solving approach to mathematics teaching gives rise to a reflective, creative attitude on the students' part if developed with an appropriate methodology. Schoenfeld [2] describes different categories of knowledge and behaviour that are involved in the mathematical activity of problem solving: the base knowledge (resources), problem-solving strategies (heuristics), metacognitive aspects (the knowledge of one's own thinking process or self-regulation during problem solving), and affective aspects.

The problem-solving theoretical framework, according to this author, is based on numerous results provided by cognitive psychology, as well as on cultural aspects. That is, learning is culturally modelled: people develop their understanding of any activity based on their participation in what has been called the 'practice community' in which that activity is

Table 1. Abilities that need to be enhanced in mathematics courses for future bioengineers.

Problem solving: Searching for information, selecting and classifying data, enunciating hypotheses, designing strategies, constructing and using mathematical models, evaluating different mathematical procedures, making decisions, applying mathematical concepts and properties, conjecturing solutions, estimating results, applying algorithms, calculating, using computer technology, validating solutions considering both the mathematical and the contextual angles, analysing alternative solutions, others.
Thinking and reasoning: Following chains of mathematical arguments of different kinds, justifying the steps in a proof, evaluating an argument proving it is true or false, understanding the different mechanisms used for proofs (direct, by *reductio ad absurdum*, counter reciprocal), using counter examples to rebut a mathematical proposition, among others.
Autonomous learning: Taking conscious control of learning, planning and selecting strategies, self-regulating and self-evaluating the process.
Working with others, communicating processes and results: Expressing concepts, procedures and results using mathematical language in both oral and written forms, understanding its relationship with natural language, translating from natural to symbolic language, understanding the meaning of mathematical expressions, manipulating symbolic and numerical expressions, using and interpreting a mathematical concept or object using different representations (graphical, symbolic, numerical), conveying mathematical ideas with computer support.

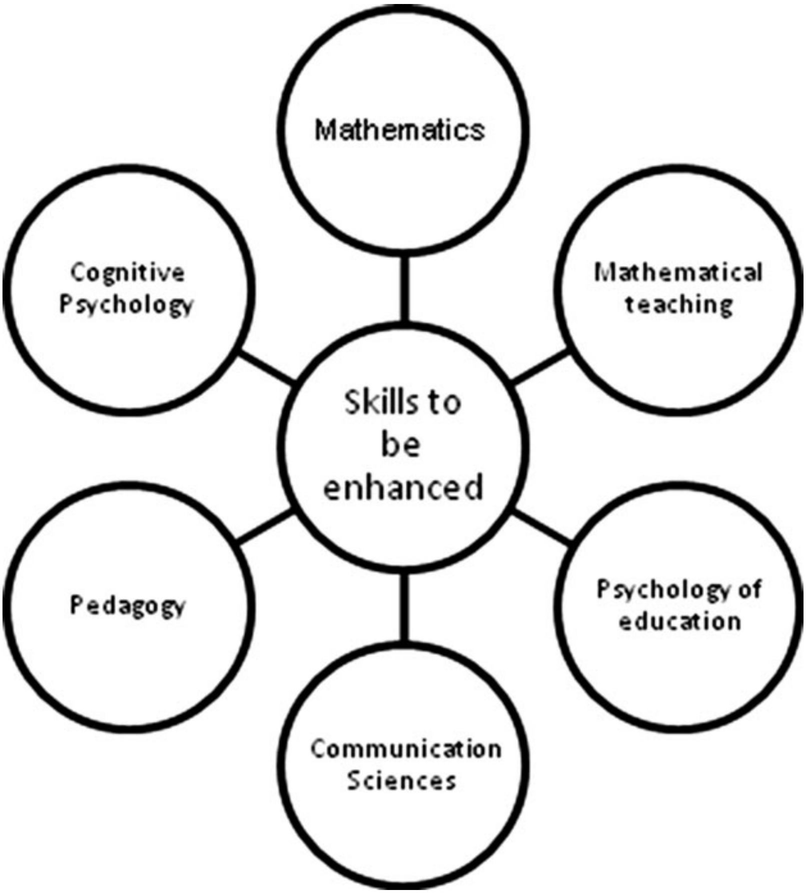


Figure 1. Perspectives from different disciplines to be considered in order to design teaching strategies for developing the skills described in Table 1.

carried out. From a socio-cultural perspective of learning, social interaction and discourse are basic elements to develop higher cognitive processes. Learning is a social phenomenon in which students acquire the necessary elements to learn through interaction with peers, teachers and educational material. Moreover, Vigotsky [3] highlights the concept of a zone of proximal development determined by 'the distance between the development level, defined by the ability to independently solve a problem, and the potential development level, determined by the ability to solve a problem under the guidance of an adult or in collaboration with a more skilled classmate.'

In adopting our theoretical framework, we were especially concerned about how students can reflect on their own learning so as to improve it, and about the best way to encourage such reflection. Cognitive psychology provided us with the concept of metacognitive knowledge, which refers to the knowledge of how we learn, reason and remember. The word metacognition literally means 'cognition about cognition' or 'cognition about knowledge.' The term was introduced by John Flavell [4] in the early 1970s based on his research on the development of memory processes. It has been posited [5] that the educator can promote students' awareness of their own learning process by incorporating metacognitive activities, that is, activities that generate a control of one's own knowledge, of one's own comprehension. As to how this can be achieved best, we were particularly convinced by the many reports pointing to the epistemic potential of writing [6–8] and reading.[9–12] Dealing with university students' reading and writing skills in each course is necessary because both are tools involved in the comprehension and elaboration of knowledge; they are learning 'strategies' and, as such, they must be acquired with the help and guidance of teachers from the specific discipline.[13] We decided that reading/writing activities (such as interpreting more or less complex formulations, or writing reports) would be used as tools with a metacognitive content as part of our methodological framework.

3. Methodological framework

Our group adopted A-R as its methodology. A-R can be defined as the study of a situation (in this case the teaching and learning of mathematics in the second year of the bioengineering course of study) to try and improve action within the community involved (in this case, teachers and students).[14–18]

The term A-R was coined by the social psychologist Kurt Lewin in 1947. The methodology Lewin developed did not focus specifically on education, but on social processes in general. During the 1970s, British and Australian researchers proposed an alternative paradigm of education research that retakes Lewin's model. Both John Elliot and Lawrence Stenhouse in Britain and Stephen Kemmis in Australia sought to develop new ways of producing knowledge about teaching–learning processes and about curriculum development and implementation.[19] In Latin America the research lines developed by Paulo Freire, Orlando Fals Borda and João Bosco Pinto stress popular education and non-formal education.[20] Currently A-R as a movement occupies a relevant place in countries like Britain, Australia, the United States, Canada, Spain, Germany, Austria, Brazil, Colombia, El Salvador and Argentina, among others. Different work lines have been developed with various shades. According to Elliott,[14] A-R focuses on situations (in which teachers are involved) that are problematic and require a practical response. Kemmis and McTaggart [18] claim that A-R does not aim to design teaching procedures but to propose action procedures that help overcome problems, making decisions that affect the professional practice itself. It is what is usually called an active research, aimed at improving the quality of education. In this methodology, the participation of those involved in the problem to investigate becomes the basic articulating axis. From this viewpoint, each teacher is a researcher. Also, the methodology allows a problem to be dealt within a given context, by virtue of which knowledge about the environment in which the research is conducted increases during the experience.

A-R is conducted through a continuous, spiralling process, in which the problem is defined, actions are planned and implemented, results obtained are observed and analysed and the whole experience is reflected about with the aim of starting a new cycle in which it is sought to improve certain unsatisfactory aspects of the previous cycle, or to solve a new problem that arose during the experience. This process is illustrated in Figure 2.

In a study of seven different teaching methodologies, Case and Light [21] highlight Jørgensen and Kofoed's [22] description of A-R as a particularly useful tool for providing engineering students with knowledge about innovation. The formal incorporation of courses in A-R into teacher-training programmes, however, may not guarantee the actual use of the methodology in practice.[23] In this sense, Somekha and Zeichnerb [24] describe various ways in which use of A-R can arise in a given context. Our adoption thereof follows the pattern of a university-born initiative.

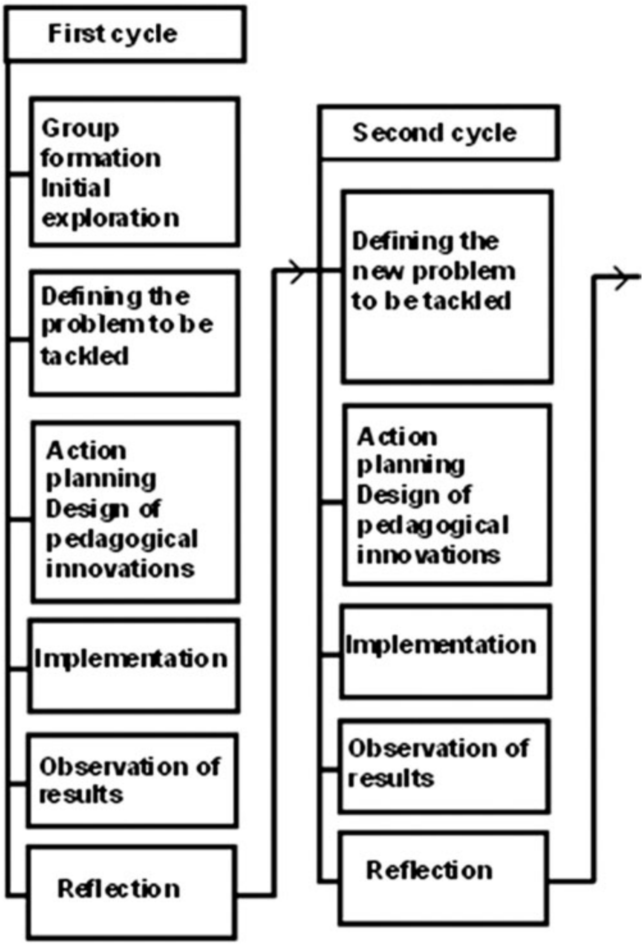


Figure 2. The action-research process is conducted through continuous cycles of action, result analysis, reflection on the latter that gives rise to a new cycle.

4. Our implementation of A-R

In our case, the community is composed of second-year students of the bioengineering course of study that take vector calculus courses; teachers of these subjects with diverse graduate and postgraduate trainings (engineers from different fields); the school's pedagogical adviser; and teachers from the humanities and language department. The latter are specialists in education science and languages. They also have or have had contact with second-year students through the counselling workshops for freshest given to first-year students, the language courses, the reading comprehension and written production courses. It is because of this link with the students that they are members of the group and not 'external advisers.'

The community is also composed of teaching assistants (advanced students and recent graduates), and peer tutors, who are students from the course of study who have been trained in the fields of pedagogy and psychology. This training is provided through workshops organized and coordinated by the vocational orientation and pedagogic counselling area.

4.1. The initial diagnosis and strategy design

Teachers of mathematics at the School of Engineering of the National University of Entre Ríos had observed that the students' previous training was inadequate. Also, the students did not keep a regular studying pace, which prevented them from fully benefitting from the classes.

Their participation in the classroom was scarce. They were perceived as passive as there was no partner for the dialogue proposed by the educators.

Teachers in problem-solving classes stated that oftentimes groups of students attended the class without the basic textbook. They were not used to reading a mathematics book. They preferred to use class notes, often prepared by their classmates, without ascertaining their accuracy.

They had trouble translating everyday language into symbolic mathematical language, as well as doing three-dimensional (3D) graphical representations. They did not base their deductions on theorems or properties, and also they did not specify the steps taken in solving a problem, or verify the solution attained.

This analysis allowed us to detect mathematics teaching–learning issues that needed to be addressed. These were categorized into: responsible participation of the student in their own learning process; early error detection; development of communicational skills; and strengthening of general mathematical skills.

With these categories in mind, and considering aspects related to a bioengineer's expected professional profile, we determined the student skills or abilities that needed to be developed or improved.

Thus arose new A-R axes. We picked up McNiff's [25] conception of A-R, but adapted it to our reality through a nest-like interconnection of the axes. The central axis gave rise to: (1) an axis focusing on the students achieving a higher measure of participation and academic autonomy; (2) an axis focusing on fostering the development of problem-solving skills; (3) an axis focusing on early error detection and the design of specific teaching strategies to improve the acquisition of those concepts or methods in which the error arises frequently.

These are intertwined in such a way as to allow a two-way communication, as indicated in Figure 3. A-R develops, thus, features dependent on the local context.

4.2. Actions implemented

In the mentioned A-R framework, various actions have been implemented. These actions might at first appear to be just small changes, but they have all been useful to generate and feed a dynamic, participative process of reflection and improvement of our teaching practice. We might mention as the starting point the joint elaboration of the theoretical framework and the study of the research methodology adopted. After that, several actions were designed and implemented in the vector calculus courses, such as:

- A *weekly report* the students are required to present on three assigned problems that includes a metacognitive activity.
- *Computer lab assignments* that incorporate interdisciplinary aspects.
- *Peer tutorship* in problem-solving classes.

All three are described below.

(1) The weekly report on three assigned problems

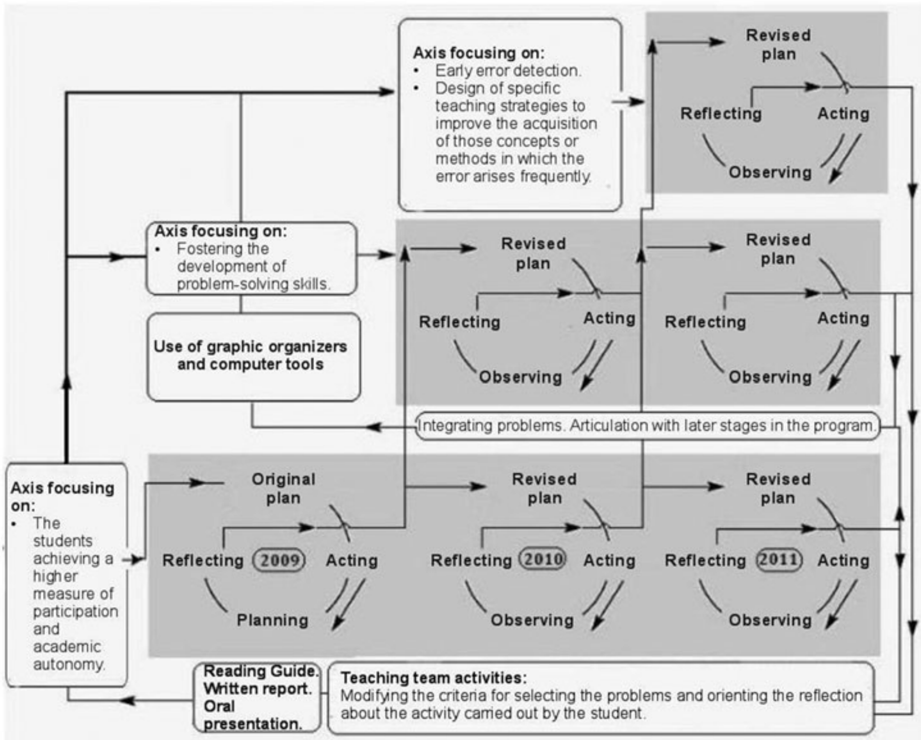


Figure 3. Articulation of the different action-research axes: (I) autonomy, self-regulation and communication; (II) problem-solving skills; (III) early error detection and specific teaching strategies.

The activity consists of completing a weekly group assignment, writing a report on it and orally defending the report in class. Each three-member group solves three problems assigned by the professor. Emphasis is placed on the students attempting to solve the problems and being able to explain their reasoning, rather than on the accuracy of the results obtained. The students are expected to describe not only the information used, the concepts required, the methods selected, the justification for such selection, the description of the algorithms used, the results and conclusions drawn, but also their reflections on the work done and the difficulties they met. So as to facilitate the latter activity, the teachers propose a questionnaire that is answered by each group at the end of each report. This list of questions has suffered changes along the A-R process which have arisen from the experiences conducted.

A few of the questions devised and the different categories in which they have been classified are presented below.

Visualizing the interrelationship between concepts: What new concepts or properties have I used? What concepts from previous courses have I applied? What new problems can I solve with concepts from this unit that I could not have tackled before?

Observing the process itself: What strategies have I used to understand each problem formulation? Which ones for planning the path to the solution? Did I meet difficulties? Which ones? How did I try and solve them?

Self-evaluating the solution process: Did I review theoretical concepts from previous courses? Did I draw a diagram explaining the successive stages of the solution process?

Did I justify each step taken? Did I verify the intermediate results? Did I discuss alternative methods for any stage of the process or for the general procedure?

> *Detecting doubts:* Did I understand the concepts I used? Was I able to apply them? What doubts are left unanswered? In which topic do I need a new explanation from my teacher?

The objective of this task, included since 2008, is to develop the students' reflective attitude, to boost metacognition, to favour an exercise in self-evaluation and to generate a higher degree of learning autonomy. Reports are discussed and analysed in groups in practical (i.e. problem-solving) classes, in which the teacher acts as a coordinator. Also, this activity allows teachers to better know the students' learning process, identify the most frequent doubts or errors, and act consequently.

(2) Computer-lab assignments

In the computer-lab assignment, bioengineering-related problems are given to be solved through the computational tool. It is sought to take advantage of the numerical, graphical, symbolic and calculational capabilities of the mathematical software available.

These assignments are designed in such a way that they require the use of general mathematical procedures that are an integral part of an engineer's training, such as modelling, simulating, plotting, calculating, comparing, algorithmizing, solving, interpreting; but also the application of definitions, the enunciation of hypotheses, the rebuttal of ideas, permanently linking theory and practice with bioengineering applications suitable for a second-year student.

The first of these assignments has the set goal of acquiring a good command of the mathematical software, so that in the experiences that follow the students can tackle problems that require a holistic approach with greater ease.

Students are free to take as much time as they need to complete the assignment in three-member groups, the only restriction being a deadline that coincides with the midterm examination covering the topics involved. Thus, completing the assignment contributes to learning the contents that will be evaluated.

(3) Incorporation of tutors to the practical (i.e. problem-solving) classes

Tutors are advanced bioengineering students who, starting in 2010, have been incorporated to the practical (i.e. problem-solving) classes with the aim of helping teachers in the personalized monitoring of the students' learning process. They are different from teaching assistants in that they do not conduct classes but provide support to students experiencing difficulties, such as a longer learning time than their peers, communication difficulties with their classmates and teachers, problems working in a team, uneasiness with oral presentations, inadequate learning strategies and lack of certain basic abilities to tackle a problem. For instance, a concrete task conducted by the tutors is to accompany and orient students in metacognitive and reflective activities included in the previously described weekly reports. Thus, the tutors' activity complements that of the teachers.

The implementation of these activities (i.e. the design of written assignments and lab reports and the way classroom discussions were conducted) was thus successively modified in agreement with the reflection–action cycles depicted in Figure 3.

5. Error detection

One of the most valuable aspects of the methodology used is that it helps the students to become aware of their misconceptions and procedural errors at an early stage when studying a topic. We shall illustrate this with three examples from the vector calculus course.

(1) Let $F(x, y) = \vec{i} + x^2\vec{j}$ be a vector field representing a force.

Plot the vector field on the x axis, the y axis, the $x = 1$ line and the $y = 1$ line.

Let C be the closed, counterclockwise-oriented trajectory defined by the sides of the square: $(0, 0), (1, 0), (1, 1), (0, 1)$. Plot the trajectory. Observe the graph and without making any calculations indicate whether the work performed by the force over the curve C is positive, negative or zero. Justify your answer.

Is $F(x, y)$ conservative? Answer using two different criteria.

Essentially, this problem asks the student to make a conjecture about the work performed by a force that has a constant x-component and an x-dependent y-component, along the sides of a 1×1 square with a corner at the origin (see Figure 4). The amounts of work performed along the upper and lower sides cancel each other, but those along the left and right sides do not, because the y-component is x-dependent.

The unexpected answer from most student groups was that the work was zero because the trajectory was closed. After analysis, we concluded that this stems from two facts: (1) a theorem exists about the work performed by a conservative force along a closed curve totalling zero; (2) many textbook exercises recommended to the students illustrated this theorem, while few, if any, illustrated the different behaviours of conservative and non-conservative vector fields, leading students to think that work performed by all forces along all closed curves is zero.

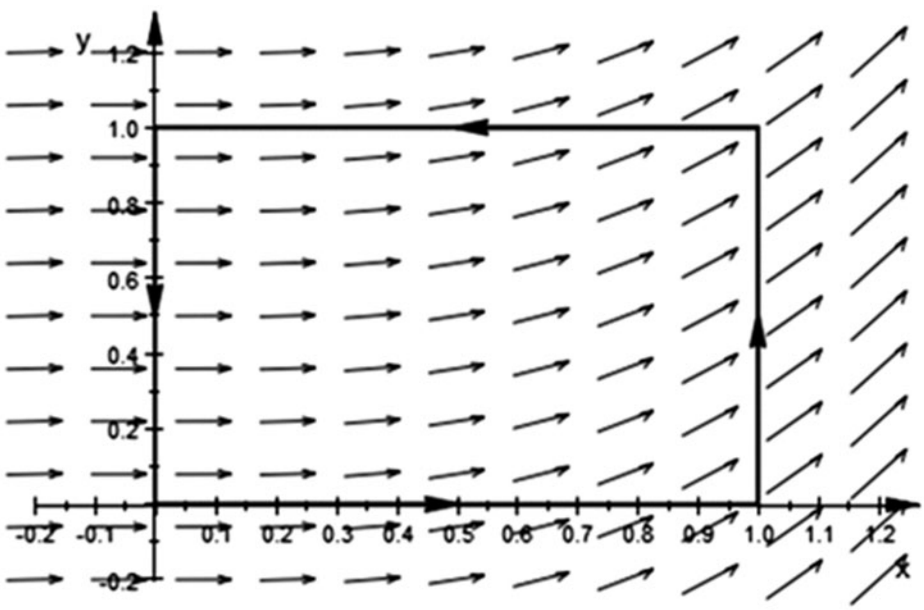


Figure 4. Representation of force and trajectory.

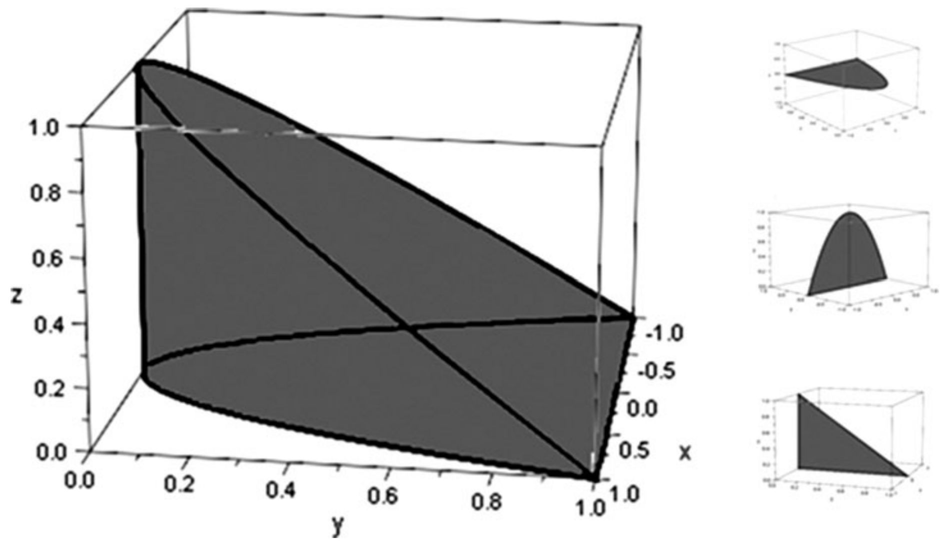


Figure 5. Parabolic wedge and its intersection with coordinate planes. Intersection with *xy*-plane (top), *xz*-plane (middle) and *yz*-plane (bottom).

Thus, the input received from the students' oral presentation of their reports gave us an invaluable opportunity to set the record straight before the misconception settled.

(2) The volume V of a solid E can be expressed through the triple integral.

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dzdydx$$

Is E defined as a Type 1 solid? Justify your answer.
Plot E. Explain what information you used to do the graph.
Express E in all possible ways using a rectangular coordinate system.
Express V by means of triple integrals; indicate all possible forms using rectangular coordinates.

The integral describes a parabolic wedge (Figure 5). When changing the order of the differentials, a majority of the students failed to obtain the forms:

$$\int_{-1}^{1}\int_{0}^{1-x^2}\int_{x^2}^{1-z} dydzdx \wedge \int_{0}^{1}\int_{0}^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dzdydx$$

After discussing their failed attempts to write both these integrals, it became clear they were puzzled by two facts: (1) the different mathematical forms one and the same boundary surface adopts according to which coordinate plane we project the region onto; (2) the fact that a 'round' boundary surface (in this case, the parabolic side) can be projected as a 'straight' region (in this case a triangle) on a certain coordinate plane. As in the previous example, it was only through the oral presentation that the difficulty became apparent, leading the teachers to a more intuitive, if less academic, approach to presenting

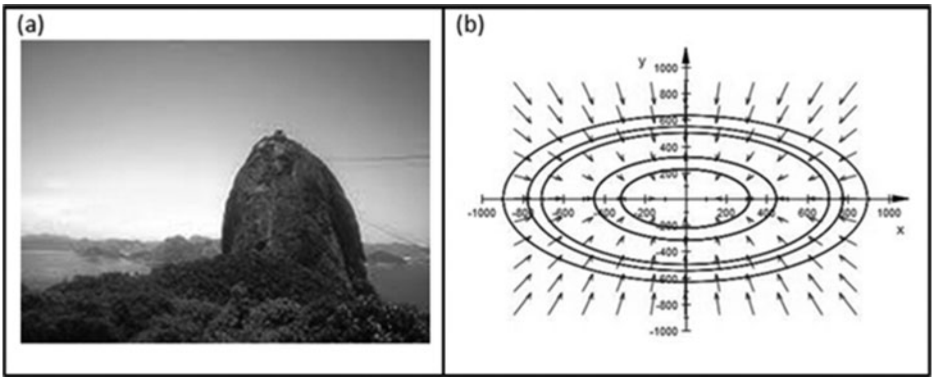


Figure 6. (a) Rio de Janeiro's Pão de Açúcar and (b) level curves with gradient vector of the mathematic model.

the problem. ('Think of this wedge as a piece of cheese. Now let's stick a knitting needle into it. Where does the needle get into the cheese, and where does it come out of it?') It is important to stress that these intuitive explanations make much more sense after the student has tried, and failed, to write the integral.

(1) Suppose you are climbing a hill the shape of which is given by the equation

$$z = 1000 - 0.01x^2 - 0.02y^2$$

and that you are at the point (60, 100, 764).
In what direction must you initially move to get to the top of the hill more quickly?
If you climb in that direction, what will your initial angle of ascent be?

Two issues arose with this problem. One stemmed from the uncertainty of colloquial language as opposed to the preciseness of mathematical symbols. Exactly what do we mean by 'in what direction'? Many students argued, not unreasonably, that the answer should be a 3D vector, not a 2D one, since the movement takes place in R^3. The misunderstanding served to make them aware of the meaning of 'directional' in 'directional derivative.' It refers to the direction we are taking on the coordinate plane of the variables. To illustrate this, we pointed to a similarly-shaped real-life mountain. Figure 6 represents Rio de Janeiro's Pão de Açúcar. If you are located on this mountain, you can be asked to move north–northwest, which will mean a direction in the plane of the variables (i.e. the earth's surface, discounting roundness), and this is not incompatible with the fact that your dependent variable z (i.e. the altitude) will also change. This is an example of how the oral presentation of a report, with the rich discussion that follows, can help the whole class fully understand a definition that had not been totally grasped.

The second issue was the mathematical meaning of the angle of ascent. The students were at a loss as to how it is measured. In the discussion, they were first encouraged to think of a one-variable function. If you move along the curve that represents it, can you measure the angle of ascent at a given point? The concept of the tangent line and its slope, and, thus, the derivative of the function, soon arose. Now in the case of this mountain we are moving on a surface, true; but not in any direction, but in that of the vector $\vec{u}$. This determines that you can only move within a vertical plane in the direction of $\vec{u}$, but since you are also on the mountain, you can only move in the intersection of the plane and the mountain. And the intersection of two surfaces is…? Right, a curve. And if we are moving along this curve, what would our angle of ascent be? Right, the angle formed by the tangent line with a horizontal line within the $\vec{u}$-oriented plane whose slope can be found by derivating the function in the direction of $\vec{u}$. The students rediscovered, thus, the concept of directional derivative.

While the examples provided are classic, the experiences above do not attempt to illustrate how to explain a hard-to-learn topic, but how to make the difficulties and misconceptions arise spontaneously, rather than being addressed by the teacher before the student actually experiences them, as would be the case in the traditional scheme of a teacher explaining a problem as the class nods and takes notes.

6. Results and analysis

Figure 7 shows the results for vector calculus students since A-R was adopted. While the number of students who had to retake the course has remained stable, there has been a sharp increase in the figures for students who passed the subject without having to take a final examination, which requires getting a B + in the two midterm examinations (as well as other requirements). This represents an important qualitative improvement over the years.

The experience was extended to the differential equations course, which comes immediately after vector calculus in the programme and is taught by the same team.

6.1. Student performance contrasted with comparable experiences

In order to evaluate the impact A-R has had on our students' performance, it appears to be useful to contrast it with the results obtained in other experiences in which A-R was not implemented.

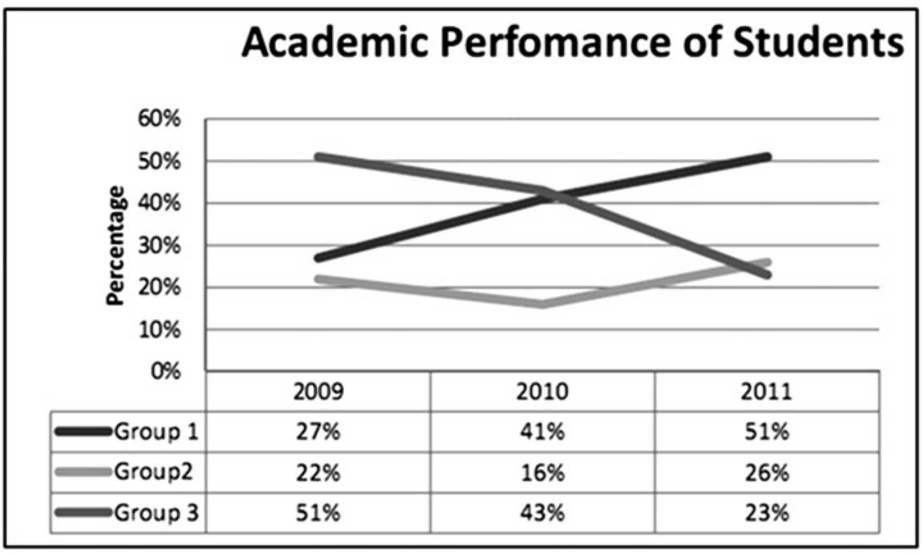


Figure 7. Academic performance of students who took vector calculus in the second semester of the years 2009, 2010 and 2011. Group 1 represents students who passed the subject without sitting a final exam, group 2 represents students who must retake the course and group 3 represents students who passed the subject but had to sit the exam.

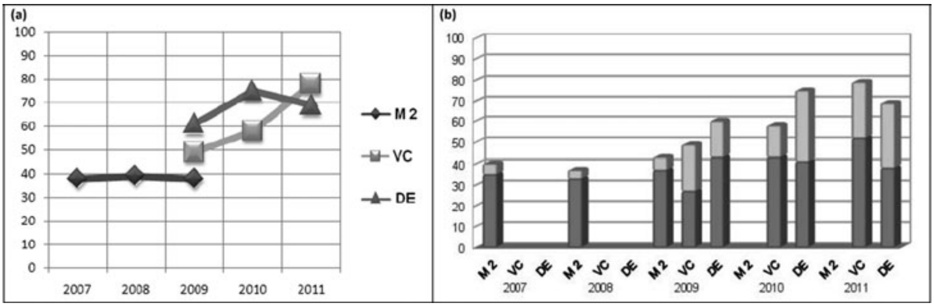


Figure 8. M2: mathematics 2, VC: vector calculus and DE: differential equation. (a) Impact of actions taken on the percentage of regular students before and after the implementation of A-R. (b) The light grey bars show the percent of students who passed the midterms but had to sit the final exam. The dark grey bars show the per cent of students who passed the subject without sitting a final exam.

In what follows, we shall adopt the Argentinian educational jargon, in which a 'regular' student is one who has passed typically two midterm tests among other requirements.

In our case, in addition to passing the midterm tests, students must participate in 80% of the weekly discussions and pass the practical laboratory works. These students may still have to take a final exam, depending on their grades in the midterms. We will make three kinds of comparisons.

6.1.1. *Comparison with courses taught by the same team before A-R was adopted*

Up to 2008, our teaching team taught the year-long subject of mathematics II, which was split into the semester-long courses of vector calculus and differential equations in 2009. In the first year in which the change was implemented, both systems coexisted, but only the students taking the semester-long courses were included in the A-R scheme.

Figure 8 shows the percentage of regular students for all three courses from 2007 to 2011. As can be seen, 2007 and 2008 were characterized by a low percentage of students attaining regular status which is what prompted our teaching approach change in the first place. The hinge year 2009 still shows poor results for mathematics II, but a dramatic improvement for vector calculus and differential equations, the new courses replacing it in which A-R was first adopted. Not only that, but the grades in the midterms were also far better, as illustrated by the number of students who did not need to take a final exam. The trend continued through 2010 and 2011.

6.1.2. *Comparison with other courses from the same knowledge field taught in the same year*

Figure 9 lists the results for one and the same contingent of students who took vector calculus together with advanced programming, and, in the second semester, differential equations together with probability and statistics. All these courses belong to the Department of Mathematics. The percentage of students attaining regular status is clearly lower for the two subjects in which A-R was not adopted.

6.1.3. *Comparison with all other second-year subjects*

Figure 10 lists the percentage of regular students for all subjects from the second year of the bioengineering programme. Once again we notice that student performance for both courses

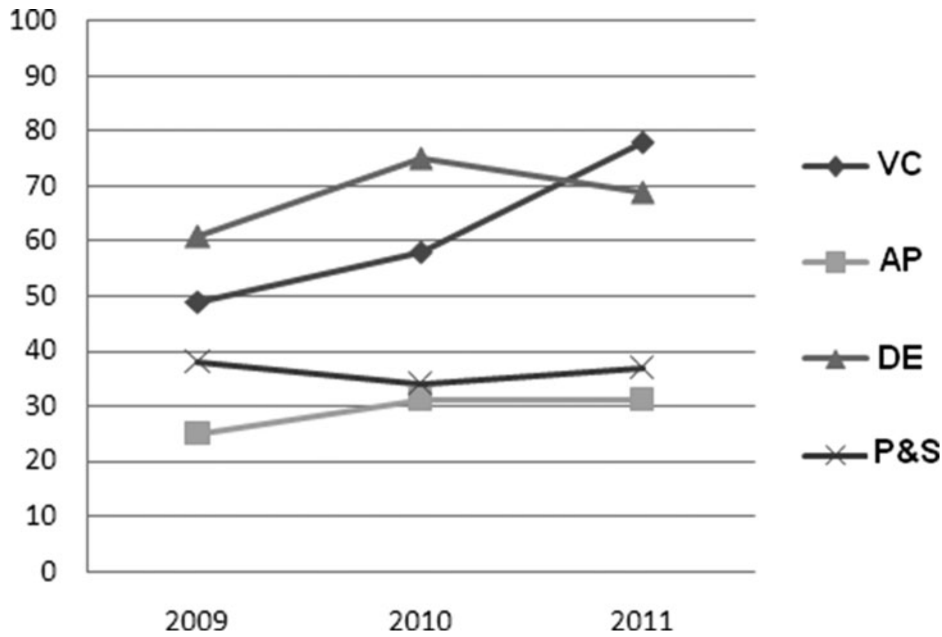


Figure 9. Percentage of regular students for different courses from the mathematics field. A-R was adopted in the vector calculus and differential equations courses only. VC, vector calculus; AP, advanced programming; DE, differential equation; P&S, probability and statistic.

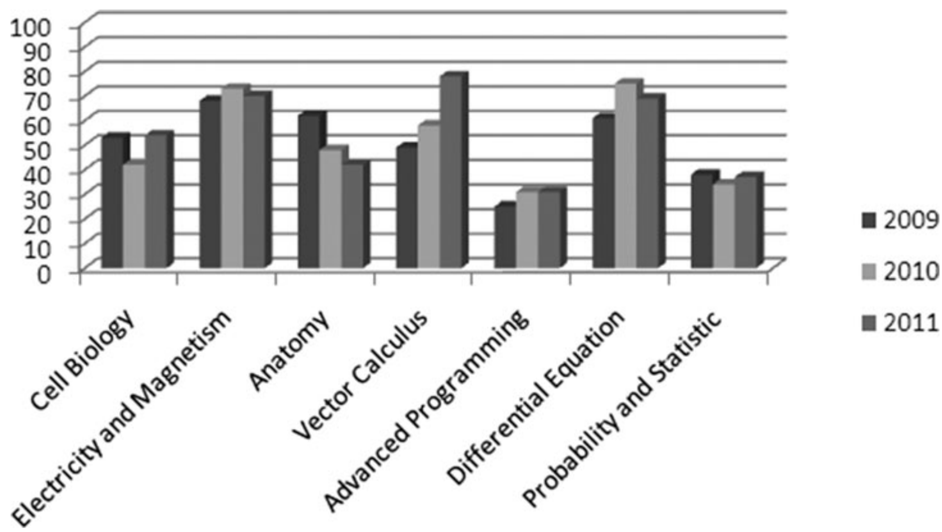


Figure 10. Percentage of regular students for all second-year subjects from the bioengineering programme.

taught along the lines described in this paper was better than that for other courses, save for electricity and magnetism, which (a) used a similar student-monitoring methodology, albeit outside the framework of an A-R project, and (b) benefitted heavily from the improvements in the vector calculus course, on which it relies for the mathematical tools needed.

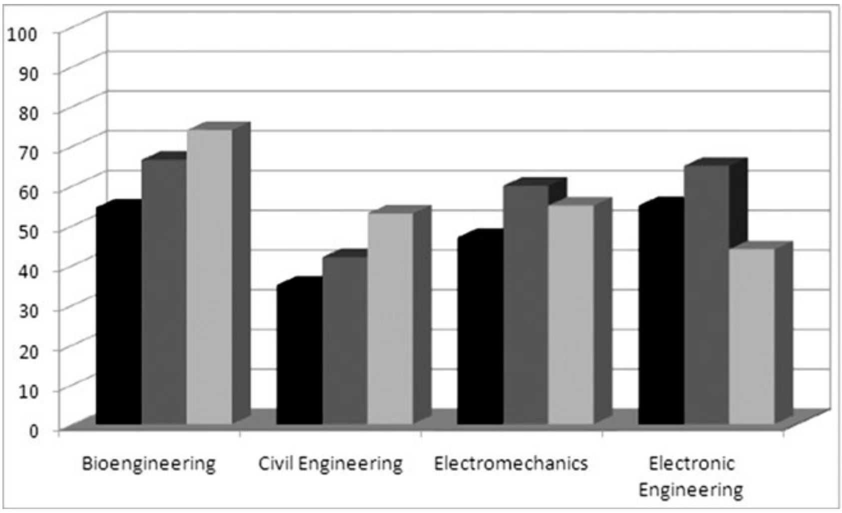


Figure 11. Student performance for bioengineering students (under an A-R teaching scheme) and for students of other engineering programmes in the same metro area (in which A-R was not adopted).

6.1.4. Comparison with other engineering programmes from the same geographical area

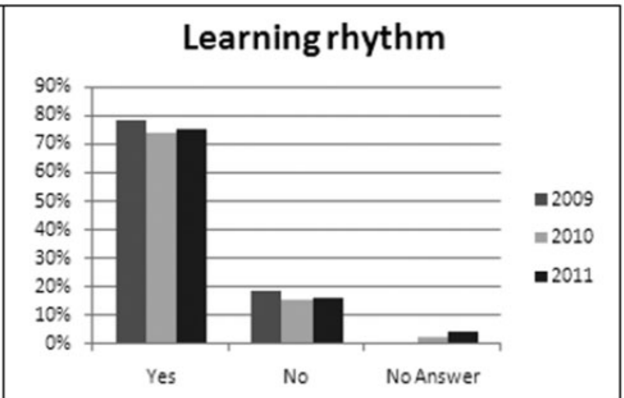
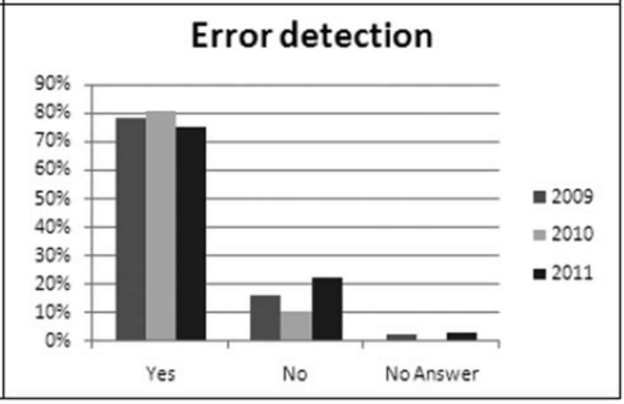
Our results can also be confronted with those obtained by students of similar courses from the same geographical area, but in which A-R was not adopted. Figure 11 shows the percentage of regular students for all three engineering programmes at the National Technological University (UTN) in Paraná, Argentina, in the same metro area. The subject considered is mathematical analysis II, which encompasses topics taught at the vector calculus and differential equations courses at our university. As can be seen, there exists a rather wild variation in the UTN students' performance from year to year, with no clear trend, and the regular student figures are definitely lower than those for Vector Calculus and Differential equations at our university since A-R was adopted.

6.2. Student feedback

In order to evaluate the students' perception about the weekly report requirement, a survey was conducted among the students in 2009, 2010 and 2011. The questions and results are shown in Table 2. On average, 85% of the students considered the experience useful and necessary for a better performance since it helped them understand concepts and correct mistakes; whereas 15% deemed it not positive essentially because it does not respect the students' individual rhythm of study and it requires too much time.

A few testimonies were received from the teachers of practical classes, the teaching assistants and the tutors. The teachers highlight positive changes: '*almost no student comes to class without having tried to solve the problems*', '*gradual, continuous learning is observed; students get more deeply into concepts seen in the theoretical classes.*' As regards negative aspects or difficulties implementing this activity, the teachers raised such issues as: '*it becomes difficult to distribute time between the control of individual work and the collective discussion to explain topics in which generalized problems are observed in the group*' and '*the high number of students per class makes a more personalized monitoring impossible to implement.*'

Table 2. Results from surveys conducted to evaluate the students' perception of the weekly report.

¿Have you been able to regulate your learning rhythm?	
¿Have you been able to detect your procedural errors and misconceptions?	

The teachers and teaching assistants who help the students complete the computer-lab assignments have made such observations as: '*We as trainers deem it very valuable for the students to be assigned problems in which they can use mathematical concepts in concrete problems representing real situations*'; '*They are motivating, they arouse the students' interest in abstract, purely mathematical topics necessary for understanding problems related to activities in their future professional life.*'

Also, the tutors make a positive evaluation of their own activities. In the reports they have presented they state that tutorship serves for '*the student not to become a number, and to recognize what their difficulties are and help them overcome them*', and to acknowledge that '*all of us can learn in different ways, and each of us has their own times and mechanisms.*' They also observe that '*towards the end of the semester, the students dared to discuss among themselves the different solving processes, which means they were in fact reasoning and applying skills.*'

7. Reflections and conclusions

Learning processes include errors. Error is an object of study in mathematical education. Ample research has been carried out aiming at finding a clear scheme for interpreting and anticipating errors. Numerous studies have also attempted to analyse them, as well as their causes and the elements that explain them with a view to obtaining a classification. Another line of research emphasizes errors as a platform to foster the study of mathematical contents.

Our research stresses the creation of spaces where errors can appear and manifest themselves in such a way that they can be corrected early. Our class assignments have been thus designed in such a way that students present on the blackboard their solutions to the problems proposed in the weekly report, indicating the respective theoretical justifications.

The teacher's task, in this case, is to create a climate of openness, dialogue and respect between peers that allows students to express doubts and difficulties without fear, so as to clarify concepts, detect errors, make corrections and also put forward suggestions about the clarity and organization of the solution presented. This is a space in which error is not punished through a grade; the only consideration is if the assignment was presented and if the student team was in class and participated in the problem discussion. We do not evaluate if the correct result for a problem was obtained, but the student's personal commitment to their own learning in doing the assignment, and their participation in the general discussion about each problem.

Teachers then take weekly reports into account for an analysis of the errors that arise in each course so that this information can be used both in pre-emptive teaching strategies and in the training of teaching assistants.

The latter aspect is key to creating a critical mass of human resources. As stated above, research suggests that presenting A-R in teacher-training programmes does not per se mean that those teachers will adopt the methodology; by incorporating young assistants into our teaching practice, we ensure that A-R will be passed on to future generations of educators.

As a final reflection we highlight that despite the complexities of conducting an A-R process, given the nature of this methodology that implies the participation of all those involved, with different motivations and ways of understanding phenomena and diverse discipline-based perspectives, it has been precisely the combination of A-R and interdisciplinary work that has given rise to changes in the teaching–learning processes. These have not been undertaken based on the discipline of mathematics alone – a fact that has led teachers to change their point of view by incorporating other ways of analysing such processes, for example, from a pedagogical and linguistic perception, and with a sense of research that involves continually formalizing, systematizing and reflecting.

This report has attempted to show how A-R has allowed an interdisciplinary group of teachers to make changes in their teaching and evaluation methodology based on a conceptual framework that has been discussed and constructed, and within which those changes are evaluated. We conclude that it is a useful tool for professionalizing the task of teaching at a university.

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