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A multiproduct batch plant design model incorporating production planning and scheduling decisions under a multiperiod scenario

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ABSTRACT

In this study, we propose a multiperiod mixed-integer linear programming model, which integrates several decisions related to multistage multiproduct batch plants. In general, plant designs are solved without considering operation decisions, whereas the proposed approach considers production planning as well as scheduling decisions. The time horizon comprises several periods where deterministic variations in prices, product demand limits, costs, and the availability raw materials are considered. The plant operates using different production campaigns throughout each time period. The proposed model allows the optimal plant structure (unit sizes and its duplication in parallel at each stage) to be obtained, as well as the detailed production plan for every time period. Thus, the proposed method allows assessments of the trade-offs between the different decision levels involved by considering fluctuations throughout the time horizon.

1. Introduction

Due to the high flexibility of batch plants in processing facilities, they are the preferred production mode for a large number of chemicals. This production flexibility facilitates faster responses to satisfy market requirements, which are subject to fluctuations over time. However, most previous studies of the optimal design of multiproduct batch plants have utilized models with a single long time horizon and constant conditions, but without considering variations due to market or seasonal fluctuations. These formulations are unsuitable for use in a highly dynamic environment where the problem data vary among periods, so some multiperiod formulations have also been developed.

In addition, most previous studies have focused on a specific decision level. In particular, batch plant design has been solved by considering several assumptions related to planning and scheduling. In general, these assumptions are made to simplify the model formulation and resolution, as well as for use in more typical or normal scenarios. Thus, previous investigations of batch plant design have employed this approach, but a more appropriate problem representation can be developed if the trade-offs among different decision levels can be incorporated, as proposed by Lee et al. [1] for the simultaneous lot-sizing and supplier selection problems, and by Ramezanian and Saidi-Mehrabad [2] for the integrated lot-sizing and scheduling approach.

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Nomenclature

Indices

r	raw materials
i	products
j	stages
k	units
l	slots
m	discrete options for the number of repetition of the campaign
n	number of batches of a product
p	discrete sizes for the units
t, τ	time periods

Parameters

co_{it}	operating cost coefficient of product i in time period t.
cp_{it}	cost coefficient for late delivery of product i in time period t.
DE_{it}^{L}	minimum demand of product i at time period t.
DE_{it}^{U}	maximum demand of product i at time period t.
F_{rit}	conversion of raw material r to produce i at time period t.
H	global time horizon.
H_t	net available production time for all products at time period t.
IM_{r0}	initial inventory of raw material r.
IP_{i0}	initial inventory of product i.
K_j	maximum number of available identical parallel units at batch stage j.
L_{kjt}	number of slots postulated for unit k of stage j during period t.
M_b	big-M constants for $b = 1, 2, 3$.
np_{it}	price of product i at period t.
N_t	number of discrete values proposed by the number of repetitions of the campaign in period t.
NBC_{it}^{U}	maximum number of batches of product i in the campaign of period t.
NN_t^{L}	minimum number of times that the campaign of period t can be repeated.
NN_t^{U}	maximum number of times that the campaign of period t can be repeated.
P_j	number of discrete sizes available for batch stage j.
q_{it}^{L}	lower bound on production level of product i in period t.
q_{it}^{U}	upper bound on production level of product i in period t.
SF_{ijt}	size factor of product i in stage j for each time period t.
pt_{ijt}	processing time of product i in batch stage j in time period t.
T_{mt}	discrete value m for the number of repetitions of the campaign of period t.
VF_{jp}	standard volume of size p for batch unit at stage j.
wp_{it}	waste disposal cost coefficient per product i.
wr_{rt}	waste disposal cost coefficient per raw material r.
α_j	cost coefficient for a batch unit in stage j.
β_j	cost exponent for a batch unit at stage j.
ε_{rt}	inventory cost coefficient for raw material r in time period t.
κ_{rt}	price for the raw material r in time period t.
σ_{it}	inventory cost coefficient for product i in time period t.
ζ_r	time periods during which raw materials have to be used.
χ_i	time periods during which products have to be used.

Binary Variables

d_{mt}	specifies if the campaign is repeated T_{mt} times over time period t.
X_{jklt}	indicates if slot l of unit k at stage j is employed in time period t.
u_{jk}	indicates if unit k of stage j is used.
v_{jp}	specifies if the units at stage j have size p.
x_{int}	denotes if n batches of product i are processed in the campaign of time period t.
Z_{ilt}	indicates if product i is assigned to slot l in time period t.

Continuous Variables

B_{it}	batch size of product i in time period t.
C_{rt}	amount of raw material r purchased in time period t.

CE_{jk}	investment cost of batch unit k of stage j.
CTC_t	campaign cycle time in time period t.
e_{jkp}	product of the binary variables u_{jk} v_{jp}.
IM_{rt}	inventory of raw material r at the end of time period t.
IP_{it}	inventory of final product i at the end of time period t.
NB_{it}	total number of batches of product i processed in time period t.
NBC_{it}	number of batches of product i included in the campaign of time period t.
NN_t	number of times a campaign is cyclically repeated over time period t.
PW_{it}	product i wasted at time period t due to the limited product lifetime.
q_{it}	amount of product i to be produced in time period t.
QS_{it}	amount of product i sold at the end of time period t.
RM_{rt}	raw material r used for production in time period t.
RW_{rt}	raw material r wasted at time period t due to the limited product lifetime.
TF_{jklt}	final processing time of slot l in unit k of stage j in time period t.
TI_{jklt}	initial processing time of slot l in unit k of stage j in time period t.
V_j	size of a batch unit at stage j.
w_{ijpnt}	variable that represents the product of variables q_{it} v_{jp} x_{int}.
ww_{mt}	variable that represents the product of variables CTC_t d_{mt}.
Y_{ijklt}	continuous variable on interval [0, 1] that indicates if product i is assigned to slot l of unit k at stage j in time period t.
ϑ_{it}	amount of late delivery for product i in time period t.

The decisions adopted during the design phase have a significant influence on the operation of the plant. In particular, purchasing, inventory, transport, and distribution are affected severely depending on the production flow determined from the plant design. These decisions are more critical when several products are produced by sharing resources. Therefore, it is crucial to know in advance whether the plant behavior is subject to different conditions.

From a scheduling perspective, most previous design approaches have assumed that plants use single product campaigns (SPCs) for production. In this policy, all of the batches of a product are produced without overlapping with other products during each period. This assumption simplifies the problem and its solution by reducing the formulation size. Multiperiod approaches allow more flexible production programs to be obtained, but SPCs are not suitable because they can overestimate the time requirements, thereby leading to buildups of materials in the inventory, which may be impractical when perishable products are considered. In addition, from an operational perspective, the production flow is not estimated and thus the purchasing, inventory, and distribution cannot be assessed.

Mixed product campaigns (MPCs) can be employed to overcome these drawbacks and to improve productivity in multiproduct batch facilities. The production campaign is repeated in a cyclical manner during the production horizon. A more steady supply of raw materials and products can be achieved given that production campaign comprises a set of batches of the different involved products. Thus, more efficient capacity requirement planning can be assured. However, despite these advantages, the incorporation of constraints for MPCs requires a more complex formulation [3].

Therefore, the approach proposed in the present study considers two elements simultaneously. First, a multiperiod perspective is considered where seasonal fluctuations are incorporated in an appropriate manner. Second, integration among decision levels is considered by a formulation that embodies several planning and scheduling features. In particular, MPCs are introduced in order to evaluate the operation flow. Thus, the plant behavior can be assessed and related decisions (inventory, distribution, purchasing, etc.) can be suitably weighted. It should be mentioned that scheduling is not applied from a short-term perspective but instead it is employed in order to estimate production flows.

Planning and scheduling integration have been addressed in many studies using different modeling approaches and solution strategies, although only from short- or medium-term perspectives. For example, Petkov and Maranas [4] solved these problems by allowing for uncertain product demand. Józefowska and Zimniak [5] implemented a decision support system using a multi-criteria genetic algorithm. Verderame and Floudas [6] analyzed planning and scheduling as inter-related activities that involve the allocation of plant resources. Li and Ierapetritou [7] used Lagrangian relaxation because of the intractable model size. Susarla and Karimi [8] considered planning and scheduling decisions simultaneously, where they presented a mixed-integer linear programming (MILP) formulation in order to integrate resource allocation and production planning in multiproduct batch plants. This model facilitates decision support related to batch scheduling, sequence-dependent changeovers, key resource allocations, maintenance, inventory profiles with safety stock limitations, and new product introductions.

Design and planning decisions have been simultaneously considered in several previous studies. In addition, by considering the problem size, several studies have resorted to resolution strategies using decomposition-based methods. For example, van den Heever and Grossmann [9] proposed a general disjunctive multiperiod nonlinear optimization model, which incorporates design as well as operation and expansion planning, and it considers the corresponding costs incurred during each time period for the multiproduct batch plant design problem. Two algorithms for the resolution of these problems were proposed: a logic-basic outer approximation algorithm and a bilevel decomposition algorithm. Moreover, Moreno et al. [10] presented a multiperiod scenario with different structural options for plant design, but without applying decomposition techniques. Later, Moreno and Montagna [11] developed a more flexible formulation where the plant configuration can be different during every time period, as well as introducing a new structural option using duplication in series for each operation [12].

Scheduling decisions were also incorporated in the design model. One of the first studies to address MPCs was reported by Birewar and Grossmann [13], who included scheduling restrictions for the case of multiproduct batch plants. However, they simplified their approach by assuming only one unit in each stage. The cycle time of the campaign was minimized and two different operation policies were considered: zero wait and unlimited intermediate storage. Later, Dietz et al. [14] proposed a multicriteria design method for multiproduct batch plants, where the design variables comprised the size of the equipment items as well as the operating conditions. This formulation considered the composition of the production campaigns. Given the important combinatorial characteristic of the problem, the proposed approach involved coupling a stochastic algorithm, specifically a genetic algorithm, with a discrete-event simulator. Working with similar plants, Corsano et al. [15] developed a multiperiod formulation in order to optimize design and production planning simultaneously, where they employed MPCs to solve the production scheduling. Using a mixed-integer non linear programming (MINLP) formulation, a set of possible production campaigns were employed, which were handled by predetermined scheduling constraints, where this approach was applied to a fermentation network. Fumero et al. [16] proposed a novel MILP formulation for multiproduct batch plant design based on MPCs, which allowed the plant configuration and the production flow to be determined. Furthermore, scheduling and design decisions were integrated in the context of the supply chain [17].

The MILP model proposed in the present study integrates several decisions such as design and production planning with MPCs for multistage multiproduct batch plants. A multiperiod context is assumed in order to consider market or seasonal variations. Thus, deterministic fluctuations in several problem parameters, such as costs, product demands, prices, and the availability of raw materials, are specifically considered in this approach. In order to assess the optimal performance of the model, the net present value is maximized by considering incomes (product sales) and expenses (investment, resources, operation, waste disposal, inventories, and penalty costs due to late delivery). Production planning is addressed using MPCs; thus, for each time period, the optimal solution determines the campaign composition, the assignment of batches to units, and the batches sequencing in each unit. Therefore, the design and planning problems are simultaneously integrated in the overall multiperiod model, where the different tradeoffs among the involved variables can be assessed. Production flows are evaluated given that MPCs are included in the model, so relevant information about the operation behavior can be obtained.

The remainder of this study is organized as follows. First, the integrated design, production planning, and scheduling problem is described in Section 2. The mathematical formulation is developed in Section 3. In Section 4, we solve numerical examples using the proposed model. Finally, we give some concluding remarks in Section 5.

2. Problem description

The problem of simultaneous design, planning, and scheduling optimization for a multiproduct batch plant within a multiperiod environment is described as follows. A batch processing plant producing I products with similar recipes is considered, each of which is elaborated following the same arrangement of J batch stages and using R raw materials. Parallel duplication of units working out-of-phase is allowed at batch stage j. Thus, stage j may comprise K_j units of identical size.

Given that the problem addressed involves plant design, the sizes of the batch units at stage j, V_j, must be determined. According to the habitual commercial procurement of units, equipment sizes are considered to be available from a set $SV_j = \{VF_{j1}, VF_{j2}, \ldots, VF_{jP_j}\}$ of discrete sizes, where the parameter P_j corresponds to the number of sizes offered at stage j.

No intermediate storage tanks allocation are allowed. Furthermore, each batch processed at any unit in stage j is transferred without delay to a unit in stage $j + 1$. Thus, a zero wait transfer policy is adopted.

As mentioned earlier, the plant operates in a multiperiod context where the total planning horizon H is split into T time periods, which may or may not have the same length H_t. The processing time pt_{ijt} and the size factor SF_{ijt} for each product i at stage j in every period t are plant data. In addition, the upper and lower bounds are known for the demands of each product i in every period t, DE^L_{it} andDE^U_{it}. The amounts of raw materials consumed are determined by the mass balances, where a given parameter F_{rit} represents the conversion of the raw material r into product i during period t in the batch process. The costs of raw materials and their availability differ for each period and they are model parameters. Moreover, the prices of the final products in each time period and their maximum storage capacities are problem data.

During each time period t, the plant operates through MPCs with cyclic execution, i.e., the production campaign comprising a number of batches of different products manufactured in the period is repeated in a cyclic manner over H_t. It should be mentioned that for each period, the number of batches of each product i is a decision variable in the model, and thus the campaign composition and its cycle time are not known a priori. Only upper limits are imposed on the number of batches of each product i in the campaign of time period t, NBC^U_{it}. In order to allocate batches to units, an asynchronous slot-based continuous-time formulation is employed. This asynchronous representation allows us to set an appropriate number of time intervals (slots) with unknown durations for each processing unit of a stage, which provides more flexibility in terms of timing decisions.

The number of times that the campaign in period t is repeated cyclically throughout the available time interval, which is denoted by NN_t, is a discrete variable of the model. In order to obtain a linear model and avoid significant and unnecessary computational efforts, we propose an appropriate discretization of this variable on the interval $[NN^L_t, NN^U_t]$. The interval endpoints represent the minimum and maximum values that the variable NN_t can take and they are adequately suggested.

The initial amounts of both the product and raw material inventories, IP_{i0} and IM_{r0}, at the beginning of the global horizon H, are assumed to be given.

In every time period t, the decisions involved in production planning consist in determining for each product i, the quantity to be produced q_{it}, the number of batches, and their sizes at the optimal campaign, i.e., NBC_{it} and B_{it}, respectively, and the total sales QS_{it}. The amounts of raw material purchased, C_{rt}, and used in the process, RM_{rt}, during each time period are also obtained. Furthermore, the inventory levels of every final product IP_{it} and raw material IM_{rt} are determined at the end of each period considered. If equal length periods are employed, waste management is also added to the formulation by considering waste due to the product that has passed its shelf life PW_{it} and due to the limited raw material lifetime RW_{rt}. Finally, late deliveries ϑ_{it} that occur in all periods are estimated.

By focusing on the scheduling decisions, for every time period t, this formulation allows us to determine the campaign composition, the assignment of batches to equipment items in each stage and its sequencing, the initial and final times of the batches processed in each unit, the campaign cycle time, CTC_t, and the number of campaign repetitions over the time period H_t.

The objective function considered is the maximization of the net present value of the profit over the time horizon, which is given by the incomes (product sales) minus the costs (investment, raw materials, storage, penalties for late deliveries, and waste disposal).

3. Model formulation

3.1. Plant design constraints

The batch size B_{it} and the number of batches NB_{it} of product i in time period t allow us to determine the amount of product i produced during that period, q_{it}, as follows:

$$q_{it} = B_{it}\, NB_{it} \quad \forall i, t. \tag{1}$$

For a multiperiod approach, the constraint that calculates the unit size at each stage j is:

$$V_j \geq SF_{ijt}\, B_{it} \quad \forall i, j, t, \tag{2}$$

where the parameter SF_{ijt}, which is also known as the size factor at stage j for product i, may differ in each period t because of seasonal variations. It specifies the size needed for stage j to process one unit mass of product i at the end of the production process.

Let NBC_{it} be the number of batches of product i processed in the production campaign of period t and NN_t is the number of times that the MPC is repeated cyclically throughout that period. Then, the total number of batches of product i processed in time interval t is defined by Eq. (3),

$$NB_{it} = NBC_{it} NN_t \quad \forall i, t. \tag{3}$$

By combining Eqs. (1) and (3), Eq. (2) takes the following form:

$$V_j \geq \frac{SF_{ijt}\, q_{it}}{NBC_{it}\, NN_t} \quad \forall\, i, j, t. \tag{4}$$

As mentioned earlier, the sizes of batch equipment V_j are available in discrete sizes VF_{jp}. Thus, the values of the variable V_j belong to the set $SV_j = \{VF_{j1}, VF_{j2}, \ldots, VF_{jPj}\}$. To handle this restriction, a binary variable v_{jp} is employed, the value of which is equal to one if the units at batch stage j have size p; otherwise, it is equal to zero. Then, this variable can be expressed by Eq. (5) as the summation of the P_j possible values that V_j can take multiplied by the binary variable v_{jp}. Eq. (6) ensures that only one binary variable can be nonzero, which guarantees that only one value of set SV_j is given to variable V_j,

$$V_j = \sum_{p=1}^{P_j} \upsilon_{jp} VF_{jp} \quad \forall\, j, \tag{5}$$

$$\sum_{p=1}^{P_j} \upsilon_{jp} = 1 \quad \forall\, j. \tag{6}$$

In addition, the binary variable x_{int} is used to determine the number of batches of product i that comprised each campaign during the time period t, NBC_{it}. The variable x_{int} is 1 only when n batches of product i are processed in the campaign during time period t. Constraint (7) ensures the selection of only one option,

$$\sum_{n=0}^{NBC_{it}^U} x_{int} = 1 \quad \forall\, i, t. \tag{7}$$

Therefore,

$$NBC_{it} = \sum_{n=0}^{NBC_{it}^U} n x_{int} \quad \forall\, i, t. \tag{8}$$

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Note that the subscript n represents an integer number from the interval $[0, NBC_{it}^U]$. In particular, when variable x_{i0t} is equal to 1, then no batches of product i are produced in time period t, and thus its production is zero. Then, assuming that q_{it}^L and q_{it}^U are the known lower and upper bounds on the production level of product i in time period t, the inequalities in (9) ensure the previous assumption and they are redundant when at least one batch of product i belongs to the campaign in that period,

$$(1 - x_{i0t})q_{it}^L \le q_{it} \le (1 - x_{i0t})q_{it}^U \quad \forall i, t. \tag{9}$$

By introducing Eqs. (5) and (8) into Eq. (4) the following equation is obtained, which is valid if the integer subscript n is nonzero $(1 \le n \le NBC_{it}^U)$,

$$NN_t \ge \sum_{p=1}^{P_j} \sum_{n=1}^{NBC_{it}^U} \frac{SF_{ijt}\, q_{it}}{VF_j\, n} v_{jp} x_{int} \quad \forall\ i, j, t. \tag{10}$$

The product of the continuous and binary variables introduces nonlinearity in Eq. (10). To reformulate constraint (10) as a linear constraint, a new continuous variable w_{ijpnt} allows us to remove the product $q_{it}\ v_{jp}\ x_{int}$. The variable w_{ijpnt} is equal to q_{it} if variables v_{jp} and x_{int} are simultaneously 1; otherwise, w_{ijpnt} is equal to zero. For this new variable, the following constraints are incorporated,

$$\sum_{p=1}^{P_j} w_{ijpnt} \le q_{it}^U x_{int} \quad \forall\ i, j, n, t, \tag{11}$$

$$\sum_{n=0}^{NBC_{it}^U} w_{ijpnt} \le q_{it}^U v_{jp} \quad \forall\ i, j, p, t, \tag{12}$$

$$\sum_{n=0}^{NBC_{it}^U} \sum_{p=1}^{P_j} w_{ijpnt} = q_{it} \quad \forall\ i, j, t. \tag{13}$$

Therefore, Eq. (10) can now be expressed as follows:

$$NN_t \ge \sum_{p=1}^{P_j} \sum_{n=1}^{NBC_{it}^U} \frac{SF_{ijt}}{VF_{jp}\, n} w_{ijpnt} \quad \forall\ i, j, t. \tag{14}$$

Furthermore, as mentioned earlier, unit duplication is allowed for every stage j. Therefore, a binary variable u_{jk} is used to determine whether unit k at stage j is utilized to process some batch $(u_{jk} = 1)$.

To avoid alternative optimal solutions, the units are included sequentially,

$$u_{jk} \ge u_{jk+1} \quad \forall\ j, 1 \le k < K_j. \tag{15}$$

3.2. Production planning constraints

The proposed model assumes that the production of each final product i requires $r = 1, 2, \ldots, R$ ingredients. Thus, Eq. (16) describes the raw material inventory for ingredient r at the end of a time interval t, IM_{rt}, which is equal to the stock in the previous period, IM_{rt-1}, plus the amount acquired during period t, C_{rt}, minus the quantity consumed in the process, RM_{rt}, and minus the wastes due to the limited product lifetime, RW_{rt}. In a similar manner, Eq. (17) sets the level of the final product i stored at the end of period t, IP_{it}, which is equal to the amount in storage at the end of the previous period, IP_{it-1}, plus the production during this period, q_{it}, minus the amount sold QS_{it} and minus the waste due to the expired product shelf life, PW_{it}, where the sold amount is bounded by the maximum demand DE_{it}^U,

$$IM_{rt} = IM_{rt-1} + C_{rt} - RM_{rt} - RW_{rt} \quad \forall r, t, \tag{16}$$

$$IP_{it} = IP_{it-1} + q_{it} - QS_{it} - PW_{it} \quad \forall i, t. \tag{17}$$

When the lengths of time periods are equal, Eq. (18) ensures that the amounts of raw materials stored in a given period are not to be used after the next ζ_r time periods. Similarly, Eq. (19) checks this condition for stored products after the next χ_i time periods,

$$IP_{it} \le \sum_{\tau=t+1}^{t+\chi_i} QS_{i\tau} \quad \forall i, t, \tag{18}$$

$$IM_{rt} \le \sum_{\tau=t+1}^{t+\zeta_r} RM_{r\tau} \quad \forall r, t. \tag{19}$$

Furthermore, constraints (20) and (21) ensure that the stocks of product i and raw material r stored during period t do not exceed their respective maximum available capacities, IP_{it}^U and IM_{rt}^U,

$$0 \le IP_{it} \le IP_{it}^U \quad \forall i, t, \tag{20}$$

$$0 \le IM_{rt} \le IM_{rt}^U \quad \forall r, t. \tag{21}$$

This problem assumes that the initial amounts of both the raw materials and products in stock, IM_{r0} and IP_{i0}, are known at the beginning of the time horizon. The mass balance (22) determines the amount consumed of raw material r to make product i during period t, RM_{rit}, where parameter F_{rit} denotes the conversion of raw material r into product i in the process during period t,

$$RM_{rit} = F_{rit} q_{it} \quad \forall r, i, t. \tag{22}$$

Constraint (23) specifies the total raw material consumption for production in period t,

$$RM_{rt} = \sum_i RM_{rit} \quad \forall r, t. \tag{23}$$

Late deliveries of products are penalized. Eq. (24) determines the late delivery ϑ_{it} that occurs when the sale of product i does not meet the agreed minimum product demand DE_{it}^L on time in every time period t [18]. A penalty cost term is included in the objective function to consider the expenses incurred if this failure occurs,

$$\vartheta_{it} \ge \vartheta_{it-1} + DE_{it}^L - QS_{it} \quad \forall i, t. \tag{24}$$

3.3. Production scheduling constraints

The scheduling model embedded in the integrated problem corresponds to a continuous time, slot-based formulation for a multistage multiproduct batch plant. The production scheduling constraints at each time period are largely based on Fumero et al. [3]. A detailed description of the assumptions regarding the units and slots utilization at each plant stage, which allow the search space to be reduced as well as eliminating alternative solutions, can be found in the previous study. However, in order to facilitate the readability of the model, the assignment and timing main constraints are described in the present study.

3.3.1. Allocation constraints

The assignment variables given by Fumero et al. [3] are extended to all time periods. At every time period, the binary variable Y_{ijklt} defines whether the batch of product i is allocated to a slot l of unit k at stage j. This variable is sufficient for slot-based formulations, but the binary variables X_{jklt} and Z_{ilt} are introduced in order to improve the model's computational performance. The variable X_{jklt} is only equal to 1 if slot l of equipment k at stage j is used during period t, while Z_{ilt} takes a value of 1 if some batch of product i is processed in slot l during period t. The following relations among the binary variables are stated:

$$Y_{ijklt} \le Z_{ilt} \quad \forall\, i, j, k, 1 \le l \le L_{kjt}, t, \tag{25}$$

$$Y_{ijklt} \le X_{jklt} \quad \forall i, j, k, 1 \le l \le L_{kjt}, t, \tag{26}$$

$$Y_{ijklt} \ge X_{jklt} + Z_{ilt} - 1 \quad \forall\, i, j, k,\ 1 \le l \le L_{kjt}, t, \tag{27}$$

$$\sum_{\substack{k \\ 1 \le k \le K_j \\ k/l \le L_{kjt}}} Y_{ijklt} = Z_{ilt}, \quad \forall i, j, 1 \le l \le L_{jkt}, t, \tag{28}$$

$$\sum_i Y_{ijklt} = X_{jklt}, \quad \forall j, 1 \le k \le K_j, 1 \le l \le L_{kjt}, t, \tag{29}$$

where L_{kjt} represents the number of postulated slots for unit k of stage j in time period t.

Eqs. (25) and (26) are active when variable Y_{ijklt} is equal to 1, or when the variables X_{jklt} or Z_{ilt} are zero. Eq. (27) ensures that the variable Y_{ijklt} is equal to 1 when both variables X_{jklt} and Z_{ilt} take a value of 1. Therefore, the variable Y_{ijklt} can be defined as continuous on the interval [0, 1], thereby reducing the number of binary variables.

In addition, the following constraints are imposed for these variables. If unit k is not allocated at stage j, i.e., $u_{jk} = 0$, then none of their slots are utilized to process products (Eqs. (30) and (31)). Otherwise, at least one batch of product is processed in some slot of that unit according to Eq. (32),

$$Y_{ijklt} \le u_{jk}, \quad \forall i, j, 1 \le k \le K_j, 1 \le l \le L_{kjt}, t, \tag{30}$$

$$X_{jklt} \le u_{jk}, \quad \forall j, 1 \le k \le K_j, 1 \le l \le L_{kjt}, t, \tag{31}$$

$$\sum_{i} \sum_{\substack{l \\ 1 \le l \le L_{kjt}}} Y_{ijklt} \ge u_{jk}, \quad \forall j, 1 \le k \le K_j, t. \tag{32}$$

In this study, we make several assumptions regarding the use of slots and units to reduce the search space. Without loss of generality, constraints (33)–(37) are imposed,

$$\sum_{i} Z_{ilt} \ge \sum_{i} Z_{il+1t}, \quad \forall 1 \le l \le L_{kjt} - 1, t, \tag{33}$$

$$Y_{i'jk'lt} \le 1 - Y_{ijklt}, \quad \forall i, i', j, 1 \le l \le L_{kjt}, 1 \le l \le L_{k'jt}, \; 1 \le k \le K_j, 1 \le k' \le K_j, (k \ne k'), t, \tag{34}$$

$$X_{jk'lt} \le 1 - X_{jklt}, \quad \forall j, 1 \le l \le L_{kjt}, 1 \le l \le L_{k'jt}, \; 1 \le k \le K_j, 1 \le k' \le K_j, (k \ne k'), t \;, \tag{35}$$

$$\sum_{i} \sum_{\substack{k \\ 1 \le k \le K_j \\ k/l \le L_{kjt}}} Y_{ijklt} \le 1, \forall j, 1 \le l \le L_{kjt}, t, \tag{36}$$

$$\sum_{i} Z_{ilt} \le 1, \forall 1 \le l \le L_{kjt} - 1, t. \tag{37}$$

Constraint (33) ensures that the slots are used sequentially in each stage, i.e., a slot is occupied only if the previous slot has been used for processing a batch on some unit in this stage. Constraints (34) and (35) guarantee that if one batch is processed in slot l of unit k in stage j, then this slot is no longer available for other units in this stage. Moreover, Eqs. (36) and (37) ensure that slot l can be only allocated for processing at most one product in each stage at the plant.

In order to avoid alternative solutions, we impose a decreasing succession formed by the weighted sum of the slots occupied in each unit of a stage during every period,

$$\sum_{\substack{l \\ 1 \le l \le L_{kjt}}} 2^l X_{jklt} \ge \sum_{\substack{l \\ 1 \le l \le L_{k+1jt}}} 2^l X_{jk+1lt}, \forall j, 1 \le k < K_j - 1, t. \tag{38}$$

As shown previously [3], the resolution process can be expedited by imposing a pre-ordering constraint during scheduling. The proposed pre-ordering ensures that in each time period, a given batch of product i is processed in the same slot in all stages,

$$\sum_{i} \sum_{\substack{k \\ 1 \le k \le K_j \\ k/l \le L_{kjt}}} iY_{ijklt} = \sum_{i} \sum_{\substack{k \\ 1 \le k \le K_{j'} \\ k/l \le L_{kj't}}} iY_{ij'klt} \forall j, j', (j < j'), 1 \le l \le L_{kjt}, t. \tag{39}$$

The computational burden is reduced by including constraint (39), although suboptimal solutions may be obtained. This assumption provides a good solution that coincides with the global optimum of the exact scheduling model in most of the cases solved [16].

As an alternative to Eq. (8), variable Z_{ilt} allows us to express the number of batches of product i included in the campaign during period t, NBC_{it}, as follows:

$$\sum_{l} Z_{ilt} = NBC_{it}, \quad \forall i, t. \tag{40}$$

Finally, for each time period t when no batch of product i is processed, the variable Z_{ilt} is zero for all slots. Although the following constraint is redundant, the computational performance can be improved:

$$Z_{ilt} \le 1 - x_{i0t}, \quad \forall i, l, t. \tag{41}$$

3.3.2. Timing constraints

The timing decisions are modeled by the following constraints:

$$TF_{jklt} = TI_{jklt} + \sum_{i} pt_{ijt} Y_{ijklt} \quad \forall j, 1 \le k \le K_j, 1 \le l \le L_{kjt}, t, \tag{42}$$

$$TF_{jklt} \le TI_{jkl+1t} \quad \forall\, j, 1 \le k \le K_j, 1 \le l < L_{kjt}, t, \tag{43}$$

$$TF_{jklt} - TI_{jkl+1t} \ge -M_1 X_{jkl+1t} \quad \forall\, j, 1 \le k \le K_j, 1 \le l < L_{kjt}, t, \tag{44}$$

$$TF_{jklt} - TI_{j+1k'lt} \geq M_2(X_{jklt} + X_{j+1k'lt} - 2) \quad \forall\ j, 1 \leq k \leq K_j, 1 \leq k' \leq K_{j+1}, 1 \leq l \leq \min\left\{L_{kjt}, L_{k'j+1t}\right\}, t, \tag{45}$$

$$-TF_{jklt} + TI_{j+1k'lt} \geq M_2(X_{jklt} + X_{j+1,k'lt} - 2) \quad \forall\ j, 1 \leq k \leq K_j, 1 \leq k' \leq K_{j+1}, 1 \leq l \leq \min\left\{L_{kjt}, L_{k'j+1t}\right\}, t. \tag{46}$$

For every time period, Eq. (42) derives the final processing time TF_{jklt} of each proposed slot at unit k in stage j from its initial time TI_{jklt} and the processing time of the assigned product in that slot. If no product is processed in slot l, the initial and final times are the same. In order to avoid slots overlapping on each unit of a stage, the processing of slot l must be finished before the initial time of slot $l + 1$, which is expressed by Eq. (43). When no batch is allocated to slot $l + 1$, the Big-M constraint (44) becomes active and by considering Eq. (43), the final and initial times of slots l and $l + 1$, respectively, are forced to be the same. M_1 is a sufficiently large parameter to make the constraint redundant when one product is assigned to slot $l + 1$.

The batch transfer policy employed in this method is zero wait, which assumes that a batch must be transferred to the next stage as soon as its processing has finished in the current stage. The Big-M constraints (45) and (46) allow us to express this transfer policy. Parameter M_2 is sufficiently large to relax these constraints when the product processed in slot l does not employ equipment k in stage j or k' in stage $j + 1$.

In order to calculate the cycle time of the campaign during period t, CTC_t, the last slot of each unit k in stage j, L_{kjt}, and the first slot effectively assigned to unit k in stage j during period t, $\tilde{l}_{jkt}$ ($\tilde{l}_{jkt} = \min\{1 \leq l \leq L_{kjt} / X_{jklt} = 1\}$), are considered as follows:

$$CTC_t = \max_j \left\{ \max_{1 \leq k \leq K_j} \left\{ TF_{jkL_{kjt}t} - TI_{jk\tilde{l}_{jkt}t} \right\} \right\}. \tag{47}$$

Using Big-M constraints, the above expression can be represented as:

$$CTC_t - TF_{jkL_{kjt}t} + TI_{jklt} \geq M_3 \left((X_{jklt} - 1) - \sum_{\substack{l' \\ 1 \leq l' < l}} X_{jkl't} \right), \quad \forall\ j,\ 1 \leq k \leq K_j,\ 1 \leq l \leq L_{kjt}, t, \tag{48}$$

where M_3 is a large parameter that makes Eq. (48) redundant for all the preceding and succeeding slots, if any, until the first non-empty slot in equipment k for stage j during period t.

Significant reductions in the resolution time are achieved by establishing the following lower limit on the campaign cycle time. Assuming that the idle time in each unit during the processing of the campaign at period t is zero, then:

$$CTC_t \geq \sum_i \sum_{l=1}^{L_{kjt}} pt_{ijt} Y_{ijklt} \quad \forall\ j, 1 \leq k \leq K_j, t. \tag{49}$$

The total time required to produce all batches corresponding to time interval t cannot exceed its length H_t. In order to satisfy this requirement, the product between the campaign cycle time and the number of campaign repetitions in every period must be less than or equal to H_t,

$$CTC_t\, NN_t \leq H_t \quad \forall\ t. \tag{50}$$

Constraint (50) is nonlinear because of the product $CTC_t\, NN_t$. However, in contrast to the problem addressed by Fumero et al. [16], the production requirements during each time period are optimization variables. Therefore, the reformulation used by Fumero et al. [16] does not avoid the nonlinearity of this expression.

In the present study, to reformulate Eq. (50) as a linear equation, the original variable NN_t is assumed to be restricted to taking values from a set $RN_t = \{T_{1t}, T_{2t}, \ldots, T_{N_t t}\}$, where T_{mt} represent the discrete value m in time period t and N_t is the number of discrete values proposed by the designer.

The binary variable d_{mt} is defined to select the number of production campaign repetitions in period t. If the campaign is repeated T_{mt} times in period t, then this variable d_{mt} is equal to 1; otherwise, the value is zero.

If some product is elaborated at period t, then constraints (51) and (52) ensure the selection of exactly one option,

$$\sum_{m=1}^{N_t} d_{mt} \leq 1 \quad \forall\ t, \tag{51}$$

$$\sum_{m=1}^{N_t} d_{mt} \geq 1 - x_{i0t} \quad \forall\ i, t. \tag{52}$$

However, if no product is produced at period t, i.e., $x_{i0t} = 1$ for all i, then the binary variable d_{mt} takes a value of zero for all m,

$$d_{mt} \leq I - \sum_{i=1}^{I} x_{i0t} \quad \forall\ m, t. \tag{53}$$

Thus, the number of times that the campaign is repeated in a cyclical manner throughout the time period t can be calculated as follows:

$$NN_t = \sum_{m=1}^{N_t} T_{mt} d_{mt} \quad \forall\, t. \tag{54}$$

By introducing the above expression into Eq. (50), a nonlinear constraint is obtained,

$$CTC_t \sum_{m=1}^{N_t} T_{mt}\, d_{mt} \le H_t \quad \forall\, t. \tag{55}$$

CTC_t does not depend on the subscript m, so Eq. (55) can be rewritten as follows:

$$\sum_{m=1}^{N_t} T_{mt}\, CTC_t\, d_{mt} \le H_t \quad \forall\, t. \tag{56}$$

To eliminate the bilinear product $CTC_t\, d_{mt}$, a nonnegative variable ww_{mt} is introduced. Then, the following linear expressions are used to represent Eq. (56):

$$\sum_{m=1}^{N_t} T_{mt}\, ww_{mt} \le H_t \quad \forall\, t, \tag{57}$$

$$\sum_{m=1}^{N_t} ww_{mt} = CTC_t \quad \forall\, t, \tag{58}$$

$$ww_{mt} \le CTC_t^U d_{mt} \quad \forall\, m, t, \tag{59}$$

where CTC_t^U is an upper bound for the campaign cycle time of period t.

3.4. Objective function

The objective function of the model given by Eq. (60) aims to maximize the net present value (NPV) of the profit throughout the entire time horizon,

$$NPV = \sum_t \sum_i np_{it} QS_{it} - \sum_t \sum_r \kappa_{rt} C_{rt} - \sum_j \sum_k CE_{jk} - \sum_t \left[\sum_r \varepsilon_{rt} \left(\frac{IM_{rt-1} + IM_{rt}}{2} \right) H_t + \sum_i \sigma_{it} \left(\frac{IP_{it-1} + IP_{it}}{2} \right) H_t \right]$$
$$- \sum_t \sum_i (co_{it} q_{it} + cp_{it} \vartheta_{it} + wp_{it} PW_{it}) - \sum_t \sum_r wr_{rt} RW_{rt}. \tag{60}$$

This economic criterion is calculated based on the difference between the revenue due to product sales and the overall costs, which include the purchases of raw materials, investments, inventories, operation, late delivery penalties, and waste disposal costs. To determine the revenues, the product price, np_{it}, is multiplied by the product amount sold in each time period. Parameter κ_{rt} denotes the price of the raw material r used to manufacture the products in time period t, while ε_{rt} and σ_{it} are the inventory costs per unit of raw material and final product, respectively. Furthermore, wp_{it} and wr_{rt} are the unit costs due to expired products and raw materials, respectively. Parameter co_{it} denotes the operating cost coefficient and cp_{it} represents the late delivery cost coefficient. All of the cost coefficients described above consider the time value of money, i.e., they are discounted prices for each time period with a specified interest rate.

The third term in Eq. (60) corresponds to the investment cost of each batch unit k in every stage j, CE_{jk}, which is calculated in Eq. (61) by a power law expression of its size V_j, according to the expression proposed by Voudouris and Grossmann [19]. The parameters α_j and β_j denote specific cost coefficients, which depend on the type of equipment at stage j, and the binary variable u_{jk} expresses whether unit k is used at stage j,

$$CE_{jk} = u_{jk}\, \alpha_j V_j^{\beta_j} \quad \forall j, 1 \le k \le K_j. \tag{61}$$

Note that constraint (61) contains a nonlinear product, which can be eliminated by substituting the sizes of batch equipment at stage j, V_j, for the appropriate discrete sizes in Eq. (5), thereby yielding the following constraint:

$$CE_{jk} = \sum_p u_{jk}\, \alpha_j VF_{jp}^{\beta_j} v_{jp} \qquad \forall j, 1 \le k \le K_j. \tag{62}$$

However, the expression given above still exhibits nonlinearities due to the cross-product $u_{jk} v_{jp}$. Therefore, the nonlinear terms in this constraint can be replaced by linear terms after defining a new continuous variable e_{jkp} through the following constraint:

$$e_{jkp} \ge u_{jk} + v_{jp} - 1 \quad \forall j, 1 \le k \le K_j, 1 \le p \le P_j. \tag{63}$$

It should be emphasized that only when the binary variables u_{jk} and v_{jp} are simultaneously equal to one, variable e_{jkp} is equal to one; otherwise, it takes a value that is equal to zero. In order to force the variable e_{jkp} to lie within 0 and 1, the following bounds are added:

$$0 \le e_{jkp} \le 1 \quad \forall j, k, p. \tag{64}$$

In this manner, the investment cost in Eq. (62) can be expressed as a linear constraint,

$$CE_{jk} = \sum_{p} \alpha_j VF_{jp}^{\beta_j} e_{jkp} \quad \forall j, k. \tag{65}$$

3.5. Formulation summary

In summary, the proposed mathematical formulation for the integrated design, planning, and scheduling of multistage batch plants involves maximizing the objective function represented by Eq. (60) using Eq. (65) as the term of investment cost, subject to constraints (5)–(9), (11)–(46), (48) and (49), (51)–(54), (57)–(59), and (63) and (64).

4. Application of the proposed approach

In this section, we present two examples to illustrate the applicability of the proposed MILP model. Each example was solved with a 0% optimality gap in GAMS [20] using the CPLEX 12.5 solver on an Intel Core i7 CPU at 3.4 GHz, with 8GB of RAM.

4.1. Example 1

The first example involves a multiproduct batch plant producing three products from two different raw materials. Each product recipe requires four batch stages in the facility. In order to reduce idle times, parallel equipment can be considered in each stage, and thus stages 1, 2, and 3 can include up to three, two, and two identical units operating out-of-phase, respectively. A global horizon time of 1 year (6000 h) is assumed, which is divided into four planning periods H_t, each of which is equal to 1500 h.

The processing times, size, and conversion factors were assumed to be equal for all time periods and their values are shown in Table 1. For each period, the prices of raw materials and the final products, as well as the maximum demand values for all of the products are given in Table 2. A minimum product demand of 50% must be satisfied in each period. For each stage, five discrete sizes were available to select the dimensions of the process units. Table 3 shows these sizes and the associated cost coefficients.

The lifetime of the raw materials was equal to two periods whereas that for the products was equal to three periods. The inventory cost coefficients for both the final products and raw materials were 0.1 $/(ton h) and 0.05 $/(ton h), respectively. These values were assumed to be the same for all time periods. The unit costs for late delivery were assumed to be 50% of the product sales price. We considered an annual discount rate of 10% in this study.

As mentioned earlier, in every time period, the number of batches of each product in the production campaign is a decision variable, and thus an upper bound for this value must be proposed. In this example, a maximum of three batches in the campaign composition was assumed for each product i in all time periods, i.e., $NBC_{it}^{U} = 3$.

The lower and upper bounds for the variable NN_t, which represents the number of repetitions of the campaign over H_t, are proposed by the designer based on a consideration of the two extreme types of campaigns that can arise in each time period, i.e.,

Table 1
Example 1 – model parameters.

	Processing time, pt_{ijt} (h)				Size factors, SF_{ijt} (L/kg)				Conversion factors, F_{rit}	
Products	J1	J2	J3	J4	J1	J2	J3	J4	R1	R2
I1	9.3	5.4	4.2	2.0	5.0	2.6	1.6	3.6	0.5	1.5
I2	8.5	5.8	4.1	2.5	4.7	2.3	1.6	2.7	1.0	1.2
I3	9.7	5.5	4.3	2.1	4.2	3.6	2.4	4.5	0.7	1.0

Table 2
Example 1 – economic data and demands.

	Raw material costs, κ_{rt} ($/kg)		Products prices, np_{it} ($/kg)			Maximum demands, DE_{it}^{U} ($\times 10^3$ kg)		
t	R1	R2	I1	I2	I3	I1	I2	I3
1	1.0	0.5	2.05	2.60	2.00	48.0	41.8	38.2
2	1.5	0.8	2.25	2.60	2.20	53.1	47.8	44.1
3	1.5	0.5	2.25	2.40	2.20	64.0	59.3	50.0
4	1.0	0.8	2.05	2.40	2.00	76.7	63.5	60.0

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Table 3
Example 1 – available standard sizes and unit cost data.

Stages	Discrete units sizes, VF_{jp} (L)					Cost coefficients	
	P1	P2	P3	P4	P5	α_j	β_j
J1	2000	2500	3000	4000	5000	135	0.6
J2	1500	2000	2500	3000	3500	148	0.6
J3	1000	1500	2000	2500	3000	140	0.6
J4	500	1000	2000	3000	4000	150	0.6

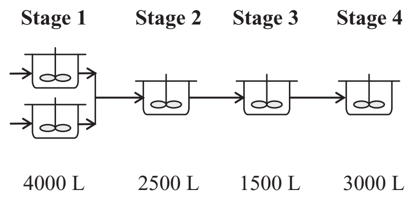

Fig. 1. Example 1 – optimal plant configuration.

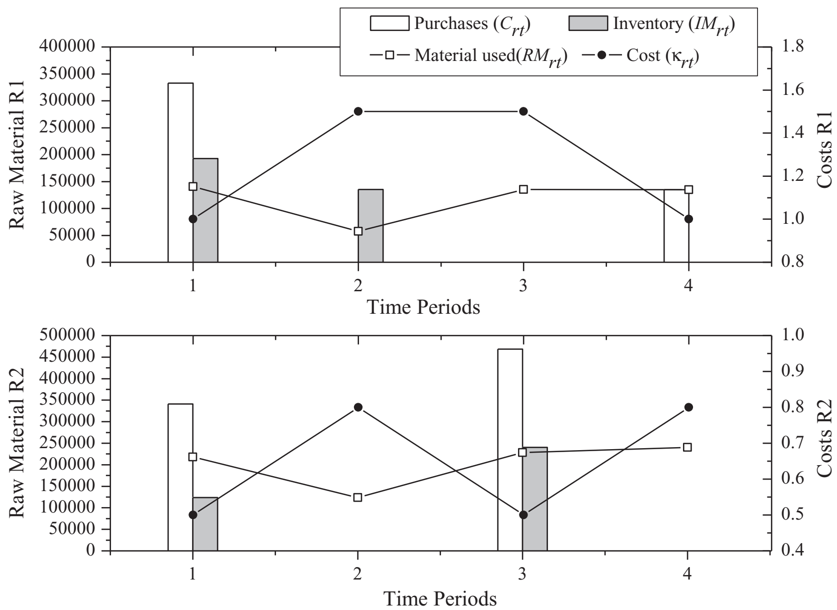

Fig. 2. Example 1 – profile for raw materials.

that with the minimum cycle time and that with the maximum cycle time, and the length of that period. For this example, NN_t, was uniformly discretized by considering 18 discrete points over the interval $[NN_t^L, NN_t^U] = [12, 182]$. Thus, for each period, the step size was equal to 10 and the recurrence relation $T_{mt} = T_{m-1t} + 10$ for $m = 2, \ldots, 18$ with $T_{1t} = 12$, which allowed us to define the value for the variable NN_t.

Under these assumptions, the model comprised 14,371 linear constraints, 3427 continuous variables, and 580 binary variables. The optimal solution had a value of $321,947.48 and it was obtained in 582.23 CPU seconds.

The optimal plant configuration for Example 1 is depicted in Fig. 1. All of the stages were designed with one unit except in stage 1, where two identical parallel units were installed. The unit sizes selected for each stage were 4000 L, 2500 L, 1500 L, and 3000 L, respectively.

Figs. 2–5 show the optimal production planning variables for products and raw materials. Fig. 2 comprises two diagrams, which correspond to raw materials R1 and R2, respectively. Raw material R1 was purchased in periods 1 and 4 whereas R2 was bought in periods 1 and 3. Table 2 shows that both raw materials were acquired when their costs reached the lowest value. For raw material R1, the extra material purchased in period 1 was kept as inventory to meet the production needs in the next two periods when its price was high. Analogously, for raw material R2, the extra material purchased in periods 1 and 3 was kept as inventory for production in subsequent periods.

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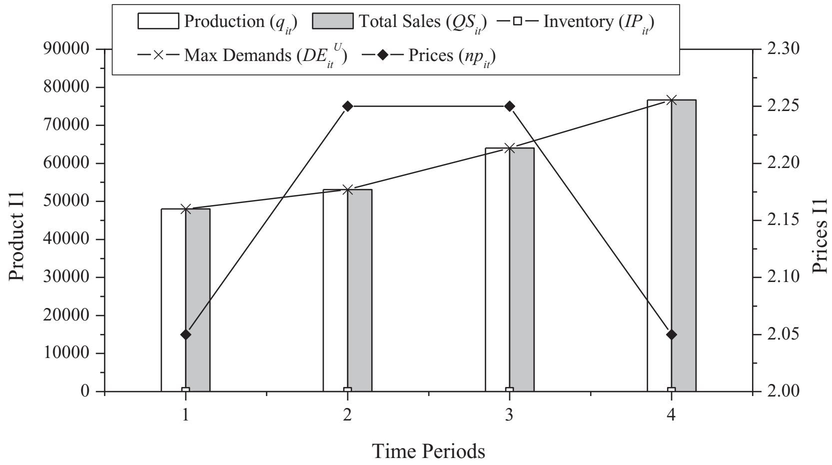

Fig. 3. Example 1 – profile for product I1.

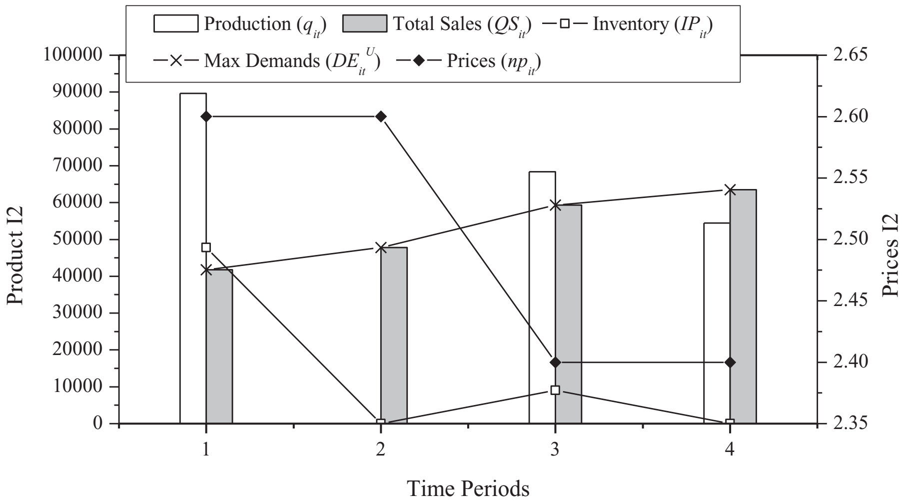

Fig. 4. Example 1 – profile for product I2.

As shown in Figs. 3 and 5, there was no inventory for the final products I1 and I3 because they were produced in all time periods and the amounts produced in each period met the maximum demand. The results for product I2 are shown in Fig. 4, which indicate that an extra amount was produced in time periods 1 and 3, and this was kept as inventory to satisfy the maximum demand in the subsequent periods, but it was not produced in period 2 and its production was lower than the maximum demand in period 4.

The products satisfied the minimum product demands in all time periods, so late deliveries did not occur in any of them. It should be noted that there was no wastage of products or raw materials.

For each period, Table 4 shows the composition of the optimal production campaign, its cycle time, and the number of campaign repetitions throughout the planning horizon. Fig. 6 illustrates the production sequence in the different stages for all time periods.

Product I2 was not produced in period 2, so the campaign for this period only comprised one batch of each of the other products, which was sufficient to meet the production plan. However, in period 4, due to an increase in the production levels of I1 and I3 compared with the previous period, the campaign in period 3 could not be applied successfully. Using the total time horizon, that campaign could not fulfill the production requirements for products I1 and I3. If the batch sizes of products I1 and I3 are increased using the maximum available unit capacities, and the campaign is repeated 42 times over the time horizon, the production levels achieved are lower than the required demand. Thus, the number of batches of products I1 and I3 in the campaign during period 4 is increased by one unit compared with the previous period in order to meet the production levels.

Finally, the economic results are summarized in Table 5.

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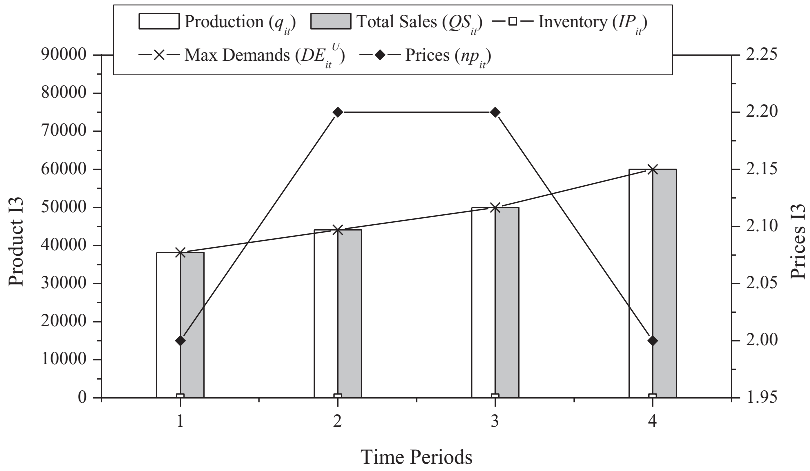

Fig. 5. Example 1 – profile for product I3.

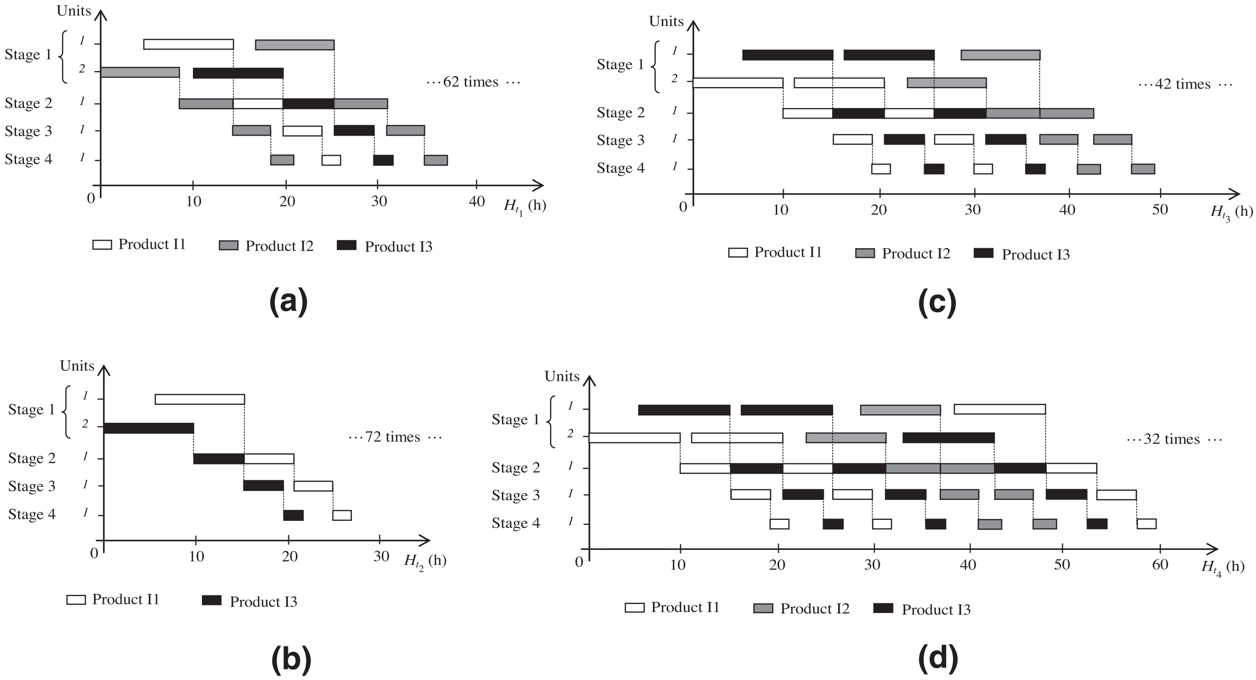

Fig. 6. Gantt chart of the optimal MPC for periods: (a) 1, (b) 2, (c) 3, and (d) 4.

Table 4
Example 1 – optimal production campaign for each time period.

t	NBC_{it} I1	NBC_{it} I2	NBC_{it} I3	CTC_t (h)	NN_t
1	1	2	1	22.5	62
2	1	0	1	10.9	72
3	2	2	2	33.4	42
4	3	2	3	44.3	32

Table 5
Example 1 – economic evaluation results ($).

Description	Optimal value
Sales income	1361277.54
Raw material cost	832573.52
Investment cost	84882.53
Raw material inventory cost	49058.79
Product inventory cost	8165.23
Operating costs	64650.00
Waste disposal cost	0.00
Late delivery penalty	0.00
Total	321947.48

Table 6
Example 2 – model parameters.

	Processing time, pt_{ijt} (h)				Size factors, SF_{ijt} (L/kg)				Conversion factors, F_{rit}	
Products	J1	J2	J3	J4	J1	J2	J3	J4	R1	R2
I1	14	25	7	6	0.7	0.6	0.5	0.5	0.5	1.5
I2	16	18	5	5	0.6	0.7	0.45	0.7	2.0	0.0
I3	10	29	8	4	0.7	0.65	0.5	0.6	0.5	1.5

Table 7
Example 2 – economic data and bounds on demands.

	Raw material costs, κ_{rt} ($/kg)		Products prices, np_{it} ($/kg)			Maximum demands, DE^U_{it} ($\times 10^3$ kg)		
t	R1	R2	I1	I2	I3	I1	I2	I3
1	1.0	0.8	2.25	2.60	2.25	55.0	72.0	20.0
2	1.5	0.5	2.00	2.40	2.00	125.0	144.0	20.0
3	1.7	0.5	2.25	2.40	2.25	85.0	168.0	60.0
4	1.1	0.8	2.00	2.60	2.00	160.0	96.0	20.0

Table 8
Example 2 – available standard sizes and unit cost data.

	Discrete units sizes, VF_{jp} (L)					Cost coefficients	
Stages	P1	P2	P3	P4	P5	α_j	β_j
J1	650	1300	2600	5200	7800	350	0.6
J2	700	1400	2800	5600	8400	350	0.6
J3	250	500	1000	2000	3000	550	0.7
J4	400	800	1600	2400	4800	550	0.7

4.2. Example 2

This example considers a batch facility with four stages, which could produce three products from two raw materials. We assumed that the first and second stages could include up to three parallel units operating out of phase, whereas the last two stages admitted up to two identical units. The process data for this example are presented in Table 6, which show that product I2 was manufactured only by using raw material R1. Some economic data and the maximum demand forecasts over each period are provided in Table 7. Similar to Example 1, a minimum of 50% of the maximum product demand must be satisfied in each period.

Like Example 1, a global planning horizon of one year (6000 h working) was considered, which was divided into four equal time periods, i.e., 1–4, which each corresponded to 3 months (1500 h). The sets of available discrete sizes for the units in each stage and the unit cost coefficients are given in Table 8.

In this example, the maximum numbers of batches for all products in the campaign composition were different and they were proposed by the designer according to their experience. In particular, for every time period, the maximum admissible value was two for product I1, three for product I2, and two for product I3. Moreover, for each time period, the number of campaign repetitions was uniformly discretized between $NN_t^L = 11$ and $NN_t^U = 101$, with a step size length of 10 units. The recurrence relation $T_{mt} = T_{m-1t} + 10$ for $m = 2, \ldots, 10$, with initial condition $T_{1t} = 11$, defines the elements in the discrete options set. All other data necessary to solve this example were the same as those used in Example 1.

The optimal solution for this example was obtained in 710.75 CPU seconds. This solution had a net present value of $ 67,099.22. The example comprised 474 discrete variables, 2349 continuous variables, and 13,307 constraints.

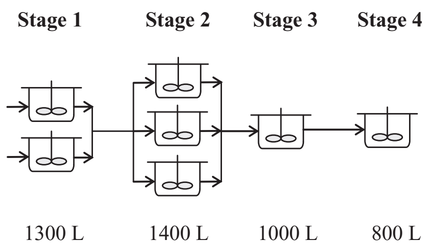

Fig. 7. Example 2 – optimal plant structure.

Table 9
Example 2 – optimal production planning variables for each time period.

	I1 (×10³ kg)			I2 (× 10³ kg)			I3 (×10³ kg)			R1 (×10³ kg)		R2 (×10³ kg)	
t	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	C_{rt}	IM_{rt}	C_{rt}	IM_{rt}
1	55.0	55.0	0.0	93.7	72.0	21.7	42.0	20.0	22.0	678.9	443.0	145.5	0.0
2	128.4	125.0	3.4	93.7	89.7	25.7	0.0	20.0	2.0	0.0	191.4	192.6	0.0
3	81.6	85.0	0.0	58.3	84.0	0.0	68.0	60.0	10.0	0.0	0.0	464.4	240.0
4	160.0	160.0	0.0	58.3	58.3	0.0	0.0	10.0	0.0	196.6	0.0	0.0	0.0

Table 10
Example 2 – optimal production campaign for each time period.

	NBC_{it}			CTC_t (h)	NN_t
t	I1	I2	I3		
1	1	2	1	36	41
2	2	2	0	36	41
3	1	1	1	29	51
4	2	1	0	28	51

Table 11
Example 2 – economic evaluation results ($).

Description	Optimal value
Sales income	1767659.16
Raw material cost	1281172.80
Investment cost	261236.67
Raw material inventory cost	62145.47
Product inventory cost	12105.00
Operating costs	83900.00
Waste disposal cost	0.00
Late delivery penalty	0.00
Total	67099.22

The optimal plant configuration and unit sizes are illustrated in Fig. 7, which shows that two and three units are selected in parallel for stages 1 and 2, respectively, whereas the other stages have a unique equipment item. The unit sizes selected for each stage were 1300 L, 1400 L, 1000 L, and 800 L, respectively.

For each period, Table 9 summarizes the amounts of final products produced and sold, amounts of raw materials purchased for producing all products, and the inventory levels of both the raw materials and products.

The following conclusions can be obtained from Tables 7 and 9. For raw materials, the purchases of raw material R1 were made only in time periods 1 and 4, because the costs had the lowest value. The extra amount acquired in period 1 was held as inventory for the production of final products in the following two periods. In addition, purchases of the raw material R2 were performed in all periods except for the last one when its cost increased. The excess material purchased in period 3 was held as raw material inventory, thereby allowing the production of the final products in the subsequent time period. All of the products were produced in all time periods, except for product I3 in the second and fourth periods, where in period 2, the maximum demand was satisfied by the amount stored from the previous period, and for period 4, the minimum demand was satisfied using the product stored from period 3. Also, note that the production of product I2 in time periods 1 and 2 was higher than the amount sold in the same interval. Thus, the extra amount was kept as inventory to satisfy requirements in the following intervals. A similar behavior was observed for product I1 in time period 2.

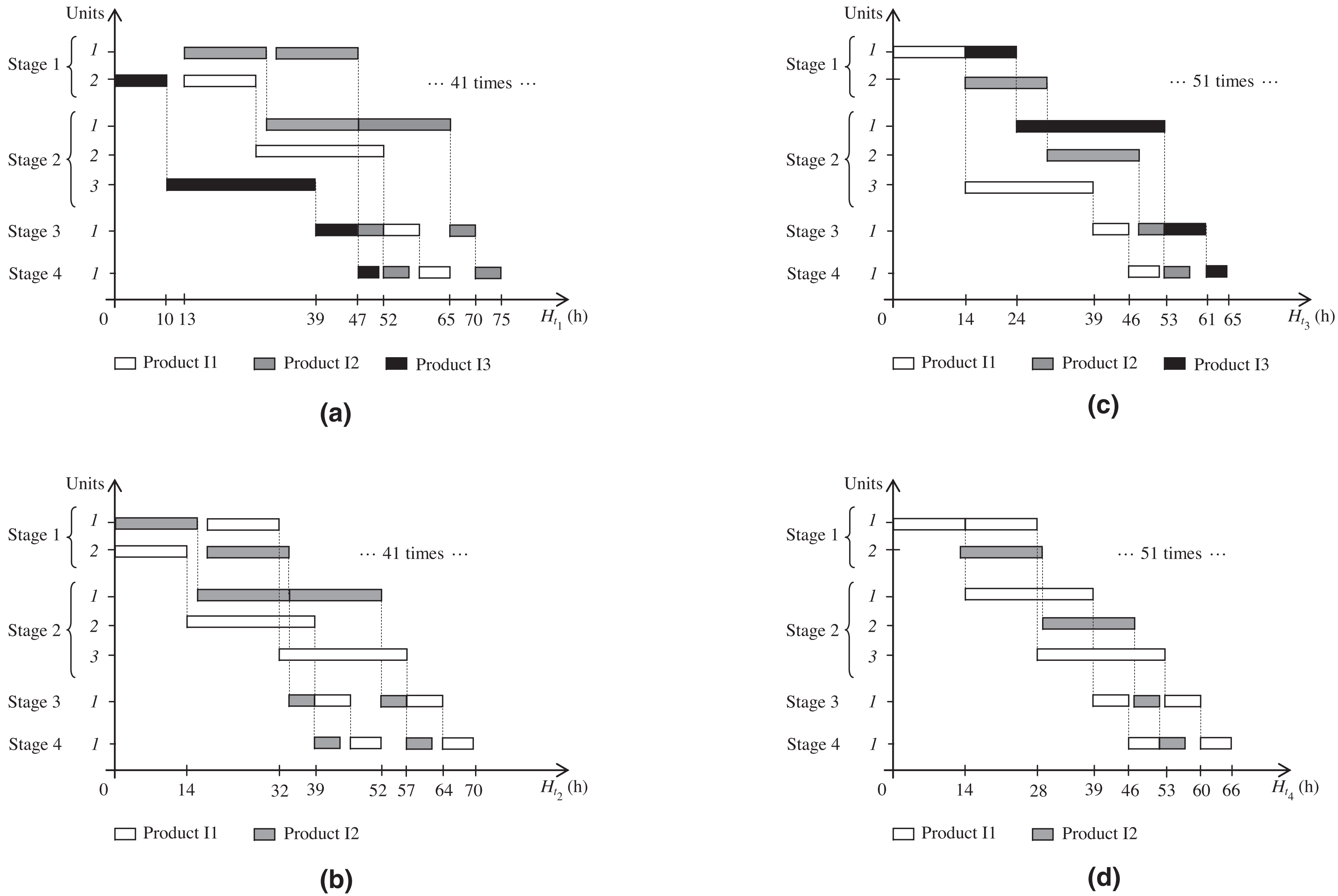

Fig. 8. Gantt chart of the optimal MPC for periods: (a) 1, (b) 2, (c) 3, and (d) 4.

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For each time period, Table 10 shows the optimal campaign composition, the campaign cycle time, and the number of times that this production sequence was repeated over the time interval. It may be noted that the campaign composition was different in all of the time periods. Fig. 8 illustrates the production sequence in the different stages for all time periods.

Finally, an economic summary of this problem is provided in Table 11.

5. Conclusions

In this study, our proposed approach integrates design, production planning, and scheduling decisions in a multiperiod context by addressing a detailed description of the problem. The obtained MILP formulation guarantees the global optimality in a reasonable computational time and it allows us to assess the trade-offs among the different decision variables of the problem.

An operation mode that includes MPCs is assumed. This approach is usually more appropriate under stable contexts, but from a strategic viewpoint, this operation mode provides a more realistic approximation of the problem than SPCs in any scenario. Thus, considering the production flows obtained, new assessment criteria can then be incorporated. They are not included in the proposed formulation, but other aspects of the business, such as logistics, purchasing decisions, and distribution policies, can also be considered and evaluated.

For design decisions, a realistic case is considered, where discrete unit sizes are available for the potential equipment to be installed. Due to seasonal or market fluctuations, variations in different elements of the problem during every time period are considered based on deterministic values for planning decisions. Finally, scheduling decisions that allow the optimal production campaign and its sequence to be obtained for each time period are modeled using a continuous-time, slot-based representation.

We presented two examples to illustrate the use and the capabilities of the proposed model. The results obtained demonstrate that the problem variables evidently interact to generate different production policies and schedules in order to adapt them to the context conditions. Thus, the proposed approach provides a valuable tool for guiding design, planning, and scheduling decisions in batch plants.

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