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Offset-free multi-model economic model predictive control for changing economic criterion

Antonio Ferramosca[a,*], Alejandro H. González[b], Daniel Limon[c]

[a] CONICET – Universidad Tecnológica Nacional (UTN), Facultad Regional de Reconquista, 27 de Abril, 1000, 3560 Reconquista, Santa Fe, Argentina
[b] Institute of Technological Development for the Chemical Industry (INTEC), CONICET-Universidad Nacional del Litoral (UNL), Güemes 3450, 3000 Santa Fe, Argentina
[c] Departamento de Ingeniería de Sistemas y Automática, Escuela Superior de Ingenieros, Universidad de Sevilla, Camino de los Descubrimientos s/n., 41092 Sevilla, Spain

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ABSTRACT

Economic Model Predictive Controllers, consisting of an economic criterion as stage cost for the dynamic regulation problem, have shown to improve the economic performance of the controlled plant. However, throughout the operation of the plant, if the economic criterion changes – due to variations of prices, costs, production demand, market fluctuations, reconciled data, disturbances, etc. – the optimal operation point also changes. In industrial applications, a nonlinear description of the plant may not be available, since identifying a nonlinear plant is a very difficult task. Thus, the models used for prediction are in general linear. The nonlinear behavior of the plant makes that the controller designed using a linear model (identified at certain operation point) may exhibit a poor closed-loop performance or even loss of feasibility and stability when the plant is operated at a different operation point. A way to avoid this issue is to consider a collection of linear models identified at each of the equilibrium points where the plant will be operated. This is called a multi-model description of the plant. In this work, a multi-model economic MPC is proposed, which takes into account the uncertainties that arise from the difference between nonlinear and linear models, by means of a multi-model approach: a finite family of linear models is considered (multi-model uncertainty), each of them operating appropriately in a certain region around a given operation point. Recursive feasibility, convergence to the economic setpoint and stability are ensured. The proposed controller is applied in two simulations for controlling an isothermal chemical reactor with consecutive-competitive reactions, and a continuous flow stirred-tank reactor with parallel reactions.

1. Introduction

The main goal of advanced control strategies is to operate the plants as close as possible to the economically optimal operation point, while ensuring stability. In the process industries, this objective is achieved by means of a hierarchical control structure [28,11]: an economic optimization level – usually referred as Real Time Optimizer (RTO) – sends the economically optimal setpoints to an MPC layer, which calculates the optimal control action to be sent to the plant, in order to regulate it as close as possible to the setpoint, taking into account a dynamic model of the plant, constraints, and stability requirements [23,31].

The time-scale separation between the RTO and MPC layers that this hierarchical control scheme produces has two main consequences on the economic performance of the plant. The first one is that the economic setpoint calculated by RTO may be inconsistent or unreachable with respect to the dynamic layer [17]. A solution to this issue is to add a new optimization level in between of RTO and MPC, referred as the steady state target optimizer (SSTO). The SSTO calculates the steady state to which the system has to be stabilized, solving a linear or quadratic programming and taking into account information from the RTO and the linear model used in the MPC [29,34,21].

The second consequence is produced by the way the MPC controller is designed. Usually the MPC control law is designed to ensure asymptotic tracking of the setpoint, without taking into account the issue of transient costs [5]. This way to operate, which is practically optimal when the setpoint does not change with respect to the dynamic of the system, may provide poor economic

* Corresponding author.
E-mail addresses: ferramosca@santafe-conicet.gov.ar (A. Ferramosca), alejgon@santafe-conicet.gov.ar (A.H. González), dlm@us.es (D. Limon).

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performance in those applications (usually characterized by frequent changes in the economic criterion) for which the cost in the transient is more significant than the cost at the steady state. Economic MPC [30] is the solution proposed in the last few years to overcome these drawbacks.

Different ways to approach this control problems have been proposed in literature: Dynamic Real Time Optimizer (D-RTO) [9,17,33] solves a dynamic economic optimization and delivers target trajectories (instead of target steady state) to the MPC layer; the one-layer MPC strategies integrate the RTO economic cost function as a stationary part of the MPC cost function [1,35,2]; Economic MPC considers the nonlinear economic cost of the RTO as the stage cost of the dynamic regulation problem. This method has been widely studied in the last few years, and Lyapunov stability has been proved for different cases [9,16], resorting to strong duality assumptions [10,12], and to dissipativity assumptions [5,3,15,24].

From a practical point of view, the economic criterion to be minimized may vary during the operation of a plant, for both: (i) market fluctuations, which may cause changes in the cost function and in the prices that parameterize this function, and (ii) variations in disturbances estimation or constraints, due to data reconciliation algorithms. Thus, EMPC formulations characterized by either time-varying or parameter-varying stage costs seem to be more suitable [6,7,12].

In general, in industrial applications, a nonlinear description of the plant may not be available, since identifying a nonlinear plant could be a very difficult task. On the other hand, the methods for the identification of linear models are very easy to implement, and are common practice in industries. For this reason, MPC controllers are designed using a linear model, which may not represent the behavior of the plant in all its operation points. For the case of petrochemical processes, for instance, the plant to be controlled is nonlinear, but has sparse operation points with different economic behaviors. The nonlinear behavior of the plant makes that the controller designed using a linear model (identified at certain operation point) may exhibit a poor closed-loop performance or even loss of feasibility and stability when the economically optimal operation point is changed. A convenient form of representing these plant-model uncertainties is by considering a finite family of linear models identified at each of the equilibrium points where the plant will be operated. This way, each operating point allows one to obtain a linear model sufficiently accurate to describe the system, and which operates appropriately in a certain region around such equilibrium point. Furthermore, since not many operating points are considered in the operation of this kind of systems, only few linear models could be required to describe the complete operation of the plant. This approach, called multi-model description of the plant, is a formulation of robust MPC which has shown to be of interest from a theoretical point of view [8,14,13,22] as well as for practical implementation [28].

In this work, the economic MPC for changing economic costs presented in [12], has been extended to the case of a multi-model representation of the plant and offset-free estimation. To this aim, a finite family of linear models is obtained, which describes the behavior of the plant to be controlled, in different operation points. These points are economically optimal steady states for the plant under a certain choice of the economic criterion. Following the idea of [8], the models are required to share the same applied control actions and a contractive constraint is imposed. However, to the aim of reducing conservativeness, the models are required to have only a few control actions (in the optimal control) equal to each other, while the remaining controls in the sequence are free degrees of freedom for each model. Recursive feasibility, stability and convergence to the optimal operation point are always ensured, no matter which model of the family represents the true plant.

The proposed controller is applied in two simulations for controlling an isothermal chemical reactor with consecutive-competitive reactions, and a continuous flow stirred-tank reactor with parallel reactions. The results of such simulations show how the multi-model approach ensures feasibility and stability when the economically optimal operation point changes, as well as better economic performance than nominal offset-free controllers.

The work is organized as follows. In Section 2 the problem is stated. In Section 3 the proposed multi-model Economic MPC is presented. In this section, Lyapunov stability of the proposed controller is proved, and its main properties are presented. Finally, illustrative examples and conclusions of this study are provided in Sections 4 and 5.

2. Problem statement

Consider a system described by an nonlinear discrete time-invariant model

$$x_p^+ = f(x_p, u) \tag{1}$$

where $x_p \in \mathbb{R}^n$ is the measured state of the plant to be controlled, $u \in \mathbb{R}^m$ is the current control vector, and x_p^+ is the successor state. Function $f(x, u)$ is assumed to be continuous and differentiable at any equilibrium point. The solution of this system for a given sequence of control inputs $\mathbf{u}$ and initial state x_p is denoted as $x_p(j) = \phi(j; x_p, \mathbf{u})$ where $x_p = \phi(0; x_p, \mathbf{u})$. The state of the system and the control input applied at sampling time k are denoted as $x_p(k)$ and $u(k)$ respectively. The system is subject to hard constraints on state and control:

$$x_p(k) \in \mathcal{X}, \quad u(k) \in \mathcal{U} \tag{2}$$

for all $k \geq 0$.

Assumption 1. $\mathcal{X}$ and $\mathcal{U}$ are convex and compact, and both sets contain the origin in their interior.

The steady-state conditions of the plant (x_s, u_s) are such that (1) is fulfilled, i.e.

$$x_s = f(x_s, u_s) \tag{3}$$

Consider now, a nonlinear function $f_{eco}(x, u, \rho)$ that is a measure of the economic objectives of the plant. The parameter ρ describes prices, costs or production goals, that might be varying during the operation of the plant. This parameter has to be considered as an input to the RTO layer, resulting from the economic scheduling and planning, and may be time-varying due to market fluctuations or data reconciliation. Thus, let us define the RTO problem as follows:

Definition 1. The optimal operation of the plant is given by the steady state (x_s, u_s), which satisfies

$$(x_s, u_s) = \arg\min_{(x,u)} f_{eco}(x, u, \rho) \tag{4}$$

$$\begin{aligned} s.t. \quad & x \in \mathcal{X}, \qquad u \in \mathcal{U} \\ & x = f(x, u) \end{aligned}$$

Notice that the optimal operation point depends on the value of ρ, that is $(x_s(\rho), u_s(\rho))$. However, for the sake of clarity, in what follows, we will use the notation (x_s, u_s).

Assumption 2. The cost $f_{eco}(x, u, \rho)$ is locally Lipschitz continuous in (x_s, u_s); that is there exists a constant $\Gamma > 0$ such that,

$$|f_{eco}(x, u, \rho) - f_{eco}(x_s, u_s, \rho)| \leq \Gamma |(x, u) - (x_s, u_s)|$$

for all ρ and all $(x, u) \in \mathcal{X} \times \mathcal{U}$ such that $|x - x_s| \leq \varepsilon$ and $|u - u_s| \leq \varepsilon$, $\varepsilon > 0$.

2.1. Multi-model description of the plant

In industrial application of MPC, in general, function $f(x, u)$ may not be available, may not be identified or may not be used by the controller (for instance due to its complexity). For these reasons, linear models are usually utilized for prediction. These models are generally identified at a certain operation point of the plant. The offset-free design of the MPC is a well-know solution to robustify the controller and to reduce the gap between the model and the plant [27,26,19,20]: a disturbance model is added to the prediction model in order to estimate and cancel the difference between predictions and measurements.

However, the linear model used for prediction represents the behavior of the plant only around a certain operation point, i.e. around the point at which the plant has been identified. If the optimal operation of the plant changes, that is if ρ changes, the optimal steady state solution to Problem (4) also changes. Hence the linear model used for prediction may not describe the behavior of the plant in this new point anymore and the offset-free strategy may fail to accomplish its task.

The main idea of the proposed work is to incorporate, in the MPC design, a multi-model description of the plant, in order to cope with model uncertainties. To this aim, a collection Π of L linear models is supposed to be known [8,14]. These models are identified at different (possible) operating points of the real plant, in such a way that model uncertainties are represented.

Let us define the set of possible linear plants as $\Pi = \{\pi_1, \ldots, \pi_L\}$, where π_i corresponds to the particular plant (A_i, B_i), $i \in \mathbb{I}_{1:L}$. Following the idea of the offset-free control, for each model $\pi_i \in \Pi$, $i \in \mathbb{I}_{1:L}$, we propose the following augmented system for predictions:

$$\begin{bmatrix} x_i^+ \\ d_i^+ \end{bmatrix} = \begin{bmatrix} A_i & I_n \\ 0 & I_n \end{bmatrix} \begin{bmatrix} x_i \\ d_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i \tag{5}$$

where d_i represents the disturbance corresponding to model π_i. Notice that, the equilibrium condition for the augmented system (5) is such that

$$\begin{bmatrix} A_i - I, & B_i \end{bmatrix} \begin{bmatrix} x_i^\infty \\ u_i^\infty \end{bmatrix} + d_i^\infty = 0 \tag{6}$$

is satisfied.

Moreover, let us assume that, for any time k, there exists one model in the family that best describes the plant at the current operation point. This model will be denoted as $\pi_r = (A_r, B_r) \in \Pi$. It is assumed that it is not know which model of the family Π is π_r. The only known fact is that $\pi_r \in \Pi$. On the other hand, let us define as $\pi_{no} \in \Pi$, the nominal model, that is the model that is used to design the controller.

Assumption 3. The pair (A_i, B_i) is controllable and each model π_i is subject to constraints (2).

Under Assumption 3, we define the sets of admissible equilibrium states and inputs of system (5) as

$$\mathcal{Z}_{s,i} = \{(x_i, u_i) \in \eta(\mathcal{X} \times \mathcal{U}) \mid x_i = A_i x_i + B_i u_i + d_i\} \tag{7}$$

$$\mathcal{X}_{s,i} = \{x_i \in \mathcal{X} \mid \exists u_i \in \mathcal{U} \text{ such that } (x_i, u_i) \in \mathcal{Z}_{s,i}\} \tag{8}$$

where $\eta \in [0, 1)$ is a constant arbitrarily close to 1, added in order to avoid possible losses of controllability caused by active constraints.

Remark 1. The models π_i that compose the family Π are strictly related to the parameter ρ of function $f_{eco}(x, u, \rho)$. In fact, they are identified at different operation points of the nonlinear plant (1), which are the minimizers of Problem (4) for different values of ρ. This implies that the values that ρ may assume, have to be known a priori in order to generate the family of linear models Π.

2.2. Observer

The estimation of the state and the disturbance of each model $\pi_i \in \Pi$ are done by means of the following observers of the augmented system:

$$\hat{x}_i(k+1) = A_i\hat{x}_i(k) + B_i u_i(k) + \hat{d}_i(k) + L_i^x(\hat{x}_i(k) - x_p(k) + \hat{d}_i(k)) \tag{9}$$

$$\hat{d}_i(k+1) = \hat{d}_i(k) + L_i^d(\hat{x}_i(k) - x_p(k) + \hat{d}_i(k)) \tag{10}$$

where $\hat{x}_i(k) \in \mathbb{R}^n$ is the observed state at time k, $\hat{d}_i(k) \in \mathbb{R}^n$ is the estimated disturbance at time k, and $x_p(k)$ is the measured state of the real plant at time k. Furthermore, L_i^x and L_i^d are the observer gains of the state and disturbance estimation respectively, and they have to be chosen in such a way that each observer is stable [27,26,19].

Remark 2. [Observability] Conditions for observability of model (5) are given in [27,19]. In these works, the augmented system is of the form

$$\begin{bmatrix} x(k+1) \\ d(k+1) \end{bmatrix} = \begin{bmatrix} A & B_d \\ 0 & I_{n_d} \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(k) \tag{11}$$

$$y(k) = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} x(k) \\ d(k) \end{bmatrix} \tag{12}$$

where $B_d \in \mathbb{R}^{n \times n_d}$ is a state disturbance matrix, with $d(k) \in \mathbb{R}^{n_d}$, and $y(k) \in \mathbb{R}^p$ being the output of the system. System (11) is observable if and only if (C, A) is observable and

$$rank\left(\begin{bmatrix} A - I_n & B_d \\ C & C_d \end{bmatrix}\right) = n + n_d \tag{13}$$

This last condition is satisfied if and only if the dimension of the disturbance is taken smaller than or equal to the dimension of the measurements, that is if $n_d \leq p$ [19, Proposition 1].

In the present work, we are considering that the complete state of the nonlinear plant (1) is measurable, with $B_d = I_n$, $C = I_n$, $C_d = 0_{n\times n}$, $p = n$, $n_d = n$, and $A \neq 2I_n$. Hence, condition (13) is satisfied.

Remark 3. Note that the equilibrium condition of the observer satisfies Eq. (6). In fact, at the equilibrium, the estimated disturbance $\hat{d}$ converges and, since we are assuming that observer is stable, this implies that $\hat{x}_i^\infty - x_p^\infty + \hat{d}_i^\infty = 0$. Then,

$$\begin{bmatrix} A_i - I & B_i \end{bmatrix} \begin{bmatrix} \hat{x}_i^\infty \\ u_i^\infty \end{bmatrix} + \hat{d}_i^\infty = 0 \tag{14}$$

for all $i \in \mathbb{I}_{1:L}$.

For the sake of clarity let us define the following extended vectors of states and disturbances:

$$z = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix}, \quad w = \begin{bmatrix} d_1 \\ \vdots \\ d_L \end{bmatrix} \tag{15}$$

where $z \in \mathbb{R}^{Ln}$, and $w \in \mathbb{R}^{Ln}$.

3. Multi-model economic MPC formulation

In this section, the economic MPC formulation, based on a multi-model approach, is proposed. As standard in economic MPC literature [30,7], the economic cost function f_{eco} is used as stage cost of the MPC controller. In particular, in the present work we extend the formulation presented in [12] to take into account a multi-model description of the plant. This particular formulation is suitable for the case of changes in the economic criterion (i.e. changes in ρ) since it always ensures feasibility thanks to a slightly modified cost and a relaxed terminal constraint, that accounts for any admissible steady state. Let us define the following economic dynamic cost:

$$V_{N_i}^{dyn}(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i) = \sum_{j=0}^{N-1} f_{eco}(x_i(j) - x_i(N-1) + x_s, u_i(j) - u_i(N-1) + u_s, \rho) \quad (16)$$

for all $i \in \mathbb{I}_{1:L}$. The cost function of the proposed MPC is then given by:

$$\Sigma_N^e(\hat{z}, \rho; \mathbf{v}) = V_{N_{no}}^{dyn}(\hat{x}_{no}, \hat{d}_{no}, \rho; \mathbf{u}_{no}) + \sum_{i=1}^{L} \ell(x_i(N-1), u_i(N-1)) \quad (17)$$

where $\hat{z}$ is the observed state at time k, $x_i(j)$ is the prediction based on the model π_i and it is such that $x_i(0) = \hat{x}_i$. The couple $(x_i(N-1), u_i(N-1))$ represents the prediction, for each model $i \in \Pi$, of $(x_i(j), u_i(j))$ for $j = N-1$; these predictions are forced, in the optimization problem, to be an admissible equilibrium point, that is $(x_i(N-1), u_i(N-1)) \in \mathcal{Z}_{s,i}$, $i \in \mathbb{I}_{1:L}$. This way, the terminal state is let as an extra degree of freedom, in order to always ensure feasibility. Moreover, note that, when $x_i(N-1)$ converges to x_s, then the stage cost function $V_{N_i}^{dyn}(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i)$ becomes equals to the economic cost $f_{eco}(x, u, \rho)$.

Function $\ell(x_i, u_i)$ is the so-called offset cost function, and is defined as follows:

Definition 2. Let $\ell(x_i, u_i)$ be a positive definite convex function such that the unique minimizer of

$$\min_{(x_i, u_i) \in \mathcal{Z}_{s,i}} \ell(x_i, u_i) \quad (18)$$

is (x_s, u_s). □

Assumption 4. There exist positive constants γ_i such that

$$\ell(x_i, u_i) - \ell(x_s, u_s) \geq \gamma_i \|x_i - x_s\| \quad (19)$$

□

Let $\mathbf{v} = \{\mathbf{u}_1, \mathbf{u}_2 \ldots, \mathbf{u}_L\}$ be the set of control sequences to be calculated. This set is such that the first M elements of each sub-sequence $\mathbf{u}_i$, with $1 \leq M \leq N$, are forced to be equal for all models in Π. The remaining $N-M$ elements are free degrees of freedom for each model. This means that at least the first element is unique for all models in Π. Notice also that, if $M = N$, then the whole sequence is unique for all models.

At time k, the optimization problem $P_N^e(\hat{z}, \hat{w}, \rho, \tilde{\mathbf{v}})$ to be solved is given by:

$$\begin{aligned}
\min_{\mathbf{v}} \quad & \Sigma_N^e(\hat{z}, \rho; \mathbf{v}) \\
s.t. \quad & x_i(0) = \hat{x}_i, \quad d_i(0) = \hat{d}_i, && i \in \mathbb{I}_{1:L} \\
& x_i(j+1) = A_i x_i(j) + B_i u(j) + d_i(j), \quad j \in \mathbb{I}_{0:N-1}, && i \in \mathbb{I}_{1:L} \\
& d_i(j+1) = d_i(j), \quad j \in \mathbb{I}_{0:N-1}, && i \in \mathbb{I}_{1:L} \\
& x_i(j) \in \mathcal{X}, \quad u_i(j) \in \mathcal{U}, \quad j \in \mathbb{I}_{0:N-1}, && i \in \mathbb{I}_{1:L} \\
& u_i(j) = u_m(j), \quad j \in \mathbb{I}_{0:M-1}, \quad i \neq m, && i, m \in \mathbb{I}_{1:L} \\
& (x_i(N-1), u_i(N-1)) \in \mathcal{Z}_{s,i}, && i \in \mathbb{I}_{1:L} \\
& V_{N_i}^e(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i) \leq V_{N_i}^e(\hat{x}_i, \hat{d}_i, \rho; \tilde{\mathbf{u}}_i), && i \in \mathbb{I}_{1:L}
\end{aligned} \quad (20)$$

where the last constraint, is a robustness constraint [8], and

$$V_{N_i}^e(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i) = V_{N_i}^{dyn}(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i) + \ell(x_i(N-1), u_i(N-1)) \quad (21)$$

for all $i \in \mathbb{I}_{1:L}$.

The parameter of problem (20), $\tilde{\mathbf{v}}$, is a feasible solutions to problem (20), based on a solution to the same problem at time $k-1$. That is, defining as $\mathbf{v}^0(k-1)$ the optimal solution to Problem (20) at time $k-1$, then

$$\tilde{\mathbf{v}}(k) = \{\tilde{\mathbf{u}}_1(k), \tilde{\mathbf{u}}_2(k), \ldots, \tilde{\mathbf{u}}_L(k)\}$$

where $\tilde{\mathbf{u}}_i(k) = \{u_i^0(1; k-1), u_i^0(2; k-1), \ldots, u_i^0(N-1; k-1), u_i^0(N-1; k-1)\}$ is a sequence made by shifting one step ahead the optimal sequence $\mathbf{u}_i^0(k-1)$ and adding in the tail the admissible equilibrium input at time $k-1$.

Note that, if $M = N$, then $\mathbf{u}_i = \mathbf{u}$ for all $i \in \mathbb{I}_{1:L}$. Therefore $\mathbf{v} = \{\mathbf{u}, \mathbf{u} \ldots, \mathbf{u}\}$, and $\tilde{\mathbf{v}} = \{\tilde{\mathbf{u}}, \tilde{\mathbf{u}} \ldots, \tilde{\mathbf{u}}\}$.

The optimal value of the decision variables will be denoted as $\mathbf{v}^0$ and $\mathbf{u}_i^0$. The control law, in the receding horizon fashion, is given by $\kappa_N^e(\hat{z}, \hat{w}, \rho, \tilde{\mathbf{v}}) = u_i^0(0; \hat{z}, \hat{w}, \rho, \tilde{\mathbf{v}})$. Notice that this control action is unique for all models in Π, since M must be at least equal to 1.

The feasible region of Problem (20), $\mathcal{X}_N$, is the set of state of the real plant, x_p, for which a feasible solution to Problem (20) exists, that is

$$\begin{aligned}
\mathcal{X}_N = \{x_p \in \mathcal{X} \mid \exists \mathbf{v} \text{ such that for } i \in \mathbb{I}_{1:L}: \; x_i(j) \in \mathcal{X}, \\
u_i(j) \in \mathcal{U}, \quad j \in \mathbb{I}_{0:N-1}; (x_i(N-1), u_i(N-1)) \\
\in \mathcal{Z}_{s,i}; V_{N_i}^e(\hat{x}_i, \hat{d}_i, \rho; \mathbf{u}_i) \leq V_{N_i}^e(\hat{x}_i, \hat{d}_i, \rho; \tilde{\mathbf{u}}_i)\}
\end{aligned}$$

Remark 4. The idea of optimizing over a set of control sequences with just M elements in common, with $1 \leq M \leq N$, represents a different solution than the one proposed in [8]. In this way, one can choose the degrees of freedom of each model in the family, reducing conservativeness. Indeed, the choice of M is a trade-off between conservativeness and computational burden. In fact, with this solution, the number of optimization variables of Problem (20) is $(M + L(N-M))m$; so, the smallest the value of M, the less conservative the control problem, but the higher the number of optimization variables.

Remark 5. The constraint $(x_i(N-1), u_i(N-1)) \in \mathcal{Z}_{s,i}$ is equivalent to imposing $x_i(N) = x_i(N-1)$. In other words, the triplets $(x_i(N-1), u_i(N-1), d_i(N-1))$ define admissible equilibrium points, such that $x_i(N) = x_i(N-1) = A_i x_i(N-1) + B_i u_i(N-1) + d_i(N-1) \in \mathcal{X}_{s,i}$. In this way, the terminal constraint necessary for stability is relaxed, in such a way that the state at the end of the horizon is forced to be just an equilibrium point, and not the optimal steady state x_s.

Remark 6. The last constraint of Problem (20) is the key point of the proposed approach. In fact, it ensures that the obtained control sequence produces a cost decreasing for all models $\pi_i \in \Pi$. This way, a candidate Lyapunov function based on the cost given in Eq. (21), can be proposed to prove asymptotic stability.

Remark 7. Note that the proposed controller, and hence the resultant control law, are not based on a switching strategy. The ingredients of Problem (20) do not change during the operation of the plant, neither the models. The parameter ρ may by subject to changes, since it describes prices, costs or production goals, that might be varying during the operation of the plant. A change of ρ means a change in the economic objective, and hence in the economically optimal operation point, but does not determine a change of any other ingredient of Problem (20).

Remark 8. Note that if the variation of ρ is continuous and is small enough to produce a small effect on the cost, then the proposed controller is Input-to-State Stable with respect to this variation of ρ [18].

3.1. Stability

The usual way to prove asymptotic stability of an MPC controller is to use the optimal value of the cost function as a Lyapunov function. This approach, however, cannot be used in an economic MPC framework [30,5], and in particular in the MPC algorithm proposed in this work.

In this section we will make some assumptions, and definitions will be given in order to derive a candidate Lyapunov function for the proposed controller. First, let us define model $\pi_r = (A_r, B_r) \in \Pi$ as the linear model that (locally) represents the behavior of the nonlinear plant at the operation point (x_s, u_s).

Note that the steady state condition for model π_r is such that $x_s = A_r x_s + B_r u_s + d_{r,s}^{\infty}$

Then, function (21), associated to model π_r, is

$$V_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r) = \sum_{j=0}^{N-1} f_{eco}(x_r(j) - x_r(N-1) + x_s, u_r(j) - u_r(N-1) + u_s, \rho) + \ell(x_r(N-1), u_r(N-1)) \quad (22)$$

The previous cost is not necessarily positive definite and then it cannot be used as a candidate Lyapunov function. However, under the assumption of dissipativity of the system with respect to the economic cost function [10,30,5,12], a candidate Lyapunov function can be derived.

Assumption 5. There exists a function $\lambda : \mathcal{X} \to \mathbb{R}$ such that

$$\min_{x,u} f_{eco}(x, u, \rho) + \lambda(x) - \lambda(x^+) \geq f_{eco}(x_s, u_s, \rho) \quad (23)$$

$$s.t. \quad x \in \mathcal{X}, u \in \mathcal{U} \quad (24)$$

with $x^+ = A_r x + B_r u + d_r$. Moreover, defining the rotated cost function

$$F(x, u, \rho) \triangleq f_{eco}(x, u, \rho) + \lambda(x) - \lambda(x^+) - f_{eco}(x_s, u_s, \rho) \quad (25)$$

there exists a $\mathcal{K}$-functions α_F such that $F(x, u, \rho) \geq \alpha_F(\|x - x_s\|)$. □

Based on this assumption, let us define the following *rotated* cost function $\tilde{F}(x, u, p) = f_{eco}(x - x(N-1) + x_s, u - u(N-1) + u_s, \rho) + \lambda(x) - \lambda(x^+) - f_{eco}(x_s, u_s, \rho)$

This function satisfies the following properties [12]:

Property 1.

1 *For all* $(x(N-1), u(N-1)) = (x_s, u_s)$, $\tilde{F}(x, u, \rho) = F(x, u, \rho)$.
2 *For all* $(x, u) = (x(N-1), u(N-1))$, $\tilde{F}(x, u, \rho) = F(x_s, u_s, \rho) = 0$
3 $\tilde{F}(x, u, \rho) \geq \alpha_F(\|x - x(N-1)\|)$ *for certain a* $\mathcal{K}$*-function* α_F. □

Define also, the *rotated* offset cost function $\tilde{\ell}(x, u)$ as follows:

Definition 3. The *rotated* offset cost function is given by:

$$\tilde{\ell}(x, u) = \ell(x, u) + \lambda(x) - \lambda(x_s) - \ell(x_s, u_s) \quad (26)$$

where $\lambda(.)$ is the same as in (25). □

We can now define an *auxiliary* cost function for model π_r:

$$\tilde{V}_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r) = \sum_{j=0}^{N-1} \tilde{F}(x_r(j), u_r(j), \rho) + \tilde{\ell}(x_r(N-1), u_r(N-1)) \quad (27)$$

Lemma 1. *The difference between the cost functions* $\tilde{V}_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r)$ *and* $V_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r)$ *is a constant* δ *given by* $\delta = \lambda(x_r) - \lambda(x_s) - N f_{eco}(x_s, u_s, \rho) - \ell(x_s, u_s)$

The proof to this Lemma can be found in A.1 section.

Based on these results, function (27) can be considered as a candidate Lyapunov function, in order to prove asymptotic stability of the closed-loop system under the control law $\kappa_N^e(\hat{z}, \hat{w}, \rho, \tilde{\mathbf{v}})$.

Consider the following assumption on the controller parameters:

Assumption 6. The prediction horizon N is such that

$$rank(Co_{N,i}) = n, \quad i \in \mathbb{I}_{1:L}$$

where $Co_{N,i} = [A_i^{N-1}B_i \ldots A_i B_i B_i]$ is the N-controllability matrix of system (A_i, B_i). Moreover, there exists a dead-beat control gain K_i, such that $A_i + B_i K_i$, $i \in \mathbb{I}_{1:L}$, has null eigenvalues.

Theorem 1. *Consider that Assumptions 1–6 hold, and consider a given parameter* ρ *for the economic cost* $f_{eco}(x, u, \rho)$*. Assume that there exists a* $\bar{k}$ *such that for* $k \geq \bar{k}$, $\hat{d}_i(k+1) = \hat{d}_i(k)$*, that is the disturbances* d_i *have converged. Then, the closed-loop system converges asymptotically to a steady point* (x_s, u_s) *that satisfies (4), fulfilling the constraints throughout the time.*

Proof. The proof will be given separately for the two cases $M = N$ and $1 \leq M < N$.

Case $M = N$

Consider the measurement at time $k \geq \bar{k}$, $x_p(k)$, the observed state $\hat{x}_i(k)$ and the observed disturbances $\hat{d}_i(k)$, $i \in \mathbb{I}_{1:L}$. Consider also the solution to Problem (20), given by $\mathbf{v}^0(k) = \{\mathbf{u}^0(k), \ldots, \mathbf{u}^0(k)\}$, where

$$\mathbf{u}^0(k) = \{u^0(0; k), u^0(1; k), \ldots, u^0(N-1; k)\}$$

Since $M = N$, from Problem (20), this sequence is feasible for all models $\pi_i \in \Pi$. This sequence of optimal control actions, provides L optimal states sequences of the form

$$\mathbf{x}_i^0(k) = \{x_i^0(0; k), x_i^0(1; k), \ldots, x_i^0(N-1; k), x_i^0(N; k)\}$$

where $x_i^0(0; k) = \hat{x}_i(k)$, and $x_i^0(N; k) = x_i^0(N-1; k)$ (by the constraint $(x_i(N-1), u_i(N-1)) \in \mathcal{Z}_{s,i}$).

Since by assumption, $\hat{d}_i(k+1) = \hat{d}_i(k)$, from equation (10) we have that $L_i^d(\hat{x}_i(k) - x_p(k) + \hat{d}_i(k)) = 0$, and then, by stability of the observer, $\hat{x}_i(k) - x_p(k) + \hat{d}_i(k) = 0$.

This implies that, at time $k+1$,

$$\hat{x}_i(k+1) = A_i \hat{x}_i(k) + B_i u^0(0; k) + \hat{d}_i(k) = x_i^0(1; k)$$

$$\hat{d}_i(k+1) = \hat{d}_i(k)$$

for all $i \in \mathbb{I}_{1:L}$. In particular, we have that:

$$\hat{x}_r(k+1) = A_r \hat{x}_r(k) + B_r u^0(0; k) + \hat{d}_r(k) = x_r^0(1; k)$$

$$\hat{d}_r(k+1) = \hat{d}_r(k)$$

where $\pi_r = (A_r, B_r) \in \Pi$, is the model that best describes the behavior of the plant at the current operation point.

In order to prove asymptotic stability, let us define the following function on model π_r:

$J_r(\hat{x}_r(k), \hat{d}_r(k), \rho; \mathbf{u}(k)) = \tilde{V}_{N_r}^e(\hat{x}_r(k), \hat{d}_r(k), \rho; \mathbf{u}(k)) - \tilde{\ell}(x_r^0(N-1; k), u^0(N-1; k))$

The value of this function corresponding to $\mathbf{u}^0(k)$ is $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$.

Notice that function $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$ satisfies the following properties:

1 $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho) \geq \alpha(\|x_r(k) - x_s\|)$, where α is a $\mathcal{K}$-function.
In fact,

$$J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho) \geq \tilde{F}(x_r(0;k), u^0(0;k), \rho) \geq \alpha_F(\|x_r(k) - x_r^0(N-1;k)\|) \geq \alpha(\|x_r(k) - x_s\|)$$

where the second to last inequality comes from Property 1 and the last one from Lemma 2 in A.1.

2 $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho) \leq \beta(\|x_r(k) - x_s\|)$, where β is a $\mathcal{K}$-function.
In fact, since $\mathcal{X}$ is compact, $J_r^0(x_s, \hat{d}_r^\infty, \rho) = 0$, and $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$ is continuous in $\hat{x}_r(k) = x_s$, then there exists a $\mathcal{K}$-function β such that $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho) \leq \beta(\|x_r(k) - x_s\|)$, for all $x_r \in \mathcal{X}_N$, [31].

Next, we want to prove that function $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$ is non-increasing. To this aim, define the following feasible solution to problem (20), at time $k+1$, $\tilde{\mathbf{u}}(k+1) = \left\{u^0(1;k), \ldots, u^0(N-1;k), u^0(N-1;k)\right\}$

Notice that $\tilde{\mathbf{u}}(k+1)$ is a sequence made by shifting one step ahead the sequence $\mathbf{u}^0(k)$ and adding in the tale the admissible equilibrium input at time k.

Define also the associated state sequence of model π_r

$$\tilde{\mathbf{x}}_r = \left\{x_r^0(1;k), \ldots, x_r^0(N;k), x_r^0(N;k)\right\}$$

and recall that $\hat{x}_r(k+1) = x_r^0(1;k)$ and $\tilde{x}_r(N;k+1) = x_r^0(N;k)$, since $\hat{d}_r(k+1) = \hat{d}_r(k)$. The value of function $J_r(.,.,.)$ corresponding to $\tilde{\mathbf{u}}(k+1)$ and $\tilde{\mathbf{x}}_r$ is then $J_r(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1))$, with $\hat{d}_r(k+1) = \hat{d}_r(k)$. Notice that $\tilde{\ell}(x_r^0(N-1;k+1), u^0(N-1;k+1)) = \tilde{\ell}(x_r^0(N;k), u^0(N;k)) = \tilde{\ell}(x_r^0(N-1;k), u^0(N-1;k))$.

Following standard arguments in MPC literature [31], comparing this last cost with $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$, we obtain

$$\begin{aligned}\Delta J_r &= J_r(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1)) - J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)\\ &= \tilde{V}_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1))\\ &\quad - \tilde{V}_{N_r}^e(\hat{x}_r(k), \hat{d}_r(k), \rho; \mathbf{u}^0(k)) = -\tilde{F}(x_r(k), u^0(0;k), \rho)\end{aligned}$$

On the other hand, the value of the auxiliary cost, due to the optimal solution at time $k+1$, $\mathbf{u}^0(k+1)$, is $\tilde{V}_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \mathbf{u}^0(k+1))$. Comparing it with $\tilde{V}_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1))$, we obtain:

$$\begin{aligned}\Delta \tilde{V}_{N_r}^e(k+1) &= \tilde{V}_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \mathbf{u}^0(k+1))\\ &\quad - \tilde{V}_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1))\\ &= V_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \mathbf{u}^0(k+1)) + \delta\\ &\quad - V_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1)) - \delta\\ &= V_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \mathbf{u}^0(k+1))\\ &\quad - V_{N_r}^e(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1)) \leq 0\end{aligned}$$

where the last inequality comes from the last constraint of problem (20). Hence, we can state that

$$\begin{aligned}\Delta J_r^0 &= J_r^0(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho) - J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)\\ &= J_r^0(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho) - J_r(\hat{x}_r(k+1), \hat{d}_r(k+1),\\ &\quad \rho; \tilde{\mathbf{u}}(k+1)) + \big(J_r(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho; \tilde{\mathbf{u}}(k+1))\\ &\quad - J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)\big) \leq -\tilde{F}(x_r(k), u^0(0;k), \rho)\end{aligned}$$

Then, from Property 1, there exists a $\mathcal{K}$-function α such that:

$$\begin{aligned}&J_r^0(\hat{x}_r(k+1), \hat{d}_r(k+1), \rho) - J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho) \leq -\alpha(\|x_r(k)\\ &\quad - x_r^0(N-1;k)\|) \leq -\alpha_J(\|x_r - x_s\|)\end{aligned} \tag{28}$$

where the last inequality comes from $\alpha(\|x_r(k) - x_r^0(N-1;k)\|) \geq \alpha(\alpha_e(\|x_r(k) - x_s\|)) = \alpha_J(\|x_r(k) - x_s\|)$, following Lemma 2 in A.1.

Then, we can conclude that $J_r^0(\hat{x}_r(k), \hat{d}_r(k), \rho)$ is a Lyapunov function, and (x_s, u_s) is an asymptotically stable equilibrium point for the closed-loop system described by model π_r. Therefore,

$$\lim_{k\to\infty} \|x_r(k) - x_s\| = 0, \quad \lim_{k\to\infty} \|u(k) - u_s\| = 0$$

Then, since (A_r, B_r) represents the model identified at (x_s, u_s), we can state that for $k \to \infty$

$$f(x_p(k), u_s) = A_r x_s + B_r u_s + d_{r,s}^\infty$$

$$x_s = A_r x_s + B_r u_s + d_{r,s}^\infty$$

Then, $f(x_p(k), u_s) = x_s$ for $k \to \infty$, that is

$$\lim_{k\to\infty} \|x_p(k) - x_s\| = 0$$

Case $1 \leq M < N$

In this case, the optimal sequence at time k is such that

$$\mathbf{v}^0(k) = \{\mathbf{u}_1^0(k), \mathbf{u}_2^0(k), \ldots, \mathbf{u}_L^0(k)\}$$

with $u_1^0(0:M-1;k) = u_2^0(0:M-1;k) = \cdots = u_L^0(0:M-1;k) = u^0(0:M-1;k)$. The optimal control sequence for model π_r is given by

$$\mathbf{u}_r^0(k) = \{u^0(0;k), u^0(1;k), \ldots, u^0(M-1;k), u_r^0(M;k)\ldots, u_r^0(N-1;k)\}$$

Following the same steps as for the case $M = N$, we can conclude that

$$\Delta J_r^0 \leq -\tilde{F}(x_r(k), u^0(0;k), \rho)$$

Notice that, from the positive definiteness of the rotated cost, stated in Property 1, we can conclude that, for $k \to \infty$, then $u^0(0;k) \to u_r^0(N-1;k) \to u_s$, and $\hat{x}_r(k) \to x_r^0(N-1;k) \to x_s$. Lyapunov stability can be proved following exactly the same steps as in the proof of case $M = N$.□

3.2. Properties of the proposed controller

The proposed controller enjoys some interesting properties that are hereafter highlighted.

1 **Economic optimality**
Since the controller is formulated as an Economic MPC [30], that is considering an economic function as the stage cost of the MPC controller, then it ensures that the evolution of the closed-loop system will be economically optimal in the transient. Moreover, since it is assumed that there is always one of the model in the set Π that represents (locally) the behavior of the real plant and by Assumption 5, the controller also ensures convergence to the economically optimal operation point of the plant, that is the one that minimizes $f_{eco}(x, u, \rho)$.

2 **Guaranteed feasibility**
Since the constraints of Problem (20) do not depend on the optimal operation point (x_s, u_s), the controller ensures feasibility under any change of the parameter ρ. Indeed, one of the consequence of the effort to maintain feasibility is that, while $x(N-1) \not\approx x_s$, the proposed controller may be suboptimal, in the sense that its performance may differ from the economic MPC presented in [10,30]. It is shown in [12] that this suboptimality

is due to the particular stage cost and to the terminal constraint imposed to the control problem, and it can be considered as the price one has to pay for ensuring feasibility under any change of ρ.

3 **Robustness under model uncertainties**
In general, in industrial practice, the plants to be controlled are nonlinear, but have sparse operation points with different economic behaviors. A way to take into account all those different behaviors is to consider a finite family of linear models (multi-model description), which operate appropriately in a certain region around a given operating point [8,14,13]. In this context, each operating point defines a linear model sufficiently accurate to describe the system. Therefore, the proposed controller ensures stability, feasibility and economic optimality whenever the linear model associated to the operation point is one in the family of the multi-model description, although it is unknown a priori which one. Since this model changes with the operation point, and then with ρ, the resulting control law ensures robustness under changes of ρ.
Note that, as it will be shown in the illustrative examples, if the linear prediction model is not "good enough", that is, it is obtained identifying at an operation point far from the current one, the offset-free strategy by itself, that is, without robustness, may not be able to control the plant. In this case, the multi-model approach will help to get the task done.

4 **Output feedback**
Note that, if the state of the real plant $x_p(k)$ is not measurable but a vector of measured output $y_p(k)=g(x(k))$ is available, an output feedback formulation of the proposed controller can be easily implemented. This can be done by identifying the linear models as in Eqs. (11)–(12) and, provided that the observability requirement given in (13) is fulfilled, by taking an observer of the form

$$\hat{x}_i(k+1) = A_i\hat{x}_i(k) + B_iu_i(k) + B_d\hat{d}_i(k) + L_i^x(C\hat{x}_i(k) + C_d\hat{d}_i(k) - y_p(k)) \tag{29}$$

$$\hat{d}_i(k+1) = \hat{d}_i(k) + L_i^d(C\hat{x}_i(k) + C_d\hat{d}_i(k) - y_p(k)) \tag{30}$$

In this case, the equilibrium condition would be given by

$$\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_i^\infty \\ u_i^\infty \end{bmatrix} + \begin{bmatrix} B_{d,i} \\ C_{d,i} \end{bmatrix} \hat{d}_i^\infty = \begin{bmatrix} 0 \\ y_{p,\infty} \end{bmatrix} \tag{31}$$

and all the results given here would still apply.

4. Illustrative examples

4.1. Isothermal chemical reactor with consecutive-competitive reactions

The proposed controller has been tested in simulation for controlling an isothermal chemical reactor with consecutive-competitive reactions [7,30]. The process considers two reactions of the form

$$P_0 + B \rightarrow P_1$$

$$P_1 + B \rightarrow P_2$$

The nonlinear model describing this process is given by:

$$\dot{x_1} = u_1 - x_1 - \sigma_1x_1x_2 \tag{32}$$

$$\dot{x_2} = u_2 - x_2 - \sigma_1x_1x_2 - \sigma_2x_2x_3 \tag{33}$$

$$\dot{x_3} = -x_3 + \sigma_1x_1x_2 - \sigma_2x_2x_3 \tag{34}$$

Table 1
Linearization points for Simulation I scenario.

Model	x_1	x_2	x_3	x_4	u_1	u_2	ρ
π_1	4.9759	1.0097	3.5787	1.4454	10	7.4791	(7,1)
π_2	5.3813	0.8583	3.4383	1.1804	10	6.6573	(10,2)
π_{no}	3.9044	1.5612	3.7523	3.7523	10	10	(10, −1)

$$\dot{x_4} = -x_4 + \sigma_2x_2x_3 \tag{35}$$

where x_1, x_2, x_3, and x_4 are the concentrations of P_0, B, P_1 and P_2. The control inputs u_1 and u_2 are the inflow rates of P_0 and B. The values of the parameters σ_1 and σ_2 are set as 1 and 0.4, respectively.

The constraints on the states are given by $0 \le x_i \le 10$, $i=1, 2, 3, 4$, while the constraints on the inputs are $0 \le u_j \le 10$, $j=1, 2$.

In the following simulations, three different controllers have been compared: a nominal Economic MPC without robustness consideration, formulated as in [12]; a nominal offset-free Economic MPC, that is an EMPC formulated as the previous one, but considering a prediction model as in Eq. (5); and finally, an Economic MPC with the multi-model approach proposed in the present work.

In all simulations, the system is assumed to start from the point $x_0=(2.2060, 1.2665, 1.8545, 0.9395)$, $u_0=(5, 5)$. The nonlinear system (32)–(35) is taken as the real plant. The simulations have been executed in the Matlab environment, and the optimizations have been executed using the Matlab function *fmincon*.

4.1.1. Simulation I

In this first simulation, the economic objective is to maximize the concentration of product P_1, that is x_3, and to minimize the control action, plus a regularization cost. The economic cost function reads:

$$f_{eco}(x, u, \rho) = -\rho_1x_3 + \rho_2(u_1 + u_2) + \|x - x_s\|_Q^2 + \|u - u_s\|_R^2 \tag{36}$$

where $\rho=(\rho_1, \rho_2)$ are the prices on the cost function, $Q=0.1I_4$ and $R=0.1I_2$, where x_s and u_s are the economically optimal steady state and input given by the corresponding value of ρ.

The multi-model MPC has been applied by linearizing the plant in 3 different operation points, each of them representing the point that minimizes the economic cost function for a certain value of ρ. These linearization points are shown in Table 1.

The linearized models have been discretized using the zero-order hold method with a sampling time of 0.1 s.

Two changes of the economic cost have been considered, based on the following prices: $\rho^{[1]}=(7, 1)$, $\rho^{[2]}=(10, 2)$. The economic optimal costs provided by these two prices are respectively: $f_{eco}(x_s^{[1]}, u_s^{[1]}, \rho^{[1]}) = -7.5718$ and $f_{eco}(x_s^{[2]}, u_s^{[2]}, \rho^{[2]}) = -1.0680$, where $(x_s^{[1]}, u_s^{[1]})$ and $(x_s^{[2]}, u_s^{[2]})$ are the linearization points for models π_1 and π_2 given in Table 1.

The MPC controller has been setup with $N=6$. The offset cost function has been taken as a weighted ∞−norm, that is $\ell(x, u)=\alpha \| x - x_s \|_\infty$, with $\alpha=200$.

The results of this simulation are shown in Figs. 1 and 2. In particular, the time evolution of the states and input of the real plant when controlled by the multi-model EMPC with $M=N$ are plotted in black solid line, by the nominal EMPC with offset-free in red dashed line, and by the nominal EMPC without offset-free in blue dashed-dotted line. The optimal economic steady state is plotted in black dotted line. The multi-model controller is always capable to drive the plant to the optimal steady state that optimizes the economic cost function, even when the optimal operation point changes, always ensuring feasibility and stability. On the other hand, the nominal EMPC without offset-free fails to stabilize the system at the optimal steady state, while the nominal EMPC with offset-free is able to drive the system towards the optimum but cannot stabilize it. This shows the main advantage of using a

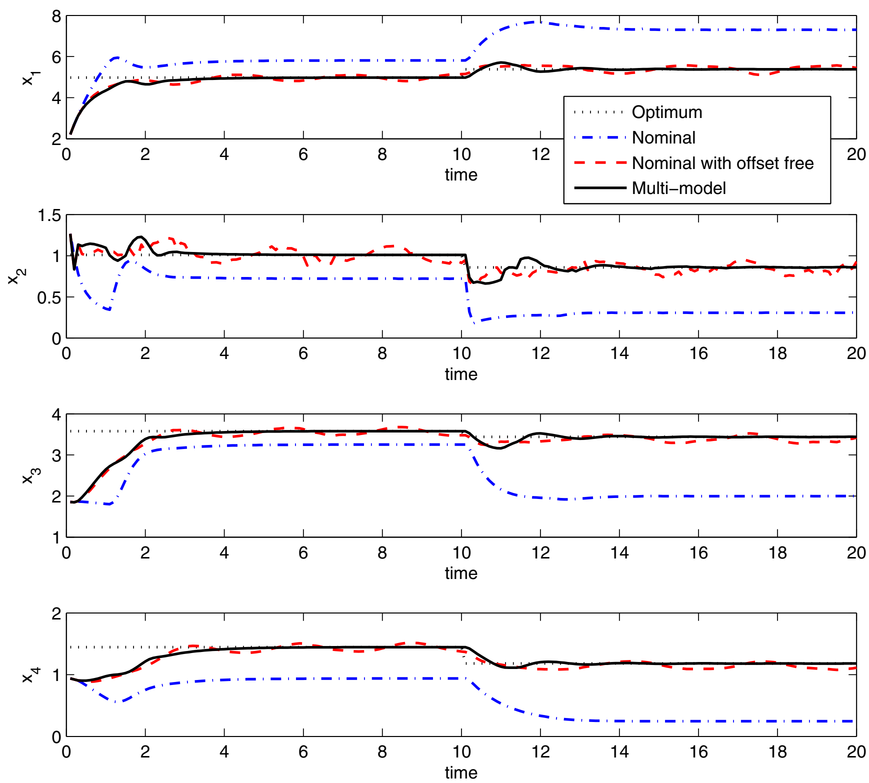

Fig. 1. Time evolution of the states: economic optimal point in black dotted line, multi-model EMPC with $M = N$ in black solid line, nominal EMPC with offset-free in red dashed line, nominal EMPC without offset-free in blue dashed-dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

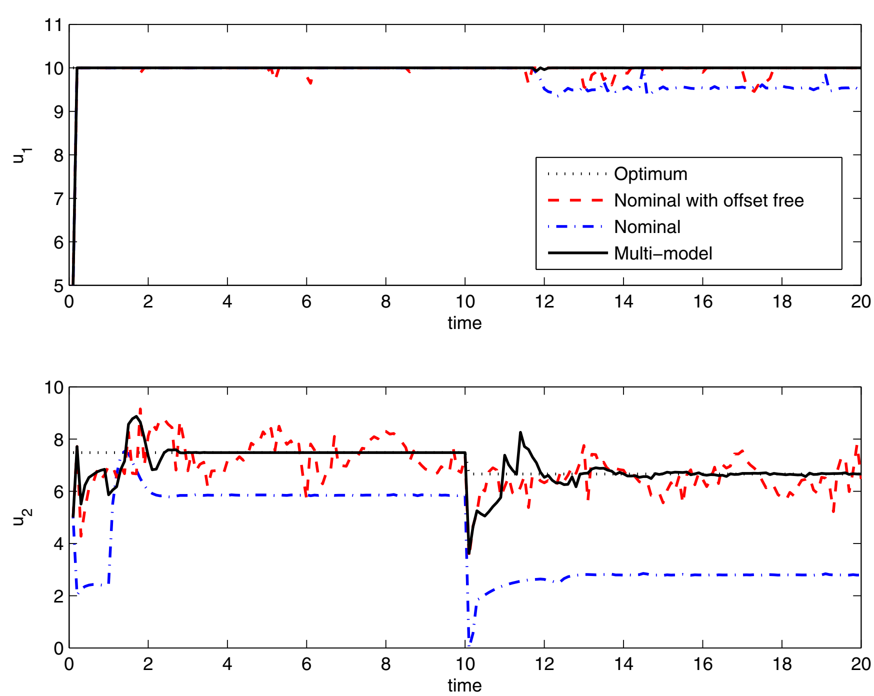

Fig. 2. Time evolution of the inputs: economic optimal point in black dotted line, multi-model EMPC with $M = N$ in black solid line, nominal EMPC with offset-free in red dashed line, nominal EMPC without offset-free in blue dashed-dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Simulation I. Transient economic performance.

	$\Psi(\rho_1)$	$\Psi(\rho_2)$
Nominal	300.16	614.75
Offset-free	191.32	54.32
Multi-model	148.39	0.79

multi-model approach: the offset-free by itself, even if it clearly improves the performance of the nominal EMPC, appears to be not enough robust in case of significant plant-model mismatches. The worse is the model, the less robust is the nominal EMPC with offset-free.

The economic performance has been assessed using the measure of the transient cost, that is:

$$\Psi(\rho) = \sum_{k=0}^{T} f_{eco}(x(k), u(k), \rho) - f_{eco}(x_s, u_s, \rho) \quad (37)$$

where T is the simulation time. A better performance is the one that minimizes the transient cost. The results are shown in Table 2. As it was expected, the multi-model approach shows better performance than the other controllers.

4.1.2. Simulation II: average nominal model

In this second test, a different nominal model has been used. This model has been obtained by linearizing the nonlinear system at the steady state provided by maintaining the inputs at their average value. This linearization point is shown in Table 3.

The other two models, π_1 and π_2 are kept as in the previous simulation. The same simulation as in the previous section has been run. The MPC controllers have been maintained with the same configuration.

Table 3
Simulation II: linearization point for the nominal model.

Model	x_1	x_2	x_3	x_4	u_1	u_2	ρ
π_{no}	2.2060	1.2665	1.8545	0.9395	5	5	-

Table 4
Simulation II. Transient economic performance.

	$\Psi(\rho_1)$	$\Psi(\rho_2)$
Nominal	194.95	143.89
Offset-free	146.77	8.42
Multi-model	134.51	0.91

The results of this second simulations are shown in Figs. 3 and 4. Note that, also in this case, the multi-model controller is capable to stabilize the system, while the nominal MPC fails in this task.

Again, the economic performance has been assessed using the index (37). The results are shown in Table 4. Also in this case, the multi-model approach shows better economic performance than the other two controllers, even if the performance of the nominal MPC gets better with the new prediction model, which indeed seems to be better than the previous one.

4.2. CSTR with parallel reactions

In this second example, we consider a continuous flow stirred-tank reactor with parallel reactions [4,25].

$$R \rightarrow P_1$$

$$R \rightarrow P_2$$

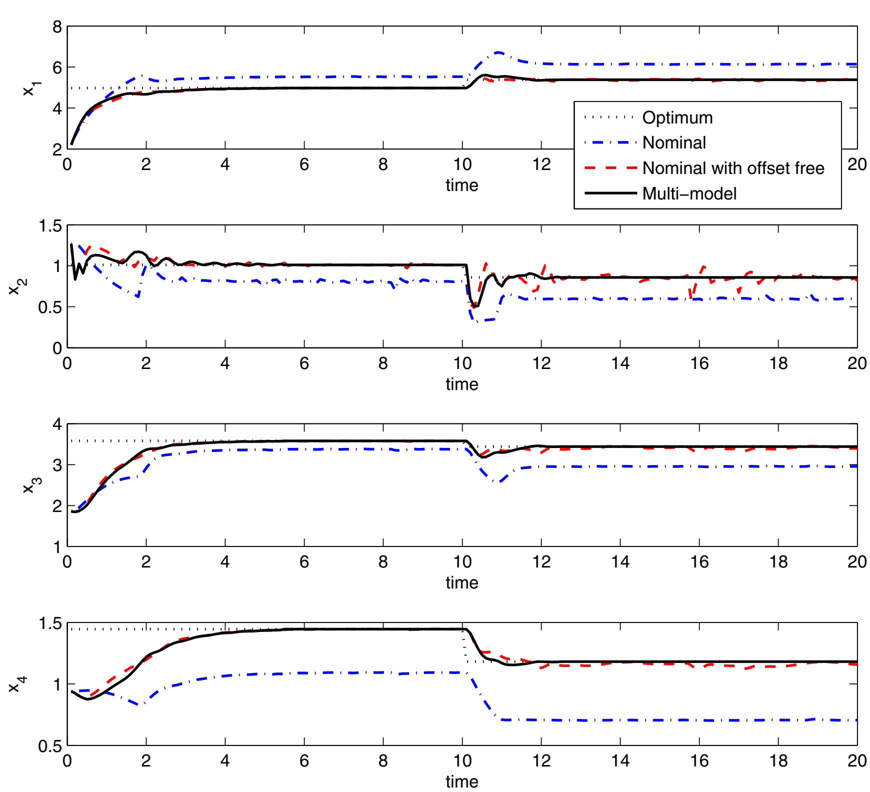

Fig. 3. Time evolution of the states: economic optimal point in black dotted line, multi-model EMPC with $M=N$ in black solid line, nominal EMPC with offset-free in red dashed line, nominal EMPC without offset-free in blue dashed-dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

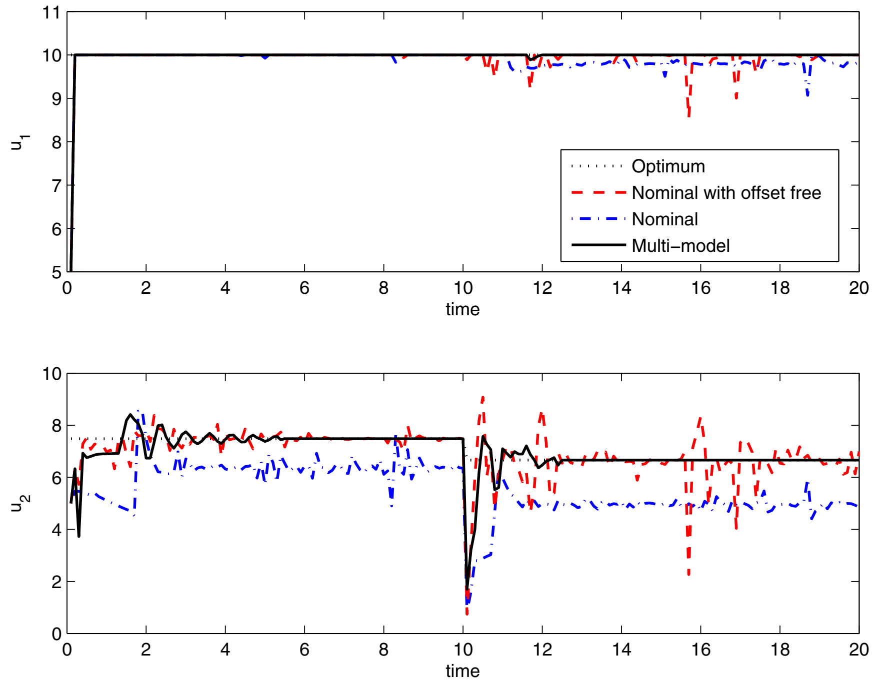

Fig. 4. Time evolution of the inputs: economic optimal point in black dotted line, multi-model EMPC with $M = N$ in black solid line, nominal EMPC with offset-free in red dashed line, nominal EMPC without offset-free in blue dashed-dotted line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

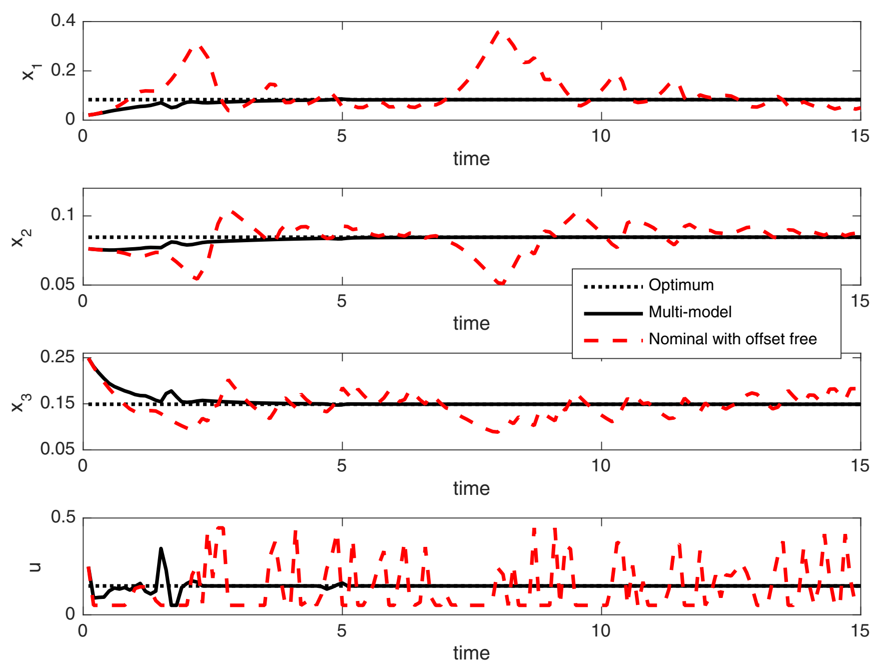

Fig. 5. Time evolution of states and input: economic optimal point in black dotted line, multi-model EMPC with $M = 1$ in black solid line, nominal EMPC with offset-free in red dashed line. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where R is the reactant, P_1 is the desired product, and P_2 is the waste product. The nonlinear model for this system is given by (from dimensionless mass balance):

$$\dot{x_1} = 1 - 10^4 x_1^2 e^{-1/x_3} - 400 x_1 e^{-0.55/x_3} - x_1 \quad (38)$$

$$\dot{x_2} = 10^4 x_1^2 e^{-1/x_3} - x_2 \quad (39)$$

$$\dot{x_3} = u - x_3 \quad (40)$$

where x_1 and x_2 are concentrations of R and P_1, while x_3 is temperature in the reactor. The control input u represents the heat flow into the reactor.

The constraints on the states are given by $0 \le x_i \le 10$, $i = 1, 2, 3$, while the constraints on the input are $0.049 \le u \le 0.449$.

In the following simulations, two different controllers have been compared: a nominal Economic MPC with offset-free, and an Economic MPC with the multi-model approach proposed in the present work.

Table 5
Linearization points for Example 2 scenario.

Model	x_1	x_2	x_3	u	ρ
π_1	0.0832	0.0846	0.1491	0.1491	1
π_2	0.9947	0	0.049	0.049	−1
π_{no}	0.0206	0.0762	0.2490	0.2490	–

In this simulation, the system is assumed to start from the point $x_0 = (0.0206, 0.0762, 0.2490)$, $u_0 = (0.2490)$. The nonlinear system (38)–(40) is taken as the real plant. The simulations have been executed in the Matlab environment, and the optimizations have been executed using the Matlab function *fmincon*.

The economic objective is to maximize the concentration of product P_1, that is x_2, plus a regularization cost. The economic cost function reads:

$$f_{eco}(x, u, \rho) = -\rho x_2 + \|x - x_s\|_Q^2 + \|u - u_s\|_R^2 \quad (41)$$

where ρ is the price on the cost function, $Q = 0.1I_3$ and $R = 0.1$, and (x_s, u_s) are the economically optimal steady state and input given by the corresponding value of ρ.

The multi-model MPC has been applied by linearizing the plant in 3 different operation points: two of them representing the point that minimizes the economic cost function for a certain value of ρ, while the nominal one is taken as an average model. These linearization points are shown in Table 5.

The linearized models have been discretized using the zero-order hold method with a sampling time of 0.1 s.

The economic cost has been setup considering: $\rho = 1$. The economic optimal steady state provided by this price is the linearization point for models π_1 given in Table 5.

Both MPC controllers have been setup with $N = 5$. The offset cost function has been taken as a weighted ∞−norm, that is $\ell(x, u) = \alpha(\|x - x_s\|_\infty + \|u - u_s\|_\infty)$, with $\alpha = 100$. The multi-model EMPC has been desigined with $M = 1$.

The results of this simulation are shown in Fig. 5. Note that, the multi-model controller is able to drive the plant to the optimal steady state, ensuring feasibility and stability. On the other hand, the nominal offset-free EMPC cannot stabilize the plant, which clearly exhibits an unstable behavior. Once again, this simulation shows the benefit of using a multi-model approach.

The economic performance has been assessed using the measure of the transient cost given by Eq. (37).

A better performance is the one that minimizes the transient cost (Ψ). The results are shown in Table 6. Once again, the multi-model approach shows a lower transient cost as one would have expected (since the nominal offset-free EMPC is not able to stabilize the plant).

Table 6
Example 2. Transient economic performance.

	$\Psi(\rho)$
Multi-model EMPC	0.2164
Nominal offset-free EMPC	0.5222

5. Conclusions

In this paper, an economic MPC based on multi-model description of the plant is proposed. A finite family of linear models has been considered (multi-model uncertainty), which operates appropriately in a certain region around a given operation point. In this way, each operation point defines a linear model, providing an enough accurate description of the system.

It has been shown that feasibility and stability conditions are preserved under changes in the economic cost function providing a better robust performance thanks to the multi-model approach. Moreover, the real plant converges to the optimal point that optimizes the economic cost function.

The proposed controller has been tested on two chemical reactors, showing better performance and robustness than standard offset-free controllers.

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Appendix A.

A.1. Proof to Lemma 1

Proof. The proof to this lemma follows similar argument as in [12, Appendix A]. Considering the definition of $\tilde{F}(x, u, \rho)$ and $\tilde{\ell}(x, u)$, we have:

$$\begin{aligned}
\tilde{V}_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r) &= \sum_{j=0}^{N-1} \tilde{F}(x_r(j), u_r(j), \rho) + \tilde{\ell}(x_r(N-1), u_r(N-1)) = \sum_{j=0}^{N-1} [f_{eco}(x_r(j) - x_r(N-1) + x_s, u_r(j) - u_r(N-1) - u_s, \rho) + \lambda(x_r(j)) \\
&\quad - \lambda(x_r(j+1)) - f_{eco}(x_s, u_s, \rho)] + \ell(x_r(N-1), u_r(N-1)) + \lambda(x_r(N-1)) - \lambda(x_s) - \ell(x_s, u_s) = \sum_{j=0}^{N-1} f_{eco}(x_r(j) - x_r(N-1) \\
&\quad + x_s, u_r(j) - u_r(N-1) + u_s, \rho) + \lambda(x_r) - \lambda(x_r(N)) - Nf_{eco}(x_s, u_s, \rho) + \ell(x_r(N-1), u_r(N-1)) + \lambda(x_r(N-1)) - \lambda(x_s) \\
&\quad - \ell(x_s, u_s) = \sum_{j=0}^{N-1} f_{eco}(x_r(j) - x_r(N-1) + x_s, u_r(j) - u_r(N-1) + u_s, \rho) + \ell(x_r(N-1), u_r(N-1)) + \lambda(x_r) - \lambda(x_s) \\
&\quad - Nf_{eco}(x_s, u_s, \rho) - \ell(x_s, u_s) = \sum_{j=0}^{N-1} f_{eco}(x_r(j) - x_r(N-1) + x_s, u_r(j) - u_r(N-1) + u_s, \rho) + \ell(x_r(N-1), u_r(N-1)) + \delta
\end{aligned}$$

where $\delta = \lambda(x_r) - \lambda(x_s) - Nf_{eco}(x_s, u_s, \rho) - \ell(x_s, u_s)$ and $x_r(N) = x_r(N-1)$ due to fact that $(x_r(N-1), u_r(N-1))$ defines an equilibrium point.

Hence,

$$\tilde{V}_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r) = V_{N_r}^e(\hat{x}_r, \hat{d}_r, \rho; \mathbf{u}_r) + \delta$$

□

A.2. Technical Lemmas

Lemma 2. *Consider model (5) subject to constraints (2). Consider that Assumptions 1–6 hold. Assume that there exists a $\bar{k}$ such that for $k \geq \bar{k}$, $\hat{d}_r(k+1) = \hat{d}_r(k)$, that is the disturbances d_r have converged. Let x_s be the optimal steady state defined in Definition 1. For all $x_r = \hat{x}_r(0) \in \mathcal{X}$ and $x_r^0(N-1) \in \mathcal{X}_{s,r}$, define the function $e(x_r) = x_r - x_r^0(N-1)$. Then, there exists a $\mathcal{K}$-function α_e such that*

$$\|e(x_r)\| \geq \alpha_e(\|x_r - x_s\|) \tag{42}$$

Proof. Notice that, due to convexity, $e(x_r)$ is a continuous function [31]. Moreover, let us consider these two cases.

1 $\|e(x_r)\|=0$ iff $x_r = x_s$. In fact, (i) if $e(x_r)=0$, then $x_r = x_r^0(N-1)$, and from Lemma 3, this implies that $x_r^0(N-1) = x_s$; (ii) if $x_r = x_s$, then by optimality $x_r^0(N-1) = x_s$, and then $x_r = x_r^0(N-1)$. Then, $\|e(x_r)\|=0$.
2 $\|e(x_r)\|>0$ for all $\|x_r - x_s\|>0$. In fact, for any $x_r \neq x_s$, $\|e(x_r)\| \neq 0$ and moreover $\|x_r - x_s\|>0$. Then, $\|e(x_r)\|>0$.

Then, since $\mathcal{X}$ is compact, in virtue of [32, Ch. 5, Lemma 6, pag. 148], there exists a $\mathcal{K}$-function α_e such that $\|e(x_r)\| \geq \alpha_e(\|x_r - x_s\|)$. □

Lemma 3. *Consider system (5) subject to constraints (2). Consider that Assumptions 1–6 hold, and consider a given parameter ρ for the economic cost $f_{eco}(x, u, \rho)$. Consider that, for $k \geq \bar{k}$, $\hat{d}_r(k+1) = \hat{d}_r(k) = d_r^\infty$, that is the disturbances reach a stationary value d_r^∞ and the optimal solution to Problem (20) is such that $x_r^0(N-1;k) = x_r(k)$, and $u_r^0(N-1;k) = u_r^0(0;k)$. Then $x_r^0(N-1;k) = x_s$, and $u_r^0(N-1;k) = u_s$.*

Proof. Consider that $(x_r^0(N-1;k), u_r^0(N-1;k))$ is the optimal solution to (20) at time k. Since by assumption $u_r^0(N-1;k) = u_r^0(0;k)$ and $x_r(k) = x_r^0(N-1;k)$, then from Property 1, $\tilde{V}_{N_r}^{e,0}(\hat{x}_r(k), \hat{d}_r(k), \rho) = \tilde{\ell}(x_r^0(N-1;k), u_r^0(N-1;k))$

Moreover, since by assumption, $\hat{d}_r(k) = d_r^\infty$, from equation (10) we have that $L_r^d(\hat{x}_r(k) - x_p(k) + d_r^\infty) = 0$, and then, by stability of the observer, $\hat{x}_r(k) - x_p(k) + d_r^\infty = 0$.

This implies that, taking into account that $(x_r^0(N-1;k), u_r^0(N-1;k))$ is a stationary point (the time dependence is removed for the sake of clarity):

$$x_r^0(N-1) = A_r x_r^0(N-1) + B_r u_r^0(N-1) + d_r^\infty = \hat{x}_r(k+1)$$

$$\hat{d}_r(k+1) = \hat{d}_r(k) = d_r^\infty$$

This implies that, model $\pi_r \in \Pi$ converges to the stationary point $x_r^0(N-1)$.

Assume now that, this stationary point is not the optimal one, that is $(x_r^0(N-1), u_r^0(N-1)) \neq (x_s, u_s)$. Then, by convexity, there exists a $\beta \in [0, 1]$ such that

$$(\tilde{x}_s, \tilde{u}_s) = \beta(x_r^0(N-1), u_r^0(N-1)) + (1-\beta)(x_s, u_s)$$

characterizes a stationary point, and moreover there exists a feasible control sequence $\tilde{\mathbf{u}}_r = \{\tilde{u}_r(0), \tilde{u}_r(1), \ldots, \tilde{u}_r(N-1)\}$ that drives the state of model π_r from $(x_r^0(N-1), u_r^0(N-1))$ to $(\tilde{x}_s, \tilde{u}_s)$. This sequence is such that, the j-th element is given by $\tilde{u}_r(j) = K_r(\tilde{x}_r(j) - \tilde{x}_s) + \tilde{u}_s$, and $\tilde{x}_r(j+1) = A_r\tilde{x}_r(j) + B_r\tilde{u}_r(j) + d_r^\infty$, $\tilde{x}_r(0) = x_r^0(N-1)$. Then, the cost to drive the system from $(x_r^0(N-1), u_r^0(N-1))$ to $(\tilde{x}_s, \tilde{u}_s)$ is given by

$$\tilde{V}_{N_r}^e(x_r^0(N-1), d_r^\infty, \rho; \tilde{\mathbf{u}}_r) = \sum_{j=0}^{N-1} \tilde{F}(\tilde{x}_r(j) - \tilde{x}_s + x_s, \tilde{u}_r(j) - \tilde{u}_s + u_s, \rho)$$

$$+ \tilde{\ell}(\tilde{x}_s, \tilde{u}_s) \leq \gamma_0 \|x_r(N-1)^0 - \tilde{x}_s\| + \ell(\tilde{x}_s, \tilde{u}_s)$$

$$+ \sigma = \gamma_0(1-\beta)\|x_r^0(N-1) - x_s\| + \ell(\tilde{x}_s, \tilde{u}_s) + \sigma$$

where $\sigma = \lambda(x_r(N-1)^0) - \lambda(x_s) + \ell(x_s, u_s)$ and

$$\gamma_0 = L(1 + |K_r|)\frac{1 - |A_r + B_r K_r|^N}{1 - |A_r + B_r K_r|}$$

Notice that, the last inequality and definitions come from Lemmas 3 and 4 in [12].

Define now $W(\beta) = \gamma_0(1-\beta)\|x_r^0(N-1) - x_s\| + \ell(\tilde{x}_s, \tilde{u}_s) + \sigma$. Using the same arguments as in [12, Lemma 5], it can be shown that there exists a value of $\beta \in [0, 1)$ such that the value $W(\beta)$ is smaller than the value of this cost for $\beta = 1$, which is $W(1) = \tilde{\ell}(x_r^0(N-1), u_r^0(N-1)) = \tilde{V}_{N_r}^{e,0}(\hat{x}_r(k), \hat{d}_r(k), \rho)$. This contradicts the optimality of the solution to Problem (20) at time k, and the fact that model π_r has already converged to a stationary point. Then it has to be that $(x_r^0(N-1), u_r^0(N-1)) = (x_s, u_s)$. So the Lemma is proved. □

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