Nonlocal chiral quark models with Polyakov loop at finite temperature and chemical potential

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Abstract

We analyze the chiral restoration and deconfinement transitions in the framework of a non-local chiral quark model which includes terms leading to the quark wave function renormalization, and takes care of the effect of gauge interactions by coupling the quarks with the Polyakov loop. Non-local interactions are described by considering both a set of exponential form factors, and a set of form factors obtained from a fit to the mass and renormalization functions obtained in lattice calculations.

I. INTRODUCTION

The detailed understanding of the behavior of strongly interacting matter under extreme conditions of temperature and/or density has become an issue of great interest in recent years. Unfortunately, even if a significant progress has been made on the development of ab initio calculations such as lattice QCD, these are not yet able to provide a full understanding of the QCD phase diagram and the related hadron properties, due to the well-known difficulties of dealing with small current quark masses and finite chemical potentials. Thus it is important to develop effective models that show consistency with lattice results and can be extrapolated into regions not accessible by lattice calculation techniques. Recently, models in which quark fields interact via local four point vertices and where the Polyakov loop is introduced to account for the confinement-deconfinement phase transition (so-called Polyakov-Nambu-Jona-Lasinio (PNJL) models [1, 2, 3, 4, 5]) have received considerable attention. Here, we consider a non-local extension of these PNJL models, which includes terms leading to the quark wave function renormalization. Two different parameterizations are used: an exponential form, and a parametrization based on a fit to the mass and renormalization function obtained in lattice calculations. In the context of this type of model the properties of the vacuum and meson sectors at $T = \mu = 0$ have been studied in Ref.[6].

This contribution is organized as follows. In Sec. 2 we introduce the model lagrangian and its parameterizations. In Sec. 3 we present and discuss our results for the behavior of some thermodynamical properties and the corresponding phase diagrams. Finally, in Sec. 4 our main conclusions are summarized.

II. THE MODEL AND ITS PARAMETRIZATIONS

We consider here a nonlocal SU(2) chiral quark model which includes quark couplings to the color gauge fields. The corresponding Euclidean effective action is given by

$$S_E = \int d^4x \left\{ \bar{\psi}(x)[-i\not{D} + \hat{m}]\psi(x) - \frac{G_S}{2}\Big[j_a(x)j_a(x) - j_P(x)j_P(x) \Big] + \mathcal{U}\left(\Phi[A(x)]\right) \right\}, \qquad (1)$$

where ψ is the $N_f = 2$ fermion doublet $\psi \equiv (u, d)^T$, and $\hat{m} = diag(m_u, m_d)$ is the current quark mass matrix, in what follows we consider isospin symmetry, that is $m_f = m_u = m_d$. The fermion kinetic term includes a covariant derivative $D_\mu \equiv \partial_\mu - iA_\mu$, where A_μ are color gauge fields. The nonlocal currents $j_a(x), j_P(x)$ are given by

$$j_a(x) = \int d^4z \; g(z) \; \bar{\psi}\left(x + \frac{z}{2}\right) \; \Gamma_a \; \psi\left(x - \frac{z}{2}\right) \; ,$$
$$j_P(x) = \int d^4z \; f(z) \; \bar{\psi}\left(x + \frac{z}{2}\right) \; \frac{i \overleftrightarrow{\not\partial}}{2 \; \kappa_p} \; \psi\left(x - \frac{z}{2}\right) \; , \tag{2}$$

Here, $\Gamma_a = (\mathbb{1}, i\gamma_5\vec{\tau})$ and $u(x')\overleftrightarrow{\partial} v(x) = u(x')\partial_x v(x) - \partial_{x'} u(x') v(x)$. The functions $g(z)$ and $f(z)$ in Eq.(2), are nonlocal covariant form factors characterizing the corresponding interactions. The scalar-isoscalar component of the $j_a(x)$ current will generate the momentum dependent quark mass in the quark propagator, while the "momentum" current, $j_P(x)$, will be responsible for a momentum dependent wave function renormalization of this propagator.

To proceed it is convenient to perform a standard bosonization of the theory. Thus, we introduce the bosonic fields $\sigma_{1,2}(x)$ and $\pi_a(x)$, and integrate out the quark fields. In what follows, we work within the mean-field approximation (MFA), in which these bosonic fields are replaced by their vacuum expectation values $\bar{\sigma}_{1,2}$ and $\bar{\pi}_a = 0$. Next, we extend the so obtained bosonized effective MFA action to finite temperature T and chemical potential μ using the Matsubara formalism. Concerning the gluon fields we will assume that they provide a constant background color field $A_4 = iA_0 = ig\,\delta_{\mu 0}\, G^\mu_a \lambda^a/2$, where G^μ_a are the SU(3) color gauge fields. Then the traced Polyakov loop, which is taken as order parameter of confinement, is given by $\Phi = \frac{1}{3}\mathrm{Tr}\,\exp(i\beta\phi)$, where $\beta = 1/T$, $\phi = iA_0$. We will work in the so-called Polyakov gauge, in which the matrix ϕ is given a diagonal representation $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$, which leaves only two independent variables, ϕ_3 and ϕ_8. Owing to the charge conjugation properties of the QCD Lagrangian [7], the mean field value of the Polyakov loop field $\bar{\Phi}$ is expected to be a real quantity. In addition, we assume as usual that ϕ_3 and ϕ_8 are real-valued fields [5], this implies that $\phi_8 = 0$, then $\bar{\Phi} = [2\cos(\phi_3/T) + 1]/3$.

Within this framework the mean field thermodynamical potential Ω^{MFA} results

$$\Omega^{\mathrm{MFA}} = -\frac{4T}{\pi^2} \sum_c \int_{p,n} \ln\left[\frac{(\rho^c_{n,\vec{p}})^2 + M^2(\rho^c_{n,\vec{p}})}{Z^2(\rho^c_{n,\vec{p}})}\right] + \frac{\bar{\sigma}_1^2}{2\,G_S} + \frac{\kappa_p^2\;\bar{\sigma}_2^2}{2\,G_S} + \mathcal{U}(\bar{\Phi}, T) \; , \tag{3}$$

Here, the shorthand notation $\int_{p,n} = \sum_n \int d^3\vec{p}/(2\pi)^3$ has been used, and $M(p)$ and $Z(p)$ are given by

$$M(p) = Z(p)\,[m_f + \bar{\sigma}_1 \; g(p)] \qquad , \qquad Z(p) = [1 - \bar{\sigma}_2 \; f(p)]^{-1} \; , \tag{4}$$

where $g(p)$ and $f(p)$ are the Fourier transform of $g(z)$ and $f(z)$, respectively. In addition, we have defined

$$\left(\rho^c_{n,\vec{p}}\right)^2 = \Big[(2n+1)\pi T - i\mu + \phi_c\Big]^2 + \vec{p}^{\,2} \ , \tag{5}$$

where the quantities ϕ_c are given by the relation $\phi = \mathrm{diag}(\phi_r, \phi_g, \phi_b)$. Namely, $\phi_c = c\ \phi_3$ with $c = 1, -1, 0$ for r, g, b respectively. At this stage we need to specify the explicit form of the Polyakov loop effective potential. Here, we used the fit to QCD lattice results proposed in Ref. [5].

$\Omega^{\rm MFA}$ turns out to be divergent and, thus, needs to be regularized. For this purpose we use the same prescription as in Ref. [8]. Namely

$$\Omega^{\rm MFA}_{(reg)} = \Omega^{\rm MFA} - \Omega^{free} + \Omega^{free}_{(reg)} + \Omega_0 \ , \tag{6}$$

where Ω^{free} is obtained from the first term in Eq.(3) by setting $\bar\sigma_1 = \bar\sigma_2 = 0$ and $\Omega^{free}_{(reg)}$ is the regularized expression for the quark thermodynamical potential in the absence of fermion interactions,

$$\Omega^{free}_{(reg)} = -4\ T \int \frac{d^3\vec{p}}{(2\pi)^3} \sum_{c,k} \ln\left[1 + e^{-\left(\sqrt{\vec{p}^{\,2}+m^2} - k\mu + i\phi_c\right)/T}\right] \ , \tag{7}$$

with $k = \pm 1$. Finally, note that in Eq.(6) we have included a constant Ω_0 which is fixed by the condition that $\Omega^{\rm MFA}_{(reg)}$ vanishes at $T = \mu = 0$.

The mean field values $\bar\sigma_{1,2}$ and $\bar\phi_3$ at a given temperature or chemical potential, are obtained from a set of three coupled "gap" equations. This set of equations follows from the minimization of the regularized thermodynamical potential, that is

$$\frac{\partial \Omega^{reg}_{\rm MFA}}{\partial \bar\sigma_1} = \frac{\partial \Omega^{reg}_{\rm MFA}}{\partial \bar\sigma_2} = \frac{\partial \Omega^{reg}_{\rm MFA}}{\partial \bar\phi_3} = 0 \ . \tag{8}$$

Once the mean field values are obtained, the (T, μ) behavior of other relevant quantities can be determined.

In order to fully specify the model under consideration we have to fix the model parameters as well as the form factors $g(q)$ and $f(q)$ which characterize the non-local interactions. Following Ref.[6], we consider two different type of functional dependencies for these form factors. The first one corresponds to the often used exponential forms,

$$g(q) = \exp[-q^2/\Lambda_0^2] \qquad , \qquad f(q) = \exp[-q^2/\Lambda_1^2] \ . \tag{9}$$

Note that the range (in momentum space) of the nonlocality in each channel is determined by the parameters Λ_0 and Λ_1, respectively. Fixing the $T = \mu = 0$ values of m_c and chiral quark condensate to reasonable values $m_c = 5.7$ MeV and $\langle \bar{q}q \rangle^{1/3} = 240$ MeV the rest of the parameters are determined so as to reproduce the empirical values $f_\pi = 92.4$ MeV and $m_\pi = 139$ MeV, and $Z(0) = 0.7$ which is within the range of values suggested by recent lattice calculations[9]. In what follows this choice of model parameters and form factors will be referred as parametrization S1. The second type of form factor functional forms we consider is given by

$$g(q) = \frac{1+\alpha_z}{1+\alpha_z\ f_z(q)} \frac{\alpha_m\ f_m(q) - m_f\ \alpha_z f_z(q)}{\alpha_m - m_f\ \alpha_z} \quad , \quad f(q) = \frac{1+\alpha_z}{1+\alpha_z\ f_z(q)} f_z(q) \ , \tag{10}$$

where

$$f_m(q) = \left[1 + \left(q^2/\Lambda_0^2\right)^{3/2}\right]^{-1} \qquad ; \qquad f_z(q) = \left[1 + \left(q^2/\Lambda_1^2\right)\right]^{-5/2} \ . \tag{11}$$

As shown in Ref.[6], with a convenient choice of parameters one can very well reproduce the momentum dependence of mass and the renormalization function obtained in a Landau gauge lattice calculation as well as the physical values of m_π and f_π. In what follows this parametrization will be referred as S2. Finally, in order to compare with previous studies where the wavefunction renormalization of the quark propagator has been ignored we consider a third parametrization (S3). In such case we take $Z(p) = 1$ (setting $f(p) = 0$) and exponential parametrization for $g(p)$. The values of the model parameters for each of the chosen parameterizations are summarized in Table I.

TABLE I: Sets of parameters.

		S1	S2	S3
m_c	MeV	5.70	2.37	5.78
$G_s\Lambda_0^2$		32.030	20.818	20.650
Λ_0	MeV	814.42	850.00	752.20
κ_P	GeV	4.180	6.034	–
Λ_1	MeV	1034.5	1400.0	–

III. RESULTS

We start by analyzing the behavior of some mean field quantities as functions of T and μ. Since the results obtained for our three different parameterizations are qualitatively quite similar we only present explicitly those corresponding to the parametrization S1. They are

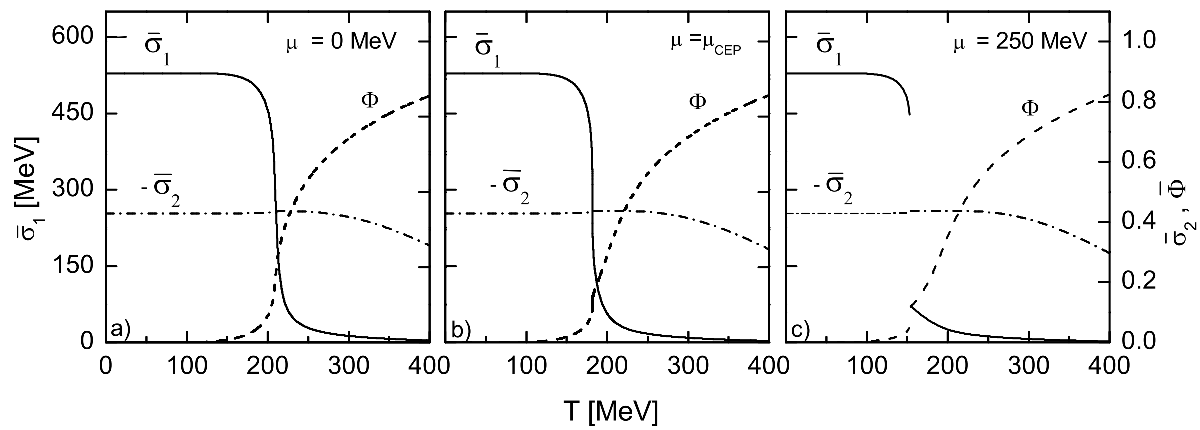

FIG. 1: Mean fields $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $\bar{\Phi}$ as functions of T for low (left), high (right) and CEP (central) chemical potentials. Note that the scale to the left corresponds to that of $\bar{\sigma}_1$ while that to the right to $\bar{\sigma}_2$ and $\bar{\Phi}$. Since $\bar{\sigma}_2$ turns out to be negative we plot $-\bar{\sigma}_2$.

given in Fig.1 where we plot $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $\bar{\Phi}$ as functions of T for some values of the chemical potential. Fig. 1a shows that at $\mu = 0$ there is a certain value of T at which $\bar{\sigma}_1$ drops rapidly signalling the existence of a chiral symmetry restoration crossover transition, its position being determined by the peak of the chiral susceptibility. At basically the same temperature the Polyakov loop $\bar{\Phi}$ increases which can be interpreted as the onset of the deconfinement transition. As μ increases there is a certain value of $\mu = \mu_{CEP}$ above which the transition starts to be discontinuous. At this precise chemical potential the transition is of second order. This situation is illustrated in Fig.1b. The corresponding values (T_{CEP}, μ_{CEP}) define the position of the so-called "critical end point". As displays in Fig.1c, for $\mu > \mu_{CEP}$ the transition becomes discontinuous, i.e. of first order. Finally, for chemical potentials above $\mu_c(T = 0) \simeq 310$ MeV the system is in the chirally restored phase for all values of the temperature. It is important to note that although $\bar{\sigma}_2$ appears to be rather constant in Fig.1, at higher values of T it does go to zero as expected. Concerning the deconfinement transition we see that as μ increases there appears a region where system remains in its confined phase (signalled by $\bar{\Phi}$ smaller than $\simeq 0.3$) even though chiral symmetry has been restored. This corresponds to the recently proposed quarkyonic phase[10].

The phase diagrams corresponding to our three different parameterizations are given in Fig.2. Here the dotted line corresponds to the line of crossover chiral transition while the full line to the line of first order chiral transition. The dashed lines correspond to the deconfinement transition (the lower and upper lines correspond to $\bar{\Phi} = 0.3$ and $\bar{\Phi} = 0.5$, respectively). Comparing those of S1 and S3 we see that the main effect of the wave function renormalization term is to shift the location of the CEP towards lower values of T and higher values μ. Concerning the lattice adjusted parametrization S2 we observe that it leads to even lower values of T_{CEP} and higher values μ_{CEP}.

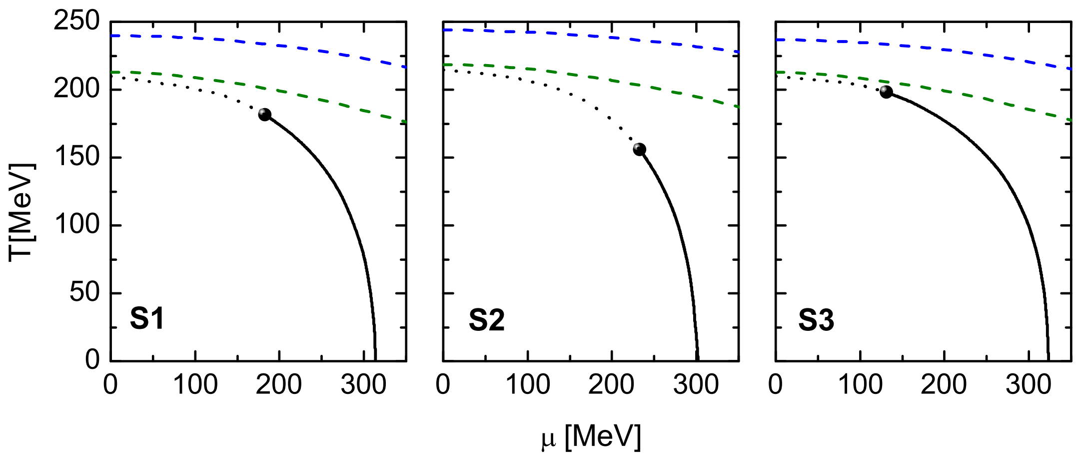

FIG. 2: Phase diagrams for the three parameterizations considered. S1 and S2 include quark wave function renormalization while S3 does not. S1 and S3 correspond to exponential form factors while S2 to lattice motivated form factors. The dotted line corresponds to the line of crossover chiral transition and the full line to that of first order chiral transition. The dashed lines correspond to the deconfinement transition (the lower and upper lines being for $\bar{\Phi} = 0.3$ and $\bar{\Phi} = 0.5$, respectively).

IV. SUMMARY AND CONCLUSIONS

A non-local extension of the PNJL model momentum which leads to momentum dependent quark mass and wave function renormalization has been studied. This model provides a simultaneous description for the deconfinement and chiral phase transition. Non-local interactions have been described by considering both a set of exponential form factors, and a set of form factors obtained from a fit to the mass and renormalization functions obtained in lattice calculations. The resulting phase diagrams turn out to be qualitative similar, the position of the critical end point being the feature which depends more crucially on each particular parametrization.

Acknowledgments

We would like to thank the members of the Organizing Committee for their warm hospitality during the workshop. This work has been supported in part by ANPCyT (Argentina), under grant PICT07 03-00818.

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