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Maximum savings approach for location and sizing of capacitors in distribution systems

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Abstract

This paper proposes a computationally efficient methodology for the optimal location and sizing of static and switched shunt capacitors in radial distribution systems. The problem is formulated as the maximization of the savings produced by the reduction in energy losses and the avoided costs due to investment deferral in the expansion of the network. The proposed method selects the nodes to be compensated, as well as the optimal capacitor ratings and their operational characteristics, i.e. fixed or switched. After an appropriate linearization, the optimization problem was formulated as a mixed-integer linear problem, suitable for being solved by means of a widespread commercial package. Results of the proposed optimizing method are compared with another recent methodology reported in the literature using two test cases: a 15-bus and a 33-bus distribution network. For the both cases tested, the proposed methodology delivers better solutions indicated by higher loss savings, which are achieved with lower amounts of capacitive compensation. To calculate the energy savings and the deferral investment cost exactly, a load flow for radial distribution network is executed before and after the compensation. The proposed method has also been applied for compensating to an actual radial distribution network served by AES-Venezuela in the metropolitan area of Caracas. A convergence time of about 4 s after 22,298 iterations demonstrates the ability of the proposed methodology for efficiently handling compensation problems.

Keywords: Capacitors; Distribution systems; Mixed-integer linear optimization

1. Introduction

After the liberalization of the power industry, the distribution business has remained as a regulated monopoly. As a consequence of the deregulation process, distribution utilities are currently facing increased pressures from shareholders and regulatory authorities to improve investment and operational efficiency. The delivery of power from sources to the consumer points is always accompanied of power losses. Some studies have determined that power losses occurred in distribution networks due to Joule effect can account for as much as 13% of the generated energy. Such non-negligible amount of losses has a direct impact on the financial results and the overall efficiency of distribution utilities. Therefore, methods for loss reductions that optimally allocate scarce financial resources and maximize firm value are essential for achieving the financial goals of distribution companies.

Basically, active losses in distribution systems can be reduced by optimal reconfigurations of the network [1–3]. The optimal reconfiguration model responds to changes in the network topology by switching the automatic breakers installed in the grid. Losses can be further reduced by connecting capacitors in series or parallel (shunt) to locally supply a considerable portion of the reactive power demanded by the consumers and thereby reducing the reactive component of branch currents [4–6].

The installation of shunt capacitors provides supplementary benefits, such as improvement of the voltage profile, the power factor and the stability of the distribution system [1].

The problem of optimal capacitor allocation and sizing for maximizing cost savings has called the attention of researchers

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for several decades. A myriad of algorithms of very different nature and degree of sophistication addressing this topic are available in the published literature. Most of them analyze the localization and sizing of static and switched capacitors in radial distribution systems. However, few of these methodologies take into account the total costs linked to reactive power compensation, i.e. installation, operation and maintenance costs. Early efforts relied on analytical approaches that delivered simple closed-form solutions to the optimization problem [7–10]. Nevertheless, the proposed analytical methods required seemingly unrealistic assumptions, such as a uniform load distribution along the feeder, uniform feeder size and radial feeder with no laterals. Due to power computing increase, the optimization problem was formulated in a more sophisticated manner and solved by numerical programming techniques. Dynamic programming [11], quadratic programming [12] and non-linear mixed-integer programming [4,13,14] have been applied to solve the optimal capacitor allocation and sizing problem. In order to cope with the problem size and overcome the inherent complexity of programming methods, some heuristic methods have been devised [6,15,16]. More recently, optimization methodologies based on artificial intelligence, e.g. genetic algorithms [17], evolutionary programming [18,19] tabu search [20], artificial neural networks [21,22] fuzzy sets and fuzzy dynamic programming [23,24], expert systems [25] and simulated annealing [26] have been proposed for optimally placing capacitive compensation in distribution networks. Some other proposals combine heuristic/metaheuristic techniques with programming methods [27]. In general, sophisticated algorithms consider in their formulations one or more of the following modeling refinements: varying loads, discrete capacitor sizes, non-linear capacitor costs, radial feeder with laterals, network operational constraints (e.g. voltage, load flow limits, etc.), switched capacitors and switching times, as well as the economical value of released feeder capacity due to compensation. For a comprehensive review of the abundant literature on the subject the reader is further referred to the literature surveys conducted in [28–30].

This paper proposes a computationally very efficient methodology for an optimal location and sizing of static and switched shunt capacitors in radial distribution networks. The optimization problem has been formulated as the maximization of the total savings produced by the reduction in energy losses and the avoided costs due to investment deferral in the expansion of the network over a considered period, subject to the whole constraint set of the optimal reactive power flow, the reactive power balance in each node of the network, and the constraints of selecting for each node only one among the various proposed capacitors bank sizes and types (fixed or switched). The costs formulation includes investment, operation and maintenance costs of the installed compensation, as well as the costs of energy losses and released capacity due to decreasing peak power losses associated to the reactive component of branch currents.

The proposed model has been refined in order to consider varying loads, discrete capacitor sizes, radial feeder with laterals, network operational constraints (e.g. voltage limits, load flow limits, etc.), switched capacitors and switching times.

The mathematical formulation of the optimization problem starts with the electrical variables of the circuit under study and its topology, which under some premises are linearized and solved through mixed-integer linear programming. Two types of variables take part in the formulation: the first group of variables represents physical magnitudes such as power flow through the network; they are real variables. The second group represents the decision of installing or not a bank of capacitors in a specific point; they are binary variables.

The methodology was developed only for radial networks because most of systems are deployed in a radial basis due to their simple operation and low investment costs. The proposed algorithm allows the evaluation of a higher number of constraints inherent to the optimization problems faced by actual utilities. Thus, besides including the basic constraints of the electricity laws, the model is flexible enough for including other constraints, such as proprietary company rules or standards, regulations requirements and subject to penalizations, operational restrictions preventing the installation of capacitors in some nodes of the network, etc.

This article is organized as follows: Section 2 provides the theoretical basis and main assumptions for building a model of optimal capacitor placement. In Section 3, the optimization problem and constraints are mathematically formulated. Section 4 presents numerical results of the comparison with other recently proposed methodology as well as for a compensation problem. In Section 5, economic assessments of the solutions provided by the algorithm are analyzed. The article is concluded in Section 6.

2. Capacitor placement

2.1. Model assumptions

The optimal location and sizing of shunt capacitors in the electric distribution network is a non-linear problem. However, as it will be seen next, this problem can be (under certain approximations) linearized and solved by using mixed-integer linear programming.

Before starting with the development of the mathematical framework, the main assumptions to be considered are listed below.

- Three-phase balanced systems.
- The phase shift between the system bus voltage angles is close to zero.
- The bus voltage magnitudes are set equal to 1.0 p.u. in the first iteration of the power flow.
- The nodes allowed to be compensated are known according to the all constrains as well as the type of bank capacity that would be installed.

2.2. Theoretical basis for the model formulation

The following expression determines the electric power losses because of the Joule effect in a line:

$$P_{\text{Loss}} = I^2 R \tag{1}$$

The magnitude of the circulating current is given as

$$I = \sqrt{I_R^2 + I_X^2} \tag{2}$$

By replacing (2) in (1), power losses can be rewritten as follows [1]:

$$P_{\text{Loss}} = (I_R^2 + I_X^2)R \tag{3}$$

The active current component I_R depends only on the circuit load. It must be delivered to the customer to cover their consumption, and it could be modified by other methodologies distinct from compensation, which are beyond the scope of this work. Therefore, only the reactive component that generates technical losses [1] will be further considered.

$$P'_{\text{Loss}} = I_X^2 R \tag{4}$$

The reactive current I_X depends on the inductive reactive power demanded by the system, and hence, it could be modified by injecting capacitive reactive power through the installation of capacitor banks. The purpose is to locate these capacitors in the points where they can improve best the technical and economic circuit performance. In order to apply mixed-integer linear programming, a suitable linearization of the technical losses in (4) is necessary. This requirement can be fulfilled according to the following procedure.

The Eq. (4) can be decomposed as follows:

$$P'_{\text{Loss}} = RI_X^2 = RI_X I_X \tag{5}$$

where

$$R = rl \tag{6}$$

One of the components I_X can be represented by the following equation:

$$I_X = \frac{Q^0}{\sqrt{3}\,V_{\text{L}}\sin\varphi} \tag{7}$$

Q^0, the initial reactive power flow, can be obtained by running an optimal power flow, or a simply approximate power flow for radial distribution systems in the actual state of the distribution circuit.

The other component of the current is replaced by the following expression:

$$I_X = \frac{Q}{\sqrt{3}\,V_{\text{L}}\sin\varphi} \tag{8}$$

where Q stands for the actual reactive power flow circulating in the branches of the network. This value can be modified by placing banks of capacitors.

After substituting the expressions (7) and (8) in (5), the power losses linearized as a function of the reactive power Q or apparent power S are expressed by the following functions:

$$P'_{\text{Loss}}(Q) = \frac{rlQ^0}{3V_{\text{L}}^2\sin^2\varphi}Q \tag{9}$$

$$Q = S\sin\varphi \tag{10}$$

$$P'_{\text{Loss}}(S) = \frac{rlQ^0}{3V_{\text{L}}^2\sin\varphi}S \tag{11}$$

The expected savings by placing capacitor banks are modeled by means of an annual cost coefficient, K that includes two parts: costs associated to annual savings due to released capacity and costs associated to annual energy losses:

$$K = \frac{12C_{\text{p}}}{\cos\varphi} + 8760F_{\text{Loss}}C_{\text{E}} \tag{12}$$

Released capacity factor is assessed through a capacity charge C_{P} in \$/(kVA month) related to the fixed cost of the network. The cost of losses is related to the energy cost C_{E} in \$/kWh.

Finally, by multiplying Eqs. (12) and (9), the global cost equation can be obtained:

$$C(Q) = K\frac{rlQ^0}{3V_{\text{L}}^2\sin^2\varphi}Q \tag{13}$$

2.3. Investment appraisal

The economic evaluation of the project must take into account the necessary investments for the installation of capacitors Inv_0 and the benefits b_k obtained in each period k from lower losses. For the economic assessment of the investment project, the project lifetime N and the discount factor d_{r} (which should adequately reflect the firm opportunity cost of capital) have to be defined according to the company's financial policy.

The compensation project will be profitable when the present value (PV) of the benefits associated to the loss reduction, over the considered project lifetime is greater than the initial investment in the capacitors banks, i.e. the net present value (NPV) is positive (see Fig. 1). The present value of loss savings B can be computed by means of the capital recovery factor (CRF) assuming uniform benefits (annuities) along the N periods, i.e. $b_1 = \cdots = b_N = b$ as follows:

$$\text{NPV} = B - \text{Inv}_0 > 0 \tag{14}$$

$$B = \frac{b_1}{\text{CRF}} = \cdots = \frac{b_N}{\text{CRF}} = \frac{b}{\text{CRF}} = \left[\frac{(1+d_{\text{r}})^N - 1}{d_{\text{r}}(1+d_{\text{r}})^N}\right] b \tag{15}$$

The equation of the project profitability (14) can be rewritten as follows:

$$b - \text{CRF}\ \text{Inv}_0 > 0 \tag{16}$$

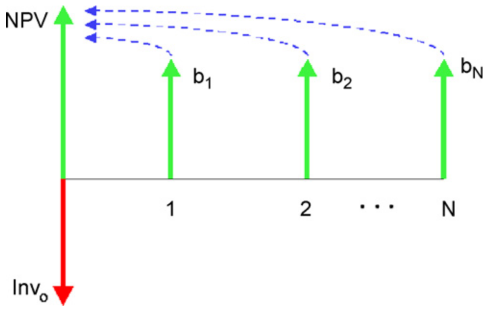

Fig. 1. NPV of the compensation investment project.

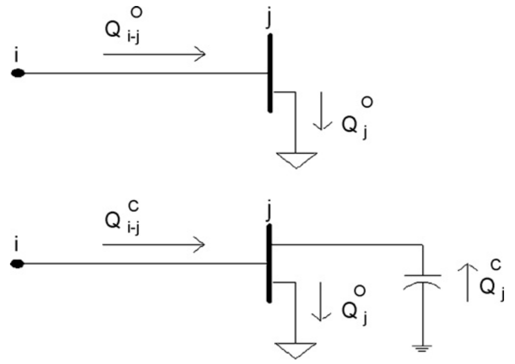

Fig. 2. Reactive power injections at node.

3. Problem formulation

The objective is to minimize the total cost of the project, which could be made by means of the maximization of the project profitability $b - \mathrm{CRF}\,\mathrm{Inv}_0$. The annual project benefits are obtained from the savings produced by the reduction in energy losses and the avoided costs due to investment deferral in the expansion of the network that can be verified as a reduction in the maximum power metered during the entire year.

In other words, the savings are the difference between the costs before compensating $C^0$and the costs after compensating C^{C}. The reactive power Q^0_{i-j} that initially flows through the network, calculated by means of the methodologies such as those proposed in [2,31,32], will be affected when a compensation in bus j, Q^{C}_i is included. Under these conditions, the inequality $Q^0_{i-j} > Q^{\mathrm{C}}_{i-j}$ holds (see Fig. 2).

In this way, we could say that the benefits to be obtained annually might be computed by the following expression:

$$b = \sum_{i=1}^{n}\sum_{j=1}^{d} C^{\mathrm{L}}_{i-j}(Q^0_{i-j} - Q^{\mathrm{C}}_{i-j}) \tag{17}$$

By replacing (17) in (16), it can be rewritten as

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{d}\frac{C^{\mathrm{L}}_{i-j}Q^{0}_{i-j}}{\mathrm{CRF}}\right]-\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{d}\frac{C^{\mathrm{L}}_{i-j}Q^{\mathrm{C}}_{i-j}}{\mathrm{CRF}}\right)+\mathrm{Inv}_0\right] > 0 \tag{18}$$

The first term of Eq. (18) corresponds to the initial power losses, which cannot be modified. Therefore, the initial objective function to be minimized will be stated as follows:

$$\min \quad \left(\sum_{i=1}^{n}\sum_{j=1}^{d} C^{\mathrm{L}}_{i-j}Q^{\mathrm{C}}_{i-j}\left[\frac{(1+d_{\mathrm{r}})^N - 1}{d_{\mathrm{r}}(1+d_{\mathrm{r}})^N}\right]\right) + \mathrm{Inv}_o \tag{19}$$

where

$$C^{\mathrm{L}}_{i-j} = K\frac{r_{i-j}l_{i-j}}{3V_{\mathrm{L}}^2\sin^2\varphi}Q^0_{i-j} \tag{20}$$

The investment cost is the aggregate cost of purchasing, transporting and installing all capacitor banks selected by the algorithm according to their capacity a, multiplied by a binary decision variable $X(i, a)$ associated to the placing decision.

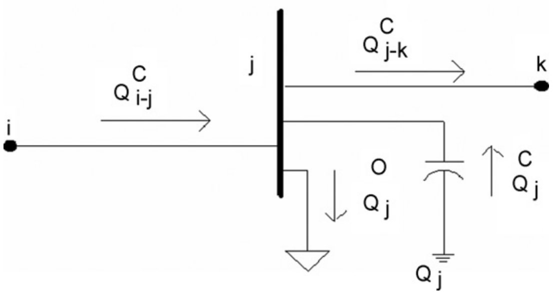

Fig. 3. Reactive power balance at node.

$$\mathrm{Inv}_0 = \sum_{i=1}^{n}\sum_{a=1}^{d} C_{\mathrm{C}}(a)X(i, a) \tag{21}$$

where $C_{\mathrm{C}}(a)$ stands for the total investment cost of the capacitors per unit of kVAr. The objective function will finally be rewritten as follows:

$$\min\left(\sum_{i=1}^{n}\sum_{j=1}^{d} K\frac{r_{i-j}l_{i-j}Q^0_{i-j}}{3V_{\mathrm{L}}^2\sin^2\varphi}Q^{\mathrm{C}}_{i-j}\left[\frac{(1+d_{\mathrm{r}})^N - 1}{d_{\mathrm{r}}(1+d_{\mathrm{r}})^N}\right]\right) + \sum_{i=1}^{n}\sum_{a=1}^{d} C_{\mathrm{C}}(a)X(i, a) \tag{22}$$

Subject to the following constraints:

3.1. Constraint 1: reactive power balance (Kirchoff's law)

For each node j, reactive power balance conditions must be fulfilled according to the following expression (see Fig. 3):

$$\sum_{\mathrm{in}} Q^{\mathrm{C}}_{i-j} - \sum_{\mathrm{out}} Q^{\mathrm{C}}_{j-k} + \sum_{a=1}^{d} Q^{\mathrm{C}}_j(a)X(j, a) = Q^0_j \tag{23}$$

3.2. Constraint 2: selection of a unique bank per node

On the other hand, the condition of installing only one capacitor bank per node is imposed; hence, the following expression

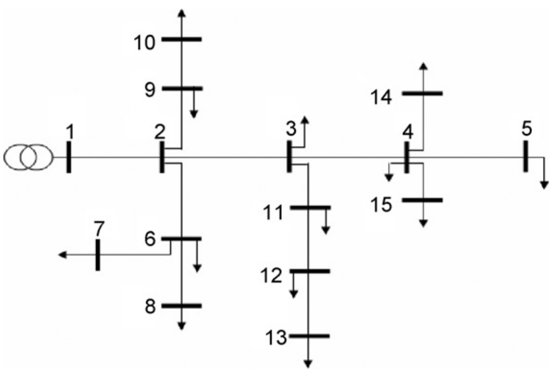

Fig. 4. Single-line diagram of the 15-bus distribution network.

needs to be satisfied:

$$\sum_{a=1}^{d} X(i,a) \leq 1 \tag{24}$$

3.3. Constraint 3: power control in lines

In order to avoid any inverse reactive power flow through the lines by over dimensioning the capacitors banks, the following constraint should be added:

$$\sum_{i}^{n}\sum_{j}^{m} Q_{i-j} \geq 0 \tag{25}$$

3.4. Constraint 4: determination of the type of capacitors (fixed or switched)

From the load curve the minimum and the maximum reactive power can be computed with the aim of determining if the capacitors being installed at a specific bus are fixed or controlled according to the following conditions:

If the capacitor bank to be installed is fixed:

$$\sum_{a=1}^{d} Q_j^{\mathrm{C}}(a) X(j,a) \leq Q_j^{\min} \tag{26}$$

If the capacitor bank to be installed is switched:

$$Q_j^{\min} \leq \sum_{a=1}^{d} Q_j^{\mathrm{C}}(a) X(j,a) \leq Q_j^{\max} \tag{27}$$

where $Q_j^{\min}$ and $Q_j^{\max}$ are minimal and maximal reactive power obtained, respectively from the load curve of the circuit.

4. Results

The proposed methodology has been initially applied to a 15-bus and a 33-bus distribution test networks. The results are compared with the solutions obtained with the methodology presented in [1]. System data are obtained from the same source than the in ref. [1]. In addition, the algorithm has been implemented for the compensation problem of a radial distribution system in the metropolitan area of Caracas served by AES Corporation in Venezuela.

Table 1
15-Bus test network: initial conditions

Main bus current before compensation [A]	96.39
System load [kVA]	$1226.4 + j1251.2$
Losses before compensation [kW]	61.8
Loss reduction [kW] according to Ref. [1]	27.7
Installed capacitors [kVAr] (at node)	[805] (3); [388](6)

4.1. 15-Bus test network

The single-line diagram of the 11 kV, 15-bus system is shown in Fig. 4. The data of the system are obtained from [33]. The comparison between the proposed methodology and the results provided by [1] are shown below.

The initial condition of the network (before compensating) is shown in Table 1. The number of variables, iterations and convergence time are presented in Table 2. The conditions of the network after installing the capacitors are presented in Table 3. The optimizing algorithm attains a loss reduction of 28.58 kW with the installation of 900 kVAr in nodes 3, 4, 6 and 11. In the simulation done in [1], a reduction of 27.7 kW is obtained with the installation of 1193 kVAr in nodes 3 and 6. The nodes where the capacitors were finally located and their compensating capacities are shown in Table 4.

Table 2
15-Bus test network: final simulation conditions

Iterations	273
Objective function [$]	8372
Total variables	100
Integers variables	60
Constraints	14
Convergence time [hh:mm:ss]	00:00:01

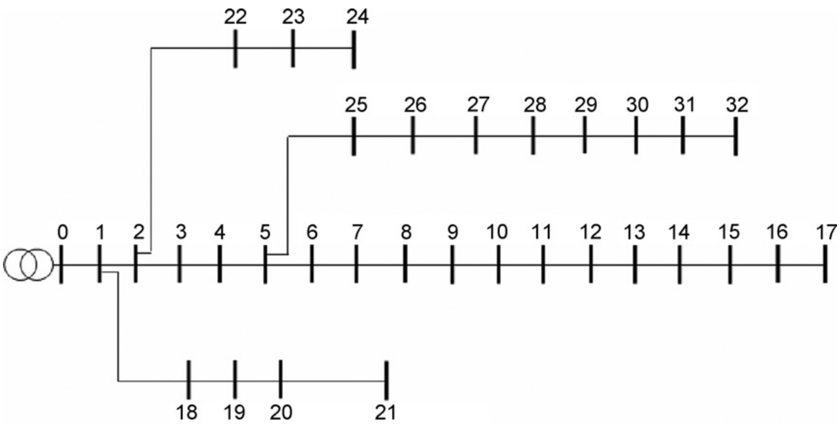

Fig. 5. Single-line diagram of the 33-bus test system.

Table 3
15-Bus test network: final conditions

Main bus current after compensation [A]	69.94
Losses after compensation [kW]	33.22
Achieved loss reduction [kW]	28.58

4.2. 33-Bus test network

The single-line diagram of the 12.66 kV, 33-bus system is illustrated in Fig. 5. The data of the system are obtained from [2]. The initial conditions of the network are shown in Table 5.

The comparison between the proposed methodology and the results obtained in [1] are shown below. The number of variables, iterations and convergence time are presented in Table 6. The final conditions of the network are shown in Table 7. By applying the proposed algorithm a loss reduction of 88.91 kW is achieved with the installation of 1350 kVAr in nodes 6, 8, 13, 23, 27, 29, and 30. In the simulations done in [1], a reduction of 79.5 kW is obtained with the installation of 1400 kVAr in nodes 12 and 29. The selected nodes where the capacitors were finally located and their respective capacities are shown in Table 8.

Table 4
15-Bus test network: location and capacity

Node	Capacity [kVAr]	Node	Capacity [kVAr]
3	150	6	300
4	300	11	150

Table 5
33-Bus test network: initial conditions

Main bus current [A]	259.42
System load [kVA]	$4715 + j2300$
Losses without capacitor [kW]	369.3
Loss reduction [kW] according to Ref. [1]	79.5
Installed capacitors [kVAr] (at node)	[1000](29); [400](12)

Table 6
33-Bus test network: final simulation conditions

Iterations	1851
Objective function [$]	9419
Total variables	132
Integer variables	99
Constraints	100
Convergence time [hh:mm:ss]	00:00:01

Table 7
33-Bus test network: final conditions

Main bus current after compensation [A]	217.91
Losses after compensation [kW]	280.39
Achieved loss reduction [kW]	88.91

Table 8
33-Bus test network: location and capacity

Node	Capacity [kVAr]	Node	Capacity [kVAr]
6	150	27	150
8	150	29	300
13	150	30	150
23	300		

Table 9
Values used for the case study

Capacity charge [$/(kVA month)]	C_P	4.91
Energy price [$/kWh]	C_E	0.035
Loss factor	F_{loss}	0.51
Project lifetime [years]	N	15
Discount rate [%/year]	d_r	20
Nominal voltage [kV]	V_L	12.47
Cost per unit of reactive capacity [$/kVAr]	$C_C(a)$	3

Table 10
State of the circuit before reactive compensation

Annual energy delivered by the circuit	39086.35 MWh/year
Power factor	0.85
Power and energy losses	694.29 MWh/year 116 kW
Nominal voltage in bus bar	12.47 kV
Maximum voltage drop	3.11%
Worst bus	87

4.3. Test case

The developed method was applied to a real network of 231 nodes that covers a zone of the metropolitan area of Caracas.

Table 11
Final condition of the simulation

Iterations	22298
Objective function [$]	12558
Total variables	564
Integers variables	423
Constraints	424
Convergence time [hh:mm:ss]	00:00:04

Table 12
State of the circuit after reactive compensation

Annual energy delivered by the circuit	39086.35 MWh/year
Power factor	0.96
Power and energy losses	285.16 MWh/year 45.26 kW
Nominal voltage in bus bar	12.47 kV
Maximum voltage drop	1.29%
Worst bus	124

Table 13
Location, capacity and type of installed capacitors

Node	Capacity [kVAr]	Nature
23	300	Fixed
50	300	Fixed
55	300	Fixed
64	300	Fixed
79	300	Fixed
94	300	Switched

Table 14
Comparison of the economic performance for the 33-bus test case

	Proposed methodology								Reference [1]		
Bus	6	8	13		23	27	29	30	12		29
Capacity (kVAr)	150	150	150	1350	300	150	300	150	400	1,400	1000
Initial losses (kW)				369.3						369.3	
Final losses (kW)				280.39						289.8	
Investment ($)				8,708						9,031	
Annual savings ($/year)				15,498						13,858	
PV savings ($)				38,542						34,463	
NPV ($)				29,834						25,432	
Payback period (years)				0.574						0.662	
Payback period (months)				6.883						7.949	
Internal rate of return (%)				168.81%						142.72%	

Table 15
Economic evaluation of the 141-bus test case

Bus	Real test case						
	23	50	55		64	79	94
Capacity (kVAr)	300F	300F	300F	1800	300F	300F	300S
Initial losses (kW)				116			
Final losses (kW)				45.26			
Investment ($)				11,611			
Annual savings ($/year)				12,331			
PV Savings ($)				93,791			
NPV ($)				82,179			
Payback period (years)				0.944			
Payback period (months)				11.330			
Internal rate of return (%)				106.20%			

This network includes all equipment of sectioning, transformation and switching of the lateral. The capacities and sizes of the capacitors that were considered in the optimization problem are the existed in stock in the warehouse of the AES electric company. These capacitors sizes are of 150, 300 and 600 kVAr. In order to accelerate the calculations, this network has been reduced to 141 nodes, by eliminating the sectioning devices. Table 9 provides the numerical values of the parameters for which the network has been simulated. Table 16 presents the network data, their connectivity and the numbering of the nodes. The single-line diagram of the network is provided in Fig. 7. The initial conditions of the network before compensating are presented in Table 10. Fig. 6 depicts the daily load pattern served by this system.

The most representative measurements of the distribution circuit and each network node have been requested to the electrical

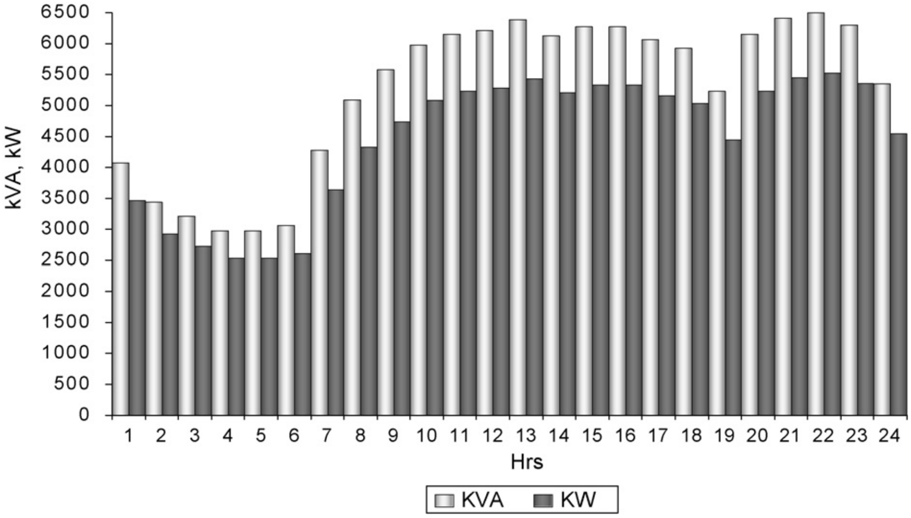

Fig. 6. Load curve of the circuit.

Table 16
Network and connectivity data

I	J	R	X	KVA	I	J	R	X	KVA	I	J	R	X	KVA	I	J	R	X	KVA
1	2	0.0577	0.0409	0	70	72	0.0700	0.0495	150	29	30	0.0342	0.0248	0	99	100	0.0033	0.0008	300
2	3	0.1725	0.1223	0	42	73	0.0231	0.0164	300	30	31	0.0128	0.0091	0	91	101	0.0231	0.0164	15
3	4	0.0009	0.0006	0	73	74	0.0030	0.0064	300	31	32	0.0347	0.0245	150	101	102	0.0578	0.0409	0
4	5	0.0092	0.0065	0	43	75	0.0379	0.0268	45	2	33	0.0443	0.0314	0	102	103	0.0889	0.0217	125
5	6	0.0068	0.0049	0	44	76	0.0552	0..0391	75	33	34	0.0020	0.0009	150	103	104	0.0629	0.0153	0
6	7	0.0469	0.0625	0	46	77	0.0516	0.0436	150	5	35	0.2274	0.0554	300	104	105	0.1170	0.0285	300
7	8	0.0736	0.0981	75	76	78	0.0167	0.0110	0	5	36	0.1265	0.1565	150	104	106	0.0114	0.0026	150
8	9	0.0649	0.0459	10	78	79	0.0415	0.0101	502.5	6	37	0.0055	0.0073	50	92	107	0.0849	0.0207	502.5
9	10	0.0507	0.0359	0	79	80	0.1003	0.0244	750	37	38	0.2036	0.1440	0	94	108	0.0612	0.0260	0
10	11	0.0116	0.0082	0	79	81	0.1513	0.0370	0	38	39	0.0938	0.0663	20	108	109	0.0452	0.0192	750
11	12	0.1291	0.0913	25	81	82	0.0033	0.0008	150	39	40	0.0347	0.0245	0	94	110	0.0033	0.0008	750
12	13	0.1227	0.0866	75	47	83	0.0085	0.0062	75	40	41	0.0918	0.0650	75	7	111	0.0719	0.0509	25
13	14	0.0488	0.0345	0	49	84	0.0517	0.0449	225	41	42	0.2318	0.1640	0	10	112	0.1070	0.0261	500
14	15	0.0957	0.0677	0	50	85	0.0147	0.0036	0	42	43	0.1207	0.0854	0	11	113	0.0347	0.0245	75
15	16	0.0860	0.0609	0	85	86	0.0037	0.0016	500	43	44	0.0443	0.0314	50	13	114	0.0623	.0.0441	0
16	17	0.0398	0.0282	150	86	87	0.0000	0.0000	150	44	45	0.0405	0.0288	0	114	115	0.0668	0.0473	0
17	10	0.0828	0.0566	0	7	88	0.0174	0.0231	75	45	46	0.0160	0.0127	0	115	116	0.0040	0.0010	300
16	19	0.0186	0.0132	0	88	89	0.0469	0.0625	65	46	47	0.0636	0.0450	0	14	117	0.0506	0.0366	65
19	20	0.0559	0.0395	75	89	90	0.0299	0.0398	0	47	48	0.0417	0.0295	125	15	118	0.0161	0.0114	0
20	21	0.0365	0.0246	75	90	91	0.0212	0.0283	0	48	49	0.0732	0.0510	150	118	119	0.0462	0.0327	110
21	22	0.0573	0.0307	0	91	92	0.0315	0.0420	0	49	50	0.0828	0.0556	0	119	120	0.0424	0.0300	0
22	23	0.0263	0.0191	75	92	93	0.0280	0.0373	0	50	51	0.0398	0.0282	125	120	121	0.0507	0.0359	0
23	24	0.0683	0.0497	0	93	94	0.0206	0.0274	110	51	52	0.0225	0.0159	75	121	122	0.0732	0.0518	0
24	25	0.0398	0.0282	0	94	95	0.0206	0.0274	0	38	53	0.0841	0.0595	100	122	123	0.0584	0.0414	100
27	26	0.0729	0.0530	150	89	96	0.0687	0.0486	150	42	54	0.0.161	0.0114	0	123	124	0.0610	0.0432	125
26	27	0.0335	0.0244	75	96	97	0.0970	0.0686	0	54	55	0.0527	0.0373	0	124	125	0.0783	0.0554	0
27	23	0.0584	0.0414	0	97	98	0.0902	0.0196	300	55	56	0.0893	0.0632	25	125	126	0.0834	0.0607	0
28	29	0.0655	0.0463	75	97	99	0.0033	0.0008	0	56	57	0.0867	0.0613	0	126	127	0.0347	0.0245	75
61	62	0.0411	0.0291	200	131	132	0.0347	0.0245	75	57	58	0.0674	0.0477	300	127	128	0.0570	0.0420	75
60	63	0.0353	0.0250	0	131	133	0.0920	0.0669	45	58	59	0.0469	0.0332	150	128	129	0.0585	0.0425	110
63	64	0.1047	0.0741	300	121	134	0.0841	0.0612	35	55	60	0.0334	0.0236	0	129	130	0.0103	0.0073	112.5
64	65	0.0674	0.0477	150	16	135	0.0527	0.0373	25	60	61	0.0327	0.0232	300	119	131	0.0355	0.0253	0
65	66	0.0302	0.0214	225	16	136	0.0302	0.0214	75	63	69	0.0366	0.0259	300	25	139	0.0950	0.0673	50
66	67	0.0456	0.0323	50	16	137	0.0584	0.0414	55	66	70	0.0231	0.0164	0	30	140	0.0519	0.0377	150
67	63	0.0218	0.0154	100	23	138	0.0769	0.0559	50	70	71	0.0120	0.0029	300	31	141	0.0584	0.0414	75

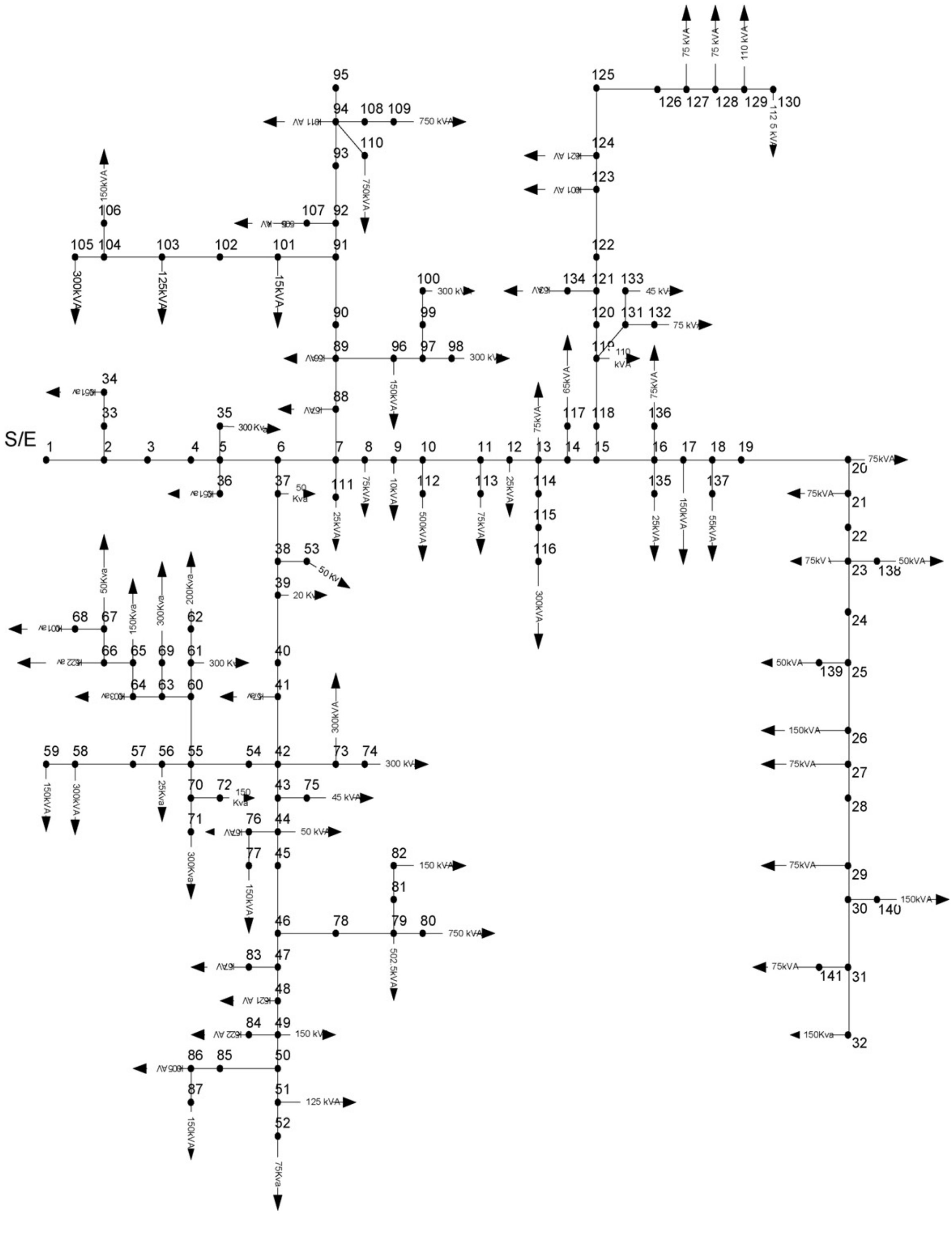

Fig. 7. Single-line diagram of a real distribution network in the metropolitan area of Caracas.

utility to carry out the optimal capacitors location study into distribution network. The large measurement campaign was promoted by the AES electrical company. The load profiles were obtained by a number of measurements and statistical evaluation to determine a daily load curve for each node and each distribution feeder circuit leaving the substation. All days of the week were considered excepted the Saturday and Sunday days, because these days introduce noise in the daily load pattern. The study was carried out in the distribution circuit belonging of the metropolitan area of Caracas, where the nodes are saturated respect to load growth. The load growth could be considered by means of an adequate demand forecast method if the network was not saturated. A new study could be carried out every 2 or 3 years to locate new capacitors in the network, reducing the energy losses and optimizing the investments in reactive power compensation.

The number of intervening variables, iterations and convergence time is included in Table 11. Table 12 presents the final condition of the network after compensating with the developed optimization model. A loss reduction of 70.74 kW is accomplished with the installation of 1500 kVAr fixed and 300 controlled distributed in the nodes 23, 50, 55, 64, 79 and 94, respectively. The nodes where capacitors were finally located and their respective capacities are included in Table 13.

5. Economic evaluation

An economic evaluation of results is performed for two of the cases presented previously: the 33-bus and the 141-bus distribution systems. In the 33-bus case, the results are compared with the results of reference [1]. In both cases the execution of a load flow is carried out with the objective of quantifying the losses before compensation. Then, the proposed methodology was applied with the aim at locating and at quantifying the optimal capacity of the capacitors banks and its best location nodes.

The power loss and released capacity savings are computed in monetary terms for the scenarios with and without reactive compensation. For the present calculations, only the investment cost in capacitors is considered. Tables 14 and 15 summarize the results of the economic assessment and the computed profitability indicators of the compensation projects for the 33-bus and 141-bus cases, respectively. In the case of the 33-bus test system, the proposed methodology delivers compensations solutions with a higher internal rate of return (168.8%/year) and shorter payback period (6.88 months) than those achieved by using the method presented in [1] (142.7%/year and 7.95 months, respectively). The high profitability and very short payback period of the compensation project in the system served by AES-Venezuela (106.2%/year and 11.33 months) proves the efficacy of the proposed method in handling problems.

6. Conclusion

This paper proposes an efficient method for the optimal location and sizing of static and switched shunt capacitors in radial distribution systems. The problem has been formulated as the maximization of the NPV of the compensation project. The objective is to maximize the global savings obtained from loss reduction and capacity release in the expansion of the network. After a suitable linearization, the optimization problem was formulated as a mixed-integer linear problem and successfully solved by using a widespread commercial package.

Power losses are a source of economic inefficiency and thus impact negatively in the business results of distribution companies. Least-cost measures for reducing power losses can notably contribute to maximize firm value. Loss reduction and deferral investments in the network could be reverted as a benefit to consumers under a suitable tariff policy.

The proposed method was tested on three distribution systems consisting of 15, 33 and 141 buses. The results on the 15-bus and 33-bus test cases were compared with the results computed with a recent methodology reported in [1]. It was found that the results of this methodology are consistently better than the results provided in the mentioned reference.

The methodology has also applied to a real case of 141 buses corresponding to a circuit that feeds a zone of the metropolitan area of Caracas attended by AES-Venezuela. A short convergence time of about 4 s after 22,298 iterations shows the effectiveness of the proposed mathematical model for solving the optimal compensation problem in a framework of high dimensionality.

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Appendix A. List of symbols

b_k	annual benefits obtained in year k
B	present value of the cumulated benefits over the project's lifetime k
C	loss costs
CRF	capital recovery factor
$C_{\mathrm{C}}(a)$	capacitor investment cost [$/kVAr]
C^{C}	loss costs after compensating
C_{E}	energy charge [$/kWh]
C_{P}	power capacity charge [$/(kVA month)]
C^0	loss costs before compensating
$C_{i,j}^{\mathrm{L}}$	loss costs in the branch between node i and node j
d	number of capacitor bank size considered
d_{r}	discount rate [%/year]
F_{loss}	loss factor
I	current magnitude that circulates through the line [A]
Inv_0	investment in year 0 (initial investment)
I_{i-j}	longitude between node i and node j
I_{K}	reactive current component [A]
I_{R}	active current component [A]
K	cost coefficient for losses and released capacity [$/(kW year)]
n	number of nodes in the network
m	number of lines in the network
N	project study period [year]
NPV	net present value of a cash flow
PV	present value of a stream of payments/savings
P_{loss}	electrical power losses due to Joule effect (kW)
Q	reactive power [kVAr]
Q_{i-j}	reactive power flow through the lines by over dimensioning the capacitors banks [kVAr]
Q^0	reactive power before compensating, obtained by mean of load flow [kVAr]
Q_j^{c}	reactive power installed in node j because of the compensation [kVAr]
Q_{i-j}^{c}	reactive power that circulates from node i to node j, after compensation [kVAr]
$Q_j^{\mathrm{c}}(a)$	reactive power installed in node j because of the compensation. This variable changes in function of the bank capacity (a) [kVAr]
$Q_j^{\max}$	maximum reactive power obtained from the load curve [kVAr]

$Q_j^{\min}$ minimum reactive power obtained from the load curve [kVAr]
Q_{i-j}^{0} reactive power that circulates from node i to node j before compensation [kVAr]
r line resistance per km [Ω/km]
R line resistance [Ω]
S apparent power [kVA]
V_{L} nominal voltage (kV)
$X(i, a)$ binary decision variable (0,1) to install a bank capacity (a) node I

Greek letter
φ impedance angle

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