Published in IET Signal Processing
Received on 30th April 2009
Revised on 3rd November 2009
doi: 10.1049/iet-spr.2009.0117

ISSN 1751-9675

Algorithm for complementary-derived orthogonal sequences applied to measurements in noisy channels

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Abstract: Complementary sequences are currently being applied to multiple fields of engineering not only when an active signal must be coded to obtain large noise immunity (proportional to the sequences length) but also when multiuser or multisensory operations need to be conducted (because of their orthogonality properties). The orthogonality between complementary sequences is only defined for sequences with the same length. This constraint limits complementary sequences implementation in cases in which all the sequences feature the same length. This paper describes the way in which orthogonal sequences of different length can be generated starting from given complementary sequences. A possible application to a noisy channel is also analysed. In the said application, a pair of complementary sequences of short length is used to code the desired information, while simultaneously an orthogonal complementary pair of larger length is used to measure the channel attenuation.

1 Introduction

Several communication and sensor-related engineering applications need to recover information contained in digital signals corrupted by noise introduced by external sources. In particular, the correlation function, as a signal processing technique, is suitable for the detection of signals with additive white Gaussian noise (AWGN) and is commonly used in communications systems, radar, active sensors and so on. Nevertheless, to maximise the advantages of this method, it is convenient to encode the signals with algorithms with an easily identifiable correlation response. The absence of side lobes in the autocorrelation function (ACF) has rendered complementary sequences [1, 2] suitable for applications in which the detection of a signal immersed in noise is required [3]. Their orthogonality property also makes them suitable in those cases in which independent information needs to be simultaneously sent in the same physical channel [4]. The complementary sequences are widely used in sensors systems [3], robotics [4], communications [5], ground penetration radars [6] and so on. Nonetheless, in the case of complementary sequences and other similar sequences, the orthogonality is limited to binary sequences of equal length. Little is known about the use of sequences of different length in the same channel and further research is worthy of consideration in this respect. Recently, a method for complementary quaternary sequences generation of different length has been developed, opening a new research line in the radar and communications field [7]. Sequences of different length are appropriate for multi-emission systems in which many users share the same physical channel but they are affected by noise in a different way, or by processes in which independent channel estimation is required [8].

This paper describes and analyses a method for generating orthogonal sets of binary sequences of different length, referred to as complementary-derived orthogonal sequences (CDOS) [8]. These sequences can be generated with larger length in relation to that of a particular complementary sequences set. The emission of two pairs of sequences of different lengths at the same time in the same transmission channel, with no interference between them, will be demonstrated using simulations as well as attenuation measurements from a noisy channel under the same conditions.

2 Complementary sequences

Complementary sequences are defined as a pair of sequences composed of two binary elements (± 1) of length $L = 2^n$ ($n0\rceil$) [1]. They can be denoted as $\{S_{w,1,n}[k], S_{w,2,n}[k]\}$, where w is a coefficient called 'generation seed' [9]. Given a pair of complementary sequences (2-CSS), their corresponding ACFs are

$$C_{S_{w,v,n}\cdot S_{w,v,n}}[k] = \sum_{i=1}^{L} S_{w,v,n}[i] \cdot S_{w,v,n}[i+k] \tag{1}$$

where i is the ith iteration of the correlation computation, with $v \in \{1, 2\}$. The sum of both ACF results in

$$C_{S_{w,1,n}\cdot S_{w,1,n}}[k] + C_{S_{w,2,n}\cdot S_{w,2,n}}[k] = 2 \cdot L \cdot \delta[k] \tag{2}$$

where $\delta[k]$ is the Kronecker delta of magnitude $2L$ without side lobes. This result is independent of the synchronous or asynchronous correlation and unattainable by other binary sequences. Complementary sequences also feature other noteworthy characteristics such as:

- concatenation of sequences in continuous emission, useful to multiplex sequences in a single emission [3];
- efficient generation and correlation processing architectures, enabling calculations reduction [9–11];
- ideal orthogonal pairs of sequences [2], which allow to detect two pairs of sequences in the same channel.

2.1 Complementary-derived orthogonal sequences

Given a 2-CSS of length L and seed w_1, there is just one orthogonal pair of the same length and seed w_2. The sum of cross-correlation functions (CCFs) between both pairs is null

$$C_{S_{w_1,1,n}\cdot S_{w_2,1,n}}[k] + C_{S_{w_1,2,n}\cdot S_{w_2,2,n}}[k] = 0 \quad \forall k \tag{3}$$

Given the orthogonal pair, referred to as 'generator pair', a pair of CDOS) $\{S'_{w2,1,p}[k], S'_{w2,2,p}[k]\}$ can be obtained by means of a recursive concatenation [8]

$$\begin{aligned} S'_{w_2,1,0}[k] &= +S_{w_2,1,n}[k] \\ S'_{w_2,1,1}[k] &= -S'_{w_2,1,0}[k] + S'_{w_2,1,0}[k-L] \\ S'_{w_2,1,2}[k] &= +S'_{w_2,1,0}[k] - S'_{w_2,1,0}[k-L] \\ &\quad - S'_{w_2,1,0}[k-2\cdot L] + S'_{w_2,1,0}[k-3\cdot L] \\ &\vdots \\ S'_{w_2,1,p}[k] &= -S'_{w_2,1,p-1}[k] + S'_{w_2,1,p-1}[k-D_{p-1}\cdot L] \end{aligned} \tag{4a}$$

$$\begin{aligned} S'_{w_2,2,0}[k] &= +S_{w_2,2,n}[k] \\ S'_{w_2,2,1}[k] &= -S'_{w_2,2,0}[k] + S'_{w_2,2,0}[k-L] \\ S'_{w_2,2,2}[k] &= +S'_{w_2,2,0}[k] - S'_{w_2,2,0}[k-L] \\ &\quad - S'_{w_2,2,0}[k-2\cdot L] + S'_{w_2,2,0}[k-3\cdot L] \\ &\vdots \\ S'_{w_2,2,p}[k] &= -S'_{w_2,2,p-1}[k] + S'_{w_2,2,p-1}[k-D_{p-1}\cdot L] \end{aligned} \tag{4b}$$

where D_p is a delay defined as $D_p = 2^p$ and p is a natural number that represents the iterations of the recursive algorithm. According to this, the length of the CDOS is $L_{\mathrm{CDOS}} = 2^p \cdot L = 2^{(n+p)}$.

As CDOS are generated from a complementary pair, the sum of CCF between $\{S_{w1,1,n}[k], S_{w1,2,n}[k]\}$ and $\{S'_{w2,1,p}[k], S'_{w2,2,p}[k]\}$ is null. Attention should be paid to the fact that the CDOS do not meet the ACF properties of complementary sequences [2]. Then the sum of ACF results in a main lobe $2L_{\mathrm{CDOS}} = 2^{(n+p+1)}$ as well as in equi-spaced side lobes. Equation (4) applies the recursive concatenation by alternating the sign of the sequences

$$S'_{w,v,p}[k] = -S'_{w,v,p-1}[k] + S'_{w,v,p-1}[k - D_{p-1}\cdot L] \tag{5}$$

Also the recursive algorithm can be applied without alternating the signs in the concatenation, that is

$$S'_{w,v,p}[k] = +S'_{w,v,p-1}[k] + S'_{w,v,p-1}[k - D_{p-1}\cdot L] \tag{6}$$

where all the parameters have been previously introduced. Applying (6), the sum of ACF will yield more side lobes with increasing amplitude, and thus be unsuitable for signal identification. Besides, sign alternation allows us to reduce the amplitude of the side lobes, offering a more easily identifiable response (Fig. 1). Note that the addition of the ACFs features a free interference window around the main lobe and also at the beginning and at the end of the ACF. This can be seen as 'zero correlation zone' sequences that exhibit interesting properties for pseudo-synchronised systems [12, 13].

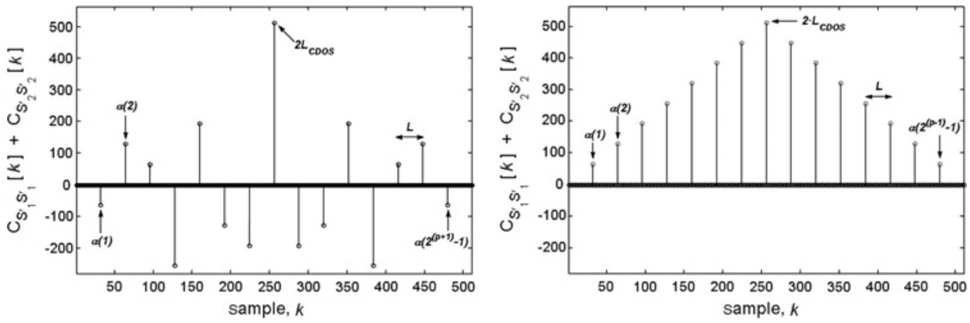

Figure 1 *Sum of ACF of CDOS for a pair of length $L_{CDOS} = 8L = 256$, using (5) for alternating signs (left) and (6) for no signs alternation (right)*

Let us refer to the CDOS generation rule as T_p, where $T_p[k] = -T_{p-1}[k] + T_{p-1}[k - D_{p-1} \cdot L]$ and $T_0 = \delta[k]$, then the side lobes amplitude of the ACF sum of a CDOS pair can be calculated applying the following expression

$$\alpha[j] = 2L \cdot \sum_{i=1}^{2^{(p+1)}-1} T_p[i] \cdot T_p[i - j \cdot L] \tag{7}$$

where $\alpha[j]$ is each side lobe resulting from the ACF sum of $\{S'_{w2,1,p}[k], S'_{w2,2,p}[k]\}$ and L is the length of the 'generator pair'. Therefore, the side lobes amplitude and the polarity can be predicted for any L_{CDOS} length from (7). These data are of great relevance if post-processing (e.g. equalisation) of correlated signal is required. A simple way to do this post-processing is by subtracting the side lobes sequence $\alpha[j]$ from the correlated signal. The $\alpha[j]$ could be attenuated by a factor obtained from the real amplitude of the detected peak, so the cancellation of the side lobes could be compensated with respect to the channel attenuation.

The feasibility of implementing systems based on complementary sequences, on a practical basis, rests on the design of efficient architectures for generation and correlation, in order to reduce the computational load. Budisin [10] and Popovic [11] developed a recursive method for an efficient generation and correlation of complementary sequences, respectively. Bearing in mind the generation rule (4), the CDOS generator can be implemented using an efficient generator and a set of delays and negative unitary gains, as it is demonstrated in a previous work [8]. The advantage of generating CDOS from an efficient generator, and of adjusting their length by repeating equal stages, is that they can be easily implemented in programmable devices such as field programmable gate arrays (FPGAs) [14], enabling a real-time signal processing.

3 Applications of CDOS in sensors and communications

Complementary sequences have been used in applications related to the identification or estimation of different transmission channels [5, 8]. CDOS have been suggested to be applied in a safety railway project (TELEVIA [3]), where two different pairs of sequences were required to make independent signal processes. The 2-CSS, also called Golay sequences, was used as the detection signal of the train's passage through specific detection points of an array sensor. The pair of CDOS, on the other hand, was used as the channel identification signal. One of the requirements was that the second pair of sequences be longer than the first one, as it was important to know the response of the measuring channel under lower signal-to-noise ratio (SNR) conditions, only attainable with longer sequences. The short length pair of sequences was used for an uninterrupted channel checking, whereas the longer one was generated with a lower frequency (or sporadically, on demand) but with a higher gain process.

To evaluate the use of CDOS and 2-CSS, this paper employs an emission/reception system based on the project above mentioned. Unlike the safety railway project, the system uses an interleaving/de-interleaving process to combine the sequences, and the emission is in baseband, without modulation (see the block diagram in Fig. 2). In this work, the physical channel has been modelled as a linear attenuation $\gamma (0 < \gamma \leq 1)$ with an external noise source. This is a good estimation for channel models neglecting or with no multipath effects [3] and a practical technique for ultrasonic and/or radar systems. A different analysis, not within the scope of this work, is necessary for channels with multipath or nonlinear effects, or for those in which factor γ is a complex number.

The transmitted signals are detected by a correlation block. The outputs of this block are the sum of CCF between Y and the 2-CSS $\{S_{w1,1,n}[k], S_{w1,2,n}[k]\}$ (denoted as Y_{CS}) and the sum of CCF between Y and the CDOS pair $\{S'_{w2,1,p}[k], S'_{w2,2,p}[k]\}$ (denoted as Y_{CDOS}). In this example, 2-CSS and CDOS are combined by means of a simple addition in baseband and they make no use of modulation schemes.

The signals received contain the sum of the 2-CSS and CDOS attenuated by the channel (γ) as well as an AWGN signal (n_o). Assuming that the transmission channel is

IET Signal Process., 2010, Vol. 4, Iss. 6, pp. 610–617
doi: 10.1049/iet-spr.2009.0117

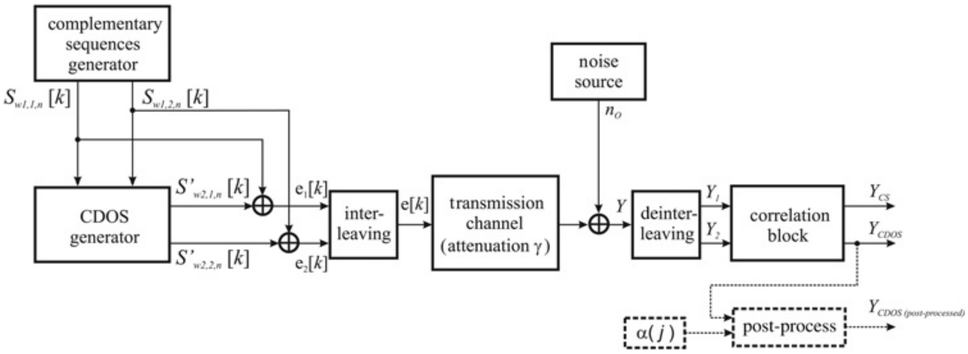

Figure 2 *Block diagram of an emission/reception system*

linear, the outputs after de-interleaving are

$$\begin{aligned} Y_1 &= \gamma \cdot (S_{w_1,1,n}[k] + S'_{w_2,1,p}[k]) + n_o[k] \\ Y_2 &= \gamma \cdot (S_{w_1,2,n}[k] + S'_{w_2,2,p}[k]) + n_o[k] \end{aligned} \tag{8}$$

The sum of CCF between $\{Y_1, Y_2\}$ and $\{S_{w1,1,n}[k], S_{w1,2,n}[k]\}$ results in a Kronecker delta with a main lobe $\gamma 2L$ (output Y_{CS}), whereas the sum of CCF between $\{Y_1, Y_2\}$ and $\{S'_{w2,1,p}[k], S'_{w2,2,p}[k]\}$ (output Y_{CDOS}) results in a main lobe $\gamma 2L_{CDOS}$ and side lobes whose amplitude and polarity are derived from (7). These results can be appreciated in Fig. 3, where $L = 256$, $L_{CDOS} = 8L$, SNR = 0 dB and attenuation $\gamma = 1$ and considering the continuous emission of the pair $\{S_{w1,1,n}[k], S_{w1,2,n}[k]\}$ and the emission and reception in baseband. The SNR was calculated taking into account the corresponding noise variance and that the amplitude of the sequences is ± 1.

Given the different lengths of both pairs of sequences ($L_{CDOS} = 2^p \cdot L$), there are 2^p pairs of 2-CSS for each pair of CDOS and, consequently, 2^p correlation peaks for each pair of CDOS.

4 Results

4.1 Simulations results

In order to demonstrate the robustness of CDOS and their possible use with 2-CSS, an analysis of the main lobe attenuation for both pairs of sequences is conducted using MATLAB®. The evolution of the correlation lobes with different channel attenuations and noise levels is analysed. The aim is to demonstrate that the transmission channel attenuation can be estimated under low SNR conditions using CDOS, with no interference with 2-CSS. Both pairs of sequences (2-CSS and CDOS) are attenuated with a

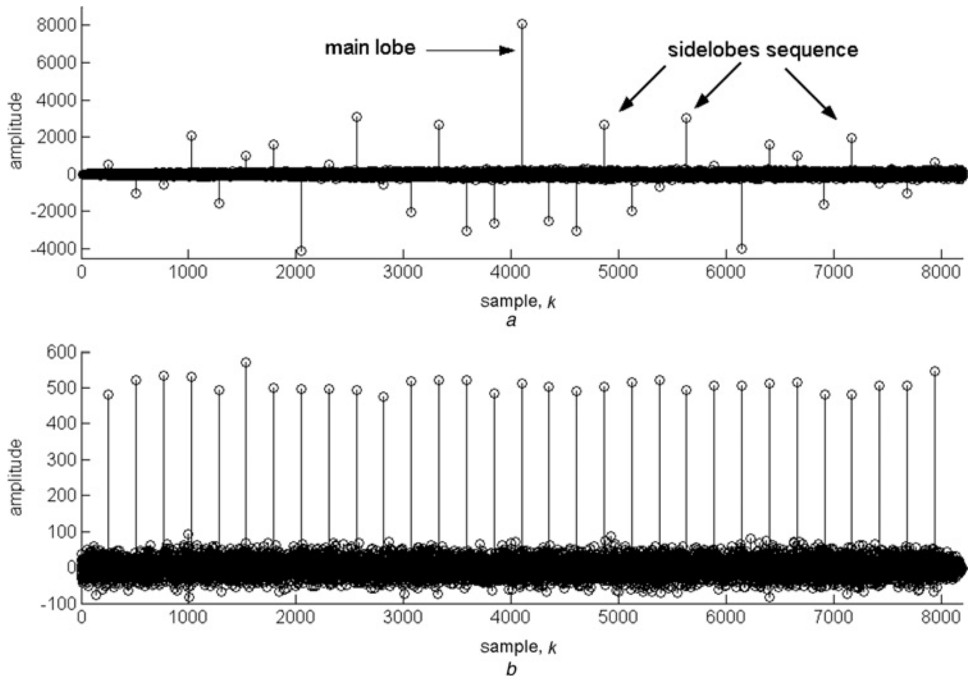

Figure 3 *Outputs*

a Y_{CDOS}
b Y_{CS}

factor γ and affected by a noise n_o, normally distributed with mean = 0 and variance $\sigma^2 = 1$. Let a 2-CSS be of length $L = 32$ $(n = 5)$ and a pair of CDOS be of length $L_{\text{CDOS}} = 1024$ $(p = 5)$, arranged as shown in Fig. 4.

The 2-CSS is arranged in such a way that the last element of the pair, $\{S_{w1,1,n}[L], S_{w1,2,n}[L]\}$, is followed by the first element of the pair, $\{S_{w1,1,n}[1], S_{w1,2,n}[1]\}$. So, 2-CSS are continuously emitted, making up a pair of larger sequences, $A[k]$ and $B[k]$. Since $L_{\text{CDOS}} = 2^p \cdot L$, there are 2^p consecutive pairs of 2-CSS for each pair of CDOS (Fig. 4). Considering that the intended purpose of CDOS is the channel attenuation measurement, they are emitted with a certain time separation, established in $2L_{\text{CDOS}}$ for this test. Then a pair of sequences $C[k]$ and $D[k]$ is obtained, as depicted in Fig. 4. Two emission sequences are created from $A[k]$, $B[k]$, $C[k]$ and $D[k]$

$$\begin{aligned} e_1[k] &= A[k] + C[k] \\ e_2[k] &= B[k] + D[k] \end{aligned} \tag{9}$$

To simplify this analysis, it is assumed that both sequences are emitted in baseband, using interleaving (see (10))

where $\otimes$ denotes said interleaving. The output of the channel is

$$Y[k] = \gamma \cdot e[k] + n_o[k] = \gamma \cdot [e_1[k] \otimes e_2[k]] + n_o[k] \tag{11}$$

Then, supposing an ideal de-interleaving, there are two different correlation outputs (Fig. 2)

$$\begin{aligned} Y_{\text{CS}}[k] = & \sum_{i=1}^{L} (\gamma \cdot e_1[i] + n_o[i]) \cdot S_{w_1,1,n}[i+k] \\ & + \sum_{k=1}^{L} (\gamma \cdot e_2[i] + n_o[i]) \cdot S_{w_1,2,n}[i+k] \end{aligned} \tag{12}$$

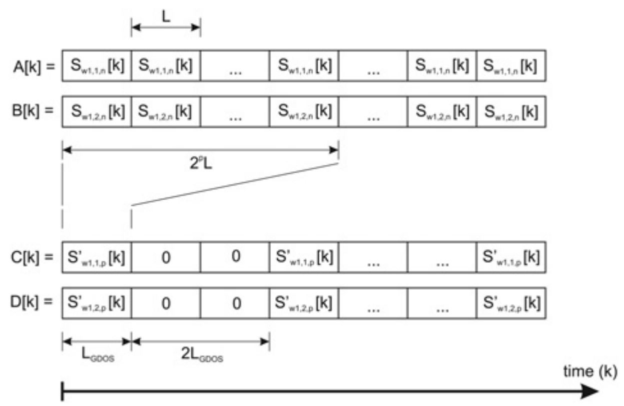

Figure 4 *Transmission scheme used for emitting 2-CSS and CDOS*

$$\begin{aligned} Y_{\text{CS}}[k] = \gamma \cdot & \left(\sum_{i=1}^{L} e_1[i] \cdot S_{w_1,1,n}[i+k] \right. \\ & \left. + \sum_{i=1}^{L} e_2[i] \cdot S_{w_1,2,n}[i+k] \right) + (C_{n_o,S_{w_1,1,n}} + C_{n_o,S_{w_1,2,n}}) \end{aligned} \tag{13}$$

$$\begin{aligned} Y_{\text{CDOS}}[k] = & \sum_{i=1}^{L_{\text{CDOS}}} (\gamma \cdot e_1[i] + n_o[i]) \cdot S'_{w_2,1,p}[i+k] \\ & + \sum_{i=1}^{L_{\text{CDOS}}} (\gamma \cdot e_2[i] + n_o[i]) \cdot S'_{w_2,2,p}[i+k] \end{aligned} \tag{14}$$

$$\begin{aligned} Y_{\text{CDOS}}[k] = \gamma \cdot & \left(\sum_{i=1}^{L_{\text{CDOS}}} e_1[i] \cdot S'_{w_2,1,p}[i+k] \right. \\ & \left. + \sum_{i=1}^{L_{\text{CDOS}}} e_2[i] \cdot S'_{w_2,2,p}[i+k] \right) \\ & + (C_{n_o,S'_{w_2,1,p}} + C_{n_o,S'_{w_2,2,p}}) \end{aligned} \tag{15}$$

where there is a term resulting from the noise C_{n0} that is the addition of CCF between the noise component and the 2-CSS or CDOS.

Afterwards, an amplitude threshold (Γ) is applied to both outputs. Under absence of noise $(n_0 = 0)$ and unitary channel attenuation conditions $(\gamma = 1)$, the correlation peaks for both pairs of sequences are proportional to the double of the sequences length ($2L$ and $2L_{\text{CDOS}}$, respectively). If $\gamma < 1$, the correlation peaks are affected by γ. Then two different static thresholds are used for both correlated outputs, each one equal to the length corresponding to each pair of sequences multiplied by γ ($\Gamma_{\text{CS}} = \gamma \cdot L$ and $\Gamma_{\text{CDOS}} = \gamma \cdot L_{\text{CDOS}}$; Fig. 5). All the correlation peaks above the threshold are averaged to obtain the attenuation measurement corresponding to the channel under low SNR conditions. An arrangement of 2-CSS of length $L = 32$ and CDOS of length $L_{\text{CDOS}} = 1024$ was employed for this analysis, (Fig. 4). Fig. 6 shows the results corresponding to a channel with a $\text{SNR} = -6$ dB, corresponding to the attenuation $\gamma = 1$.

As illustrated in Fig. 6, the use of CDOS allows us to estimate channel performance more accurately when compared with the shorter 2-CSS. The attenuation of the channel reduces the SNR of the received signals. From $\gamma = 0.6$ to $\gamma = 0.4$, the estimation obtained with 2-CSS shows a dispersion that underestimates the real attenuation of the channel at first and ends up overestimating it. This is explained by the effect of the noise, which shows that other samples in addition to the main lobe were over or

$$e[k] = e_1[k] \otimes e_2[k] = \left[\underbrace{e_1[1]}_{k=1} \quad \underbrace{e_2[1]}_{k=2} \quad \underbrace{e_1[2]}_{k=3} \quad \underbrace{e_2[2]}_{k=4} \quad \underbrace{e_1[3]}_{k=5} \quad \underbrace{e_2[3]}_{k=6} \quad \cdots \quad \cdots \right] \tag{10}$$

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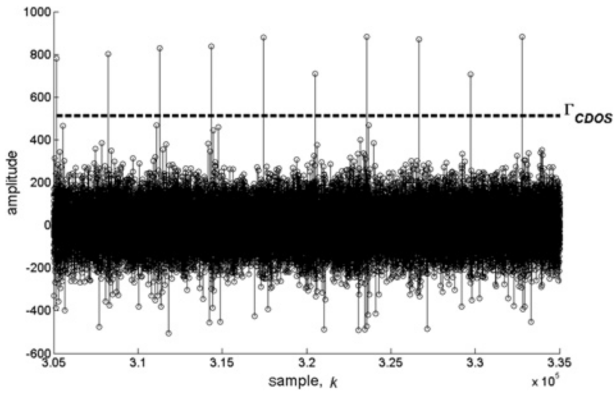

Figure 5 *Y_{CDOS} and detection threshold Γ_{CDOS} ($\gamma = 0.5$ and SNR $= -6$ dB)*

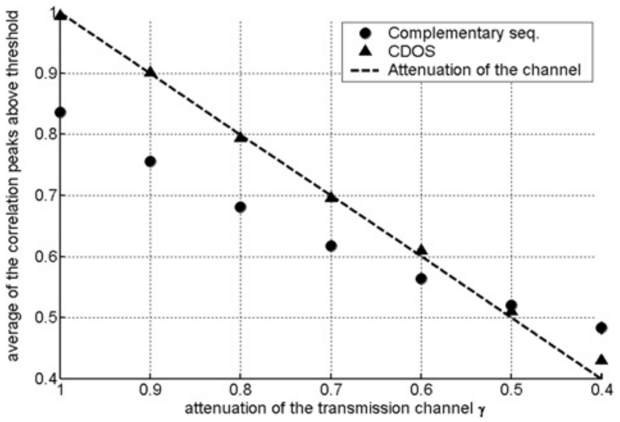

Figure 6 *Amplitude of the correlation peaks of 2-CSS and CDOS for a channel with a SNR $= -6$ dB*

below the threshold and leads to wrong detections. CDOS, in turn, estimates the attenuation with a minor error, including attenuations in the order of $\gamma = 1$, exhibiting no interferences with complementary sequences and external noise. No post-processing was considered for this analysis so better results are expected in systems or applications using some type of post-processing.

4.2 Experimental results

In order to verify in a simple way the proposed algorithms, an experimental test was conducted with a prototype implemented in FPGA. The block diagram of the complete system is shown in Fig. 2. Generators and correlators are implemented in a Xilinx FPGA Virtex XCV300-4BG352, using an 8-bit ADC for signal digitalisation. Some characteristics of the test are:

- main clock: 50 MHz;
- frequency emission: 20 kHz;
- percentage of occupancy of the FPGA: 301 slices (9%).

The details of the FPGA implementation of the generator and correlator are described in [15]. Both pairs of sequences (2-CSS and CDOS) are generated and added within the FPGA. Some tests with 2-CSS of length $L = 16$ elements and a pair of CDOS of length $L_{CDOS} = 32$ elements were carried out, using SNR ranging from 6 to -6 dB, for a signal of 400 mV. Fig. 7 depicts an oscilloscope view of both correlation outputs. An amplitude threshold was applied to both correlator outputs and all the correlation peaks above the threshold were averaged to obtain the mean correlation peak amplitude under different SNR conditions. The attenuation channel factor was unitary ($\gamma = 1$), but the results of the correlation of both pairs of sequences were slightly different. Note that the main lobes of the Y_{CS} output are notoriously affected by the noise, yet those corresponding to the Y_{CDOS} output remain almost unaltered. The relative error between the sum of ACF with regard to the SNR is displayed in Fig. 8. It can be easily observed that the relative error in the estimation of the correlation peaks increases abruptly when the SNR falls below 0 dB. In the case of SNR $= -6$ dB, the estimation

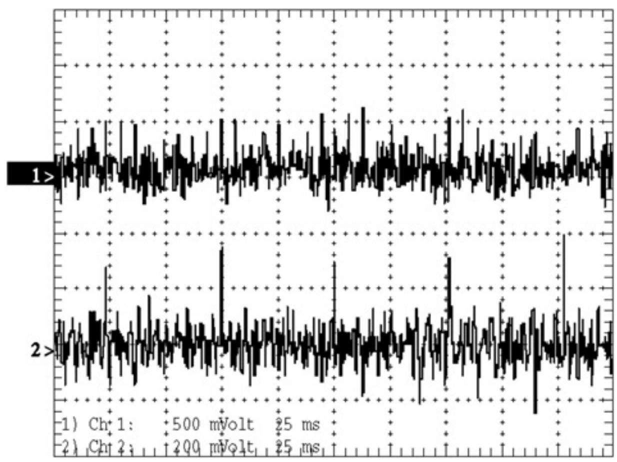

Figure 7 *Oscilloscope view of correlations outputs Y_{CS} (Ch.1) and Y_{CDOS} (Ch.2)*

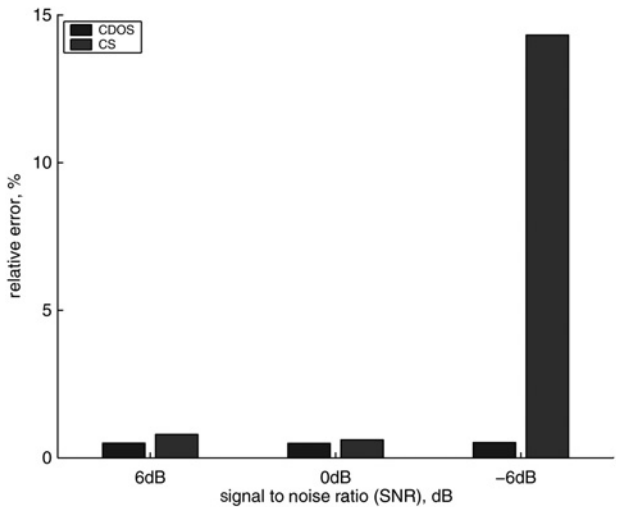

Figure 8 *Relative error obtained from the sum of ACF under different SNR conditions*

doi: 10.1049/iet-spr.2009.0117

of the amplitude with 2-CSS yields a relative error of 14.7% while the CDOS estimation error remains around 1%. If a larger CDOS is used, a minor relative error is expected. These results are consistent with those in Fig. 6. It is worth noticing that in this test a noise level contaminates the signal and two pairs of sequences are emitted at the same time in the same physical channel. This enables to verify the robustness of the proposed technique when working in noisy channels, and it further proves the feasibility of implementing orthogonal sequences of different length in the same physical channel.

5 Conclusions

This paper describes an algorithm for the generation of sets of orthogonal sequences of different length derived from complementary sequences. The likelihood of using binary sequences of different lengths in the same physical channel with no interference between them has been demonstrated, and therefore their use for different purposes admitted. The applications mentioned, along with some others, can be improved or further developed using M-CSS. Still an in-depth analysis of the effects of modulation on the mathematical properties of CDOS as well as of the way in which the traditional emission schemes can be enhanced is still pending. It should not be ruled out that the use of ZCZ sequences might improve the performance of multi-emission systems, and the combination of CDOS with ZCZ sequences is expected to allow enrichment of their applications. Several works related to the generation of ZCZ sequences support this statement [12, 13].

Successful results were reached in the estimation of channel attenuation under low SNR conditions. This technique could be widely applied especially to communications, transmission channel identification and multi-emission schemes. The proposed algorithm becomes useful for attenuation measurements in noisy channels and multi-emission schemes for sensory systems. Information regarding the impulse response estimation of the channel is a potential extension of this work. Some authors have suggested the use of M-CSS in channel identification [5], so CDOS are expected to improve that process under low SNR conditions and interferences.

Focus should be on the study of non-binary sequences so as to take advantage of their particular properties. In this regard, a promising method for the generation of complete complementary (quaternary) sequences of different length is described by Raja Durai *et al.* [7]. Furthermore, a study on the use of CDOS in channels of more complex models should be conducted to cover multipath and non-linear effects.

6 Acknowledgment

This work has been possible thanks to the financial support granted by the Universidad Nacional de Mar del Plata, ING-187/07 (Argentina), the National Agency for Scientific and Technological Promotion (ANPCyT) Grant PICT 11-473/04 (Argentina), the National Council on Scientific and Technical Research (CONICET) Grant PIP 6245/04 (Argentina) and HELEN project (High Energy Physics Latin-American-European Network).

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