Reduced architecture for simultaneous correlation of orthogonal sets of complementary sequences

M.A. Funes, P.G. Donato, M.N. Hadad and D.O. Carrica

An architecture that enables the simultaneous correlation of four sets of complementary sequences is presented. The proposal is based on the properties of orthogonal sets as well as on the use of a recursive generation algorithm. Theoretically demonstrated, it constitutes a remarkable advance in the practical application of complementary sequences in platforms with limited resources.

Introduction: Some signal coding used in radar and multiemission systems relies on the orthogonality properties between specific sets of binary complementary sequences [1, 2]. Orthogonal sequences coding has been applied to allow the independent detection of different signals with no interference. Nowadays sustained efforts are being devoted to reducing the calculations involved in the generation and/or correlation of these signals by means of recursive algorithms.

Previous work with Golay sequences [3] has demonstrated that the correlation can be thought of as an inverse generation process, thereby leading to a more simplified correlation algorithm (optimised Golay correlator (OGC)). Next, the simultaneous correlation of two orthogonal pairs of Golay sequences was demonstrated in [4], using a single correlation architecture (O^{2}GC). To generalise the OGC with respect to the M complementary set of sequences (M-CSS), a new correlation architecture is proposed in [5]. This Letter presents a recursive algorithm to simultaneously correlate 4-CSS and so obtain a similar calculation reduction to that in [4].

Basic concepts: Based on [5] the nth stage output of an M-CSS correlator can be expressed as

$$\boldsymbol{C}_{n-1}[z] = \boldsymbol{D}'_n \times \boldsymbol{W}_n \times \boldsymbol{H}_4 \times \boldsymbol{C}_n[z] \tag{1}$$

where $\boldsymbol{C}_n[z] = [C_{1,\,n}[z], C_{2,\,n}[z], C_{3,\,n}[z], C_{4,\,n}[z]]^T$ is the input correlation vector in the nth stage, H_4 is a Hadamard matrix of order 4, $\boldsymbol{W}_n$ is the matrix seed and $\boldsymbol{D}_n{}'$ is a delay matrix:

$$\boldsymbol{W}_n = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & w_{1,N} & 0 & 0 \\ 0 & 0 & w_{2,N} & 0 \\ 0 & 0 & 0 & w_{2,N} \times w_{1,N} \end{vmatrix}$$
$$\boldsymbol{D}'_n = \begin{vmatrix} z^{-3\times 4^{n-1}} & 0 & 0 & 0 \\ 0 & z^{-2\times 4^{n-1}} & 0 & 0 \\ 0 & 0 & z^{-4^{n-1}} & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{2}$$

Given a 4-CSS, there are four mutually orthogonal sets, which can be generated from different seeds in the first generation stage [2, 4]. In other words, the orthogonality is given by $w_{i,1}$ ($i = 1, 2$), all the other seed coefficients remaining equal. There are four possible values for $w_{i,1}$. So, in a first approach, it is reasonable to think about the use of four correlators to independently process all the sets. However, considering the architecture proposed in [5] a new algorithm can be developed to correlate all the sets with a single correlator.

Simultaneous correlation: Let's consider an N stages correlator with four orthogonal sets in which S^α, S^β, S^γ and $S^\in$ are present at the inputs with different time delays (denoted by τ). Each set is made up of its corresponding sequences (e.g., S_1^α is the sequence 1 of the set α). As the four sets overlap in time, the correlator input becomes:

$$\boldsymbol{C}_N = \boldsymbol{S}_N^\alpha z^{-r_1} + \boldsymbol{S}_N^\beta z^{-r_2} + \boldsymbol{S}_N^\gamma z^{-r_3} + \boldsymbol{S}_N^\in z^{-r_4} \tag{3}$$

By replacing (3) in (1) at the Nth stage, the following is obtained:

$$\begin{aligned}
C_{1,N-1} &= [S_{1,N}^\alpha z^{-\tau_1} + S_{1,N}^\beta z^{-\tau_2} + S_{1,N}^\gamma z^{-\tau_3} + S_{1,N}^\in z^{-\tau_4} \\
&\quad + S_{2,N}^\alpha z^{-\tau_1} + S_{2,N}^\beta z^{-\tau_2} + S_{2,N}^\gamma z^{-\tau_3} + S_{2,N}^\in z^{-\tau_4} \\
&\quad + S_{3,N}^\alpha z^{-\tau_1} + S_{3,N}^\beta z^{-\tau_2} + S_{3,N}^\gamma z^{-\tau_3} + S_{3,N}^\in z^{-\tau_4} \\
&\quad + S_{4,N}^\alpha z^{-\tau_1} + S_{4,N}^\beta z^{-\tau_2} + S_{4,N}^\gamma z^{-\tau_3} + S_{4,N}^\in z^{-\tau_4}] \\
&\quad \times z^{-3\times 4^{N-1}} \\
C_{2,N-1} &= [S_{1,N}^\alpha z^{-\tau_1} + S_{1,N}^\beta z^{-\tau_2} + S_{1,N}^\gamma z^{-\tau_3} + S_{1,N}^\in z^{-\tau_4} \\
&\quad - S_{2,N}^\alpha z^{-\tau_1} - S_{2,N}^\beta z^{-\tau_2} - S_{2,N}^\gamma z^{-\tau_3} - S_{2,N}^\in z^{-\tau_4} \\
&\quad + S_{3,N}^\alpha z^{-\tau_1} + S_{3,N}^\beta z^{-\tau_2} + S_{3,N}^\gamma z^{-\tau_3} + S_{3,N}^\in z^{-\tau_4} \\
&\quad - S_{4,N}^\alpha z^{-\tau_1} - S_{4,N}^\beta z^{-\tau_2} - S_{4,N}^\gamma z^{-\tau_3} - S_{4,N}^\in z^{-\tau_4}] \\
&\quad \times w_{1,N} z^{-2\times 4^{N-1}} \\
C_{3,N-1} &= [S_{1,N}^\alpha z^{-\tau_1} + S_{1,N}^\beta z^{-\tau_2} + S_{1,N}^\gamma z^{-\tau_3} + S_{1,N}^\in z^{-\tau_4} \\
&\quad + S_{2,N}^\alpha z^{-\tau_1} + S_{2,N}^\beta z^{-\tau_2} + S_{2,N}^\gamma z^{-\tau_3} + S_{2,N}^\in z^{-\tau_4} \\
&\quad - S_{3,N}^\alpha z^{-\tau_1} - S_{3,N}^\beta z^{-\tau_2} - S_{3,N}^\gamma z^{-\tau_3} - S_{3,N}^\in z^{-\tau_4} \\
&\quad - S_{4,N}^\alpha z^{-\tau_1} - S_{4,N}^\beta z^{-\tau_2} - S_{4,N}^\gamma z^{-\tau_3} - S_{4,N}^\in z^{-\tau_4}] \\
&\quad \times w_{2,N} z^{-4^{N-1}} \\
C_{4,N-1} &= [S_{1,N}^\alpha z^{-\tau_1} + S_{1,N}^\beta z^{-\tau_2} + S_{1,N}^\gamma z^{-\tau_3} + S_{1,N}^\in z^{-\tau_4} \\
&\quad - S_{2,N}^\alpha z^{-\tau_1} - S_{2,N}^\beta z^{-\tau_2} - S_{2,N}^\gamma z^{-\tau_3} - S_{2,N}^\in z^{-\tau_4} \\
&\quad - S_{3,N}^\alpha z^{-\tau_1} - S_{3,N}^\beta z^{-\tau_2} - S_{3,N}^\gamma z^{-\tau_3} - S_{3,N}^\in z^{-\tau_4}) \\
&\quad + S_{4,N}^\alpha z^{-\tau_1} + S_{4,N}^\beta z^{-\tau_2} + S_{4,N}^\gamma z^{-\tau_3} + S_{4,N}^\in z^{-\tau_4}] \\
&\quad \times w_{1,N} w_{2,N}
\end{aligned} \tag{4}$$

Based on [1], the sum and subtraction of the M sequences of a complementary set result in sequences of length $\frac{L}{M}$ and amplitude multiplied by M:

$$\begin{aligned}
S_{1,N} + S_{2,N} + S_{3,N} + S_{4,N} &= 4S_{1,N-1} \\
S_{1,N} - S_{2,N} + S_{3,N} - S_{4,N} &= 4w_{1,N} \times S_{2,N-1} z^{-4^{N-1}} \\
S_{1,N} + S_{2,N} - S_{3,N} - S_{4,N} &= 4w_{2,N} \times S_{3,N-1} z^{-2\times 4^{N-1}} \\
S_{1,N} - S_{2,N} - S_{3,N} + S_{4,N} &= 4w_{1,N} w_{2,N} \times S_{4,N-1} z^{-3\times 4^{N-1}}
\end{aligned} \tag{5}$$

By replacing (5) in (4) and simplifying,

$$\begin{aligned}
C_{1,N-1} &= 4[S_{1,N-1}^\alpha z^{-\tau_1} + S_{1,N-1}^\beta z^{-\tau_2} \\
&\quad + S_{1,N-1}^\gamma z^{-\tau_3} + S_{1,N-1}^\in z^{-\tau_4}] z^{-3\times 4^{N-1}} \\
C_{2,N-1} &= 4[w_{1,N} S_{2,N-1}^\alpha z^{-\tau_1 - 4^{N-1}} \\
&\quad + w_{1,N} S_{2,N-1}^\beta z^{-\tau_2 - 4^{N-1}} + w_{1,N} S_{2,N-1}^\gamma z^{-\tau_3 - 4^{N-1}} \\
&\quad + w_{1,N} S_{2,N-1}^\in z^{-\tau_4 - 4^{N-1}}] w_{1,N} z^{-2\times 4^{N-1}} \\
C_{3,N-1} &= 4[w_{2,N} S_{3,N-1}^\alpha z^{-\tau_1 - 2\times 4^{N-1}} \\
&\quad + w_{2,N} S_{3,N-1}^\beta z^{-\tau_2 - 2\times 4^{N-1}} + 4w_{2,N} S_{3,N-1}^\gamma \\
&\quad \times z^{-\tau_3 - 2\times 4^{N-1}} \\
&\quad + w_{2,N} S_{3,N-1}^\in z^{-\tau_4} z^{-2\times 4^{N-1}}] w_{2,N} z^{-4^{N-1}} \\
C_{4,N-1} &= 4[w_{1,N} w_{2,N} S_{4,N-1}^\alpha z^{-\tau_1 - 3\times 4^{N-1}} \\
&\quad + w_{1,N} w_{2,N} S_{4,N-1}^\beta z^{-\tau_2 - 3\times 4^{N-1}} \\
&\quad + w_{1,N} w_{2,N} S_{4,N-1}^\gamma z^{-\tau_3 - 3\times 4^{N-1}} \\
&\quad + w_{1,N} w_{2,N} S_{4,N-1}^\in z^{-\tau_4 - 3\times 4^{N-1}}] w_{1,N} w_{2,N}
\end{aligned} \tag{6}$$

For binary sequences, the seed has only two possible values (± 1). Then, the product $w_{i,n} \times w_{i,n} = 1$ and (6) is reduced to:

$$\boldsymbol{C}_{N-1} = 4[\boldsymbol{S}^{\alpha}_{N-1} z^{-\tau_1} + \boldsymbol{S}^{\beta}_{N-1} z^{-\tau_2} + \boldsymbol{S}^{\gamma}_{N-1} z^{-\tau_3} + \boldsymbol{S}^{\in}_{N-1} z^{-\tau_4}] z^{-3 \cdot 4^{N-1}} \tag{7}$$

Taking the outputs of the stage N as the input of $N-1$:

$$\boldsymbol{C}_{N-2}[z] = \boldsymbol{D}'_{N-1} \times \boldsymbol{W}_{N-1} \times \boldsymbol{H}_4 \times \boldsymbol{C}_{N-1}[z] \tag{8}$$

If the same process of (5) is applied to (8), and is simplified as in (7),

$$\boldsymbol{C}_{N-2} = 4^2[S^{\alpha}_{N-2} z^{-\tau_1} + S^{\beta}_{N-2} z^{-\tau_2} + S^{\gamma}_{N-2} z^{-\tau_3} + S^{\in}_{N-2} z^{-\tau_4}] z^{-3(4^{N-1}+4^{N-2})} \tag{9}$$

After repeating this process until the last stage, the output yields

$$\begin{aligned} C_{1,0} &= 4^N[S^{\alpha}_{1,0} z^{-\tau_1} + S^{\beta}_{1,0} z^{-\tau_2} \\ &\quad + S^{\gamma}_{1,0} z^{-\tau_3} + S^{\in}_{1,0} z^{-\tau_4}] z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \\ C_{2,0} &= 4^N[w^{\alpha}_{1,1} S^{\gamma}_{2,0} z^{-\tau_1} + w^{\beta}_{1,1} S^{\beta}_{2,0} z^{-\tau_2} \\ &\quad + w^{\gamma}_{1,1} S^{\gamma}_{2,0} z^{-\tau_3} + w^{\in}_{1,1} S^{\in}_{2,0} z^{-\tau_4}] \times w^{*}_{1,1} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \\ C_{3,0} &= 4^N[w^{\alpha}_{2,1} S^{\alpha}_{3,0} z^{-\tau_1} + w^{\beta}_{2,1} S^{\beta}_{3,0} z^{-\tau_2} \\ &\quad + w^{\gamma}_{2,1} S^{\gamma}_{3,0} z^{-\tau_3} + w^{\in}_{2,1} S^{\in}_{3,0} z^{-\tau_4}] w^{*}_{2,1} z^{-3\Sigma_{i=1}^{N} \times 4^{i=1}} \\ C_{4,0} &= 4^N[w^{\alpha}_{1,1} w^{\alpha}_{2,1} S^{\alpha}_{4,0} z^{-\tau_1} + w^{\beta}_{1,1} w^{\beta}_{2,1} \times S^{\beta}_{4,0} z^{-\tau_2} \\ &\quad + w^{\gamma}_{1,1} w^{\gamma}_{2,1} S^{\gamma}_{4,0} z^{-\tau_3} \\ &\quad + w^{\in}_{1,1} w^{\in}_{2,1} S^{\in}_{4,0} z^{-\tau_4}] w_{1,1} w^{*}_{2,1} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \end{aligned} \tag{10}$$

where $w^{*}_{i,1}$ denotes the seed values on the correlator at the last stage, and $w^{\alpha}_{i,1}$, $w^{\beta}_{i,1}$, $w^{\gamma}_{i,1}$ and $w^{\in}_{i,1}$ are the values used in their generations.

The orthogonality between different M-CSS is determined by the seeds of the first generation stage, which is in accordance with the last correlation stage. In this case, four combinations of seeds are possible: $w_{1,1} = w_{2,1} = 1$, $w_{1,1} = w_{2,1} = -1$, $w_{1,1} = -w_{2,1} = 1$ and $w_{1,1} = -w_{2,1} = -1$. Each of them allows us to generate an orthogonal set of sequences. Then, in the correlation process, each set is detected when the seed of the last correlator stage is equal to that corresponding to a particular set. When $w^{*}_{1,1} = w^{\alpha}_{1,1}$ and $w^{*}_{2,1} = w^{\alpha}_{2,1}$, all the terms affected by a different seed are cancelled and the sum of correlations is

$$\begin{aligned} \sum_{i=1}^{4} C_{i,0} &= 4^N[S^{\alpha}_{1,0} + S^{\alpha}_{2,0} + S^{\alpha}_{3,0} + S^{\alpha}_{4,0}] z^{-\tau_1} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \\ &= 4 \times 4^N z^{-\tau_1} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \end{aligned} \tag{11}$$

If $w^{*}_{1,1} = w^{\beta}_{1,1}$ and $w^{*}_{2,1} = w^{*}_{2,1}$, the sum of correlations is

$$\begin{aligned} \sum_{i=1}^{4} C_{i,0} &= 4^N[S^{\beta}_{1,0} + S^{\beta}_{2,0} + S^{\beta}_{3,0} + S^{\beta}_{4,0}] z^{-\tau_2} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \\ &= 4 \times 4^N z^{-\tau_2} z^{-3\Sigma_{i=1}^{N} \times 4^{i-1}} \end{aligned} \tag{12}$$

These equations can be extrapolated using the seed values of the sets γ and $\in$. Note that in each case a Kroenecker delta of the respective orthogonal set is obtained and the whole correlation can be realised with a single correlator architecture, Fig. 1.

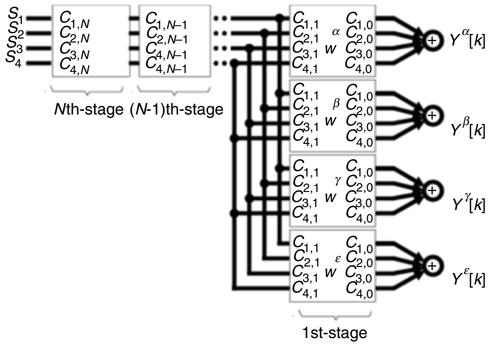

Fig. 1 *Proposed correlator*

Proposal efficiency: The contribution of this work can be appreciated considering the calculations needed to correlate the four orthogonal sets. Table 1 demonstrates that the proposed correlation scheme is more efficient than the use of four correlators (for $N > 1$), and it increases with the number of stages. Such optimisation improves the proposal presented in [5] by using the analysis presented in [4].

Table 1: Required calculations required to correlate four orthogonal 4-CSS

	Multiplications	Add/Subs	Delays
Straightforward	4^{N+3}	$64(4^N - 1)$	$2 \times 4^{N+3}$
De Marziani [2]	$64 \times N$	$128 \times N$	$32(4^N - 1)$
Funes *et al.* [5]	$16 \times N$	$32 \times N$	$8 \times (4^N - 1)$
Proposed	$4 \times (N+3)$	$8 \times (N+3)$	$2 \times (4^N + 8)$

Conclusions: The improvement applied to the correlation of orthogonal complementary set of sequences makes it possible to simultaneously correlate these sets with a single correlator. The calculation efficiency obtained with this contribution constitutes a significant step toward the practical application of these sequences.

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M.A. Funes, P.G. Donato, M.N. Hadad and D.O. Carrica (*CONICET. J.B., Justo 4302, Mar del Plata, Argentina*)

E-mail: mfunes@fi.mdp.edu.ar

References

1 Tseng, C.-C., and Liu, C.: 'Complementary sets of sequences', *IEEE Trans. Inf. Theory*, 1972, **18**, pp. 644–652

2 De Marziani, C., Urĕna, J., Hernandez, A., Mazo, M., Alvarez, F., Garcia, J., and Donato, P.: 'Modular architecture for efficient generation and correlation of complementary set of sequences', *IEEE Trans. Signal Process.*, 2007, **55**, (5), pp. 2323–2337

3 Donato, P., Funes, M., Hadad, M., and Carrica, D.: 'Optimised Golay correlator', *Electron. Lett.*, 2009, **45**, (7), pp. 380–381

4 Donato, P., Funes, M., Hadad, M., and Carrica, D.: 'Simultaneous correlation of orthogonal pairs of complementary sequences', *Electron. Lett.*, 2009, **45**, (25), pp. 1332–1334

5 Funes, M.A., Donato, P.G., Hadad, M.N., and Carrica, D.O.: 'Hardware reduction based on m-css correlation improvements'. Proc. Argentine School of Micro-Nanoelectronics, Technology and Applications IEEE Conf. EAMTA 2009, San Carlos de Bariloche, Argentina, 2009, pp. 17–21