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A Decision Making Account for the Cognitive Processing of Demorgan´S Laws

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Artículo Original

Resumen: El objetivo del presente estudio consiste en proponer una descripción preliminar del comportamiento espontáneo del pensamiento humano en tareas de razonamiento relacionadas con las equivalencias lógicas atribuidas a Augustus DeMorgan. Estas equivalencias relacionadas con la negación de conjunciones y disyunciones se han estudiado previamente sólo en relación con silogismos, pero no desde la toma de decisiones. Se realizó un estudio exploratorio *on-line* para poner a prueba dos hipótesis. La primera hipótesis sostiene que la ley de Morgan para conjunciones es más fácil de reconocer intuitivamente que la correspondiente ley para disyunciones. La segunda afirma que los errores en estas tareas son regulados por un patrón cognitivo. La evidencia obtenida resultó incompatible con la primera hipótesis y compatible con la segunda. Se propuso un heurístico para explicar la inesperada facilidad con que los sujetos reconocieron la ley de disyunciones pero no la de conjunciones. Se propusieron finalmente lineamientos para futuras investigaciones.

Palabras claves:
Conjunción; Disyunción; Negación Lógica; Leyes De Morgan; Toma De Decisiones

Abstract: The aim of this contribution is to propose a preliminary account for the intuitive recognition of the logical equivalences attributed to Augustus DeMorgan. Such equivalences concerned with the negation of conjunctions and disjunctions have been previoulsy studied only in the context of syllogistic tasks, but not from the perspective of decision making. An on-line exploratory study was conducted to test two hypotheses. The first hypothesis states that DeMorgan´s law for the negation of conjunctions is easier to recognize than the corresponding law for the disjunctions case. The second hypothesis states that spontaneous errors in the recognition of DeMorgan´s laws follow a cognitive pattern. The results obtained for the first hypothesis suggest that the disjunctions case is more intuitive than the conjunctions case. An heuristic explanation for such unexpected result is suggested. The second hypothesis testing results suggest that the observed errors are not random. Suggestions for future research are proposed.

Key Words:
Conjunction; Disjunction; Logical Negation; Demorgan´S Laws; Decision Making

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1. Introduction

The different psychological theories that try to explain human reasoning could be classified according to the architectural centrality they attribute to the formal rules of logic (Braine, 1978; Fernández Berrocal & Carretero, 1995; Martín & Valiña, 2002). If the model attributes the most critical psychological functions to mental or natural logic, such theory of reasoning can be described as syntactic or rules-driven (Rader & Sloutsky, 2001). In the opposite case, when the model rejects the centrality of rules and gives priority to psychological representations like mental models or heuristics, such theory can be considered semantic or non-rules-driven. An example of a rules-driven reasoning theory is the psychology of proof proposed by Rips (1994). On the other side, the most representative non-rules-driven theory of deduction is the mental models approach promoted by Johnson-Laird (1983; Fumero, Santamaría & Johnson-Laird, 2010). Both accounts have

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provided coherent models and profuse evidence that explain several patterns of spontaneous deductive reasoning when conjunctions, disjunctions, negations and other logical functions are included in syllogistic tasks (Santamaría, 1995).

The present contribution gives a preliminary account for a particular case of reasoning that both rules-driven and non-rules-driven models have only partially studied: the equivalences recognition for the negation of conjunctions and for the negation of disjunctions (García Madruga, Moreno, Carriedo, Gutiérrez & Johnson-Laird, 2001; Rips, 1994). The aim of this work is to provide an exploratory description of the spontaneous behavior of human thought when DeMorgan´s laws recognition are involved in experimental tasks. Although such tasks are properly deductive, the present contribution proposes a decision making account based on heuristics. This proposal preliminary covers the two fundamental criteria that organize judgment and decision making research, that is, coherence and correspondence (Dunwoody, 2009; Hammond, 1996). The former is analyzed by the comparison between the spontaneous reasoning productions and the propositional calculus of DeMorgan´s laws, while the latter is explored by proposing a plausible algorithm that describes the decision making processes involved.

2. DeMorgan´s laws

The two fundamental logical properties named after Augustus DeMorgan (DeMorgan, 1847; Muñoz García, 2005) state that: *i*) the negation of the conjunction of a collection of propositions is equivalent to the disjunction formed by the negation of each individual proposition and; *ii*) the negation of the disjunction of a collection of propositions is equivalent to the conjunction formed by the negation of each individual proposition. A formal definition of these properties is presented in Equations 1 and 2.

$$\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$$
(1)

$$\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$$
(2)

The letters p and q are referred to any propositions, that is, to declarative statements that can be considered either true or false. The operations involved in these properties are negation ($\neg$), double implication ($\Leftrightarrow$), conjunction ($\wedge$) and disjunction ($\vee$). An application example of Equation 1 might be the equivalence between the statement “It is false that: Winters are sad and summers are happy” and the statement “Winters are not sad or summers are not happy”. An application example of Equation 2 would be the equivalence between “It is false that: Napoleon was right-handed or Joan of Arc was left-handed” and the statement “Napoleon was not right-handed and Joan of Arc was not left-handed”. Both examples are taken from the experimental tasks introduced below.

These logical relations can also be formally treated as set properties when: *i*) p and q are defined as sets P and Q; *ii*) the conjunction is converted into intersection ($\cap$) and the disjunction into union ($\cup$), and Equations 1 and 2 are translated into Equations 3 and 4, respectively.

$$\overline{P \cap Q} = \overline{P} \cup \overline{Q}$$
(3)

$$\overline{P \cup Q} = \overline{P} \cap \overline{Q}$$
(4)

The upper bar in Equations 3 and 4 expresses the set operation of complementation and the equality ($=$) implies simultaneous double containment ($\subseteq$ and $\supseteq$). A formal proof of DeMorgan´s laws as expressed in Equations 3 and 4 is presented in Equations 5 and 6, where x is any arbitrary element.

$$x \in \overline{P \cap Q} \Leftrightarrow x \notin (P \cap Q) \Leftrightarrow x \notin P \vee x \notin Q \Leftrightarrow x \in \overline{P} \vee x \in \overline{Q} \Leftrightarrow x \in (\overline{P} \cup \overline{Q})$$
(5)

$$x \in \overline{P \cup Q} \Leftrightarrow x \notin (P \cup Q) \Leftrightarrow x \notin P \wedge x \notin Q \Leftrightarrow x \in \overline{P} \wedge x \in \overline{Q} \Leftrightarrow x \in (\overline{P} \cap \overline{Q})$$
(6)

When x is in $(\overline{P} \cup \overline{Q})$ because it is also in $\overline{P \cap Q}$ and vice versa, the equality presented in Equation 3 gets proved. The same occurs for Equations 6 and 4 when the respective relation is followed.

3. Previous findings concerning the psychology of DeMorgan´s laws

Although Rips (1994) included these properties in PSYCOP, an algorithm that illustrates the consistency of his natural logic theory of deductive reasoning, only a few experiments have been specifically conducted in relation to DeMorgan´s laws (Carriedo, Moreno, Gutiérrez & García Madruga, 1998; Rader & Sloutsky, 2001). The main findings have been collected for the case of disjunctive reasoning premises in syllogistic tasks. Several studies suggest that content and context are critical variables for the propositions included in the premises of a syllogism when different deductive tasks are presented (Martín & Valiña, 2002). It has also been noted that the inclusive and exclusive negations of a disjunction behave in different psychological manners (Rader & Sloutski, 2001). The inclusive interpretation, that is, *p* or *q* or *both*, was more frequent in abstract and concrete contexts rather than in threat or election contexts (Newstead, Griggs & Chrostowski, 1984). Richardson and Ormerod (1997) found that familiarity and causality in conditionals promote the reduction of logical errors in syllogistic tasks that include disjunctions. The same authors gave other significant suggestions for the present study in the context of the experimental paradigm of recognition. The tasks of this paradigm typically require the selection of the correct answer among a list of options. Such options are usually syllogism conclusions. Their evidence suggests that subjects do not spontaneously recognize the equivalence between a disjunction and a conditional that includes a negation in a premise (Richardson & Ormerod, 1997).

In a recent study, Fumero et al. (2010) found partial evidence for the hypothesis that predicts different reasoning patterns according to different personality styles. Extraversion and neuroticism as defined in NEO-PI-R personality test (Costa & McCrae, 1999) resulted critical for the increment of success in *modus tollens* syllogisms.

In the tradition of the mental models approach, an important collection of findings also suggests that the constructive scenarios triggered by inclusive and exclusive disjunctions have different psychological complexities (Johnson-Laird *et al.*, 1994). The reason for this difference is the number of mental models that each disjunction requires for the achievement of a valid syllogistic conclusion. The main findings of this research trend indicate that (Martín & Valiña, 2002): *i*) a double exclusive disjunction is easier to recognize than an inclusive double disjunction; *ii*) affirmative deductions are easier than negative ones and; *iii*) some ceiling effects might be observed in disjunction experiments when the tasks are too easy or too hard for the experimental subjects (Fumero et al., 2010).

From a memory based approach for the study of logical connectives, a *conjunctive bias* has been proposed (Rader & Sloutsky, 2001). This bias has been defined as the tendency to obtain a more accurate performance in recognition and recall for conjunctions than for disjunctions and implications. To explain such phenomenon the authors argued that working memory gets a lower load for conjunctions tasks than for other logical connectives.

These previous findings about the psychology of DeMorgan´s laws are concerned with syllogisms. The present contribution, in contrast, proposes the study of the intuitive direct matching between the equivalences expressed in Equations 1 and 2. This task might be considered harder than the one usually applied in reasoning experiments, but provides a direct evaluation of DeMorgan´s laws intuition (Gigerenzer, 2000, 2007).

In sum, previous findings suggest that DeMorgan´s laws equivalences might be harder to recognize for the disjunctive case than for the conjunctive case. This conjecture might be justified because: *i*) conjunction tasks require a lower working memory load than disjunctions; *ii*) the distinction between inclusive and exclusive disjunctions might generate diverse interpretations that lead to a successful reduction for the disjunctive case, as explained below and; *iii*) the conjunctive bias might be activated by experimental tasks that imply DeMorgan´s laws.

The aim of the present contribution focuses on the intuitive recognition rather than on the deductive processing of these logical laws. Therefore, this exploratory study is original because previous findings are exclusively concerned with syllogisms.

4. Method

An exploratory study was designed to describe the spontaneous reasoning behavior of subjects in a recognition task that includes DeMorgan´s laws. With this purpose a set of reasoning tasks was created and administered to a sample of subjects drew from the general population. Because the present study consists only in a preliminary contribution, an on-line experiment was conducted to analyse the response patterns and errors for Equations 1 and 2, or 3 and 4. A within-subjects design was applied to perform such analysis according to recent methodological recommendations (Lambdin & Shafer, 2009).

2.1. Participants

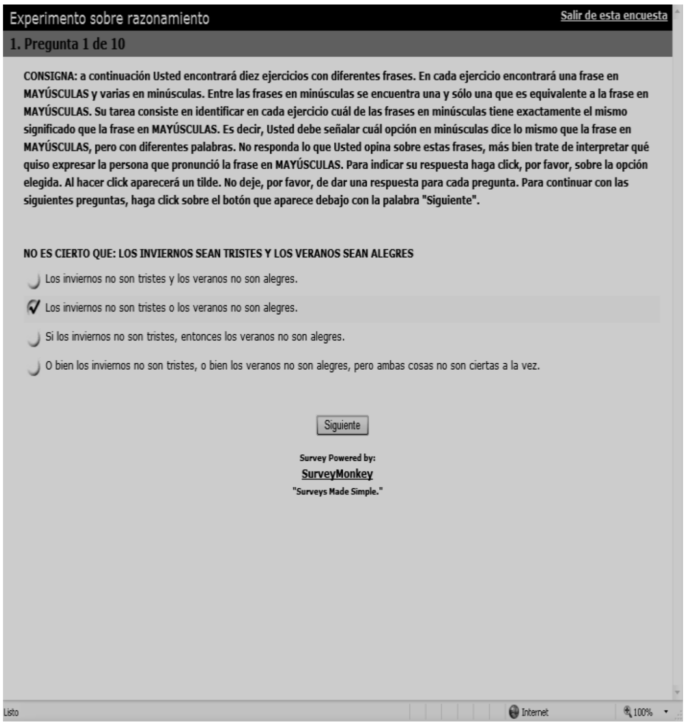
Thirty Argentinean subjects participated in this study. Seventeen female and thirteen male (mean age 25.82 +/- 2.96 years). All participants were randomly recruited from a list of university students that visited the website of the Institute for Psychological Research at Universidad del Salvador (Argentina), located in http://iipus.webs.com. Subjects were invited to participate in a reasoning experiment when visiting the "News" folder of the website. After accepting the invitation, subjects completed a ten questions test. The original sample had forty subjects, but ten did not complete the experiment and therefore were excluded from the final sample.

2.2. Materials and procedure

In thedesign of the experimental materials, the selection paradigm and the inference rules task was applied, following the taxonomy proposed by Martín and Valiña (2002). This implies that subjects received a collection of reasoning tasks. Each task consisted in recognizing the logical equivalence for a previous given statement among a list of options. The sequence of each list was randomized. Subjects were informed that only one of the four offered alternatives was correct. The task consisted in recognizing that equivalence. Ten items were included, five for the negation of a conjunction (Equations 1 and 3) and five for the negation of a disjunction (Equations 2 and 4).

Because the participants were recruited among university students from Argentina, the tasks were presented in Spanish. The general instruction stated that the experiment consisted in recognizing the equivalence between different expressions. With that purpose a target statement in capital letters was presented. A collection of other four candidate statements --in small letters-- was attached to the phrase in capital letters. The task was to recognize among the small letters statements the equivalent to the capital letters statement. A screen capture of the on-line task is presented in Figure 1.

Figure 1
Screen capture of the on-line recognition task



Note: the general instruction for the experiment was presented in bold fonts. Below, in capital letters and also in bold fonts, was presented the target statement for the recognition task. Four response options in small letters were offered. Subjetcs were instructed to recognize and select the small letters equivalent to the capital letters statement.

The response options were designed as indicated in Table 1, where the symbol $\Rightarrow$ indicates implication, $\vee$ indicates inclusive disjunction, and $\underline{\vee}$ indicates exclusive disjunction.

Table 1
Logical figures for the DeMorgan´s laws recognition tasks

DeMorgan´s laws	*Logical figures for all response options*				
	1.	2.	3.	4.	5.
$\overline{A \cap B} = \overline{A} \cup \overline{B}$	a) $\neg p \wedge \neg q$ b) $\neg p \vee \neg q$ • c) $\neg p \Rightarrow \neg q$ d) $\neg p \underline{\vee} \neg q$	a) $\neg p \Rightarrow \neg q$ b) $\neg p \underline{\vee} \neg q$ c) $\neg p \wedge \neg q$ d) $\neg p \vee \neg q$ •	a) $\neg p \vee \neg q$ • b) $\neg p \wedge \neg q$ c) $\neg p \underline{\vee} \neg q$ d) $\neg p \Rightarrow \neg q$	a) $\neg p \underline{\vee} \neg q$ b) $\neg p \Rightarrow \neg q$ c) $\neg p \vee \neg q$ • d) $\neg p \wedge \neg q$	a) $\neg p \underline{\vee} \neg q$ b) $\neg p \wedge \neg q$ c) $\neg p \vee \neg q$ • d) $\neg p \Rightarrow \neg q$
	6.	7.	8.	9.	10.
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	a) $\neg p \Rightarrow \neg q$ b) $\neg p \underline{\vee} \neg q$ c) $\neg p \vee \neg q$ d) $\neg p \wedge \neg q$ •	a) $\neg p \vee \neg q$ b) $\neg p \wedge \neg q$ • c) $\neg p \Rightarrow \neg q$ d) $\neg p \underline{\vee} \neg q$	a) $\neg p \underline{\vee} \neg q$ b) $\neg p \Rightarrow \neg q$ c) $\neg p \wedge \neg q$ • d) $\neg p \vee \neg q$	a) $\neg p \wedge \neg q$ • b) $\neg p \vee \neg q$ c) $\neg p \underline{\vee} \neg q$ d) $\neg p \Rightarrow \neg q$	a) $\neg p \Rightarrow \neg q$ b) $\neg p \wedge \neg q$ • c) $\neg p \underline{\vee} \neg q$ d) $\neg p \vee \neg q$

Note: the tasks 1 to 5 correspond to the Equations 1 and 3. The tasks 6 to 10 correspond to the Equations 2 and 4. The symbol ● indicates the correct equivalence.

The materials included in these tasks involve the participation of inclusive and exclusive disjunctions because previous studies suggested the importance of such distinction (Martín & Valiña, 2002). Care was also taken to exclude statements semantically related to extraversion and neuroticism to avoid performance differences induced by personality variables according to the recent contributions by Fumero *et al.* (2010).

2.3. Hypotheses

The experimental hypothesis *H1* states that the logical equivalence for the negation of a conjunction (NC) is easier to recognize than the equivalence for the negation of a disjunction (ND). Formally, $H1: \overline{NC} > \overline{ND}$. Both variables ND and NC are operationalized as the mean of correct recognitions for the corresponding cases. This hypothesis is formulated in coherence with previous findings that provide evidence for the harder condition of disjunctions processing when compared to conjunctions (García Madruga, Gutiérrez, Carriedo, Moreno & Johnson-Laird, 2002; Martín & Valiña, 2002). The core argument that holds for this hypothesis states that conjunction has only one truth table, but disjunction has two. The inclusive disjunction (expressed as $\vee$) is formally different than the exclusive disjunction (expressed as $\underline{\vee}$).

The experimental hypothesis *H2* states that spontaneous errors in the recognition of DeMorgan´s laws follow a specific pattern. Because three different incorrect answers were offered in the equivalence tasks as indicated in Table 1, the hypothesis *H2a* states that at least one comparison between the frequency of incorrect answers for the NC case shows a significant difference. Such difference would imply the dominance of one particular error figure (*E*). Formally,

$$H2a: \overline{ENC_i} \neq \overline{ENC}_j, \forall i \forall j / i \neq j, i = j = \{1,2,3\}.$$

The hypothesis *H2b* states the same that was expressed in *H2a*, but for errors in the ND case. Formally,

$$H2b: \overline{END_i} \neq \overline{END}_j, \forall i \forall j / i \neq j, i = j = \{1,2,3\}.$$

This exploratory conjecture emerges from the heuristics and biases approach (Kahneman & Klein, 2009; Kahneman, Slovic, & Tversky, 1982; Kahneman & Tversky, 2000). It is proposed that thought errors are not systematically random and that a coherent descriptive model for such phenomena can eventually be proposed (Tversky & Kahneman, 1974). A recent example of a heuristic account for similar error patterns in logical reasoning tasks has been provided by Macbeth, López Alonso, Razumiejczyk, Sosa, Pereyra, and Fernández (2009).

5. Results and discussion

The *H1 h*ypothesis was not supported. As predicted, a different response pattern between NC and ND was found, but not in the expected direction. The opposite pattern was observed (sign test, $p < .001$, Cliff´s δ effect size = 0.52). The number of recognitions was significantly greater for the negation of disjunctions ($\overline{ND} = 2.3, SD = 2.103$) than for the negation of conjunctions ($\overline{NC} = 0.37, SD = 1.03$). A possible interpretation for this unexpected result is related with the heuristics and biases approach to thought phenomena. Instead of a formal deductive reasoning computation, a decision making process has probably been triggered by the proposed experimental task. Subjects might have taken shortcuts to select a response option as suggested by Gigerenzer for similar reasoning tasks (Gigerenzer, 2000, 2007). If that is the case, then a possible heuristic could be the sequential representation of simultaneous possible scenarios. An intuitive criterion based on the hints proposed by Johnson-Laird (1983) to explain the structure of these processes might be the quantity of scenarios involved in the representation, that is, the cardinality of the scenarios set. This heuristic sequence might have two rules: *i*) always try to reduce the scenarios set cardinality and; *ii*) increase the scenarios set cardinality only when that is inevitable. This search for the better response option ends when the final scenario of the target proposition matches one of the offered options. The application of this heuristic leads to the observed results. For example, let $p \wedge q$ be a scenario with two objects: a plate (p) and a spoon (q) are over a table. The scenarios sequence may always start with affirmative conditions because some experiments suggest that the human mind tends to be candid or spinozian (Gilbert, Malone, & Krull, 1990; Spinoza, 1998/1677), that is, the information processing starts accepting the original true condition of all statements. Negation or falseness attribution is posterior, that is, a second instance operation (Skurnik, 1998). Under this condition, the starting scenario might be, for example, the representation of a plate and a spoon over a table. The affirmative or true condition of p is represented by the presence of the plate over the table. Hence, the negation or falseness of p has to be the absence of the plate. After the starting scenario of $p \wedge q$ is given, the negation occurs to build up the target proposition of the experimental task. If this process follows the proposed heuristic, then rules *i* and *ii* must be applied. The first one fails because the negation of the disjunction cannot preserve the cardinality of 1 and a cardinality of 2 must be accepted by rule *ii*. The result is a scenario with two possible tables, one with no plate, and the other with no spoon. After completing this process, the search for the answer ends with a cardinality of 2.

For the disjunction negation the opposite occurs. The original scenario has a cardinality of 2 because two possible tables are needed, one with a plate (no spoon) and the other with a spoon (no plate). The negation gives a new scenario that is compatible with rule *i*, that is, a cardinality reduction can be achieved. The result is a scenario with an empty table after building up the target proposition. The condition of no plate and no spoon matches faster and easier a response option in this experiment. Table 2 summarizes the proposed heuristic using the notation of the mental models approach (Fumero et al., 2010; Johnson-Laird, 1983).

Table 2
A possible heuristic that accounts for the easier recognition of disjunction negations when compared to conjunction negations

Conjunction negation	*Disjunction negation*
1. Affirmative or candid statement (#1) p q	1. Affirmative or candid statement (#2) p q
2. Negation (#2) -p -q	2. Negation (#1) -p -q

Note: the cardinality (#) increases for the negation and decreases for the disjunction after heuristic rules *i* and *ii* are applied. A smaller cardinality possibly implies that the task is easier.

Additionally, it is pertinent to comment on the argument presented to support the hypothesis *H1* related to the double truth table of the logical disjunction. Although the argument is relevant, DeMorgan´s laws apply only to the inclusive condition. The non equivalence of the exclusive disjunction expressed in Equations 2 and 4 might be observed in Tables 3 and 4.

Table 3
Truth table for the inclusive disjunction

$\neg$	$(p$	$\vee$	$q)$	$\Leftrightarrow$	$\neg$	p	$\wedge$	$\neg$	q
F	T	T	T	**T**	F	T	F	F	T
F	T	T	F	**T**	F	T	F	T	F
F	F	T	T	**T**	T	F	F	F	T
T	F	F	F	**T**	T	F	T	T	F

Note: because the double implication is always true, the relation is an equivalence or tautology.

Table 4
Truth table for the exclusive disjunction

$\neg$	$(p$	$\veebar$	$q)$	$\Leftrightarrow$	$\neg$	p	$\wedge$	$\neg$	q
T	T	F	T	**F**	F	T	F	F	T
F	T	T	F	**T**	F	T	F	T	F
F	F	T	T	**T**	T	F	F	F	T
T	F	F	F	**T**	T	F	T	T	F

Note: because the double implication is not always true, the relation is not an equivalence. The discrepancy can be appreciated in the first sentence, where the simultaneous truth of *p* and *q* generates a false exclusive disjunction. The corresponding sentence for the inclusive case generates a true disjunction in Table 3.

This restriction implies that the negation of conjunctions and the negation of disjunctions have both only one truth table for DeMorgan´s laws. Hence, the disjunctions case cannot be expected to be harder than the conjunctions case. The experimental results of the present study are consistent with this observation.

The hypothesis *H2* resulted partially coherent with the evidence. An error pattern was found for the negation of conjunctions expected in *H2a*. Only a random behavior of errors was found for the negation of disjunctions stated in *H2b*. The most frequent response error for the negation of conjunctions case had the form $(\neg p \wedge \neg q)$. The selection of this response was significantly more frequent than the selection of the alternative options indicated in Table 5.

A significant difference between errors 1 and 2 (sign test, p = .043, Cliff´s δ effect size = 1) and between errors 1 and 3 (sign test, p = .043, Cliff´s δ effect size = 1) was found. No significant difference resulted between errors 2 and 3 (sign test, p = .414, Cliff´s δ effect size = 0.08).

Table 5
Response errors frequency patterns for DeMorgan´s laws

Logic figure	*Mean (SD)*
Negation of conjuntions	
1. $(\neg p \wedge \neg q)$	21(4.24)
2. $(\neg p \Rightarrow \neg q)$	2.6(1.52)
3. $(\neg p \veebar \neg q)$	4.6(3.65)
Negation of disjunctions	
1. $(\neg p \vee \neg q)$	6.2(2.86)
2. $(\neg p \Rightarrow \neg q)$	4.6(36)
3. $(\neg p \veebar \neg q)$	6.8(4.76)

No significant difference was found between the errors for the disjunctions case (sign tests, *ps* > .05, Cliff´s δ effect sizes < 0.20), against the expectations of *H2b*. This result suggests that errors in the recognition of the equivalence for the negation of disjunctions behave in a random manner.

In sum, the results on the first hypothesis suggest that subjects achieve a better recognition for the disjunctions law than for the conjunctions law. In regards with the second hypothesis, the results suggest that the observed errors for the conjunctions law follow a pattern, but errors for the disjunctions law do not. This evidence is consistent with a mental models approach that explains why the negation of disjunctions generates a reduction of the working memory load. In contrast, the negation of conjunctions implies an amplification of the mental information amount, which leads to difficulties in the recognition of its logical equivalence.

6. General Discussion

The results obtained in the present study suggest that: *i*) the spontaneous recognition of the logical equivalence for the negation of disjunctions is more intuitive than the corresponding to the conjunctions case and; *ii*) the errors in the recognition of DeMorgan´s laws follow a pattern, but only for the negation of conjunctions.

Although the negation of disjunctions might be considered harder to process than the negation of conjunctions because the former has a more complex logical form than the latter (García

Madruga *et al.*, 2001), the results of the present contribution showed the opposite phenomenon for DeMorgan´s laws. An heuristic account was proposed to explain this unexpected result. Subjects probably followed a decision process rather than a deductive computation. It was suggested that experimental participants spontaneously tried to reduce the magnitude of the task representations and that this shortcut provides an easier way to find the correct equivalent for a given statement that satisfies the form of DeMorgan´s laws.

The main difference with previous findings is probably related with the expected activation of the *conjunctive bias* (Rader & Sloutsky, 2001). This phenomenon did not occur in the present study. This result can be justified because that bias has been studied for syllogistic tasks, but not for intuitive or non-effort recognition processing. The recent ecological approach to thought phenomena suggests that spontaneous human cognition typically activates heuristics rather than complex inferential computations to complete cognitive tasks (Gigerenzer, 2007). In that context, a decision making account concerned with a mental models reduction explains why an heuristic, rather than the conjunctive bias, was activated.

The present contribution is only exploratory and has several limitations: *i*) data were collected on-line; *ii*) although the statistical power of the tested hypotheses was set at .80, the sample of participants was small; *iii*) the response options offered in the task were limited and were not obtained from a representative sample of tasks (Dhami, Hertwig, & Hoffrage, 2004), and *iv*) the heuristic proposed to give an account for the exploratory results might be plausible, but no further experimental evidence was obtained to support it. These limitations should be reduced in further studies. The main findings obtained in the present contribution might be thoroughly tested in future experiments.

The results of the present study are preliminary contributions to the two main approaches to the psychology of thought for the intuition of DeMorgan´s laws, that is, the coherence and the correspondence criteria (Dunwoody, 2009; Hammond, 1996). The results obtained after hypothesis *H1* were discussed in the context of the normative propositional calculus as a contribution to the coherence approach. The account suggested for explaining the errors indicated in hypothesis *H2* contribute, in contrast, to the correspondence criteria because it priorizes the empirical accuracy over the normative consistency.

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