MESON MASSES AT FINITE TEMPERATURE IN A NONLOCAL SU(3) CHIRAL QUARK MODEL

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We study the temperature behavior of light scalar and pseudoscalar meson masses within a three-flavor nonlocal chiral quark model that includes the coupling of quarks to the Polyakov loop. Chiral restoration and deconfinement transitions are described.

Keywords: quark model, chiral symmetry restoration, meson masses

1. Introduction

The understanding of the behavior of strongly interacting matter under finite temperature and/or density has become an issue of great interest in recent years. In this context, it is important to study how hadron properties (masses, mixing angles, decay constants, etc.) get modified when hadrons propagate in a hot and/or dense medium. Since the origin of the light scalar and pseudoscalar mesons is related to the phenomenon of chiral symmetry breaking, the temperature and/or density behavior of their properties is expected to provide relevant information about chiral symmetry restoration.

The theoretical study of strong interactions in the nonperturbative regime can be addressed by developing effective models that are consistent with lattice QCD results and extrapolable to regions not accessible by lattice calculation techniques. Among the various models proposed so far, we concentrate here in chiral quark models that include nonlocal interactions[1,2]. These theories can be viewed as extensions of the widely studied Nambu−Jona-Lasinio model[3]. In fact, nonlocality arises naturally in the context of several successful approaches to low-energy quark dynamics, and also lattice QCD calculations indicate that quark interactions should act over a certain range in momentum space[4]. Recently, the description of confinement has been addressed within this these models through the inclusion of the Polyakov loop, which is taken as an order parameter for the deconfinement transition. The aim of the present work is to study in this context the finite temperature behavior of light scalar and pseudoscalar meson masses. Details of this analysis can be found in Ref.[5].

The basic theoretical formalism is presented in Sect. 2. In Sect. 3 we determine a compatible set of model parameter values, and analyze meson masses at zero and finite temperature. The main outcomes are summarized in Sect. 4.

2. Theoretical formalism

We consider a nonlocal covariant SU(3) quark model which includes the coupling of quarks to a background color gauge field. The Euclidean effective action for the quark sector of this model is given by[5]

$$S_E = \int d^4x \, \Big\{ \bar\psi(x)\left[-i\gamma_\mu D_\mu + \hat m\right]\psi(x) - \frac{G}{2}\left[j_a^S(x)\; j_a^S(x) + j_a^P(x)\; j_a^P(x)\right] \\ -\frac{H}{4}\, T_{abc}\left[j_a^S(x) j_b^S(x) j_c^S(x) - 3\; j_a^S(x) j_b^P(x) j_c^P(x)\right] \;+\; \mathcal{U}\left[A(x)\right]\Big\} \;, \qquad (1)$$

where $\psi \equiv (u\ d\ s)^T$, whereas $\hat m = \mathrm{diag}(m_u, m_d, m_s)$ stands for the current quark mass matrix. For simplicity we consider isospin symmetry, hence $m_u = m_d = \bar m$. The currents $j_a^{S,P}(x)$ are given by

$$j_a^{S,P}(x) \;=\; \int d^4z\; \tilde g(z)\; \bar\psi\left(x+\frac{z}{2}\right)\; \Gamma_{S,P}\; \lambda_a\; \psi\left(x-\frac{z}{2}\right)\;, \qquad (2)$$

where $\Gamma_S = \mathbb{1}$, $\Gamma_P = i\gamma_5$, $\tilde g(z)$ is a form factor responsible for the nonlocal character of the interaction, and λ_a are the eight Gell-Mann matrices, plus $\lambda_0 = \sqrt{2/3}\ \mathbb{1}$. The constants T_{abc} in Eq. (1) are given by $T_{abc} = (1/3!)\epsilon_{ijk}\epsilon_{mnl}(\lambda_a)_{im}(\lambda_b)_{jn}(\lambda_c)_{kl}$, while the effective potential $\mathcal{U}$ accounts for gauge field self-interactions.

The partition function associated with the effective action in Eq. (1) can be bosonized by introducing the scalar and pseudoscalar meson fields $\sigma_a(x)$ and $\pi_a(x)$, together with auxiliary fields $S_a(x)$ and $P_a(x)$. Then, using the Matsubara formalism, the bosonized effective action can be extended to finite temperature.

The coupling of fermions to the color gauge fields arises from the covariant derivative $D_\mu \equiv \partial_\mu - iA_\mu$ in the fermion kinetic term. We assume that the quarks move in a constant background field A_0. Then the traced Polyakov loop, taken as order parameter of confinement, is given by $\Phi = \frac{1}{3}\mathrm{Tr}\,\exp(i\beta\phi)$, where $\beta = 1/T$, $\phi = iA_0$. In the so-called Polyakov gauge the matrix ϕ is shown to be diagonal, $\phi = \phi_3\lambda_3 + \phi_8\lambda_8$, which leaves only two independent variables, ϕ_3 and ϕ_8.

Let us work within the mean field approximation (MFA), keeping only the nonzero vacuum expectation values (VEVs) $\bar\sigma_0$ and $\bar\sigma_8$ (the remaining VEVs of the bosonic fields vanish owing to charge conservation and isospin symmetry). Within the MFA the grand canonical thermodynamical potential reads[6]

$$\Omega_{\rm MFA}(T) = -\,2 \sum_{f,c} \int_{p,n} {\rm Tr}\ \ln \left[p_{nc}^2 + \Sigma_f^2(p_{nc})\right] - \frac{1}{2}\sum_f (\bar\sigma_f\ \bar S_f + \frac{G}{2}\ \bar S_f^2)\ -\ \frac{H}{4}\ \bar S_u\ \bar S_d\ \bar S_s + \mathcal{U}(\Phi,T)\ , \qquad (3)$$

where $f = u, d, s$, $c = r, g, b$, and we have used the notations $\int_{p,n} = \sum_n \int d^3p/(2\pi)^3$ and $p_{nc} = (\vec p\ , \omega_n - \phi_c)$, where ω_n are fermionic Matsubara frequencies and ϕ_c is defined by the relation $\phi = {\rm diag}(\phi_r, \phi_g, \phi_b)$. The quark constituent masses $\Sigma_f(p_{nc})$ are here momentum-dependent quantities, given by

$$\Sigma_f(p_{nc})\ =\ m_f\ +\ \bar\sigma_f\, g(p_{nc})\ , \qquad (4)$$

where $g(p)$ is the Fourier transform of the form factor $\tilde g(z)$. For convenience we have defined mean field values $\bar\sigma_f$ given by $\bar\sigma_u = \bar\sigma_d = \sqrt{2/3}\,\bar\sigma_0\ +\ 1/\sqrt{3}\,\bar\sigma_8$, $\bar\sigma_s = \sqrt{2/3}\,\bar\sigma_0\ +\ 2/\sqrt{3}\,\bar\sigma_8$ (similar relations hold for $\bar S_f$). Within the stationary phase approximation, the mean field values $\bar S_f$ and $\bar\sigma_f$ turn out to be related by[6]

$$\bar\sigma_u + G\,\bar S_u + \frac{H}{2}\,\bar S_u \bar S_s = 0\ \ , \qquad \bar\sigma_s + G\,\bar S_s + \frac{H}{2}\,\bar S_u^2 = 0\ \ . \qquad (5)$$

The effective potential $\mathcal{U}(\Phi,T)$ can be fitted by taking into account group theory constraints and lattice results. We take here the form given in Ref.[7]. In addition, it is seen that assuming that ϕ_3 and ϕ_8 are real-valued[7] one has $\phi_8 = 0$. Thus we have to determine the value of ϕ_3 together with the mean field values $\bar\sigma_u$ and $\bar\sigma_s$. This can be done through the minimization of the (regularized) thermodynamical potential $\Omega_{\rm MFA}(T)$, which leads to a set of three coupled "gap equations".

In order to obtain the meson mass spectrum one has to consider the mesonic fluctuations around the mean field values. It is convenient to introduce a new basis defined by $\xi_{ij} = [\lambda_a\,(\xi_a - \bar\xi_a)/\sqrt{2}]_{ij}$, where $\xi_a = \sigma_a, \pi_a$ and i, j run from 1 to 3. The resulting quadratic terms in S_E at finite temperature can be written as[5]

$$S_E^{\rm quad} = \frac{1}{2}\int_{q,m} \Big[G^+_{ij,kl}(\vec q^{\ 2},\nu_m^2)\sigma_{ij}(q_m)\sigma_{kl}(-q_m) + G^-_{ij,kl}(\vec q^{\ 2},\nu_m^2)\pi_{ij}(q_m)\pi_{kl}(-q_m)\Big]\ , \qquad (6)$$

where $q_m = (\vec q\ , \nu_m)$, $\nu_m = 2m\pi T$ being bosonic Matsubara frequencies. The functions $G^\pm_{ij,kl}$ in Eq. (6) are given by

$$G^\pm_{ij,kl}(\vec q^{\ 2},\nu_m^2) = C^\pm_{ij}(\vec q^{\,2},\nu_m^2)\delta_{il}\,\delta_{jk} + \big((r^\pm)^{-1}\big)_{ij,kl}\ , \qquad (7)$$

where $C^\pm_{ij}(\vec q^{\,2},\nu_m^2)$ are loop integrals[5] and $r^\pm_{ij,kl} = G\,\delta_{il}\,\delta_{jk} \pm\ (H/2)\,\epsilon_{ikh}\,\epsilon_{jlh}\,\bar S_h$. In the basis of physical fields, the quadratic action for the pseudoscalar sector reads[5]

$$S_E^{\rm quad}\Big|_P = \int_{q,m}\Big\{G_\pi(\vec q^{\ 2},\nu_m^2)\,\pi^+(q_m)\,\pi^-(-q_m)\ +\ G_K(\vec q^{\ 2},\nu_m^2)\Big[K^0(q_m)\,\bar K^0(-q_m) + K^+(q_m)\,K^-(-q_m)\Big] + \frac{1}{2}\sum_{P=\pi^0,\eta,\eta'} G_P(\vec q^{\ 2},\nu_m^2)\,P(q_m)\,P(-q_m)\Big\}\ . \qquad (8)$$

The physical fields η and η' are related to the U(3) states η_0 and η_8 through mixing angles θ_η and $\theta_{\eta'}$, which in principle are different to each other. An equivalent expression to Eq. (8) can be found for the scalar meson sector.

Now from the quadratic effective action it is possible to obtain the scalar and pseudoscalar meson masses by solving the equations

$$G_M(-m_M^2, 0) = 0 \ , \tag{9}$$

with $M = \pi$, K, σ, etc. The mass values determined by these equations correspond to the spatial "screening-masses" of the mesons' zeroth Matsubara modes, and their inverses describe the persistence lengths of these modes at equilibrium with the heat bath (i.e. they drive a behavior $\exp(-m_P\, r)$ in the conjugate 3-space coordinate r).

3. Meson masses at zero and finite temperature

Let us determine the model parameters to be used in our numerical calculations. For simplicity we consider a model with a Gaussian form factor, namely

$$g(p) \ = \ \exp\left(-p^2/\Lambda^2\right) \ . \tag{10}$$

This introduces a new free parameter Λ, which plays the rôle of an ultraviolet cut-off momentum scale. For $T \to 0$ it is seen that the coupling of fermions to the background gauge field vanishes, therefore the traced Polyakov loop is essentially determined by the effective potential $\mathcal{U}$. Thus we end up with five free parameters, namely the current quark masses $\bar{m}$ and m_s, the coupling constants G and H and the cut-off scale Λ. Here we have chosen to take the value of $\bar{m}$ as input, whereas the remaining four parameters have been fixed from the measured values of the pion, kaon and η' masses and the pion decay constant f_π. Taking $\bar{m} = 5$ MeV, we obtain

$$m_s = 119 \text{ MeV} \ , \qquad \Lambda = 843 \text{ MeV} \ , \qquad G\Lambda^2 = 13.35 \ , \qquad H\Lambda^5 = -273.7 \ . \tag{11}$$

Table 1. Model predictions and empirical values for meson masses (all in MeV)

m_P	Our Model	Empirical	m_S	Our Model	Empirical
m_π	139*	139	m_{a_0}	900	980
m_K	495*	495	m_κ	1380	1425
m_η	523	547	m_σ	566	400-1200
$m_{\eta'}$	958*	958	m_{f_0}	1280	980

(*) Input values

Our numerical results for meson masses are presented in Table I, together with the corresponding empirical values quoted by the PDG[8]. Input values are marked with an asterisk. From the table it is seen that the model predictions are in reasonable agreement with the measured values. In addition, the obtained mass ratio $m_s/m = 23.8$ is close to the corresponding current algebra prediction, namely $m_s/m = (2m_K^2 - m_\pi^2)/m_\pi^2 \simeq 25$. We notice that in the case of the κ scalar meson the equation $G_\kappa(-x^2, 0) = 0$ has no solution in the real x axis, thus we have defined the mass m_κ as the point where the absolute value of $G_\kappa(-x^2, 0)$ becomes minimal.

Now taking the parameters in Eq. (11) one can solve the gap equations to calculate the mean field values $\bar{\sigma}_u$, $\bar{\sigma}_s$ and ϕ_3 at finite temperature. As expected, it is found that there is a crossover phase transition in which the chiral symmetry is restored, and consequently one finds a sharp peak in the chiral susceptibility that can be used to define a transition temperature T_c. In our model we find $T_c = 202$ MeV, in much better agreement with lattice results, $T_c^{(\rm latt)} = 160 - 200$ MeV[9], than the value recently obtained in the local SU(3) PNJL model, $T_c^{(\rm PNJL)} = 259$ MeV[10]. In addition one finds a deconfinement phase transition, which occurs at about the same critical temperature. These results are qualitatively similar to those found in Ref.[11] in the context of an ILM-motivated nonlocal SU(3) model.

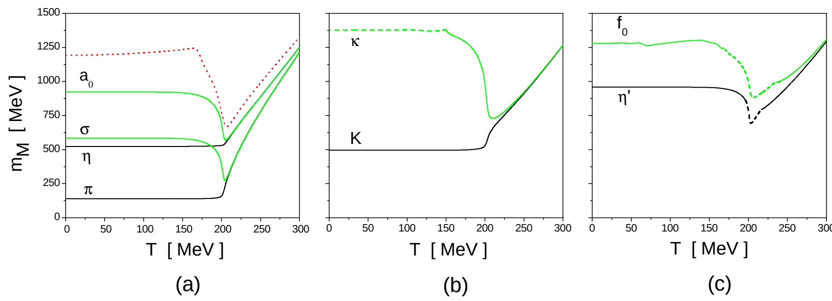

Fig. 1. Scalar and pseudoscalar meson masses as functions of the temperature.

Let us analyze the behavior of meson masses with temperature. These can be determined by solving Eqs. (9). In Fig. 1(a) we quote the curves for the masses of pseudoscalar mesons π and η and scalar mesons σ and a_0, which are chiral partners of the former. It is seen that above T_c the masses of chiral partners become degenerate, as expected from chiral restoration. When the temperature is further increased all four masses are found to rise continuously, showing that they are dominated by thermal energy. In general, it is seen that the functions $G_M(-k^2, 0)$ are well defined for low k. If k is increased, at some "pinch point" the integrals in $C^{\pm}_{ij}(-k^2, 0)$ become divergent and need to be regularized. Here we follow the prescription discussed in Ref.[6], conveniently extended to finite temperature. The pinch point can be interpreted as a threshold above which mesons could decay into two massive quarks. This threshold is represented with the dotted curve in Fig. 1(a).

In Fig. 1(b) we plot the curves for the masses of the pseudoscalar mesons K, and their scalar partners κ. The dashed stretch corresponds to the region in which $G_\kappa(-k^2, 0) = 0$ has no solution for real k. As a consequence of the large current strange quark mass, K and κ meson masses match only at $T \simeq 225$ MeV, somewhat above T_c. Finally, in Fig. 1(c) we quote the temperature dependence of f_0 and η' masses, for which the degeneracy is achieved only at $T \simeq 300$ MeV.

4. Summary

We have studied the finite temperature behavior of scalar and pseudoscalar meson masses in the context of an SU(3) nonlocal chiral model. The effect of gauge interactions has been introduced by coupling the quarks with a background gauge field, and the deconfinement transition has been studied through the behavior of the traced Polyakov loop. For a model including an exponential form factor, we have fixed the average non-strange quark mass $\bar{m}$ to a phenomenologically sound value of $\bar{m} = 5$ MeV, whereas the remaining parameters have been determined from the pion, kaon and η' meson masses and the pion decay constant f_π. Using this set of parameters we have obtained adequate predictions for the masses of the remaining scalar and pseudoscalar mesons.

For finite temperature the former model parameters have been kept fixed, while those appearing in the Polyakov loop potential have been taken from a fit to lattice results. As expected, the model shows a fast crossover phase transition, corresponding to the restoration of SU(2) chiral symmetry. The transition temperature is found to be $T_c = 202$ MeV, in agreement with lattice results. In addition one finds a deconfinement phase transition, which occurs at about the same critical temperature. Concerning the behavior of meson masses, it is seen that beyond T_c pseudoscalar masses get increased, becoming degenerate with the masses of their chiral partners, as expected from chiral restoration. The temperature at which chiral partners meet depends on the strange quark composition of the corresponding mesons.

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